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Mathematical modelling of post-mining dislocations kinetics as a quasi-random process*

The paper presents the problem of determination of one- and multi-stage prognosis of post-mining surface dislocations. The finite and chronologically ordered vector of geodetic surveys is the describing variable herein. Completed surveys show that the analyzed process can be written as a composition of both deterministic process and singular one.

Hence the quantitative description of the kinetics of the process of dislocation forming has been assigned to the class of the stochastic model. An adequate series sum in which time is the argument and random variables are the values makes up the formal definition of the model. The optimization of one-stage prognosis has been carried out for utility purposes. The Durbin-Levinson algorithm is the applied numerical procedure.

The utility fragment of this is based on verification of the defined model for certain mining-geological conditions and surveying results. The obtained analytical representation and optimal prognosis of the kinetics of vertical dislocations correspondend to surveying results, which can be testified by adequate measures of the quality of description of the process.

1. *Introduction*

Underground exploitation forces dislocations of parts of the rock mass, generally towards the selected volume of the deposit. It is not possible to forsee accurately that, at a certain moment, the selected point of the rock mass will find itself at a pre-determined place of the space-time. In this sense the dislocating process touched off by underground exploitation is a random movement of the point, hence a certain representation (time being the argument random variables — values) might be assumed as the mathematical model.

The problem of the definition of the current prognosis of post-mining dislocations is the subject-matter here; a chronologically ordered vector of the survey of dislocations of a given point of the rock mass within the area affected by mining exploitation is the describing variable here.

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Appropriate discussion will pertain to a certain class of stochastic models. Specification of the optimal prognosis of dislocations of the point will be a particularly significant issue. In order to analyze the formulated problem, selected elements of mathematical analysis, theory of probability and theory of stochastic processes will be quoted.

2. *Formulation of the problem*

The stochastic process may be presented as each representation of the following type:

$$X : T \times \Omega \rightarrow \mathbb{R} \tag{1}$$

where:

T — time,
Ω — non-empty set; $\Omega \in \Sigma$,
Σ — family of subsets of set Ω,
$\mathbb{R}$ — a certain function,

in which $\forall t \in T \; X_t = X(t, \cdot)$ is the random variable.
Each representation of the following form:

$$X(\cdot, \eta) : T \rightarrow \mathbb{R} \tag{2}$$

will be called realization of process $X = \{X_t\} \in T$.

If process $\{X_t\}$ is a stationary one in a broader sense, the following autocovariance function will be the characterizing quantity of process (1):

$$R(t-s) = \mathrm{cov}(X_t, X_s) - E(X_t - EX_t)(X_s - EX_s) \tag{3}$$

for process $\{X_t\}_{t \in T}$ function (3) assumes the following properties:

$$R(0) \geqslant 0 \tag{i}$$

$$| \; R(h) \; | \leqslant R(0) \tag{ii}$$

$$R(t-s) = R(s-t) \qquad \text{for optional } t, s \in T \tag{iii}$$

Stochastic process $\{X_t\}_{t \in T}$ is called the Gauss one if

$$Z = \sum_{i=1}^{n} \alpha_i X_{ti} \qquad \text{is the Gauss random variable.}$$

It has been proved [1] that an optional stationary process may be described as the sum of linear and singular processes. If process $\{X_t\}$ is a stationary one in a broader sense, for an optional $t \in T$ the following equality comes about:

$$X_t - \varphi_1 X_{t-1} - \ldots - \varphi_p X_{t-p} = Z_t + \eta_1 Z_{t-1} + \ldots + \eta_n Z_{t-n} \tag{4}$$

where:

φ, η — p and n degree multinomials respectively

$\{Z_t\}$ — stochastic process (white noise); $\{Z_t\} \sim WN(0, \sigma^2)$.

The main goal of this is to define the linear prognosis for process $\{X_t\}$ if part of realization of process $\{X_1, \ldots X_t)$ is given i.e.:

$$\hat{X}_{t+k} = \sum_{i=0}^{t} \Phi_i X_{t-i} \tag{5}$$

Besides, the volume of prognosed quantity X_{t+k} should satisfy the following criterion:

$$E \mid X_{t+k} - \hat{X}_{t+k} \mid^2 = \min \tag{6}$$

3. *Analysis of the problem*

Specification of optimal linear prognosis according to (5), condition (6) being taken into consideration, resolves itself down to extremum of a certain expression to be determined. Note that it will be convenient to apply here estimators of spectral density [3].

If process $\{X_k\}_{k \in T(T\text{—total})}$ is a stationary one, then

$$E(X_k \cdot X_1^*) = R_{k-1} \tag{7}$$

where:

X_1^* — conjugate,

$\{R_q\}_{q \in T}$ — autocovariance function.

