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Effectiveness of Dynamic Matrix Control algorithm with Laguerre functions

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The paper is concerned with the presentation and analysis of the Dynamic Matrix Control (DMC) model predictive control algorithm with the representation of the process input trajectories by parametrised sums of Laguerre functions. First the formulation of the DMCL (DMC with Laguerre functions) algorithm is presented. The algorithm differs from the standard DMC one in the formulation of the decision variables of the optimization problem – coefficients of approximations by the Laguerre functions instead of control input values are these variables. Then the DMCL algorithm is applied to two multivariable benchmark problems to investigate properties of the algorithm and to provide a concise comparison with the standard DMC one. The problems with difficult dynamics are selected, which usually leads to longer prediction and control horizons. Benefits from using Laguerre functions were shown, especially evident for smaller sampling intervals.

Key words: process control, model predictive control, DMC algorithm, Laguerre functions

1. Introduction

Model Predictive Control (MPC) is now an established advanced control technology, represented by numerous control algorithms and software packages applied successfully in the industrial practice, see, e.g., [1–4,6,8–14,19]. Dynamic Matrix Control (DMC) algorithm was one of the first MPC algorithms developed and applied in practice, still being one of the most popular MPC solutions in the process industries. The algorithm uses nonparametric process models in the form of discrete-time unit step responses. Such models are relatively easy to identify during the standard on-line experiments, which is the main reason of popularity of DMC. On the other hand, step response models can have long horizons of dynamics, i.e., large numbers of sampling intervals before the outputs

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stabilize after a step change of the process input. In particular, this happens when short sampling intervals are chosen, now easily implemented in modern, powerful process controllers to avoid too large additional delay stemming from the sampling procedure. This can be also caused by difficult process dynamics, e.g., by relatively short and different process delay times, when compared to the dominant time constants, to capture accurate modeling of the delays. Therefore, long prediction horizons can then result in the DMC algorithm together with also relatively long control horizons. This usually leads to increased on-line computational burden, mainly caused by increased dimensionality of the DMC optimisation problem.

Using polynomial models is a way to simplify complex dynamical modeling, the use of Laguerre functions is here a popular solution, see, e.g., [17]. It was also proposed in MPC applications, to speed up the use of models in the prediction process. The application of Laguerre functions for representation of control input trajectories can be found mainly for state-space process models, see [16, 18]. Recently, this approach has been applied in nonlinear MPC for more efficient optimization with linearized models in predictive structures of GPC type, see [5]. In this paper, the use of the Laguerre functions for the parametrization of predicted control input trajectories in the DMC algorithm will be proposed and analysed. Basing on a thorough analysis of simulation results of two benchmark MIMO problems, it will be shown that such approach may be computationally more efficient, first of all for processes with difficult dynamics and long step responses (long horizons of dynamics) and thus relatively long control horizons. The paper is a significantly extended version of the presentation at the 20th Polish Control Conference [15], in particular by addition of the analysis of the impact of the sampling interval, which occured to be crucial. Moreover, much more results concerning the first benchmark problem and all results for the second problem are added. This allowed to formulate much more representative conclusions.

The structure of the paper is as follows. In Section 2 the standard DMC algorithm will be briefly recalled. The use of Laguerre functions to parametrize the control input trajectories over the prediction horizon and the DMCL (DMC with Laguerre functions) algorithm will be formulated in Section 3. In Section 4 the efficiency of the DMCL algorithm, in comparison with the standard formulation of the DMC algorithm, will be thoroughly investigated on two MIMO benchmark example problems. Conclusions will be the last part of the paper.

2. Dynamic Matrix Control Algorithm

The DMC algorithm was one of the first model-based predictive feedback control (MPC) algorithms, it is still very popular in the industry applications. The general principle of the MPC algorithms is known, different formulations can be found in many papers and books, see, e.g., [6, 9–11, 19]. In this paper the DMC algorithm will be further developed, therefore its standard formulation will be first given.

The principle of the MPC is to evaluate optimal current process control input signals by minimizing certain performance function (cost function) over a future prediction horizon of N samples, at each sampling instant k. The following performance function is one of the most widely applied in process control implementations:

$$J(k) = \sum_{p=1}^{N} \|y^{sp}(k+p|k) - y(k+p|k)\|^2_{\mathbf{\Psi}} + \sum_{p=0}^{N_u-1} \|\Delta u(k+p|k)\|^2_{\mathbf{\Lambda}}, \qquad (1)$$

where $\|x\|^2_{\mathbf{R}} = x^T\mathbf{R}x$, $\mathbf{\Psi} \geqslant \mathbf{0}$ and $\mathbf{\Lambda} > \mathbf{0}$ are square diagonal scaling matrices, positive semi-definite and positive definite, respectively, of dimensions corresponding to the dimensions n_y and n_u of the vectors u and y representing process controlled outputs and process control inputs, respectively. A simpler formulation of (1) is often used in theoretical considerations, with one scaling scalar λ only, i.e., $\mathbf{\Psi} = \mathbf{I}$ and $\mathbf{\Lambda} = \lambda\mathbf{I}$. $N_u \leqslant N$ denotes the length of the control horizon, $y^{sp}(k+p|k)$ and $y(k+p|k)$ are predictions of the set-point (reference) and process output vectors, for a future sample $k+p$, but calculated at the current sample k, $p = 1, \ldots, N$. We assume in the paper that scaling matrices are the same for each sample over the prediction horizon, extension to variable scaling is possible, see e.g., [7] for results of the influence of sample-dependent scaling.

Control input increments on the control horizon are the decision variables of the MPC optimisation problem in most standard DMC formulations,

$$\Delta u(k+p|k) = u(k+p|k) - u(k+p-1|k), \quad p = 0, \ldots, N_u - 1.$$

Therefore, the vector of decision variables, denoted by $\Delta\mathbf{u}(k)$, is as follows

$$\Delta\mathbf{u}(k) = \left[\Delta u(k|k)^T \; \Delta u(k+1|k)^T \cdots \Delta u(k+N_u-1|k)^T\right]^T. \qquad (2)$$

The optimization of $J(k)$ can be under constraints, we assume the following simple constraints:

$$-\Delta u_{\max} \leqslant \Delta u(k+p|k) \leqslant \Delta u_{\max}, \qquad p = 0, \ldots, N_u-1, \qquad (3)$$

$$u_{\min} \leqslant u(k+p|k) \leqslant u_{\max}, \qquad p = 0, \ldots, N_u-1, \qquad (4)$$

$$y_{\min} \leqslant y(k+p|k) \leqslant y_{\max}, \qquad p = 1, \ldots, N. \qquad (5)$$

More general form of the constraints is possible, including any linear functions of all variables used, but is here