For $R_q \exists\, F\left[-\frac{1}{2}, \frac{1}{2}\right] \to R$ that

$$\forall_{q \in T} R_q = \int_{-\frac{1}{2}}^{\frac{1}{2}} \exp(2 \cdot \pi \cdot q \cdot \vartheta) dF(\vartheta) \tag{8}$$

assuming that $F'(\vartheta) = f(\vartheta)$, the following will be obtained:

$$R_q = \int_{-\frac{1}{2}}^{\frac{1}{2}} \exp(2 \cdot \pi \cdot q \cdot \vartheta) f(\vartheta) d\vartheta \tag{9}$$

As it has been mentioned before, autocovariance function is an important quantity characterizing process (1); the process is also characterized by autocorrelation function

— in the mathematical sense equivalent to autocovariance function. It is possible to prove [2] that if the autocorrelation function of process $\{X_t\}$ is an analytical one, the future status of process $X_{t+\tau}$ (prognosis) might be determined on the basis of current vector of the value of the status of process $\{X_t\}$ and its derivatives.

Herein, in order to forecast process $X_{t+\tau}$ the Durbin-Levinson algorithm has been applied, its tentative proceeding scheme looking thus.

If process $\{X_t\}$ is a stationary one with its mean m and autocovariance function $R(\cdot)$, then process $\{Y_t\}=\{X_t-m\}$, for processes $\{X_t\}$ and $\{Y_t\}$ the following dependence comes about:

$$P_{sp\{1,X_1,\ldots,X_n\}}X_{n+h}=m+P_{sp\{Y_1,\ldots,Y_n\}}Y_{n+h} \qquad (10)$$

where:

$sp\ \{1,X_1,\ldots,X_n\}$ — the smallest closed subspace H_n of the Hilbert space $L^2\ (\Omega,\ \Sigma,\ p)$ for $n\geqslant 1$
Ω — non-empty set; $\Omega\in\Sigma$
Σ — family of subsets of set Ω
P — normalized measure
h — number of steps of prognosis of the process,
X_{n+h}; Y_{n+h} — prognoses of processes (X) and (Y).

„One-step" prognoses will be defined thus:

$$\hat{X}_{n+1}:=\begin{cases}0 \\ P_{Hn}\cdot X_{n+1} & \text{for } n\geqslant 1\end{cases} \qquad (11)$$

On the basis of (5) the following may be written down:

$$\hat{X}_{n+1}=\varphi_{n1}\cdot X_n+\ldots+\varphi_{nn}\cdot X_1 \qquad n\geqslant 1 \qquad (12)$$

Applying the projection theorem [1] the folowing will be obtained:

If M is a closed subspace of Hilbert space H and $x\in H$, then: there is the only such element $\hat{x}\in M$ that

$$\|x-\hat{x}\|=\inf_{y\in M}\|x-y\| \qquad (13)$$

and

$$x\in M \text{ and } \|x-\hat{x}\|=\inf\|x-y\|\Leftrightarrow x\in M \text{ and } (x-\hat{x})x\in M^{\perp} \qquad (14)$$

$\hat{x}$ — orthogonal projection of element x onto subspace M,
$M^{\perp}$ — orthogonal complement of subspace M.
$M^{\perp}:\{x\in H \mid \forall y\in M: x\perp y\}$.

Equation (12) will be referred to as the one-step prognosis equation; it may be written down also in the following way:

$$<\sum_{i=1}^{n}\varphi_{ni}\cdot X_{n+1-i},\ X_{n+1-j}> \ = \ <X_{n+1},\ X_{n+1-j}>;\ j=1,\ldots,n \tag{15}$$

where $<.,.>$ scalar product.

Equations (15) — applying the linearity of scalar products — will assume the following form:

$$\sum_{i=1}^{n}\varphi_{ni}\cdot R(i-j)=R(j);\quad j=1,\ldots,n$$

or

$$\Xi_n\cdot\varphi_n=R_n \tag{16}$$

where:

$$\Xi_n=[R(i-j)]_{i,j=1^n},\ R_n=(R(1),\ldots,R(n));\ \varphi_n=(\varphi_{n1},\ldots,\varphi_{nn}) \tag{17}$$

The Cramer theorem implies that system of equations (16) has just one solutin $\Leftrightarrow$ if Ξ is a nonsingular matrix — then

$$\varphi_n=\Xi^{-1}\cdot R_n \tag{18}$$

For a stationary process, in accord with (12), the following will be obtained:

$$X_{r+h}=\sum_{j=1}^{r}a_j\cdot X_{j+h-1}\qquad \text{for } h>1 \tag{19}$$

hence for $n\geqslant 1$ there are real constants $a_1^{(n)},\ldots,a_r^{(n)}$ that will satisfy the following equation:

$$X_n=a^{(n)T}x_r$$
$$X_r=(X_1,\ldots,X_r);\ a^{(n)}=(a_1^{(n)},\ldots,a_r^{(n)}) \tag{20}$$

Applying (18) and (20); we might write the following:

$$R(0)=a^{(n)T}\cdot\Xi_r a^{(n)}=a^{(n)T}\mathbf{P}\Lambda\mathbf{P}^T a^{(n)} \tag{21}$$

where: $\mathbf{P}\cdot\mathbf{P}^T$ — identity matrix,

$\Lambda=\begin{bmatrix}\kappa_1 & & \\ & \ddots & \\ & & \lambda_r\end{bmatrix}$; $\lambda_1\leqslant\lambda_2\leqslant\ldots\leqslant\lambda_r$ — strictly positive own values of matrix Ξ.

Hence

$$R(0) \geqslant \lambda_1 a^{(n)T} \mathbf{P}\mathbf{P}^T a^{(n)} = \lambda_1 \cdot \sum_{j=1}^{n} (a_j^{(n)})^2 \tag{22}$$

(22) implies that $a_j^{(n)}$ for each determined value $jR(\cdot)$ is a bounded function for variable n.
Therefore

$$R(0) \leqslant \sum_{j=1}^{r} \mid a_j^{(n)} \mid \mid R(n-j) \mid \tag{23}$$

Boundedness $a_j^{(\cdot)}$ and inequality (23) make system of equation (18) be uneguivocally solved, for matrix Ξ_n is a nonsingular one.

4. *Optimization of the prognosis of the process*

If there is a given fragment of realization of process $\{X_{t\in T}\} = \{X_1, \ldots, X_t\}$, it is significant to determine the optimal linear prognosis for moment $t+k$. In accordance with (5) we have the following:

$$\hat{X}_{t+k} = \sum_{i=0}^{t} \Phi_i X_{t-i} \tag{24}$$

Taking into account the random character of (24), criterion (6) may be written down as follows:

$$E \mid X_{t+k} - \sum_{i=0}^{t} \Phi_i X_{t-i} \mid^2 = \min \tag{25}$$

Further on, applying (8), (25) will assume the following form:

$$E \mid \int_{-\frac{1}{2}}^{\frac{1}{2}} \exp(2 \cdot \pi \cdot i(t+k) \cdot \vartheta) d\vartheta - \sum_{r=1}^{t} \varphi_j \cdot \int_{-\frac{1}{2}}^{\frac{1}{2}} \exp(2 \cdot \pi \cdot ir \cdot \vartheta) f(\vartheta) d\vartheta \mid^2 \tag{26}$$

After it has been transformed:

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} \mid \exp(2 \cdot \pi \cdot i(t+k) \cdot \vartheta) d\vartheta - \sum_{r=1}^{t} \varphi_r \cdot \exp(2\pi \cdot i \cdot r \cdot \vartheta \mid^2 dF(\vartheta) \tag{27}$$

If spectral density occurs, the following may be written:

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} |\exp(2\cdot\pi\cdot i(t+k)\cdot\vartheta)d\vartheta-\sum_{r=1}^{t}\varphi_r\cdot\exp(2\pi\cdot i\cdot r\cdot\vartheta|^2 f(\vartheta)d\vartheta \tag{28}$$

where spectral density will be expressed by the following dependence:

$$f(\vartheta)=\sum_{i=-\infty}^{\infty}R_i\cdot\cos(2\pi\cdot i\cdot\vartheta) \tag{29}$$

$$R_i=\int_{-\frac{1}{2}}^{\frac{1}{2}}\cos(2\cdot\pi\cdot i\cdot\vartheta)f(\vartheta)d\vartheta \tag{30}$$

R_i is subject to estimation, defining an appropriate estimator, most effectively by means of the periodiagram.

Analysis shows that the following model is the optimal linear prognosis $\hat{X}_{n+1}$:

$$\hat{X}_{n+1}=\sum_{i=1}^{n}\varphi_{ni}\cdot X_{n+1-i};\ n=1,2,\ldots \tag{31}$$

where:

$$\Xi_n=[R(i-j)]_{i,j=1}^{n},\ R_n=R(1),\ldots,R(n));\ \varphi_n=(\varphi_{n1},\ldots,\varphi_{nn})$$

The error mean-square of the prognosis amounts to:

$$\delta_n=R(0)-R_n^T\Xi_n^{-1}R_n \tag{32}$$

System of equations (16) has just one solution $\Leftrightarrow$ if Ξ_n is a nonsingular matrix. Conditions for matrix Ξ_n to be nonsingular for each n:

If $R(0)>0$ and $R(h)\to 0$ if $h\to\infty$ covariance matrix $\Xi_n=[R(i-j)]_{i,j=1}^{n}$ for measuring series $\{X_1, X_2,\ldots,X_n\}$ is nonsingular for optional n.

It is possible to prove that the foregoing theorem is true applying contradiction-proof procedure, which will roughly look thus:

Assuming that Ξ_n is singular for a certain n, then $E(X_t)=0$; if so — there is a $k>1$ and such constants $\alpha_1, \alpha_2,\ldots, \alpha_k$ that make Ξ_n be nonsingular.

Besides $X_{k+h}=\sum_{i=1}^{k}a_i\cdot X_{i+k-1}$ for $h\geqslant 1$; then $\forall n\geqslant k+1\exists\alpha_1^{(n)}, \alpha_2^{(n)},\ldots,\alpha_k^{(n)}$ to imply

$$X_n=\alpha^{(n)T}X_k$$

Hence $R(0)=\alpha^{(n)T}\Xi_k\alpha^{(n)}$ is a bouned function of variable n for fixed value of k and $\alpha_k^{(n)}$;

thus it is possible to write the following:

$$R(0)=\mathrm{Cov}(X_n, \sum_{i=1}^{k}\alpha_j^{(n)}X_i)$$

which implies that $R(0)\leqslant\sum_{j=1}^{k} \mid \alpha_j^{(n)} \mid \mid R(n-j) \mid$ is contradictory to the assumption, which in turn marks out the end of the proof.

5. *Verification of the model*

Due to underground exploitation (development scheme of the mining area and locations of measuring points of the surface having been presented in Fig.1) an undetermined post-mining trough was forming, underground exploitation was carried out by means of the fall-of-roof wall system; average thickness of deposit 3.2 [m]; depth of deposit approx. 315 [m]. Measuring line (bench-marks) ran along a railway embankment. Exploitation development direction was roughly parallel to the longitudinal axis of the railway line. Deposit was cut into 5 walls, extraction differed during exploitation — one, two, sometimes three walls were operated by means of fall-of-roof system, the speed of progress of the forehead ranged between 25 and 80 [m/month].

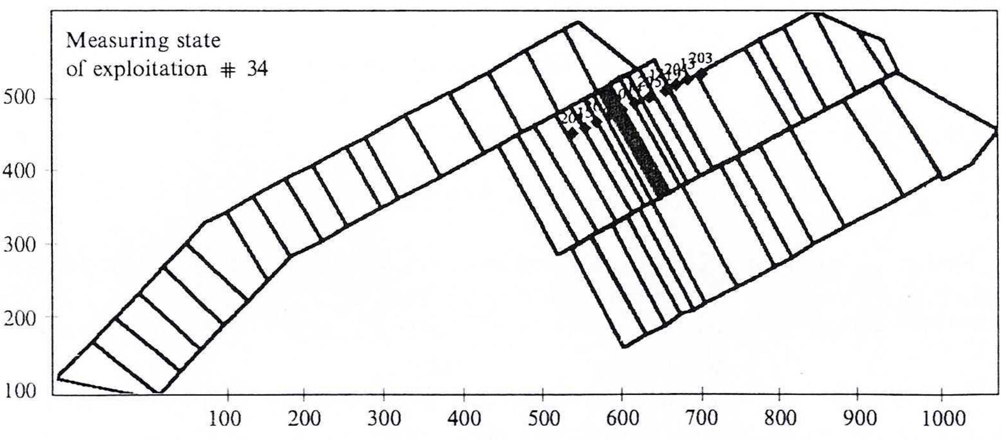

Fig. 1. Underground exploitation development scheme at measurement # 34;
◆ — points of measuring line, ——— — outline of mining area

Geodesic survey in the area in question resolved itself down to the determination of two elementary quantities: length changes of sectors of the line and the height of points of the line. Both the measuring groups in each session were referred to a fixed point. Levelling measurement was taken according to surveying instructions G-2. Length measurement was taken with the use of a steel band hanging on a stand, constant tension having been applied. Geodesic observation and adjustment procedure were carried out by a team of specialists the Academy of Mining and Metallurgy. Time intervals between adjacent measurements amounted to 13 up to 28 days — approx. 21 days on the average.

Adjustment of the values of measuring results of vertical dislocations has been specified with an accuracy of up to 0.1 [mm]. For the purposes of this work adjusted values of subsidence have been rounded up to 1 [mm]; for fragments of the measuring line measuring results have been written as respective integers and grouped in Table 1 as absolute values. Modelling results and process prognoses are also of type $|w_i|$; appropriate differences have been merely proceded by „ – " (Tables 2 and 2a).

Table 1

Vertical dislocations for 14 time horizons and for 11 measuring points

Number of measuring points

Time [days]	208	15	207	14/21	206	14	205	14/19	204	13	203
SUBSIDENCES W [mm]											
405	133	119	104	96	88	80	78	76	75	78	78
433	141	129	116	109	102	98	98	97	96	100	100
455	150	140	128	121	115	111	114	114	114	119	121
481	158	148	140	133	129	128	133	134	136	146	150
496	160	150	143	138	134	136	145	148	152	170	181
509	164	161	156	152	150	154	168	174	183	206	224
521	176	168	166	163	163	172	184	206	216	254	271
544	181	185	188	187	193	210	247	267	285	342	367
564	211	210	219	222	238	266	319	346	369	431	451
584	238	248	264	271	289	328	386	414	436	491	500
605	284	300	336	351	377	426	486	511	528	568	560
626	377	410	471	494	526	577	623	637	643	652	624
647	506	558	638	661	591	727	744	742	736	718	673
670	709	770	843	859	870	871	850	833	814	772	712

Graphic representation of measuring results, approximation and appropriate differences (Table 2 and 2a) for two measuring points has been presented in Fig. 2 and 3.

While analyzing the characteristics of the subsidence of an optional point of the surface due to mining exploitation, deterministic part W_{0i} may be specified alongside the so called random part of vertical component of dislocation ε_i, thus the following may by written:

$$W_i^p = W_{0i} + \varepsilon_i \qquad \text{(iv)}$$

W_i^p — measuring result of dislocation of a point due to exploitation.

Table 2

Approximation and one-step prognosis results (according to 31) of post-mining vertical dislocations of points of surface W [mm]

Time	Point 208			Point 16			Point 207		
[days]	measurement	model	difference	measurement	model	difference	measurement	model	difference
405	133	136.9	3.9	119	123.9	4.9	104	109.8	5.8
433	141	141.1	0.1	129	126.0	3.0	116	110.9	−5.1
455	150	147.4	−2.6	140	137.3	−0.7	128	128.4	0.4
481	158	156.7	−1.3	148	148.1	0.1	140	141.3	1.3
496	160	166.5	6.5	150	153.9	3.9	143	153.4	10.4
509	164	169.5	5.5	161	155.8	−5.2	156	147.9	−8.1
521	176	170.4	−5.6	168	168.4	0.4	166	168.6	2.6
544	181	179.7	−1.3	185	176.7	−8.3	188	177.1	−10.9
564	211	187.8	−23.1	210	199.8	−10.1	219	210.7	−8.2
584	238	220.1	−17.9	248	235.8	−12.2	264	253.4	−10.6
605	284	262.9	−21.1	300	289.9	−10.1	336	316.0	−20.0
626	377	326.3	−50.7	410	361.5	−48.5	471	423.1	−47.9
647	506	451.3	−54.7	558	537.1	−20.9	638	648.4	12.4
670	709	651.1	−57.9	770	766.4	−3.6	843	861.8	18.8
688 prognosis	*1024*	**985.6**	−39.6	*1098*	**1068.8**	−30.8	*1143*	**1121.8**	−22.8

Table 2a

Approximation and one-step prognosis results (according to 31) of post-mining vertical dislocations of points of surface W [mm]

Time	Point 14			Point 204			Point 203		
[days]	measurement	model	difference	measurement	model	difference	measurement	model	difference
405	80	88.5	8.5	75	84.8	9.8	78	88.3	10.3
433	98	98.0	0.0	96	88.1	−7.9	100	90.9	−9.1
455	111	120.1	9.1	114	120.0	6.0	121	125.6	4.6
481	128	136.0	8.0	136	137.4	1.4	150	147.6	−2.4
496	136	156.8	20.8	152	162.7	10.7	181	185.3	4.3
509	154	166.6	12.6	183	173.3	−9.7	224	219.6	−4.4
521	172	188.6	16.6	216	217.0	1.0	271	276.0	5.0
544	210	210.7	0.7	285	255.3	−29.7	367	329.2	−37.8
564	266	257.3	−8.7	369	364.8	−4.2	451	485.7	34.7
584	328	325.9	−2.1	436	476.2	40.2	500	563.6	63.6
605	426	401.8	−24.2	528	528.8	0.8	560	569.8	9.8
626	577	521.9	−55.1	643	639.7	−3.3	624	629.6	5.6
647	727	706.8	−20.2	736	781.9	45.9	673	696.7	23.7
670	871	890.6	−19.6	814	857.5	43.5	712	731.5	19.5
688 prognosis	*1049*	**1066.0**	17.0	*1028*	**1057.8**	29.8	*963*	**928.4**	−34.6

EXPONENTIAL MODEL

Description of post-mining dislocations *W* [mm]; point # 208

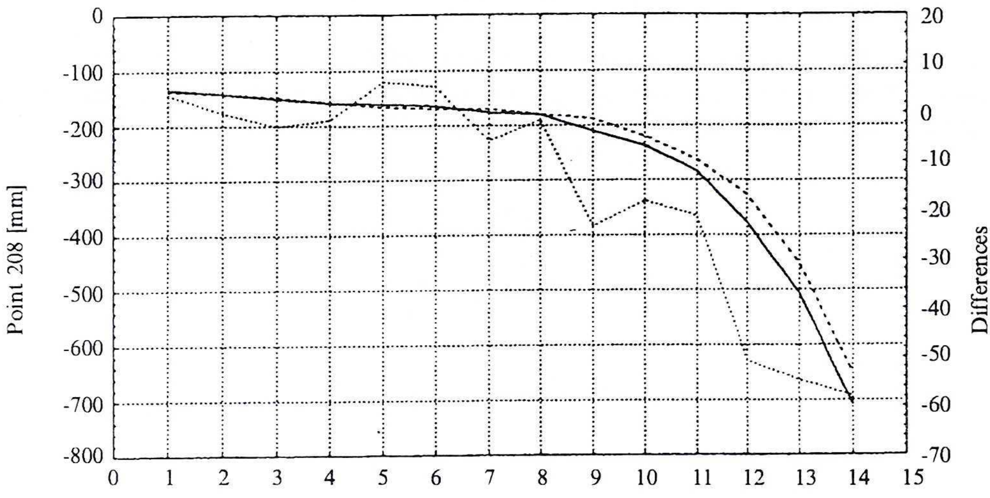

Fig. 2. "axis *t*" 2:=405 [days]; 3:=433 [days], ..., 15:=670 [days] —— P208 measurement ······ Description results —— Differences

EXPONENTIAL MODEL

Description of post-mining dislocations *W* [mm]; point # 15

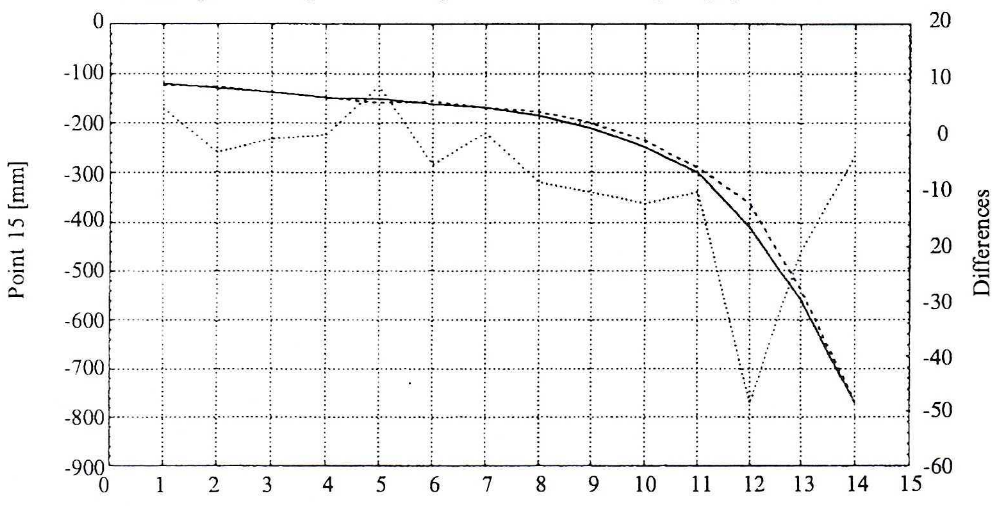

Fig. 3. "axis *t*" 2:=405 [days]; 3:=433 [days], ..., 15:=670 [days] —— P15 – measurement ······ Measuring results —— Differences

For measuring series n the following will be obtained:

$$W^p = \{W_1^p, W_2^p, \ldots, W_n^p\} \tag{v}$$

According to (iv) dependence (v) is generally a random variable, hence correlation coefficient or correlation matrix is a significant measure determining relationship between two random variables for a set of random variables — dependence (vi) [2].

$$R_{XY} = \frac{\sum_{i=1}^{N}(x_i - \hat{x})(y_i - \hat{y})}{\sqrt{\sum_{i=1}^{N}(x_i - \hat{x})^2 \cdot \sum_{i=1}^{N}(y_i - \hat{y})^2}} \tag{vi}$$

Correlation matrix for measuring results of vertical dislocations (Table 1) has been presented in Table 3 — it implies that these are strongly correlated random variables.

Table 3

Correlation matrix – subsidence measuring results $W(x, t)$

Variables	208	15	207	14/21	206	14	205	14/19	204	13	203
208	**1**	**1**	**1**	**0.99**	**0.99**	**0.98**	**0.95**	**0.94**	**0.92**	**0.88**	**0.84**
15		**1**	**1**	**1**	**0.99**	**0.98**	**0.96**	**0.95**	**0.93**	**0.89**	**0.86**
207			**1**	**1**	**1**	**0.99**	**0.97**	**0.96**	**0.95**	**0.91**	**0.87**
14/21				**1**	**1**	**0.99**	**0.98**	**0.97**	**0.95**	**0.91**	**0.88**
206					**1**	**1**	**0.99**	**0.98**	**0.96**	**0.93**	**0.90**
14						**1**	**1**	**0.99**	**0.98**	**0.95**	**0.93**
205							**1**	**1**	**1**	**0.98**	**0.96**
14/19								**1**	**1**	**0.99**	**0.97**
204									**1**	**0.99**	**0.98**
13										**1**	**1**
203											**1**

Subsidence of observation points measuring results — as sets of dislocations of 12 measuring series — are strongly correlated (Table 3). Therefore estimators of regression equations have been determined, subsidence results of point 208 having been assumed as the "independent variable", subsidences of point 15 having been assumend as "dependent variables" (Fig. 4), for point 207 respectively (Fig. 5). Regression equations have the following form:

Linear regression equation W (15:t_i=28.889+1.1423*W (208:t_i)
Correlation coefficient: r, 99951

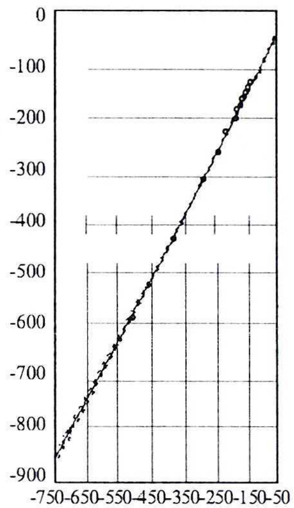

Fig. 4. Diagram of regression equation —⊖— – Regression 95 confidence percentage

Regression equation – subsidences measuring results W (207;t_i=60.201+1.3246*W (208;t_i)
Correlation coefficient: r, 99681

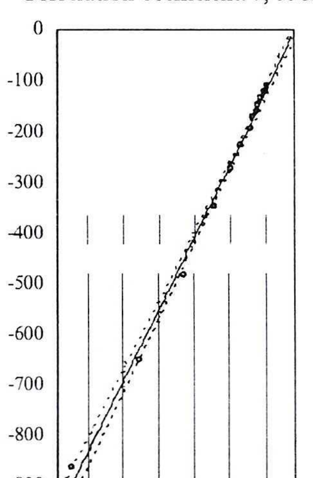

Fig. 5. Diagram of regression equation

$$W(x_{k+1})=a_{k+1}\cdot W(x_k;\ t_i)+b_{k+1};\ i=2,\ 3,\ \ldots,\ 11$$

Estimators and correlation coefficients determined for the analyzed set of measurements by means of the STATISTICA pack have been listed in Table 4.

Table 4

Points	15	207	14/21	206	14	205	14/19	204	13	203
estimator $\hat{a}$	1.142	1.325	1.382	1.434	1.473	1.451	1.420	1.382	1.276	1.140
estimator $\hat{b}$	28.9	60.2	71.2	70.1	71.6	45.2	28.3	12.7	33.4	65.8
correlation coefficient ρ_{ij}	0.99	0.99	0.99	0.98	0.976	0.952	0.937	0.922	0.877	0.843
distance $d=X_1+X_{208}$	20	36	52	72	90	110	130	146	162	182

Note that the deliberations presented herein concerning mathematical modelling have been verified upon the example of vertical post-mining dislocations. Similar verification and prognosis of an adequate process might be practically carried out for an optional set of measuring results of characteristic quantities in the line of

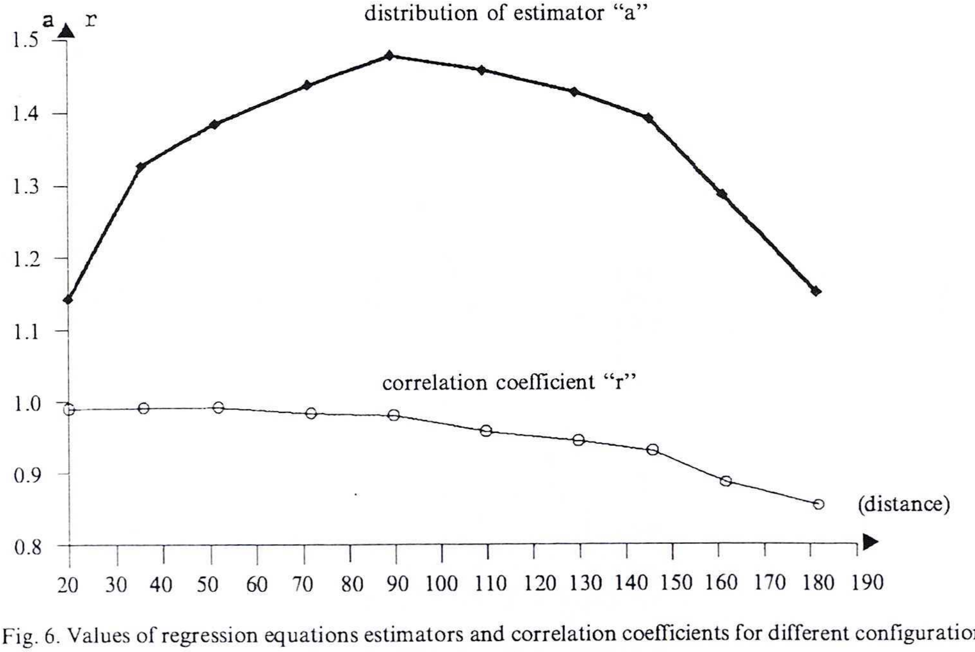

Fig. 6. Values of regression equations estimators and correlation coefficients for different configurations of variables according to Table 4

environmental protection, particulary in the sphere of approximation and current prognosis of transformations of geometry of an area.

The foregoing regression equations imply that once a nonstationary distribution of post-mining vertical dislocations of a given observation point is at our disposal, it is possible to approximate distributions of dislocations of "surface points" within a neighbourhood of the point. This is a significant property which makes it possible to restrict the number of the set of measuring points or determine subsidences at hypothetical points.

Table 5

Model	AUTOCORRELATION FUNCTION Point 15 White noise evaluation			
Delay	Autocorrelation	W. noise evaluation	Box Ljung Q	*P*
1	0.620057*	0.240906*	6.6247*	0.0101*
2	0.348553*	0.231455*	8.8925*	0.0173*
3	0.157877*	0.221601*	9.4001*	0.0244*

Table 5a

Model	AUTOCORRELATION FUNCTION Point 207 White noise evaluation			
Delay	Autocorrelation	W. noise evaluation	Box Ljung Q	*P*
1	0.653317*	0.240906*	7.3545*	0.0067*
2	0.379073*	0.231455*	10.0368*	0.0062*
3	0.174812*	0.221601*	10.6591*	0.0137*

Albeit the values of regression equations estimators are practically insignificant, equations of that kind — providing that they have been properly constructed — make it possible to minimize error, to be more precise — te determine the probability that an error bigger than the admissible one will not occur more often than it is implied by confidence level, which is quite significant. Autocorrelation function in turn makes it possible to estimate the so called random factor superimposed additively upon the values of generated subsidences — although the influence is remarkable, prediction of the subsidence process is by all means satisfactory.

6. *Final remarks*

The main objective of this article was to construct an appropriate process model for the determination of the optimal prognosis if part of observation results of a phenomenon is given, duration time of the phenomenon being the argument. It has been determined that the analyzed process in its formal sense is a random one, a certain class of stochastic models being the quantitative description. Actually, non-stationary processes, in most cases, may be analyzed as stationary in a broader sense — hence the stochastic process has been analyzed by means of autocovariance function and spectral density estimators. The so defined model makes it possible to predict the physically diversified phenomena that satisfy the mathematical conditions of the stochastic process.

The utilitarian part of the article pertains to mathematical modelling of dislocations of points of the terrain and rock mass within the area of the impact of underground exploitation. These are post-mining movements that by their mere nature are a menace to many components of environmental protection; thus this is an important problem both in cognitive sphere and in respect of application.

Deliberations presented herein provide grounds to set forth the following conclusions:

1. Analysis of surveying results of non-stationary condition of post-mining vertical dislocations implies that — in the sense of quantitive representation — an exponential convergent series may be an adequate mathematical describing model of the kinetics of subsidence of a points of the rock mass.

2. Probabilistic space and stochastic process have been defined for the deformation area. Autocovariance function is the characterizing quantity of a given process; minimum square criterion of the expected value of difference between estimated value and measuring result has been applied at determination of optimal prognosis.

3. Numerical proceeding of a defined description of dislocations area has been carried out with the use of the Durbin-Levinson algorithm. In order to obtain a high accuracy of the prognosis of the process *condition*: $\delta_w \leqslant \delta_w$dop. prediction has been limited down to one-step prognosis.

4. Verification of the formulated description has been carried out for a definite physical process — post-mining dislocations were being modelled. 14 cycles of measuring dislocation results arranged chronologically assigned to each of the 11 observation points make up the matrix of the value of the state of the analyzed process.

5. Basic deliberations pertain to determination of the stochastic relationship: *deformation of the base determined due to measurements* → *process modelling* results. Deformation of the base is represented here by sets of measuring results of geometrical transformation of the terrain. Modelling — this is application of an appropriate operator with assigned parameters in the set of measuring results.

6. Random disturbances having been additively superimposed upon both the groups of sets of descriptive variables provided a significant assumption for statistical analysis. Optimal regression model estimators have been determined for the so formulated problem.

7. Appropriate statistical measures characterizing the variables that describe the stochastic process and defining their mutual correlations have been sample-determined.

8. Stable estimators of appropriate regression equations imply that it is possible to approximate non-stationary characteristics of dislocations of "surface points" within a considerable environment of a point for which distribution of vertical post-mining dislocations has been determined due to measurements. This is a particularly important property — as it is, it is possible to reduce the size of a physical set of measuring points, thickening at the same time the deformation area to be covered by a considerably accurate prognosis.

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Wiesław Piwowarski

Modelowanie matematyczne kinetyki przemieszczeń pogórniczych jako procesu quasilosowego

Streszczenie

W pracy sformułowano problem wyznaczania jedno- i wielokrokowej prognozy pogórniczych przemieszczeń powierzchni terenu. Zmienną opisującą jest tu skończony i uporządkowany chronologicznie wektor wyników pomiarów geodezyjnych. Dokonane pomiary wskazują, że analizowany proces można zapisać jako złożenie procesu deterministycznego i procesu osobliwego. Stąd też opis ilościowy kinetyki procesu kształtowania przemieszczeń przyporządkowano do klasy modeli stochastycznych. Formalne zdefiniowanie modelu stanowi tu odpowiednia suma szeregu gdzie argumentem jest czas a wartościami zmienne losowe. Ze względów użytkowych przeprowadzono optymalizację prognozy jednokrokowej. Aplikacyjną procedurą numeryczną jest tu algorytm Durbina-Levinsona. Utylitarny fragment pracy to weryfikacja zdefiniowanego modelu dla konkretnych warunków górniczo-geologicznych i konkretnych wyników pomiarów. Uzyskane analitycznie odwzorowanie i optymalna prognoza kinetyki przemieszczeń pionowych dobrze przystają do wyników pomiarów, co potwierdzają odpowiednie miary jakości opisu procesu.

Виеслав Пивоварски

Математическое моделирование кинетики перемещений вызванных горными работами как квазислучайного процесса

Резюме

В работе представлена проблема определения одно- и многошагового прогноза перемещений местности в результате горных работ. Описывающей переменной является здесь конечный и хронологически упорядоченный вектор результатов геодезических измерений. Проведенные измерения доказывают, что анализированный процесс может быть записан как сумма детерминированного процесса и особого процесса. Поэтому количественное описание кинетики процесса формирования перемещений отнесено на счет класса стохастических моделей. Формальное определение модели является здесь соответственной суммой ряда, где аргументом является время а значениями случайные величины. Принимая во внимание эксплуатационные качества проведена оптимизация одношагового прогноза.

Аппликационной цифровой процедурой является здесь алгоритм Дурбина-Левинсона. Утилитарный фрагмент работы это верификация определенной модели для конкретных горно-геологических условий и конкретных результатов измерений. Полученное аналитическим способом отображение и оптимальный прогноз кинетики вертикалных перемещений хорошо совпадают с результатами измерений, что подтверждают соответственные меры качества описания процесса.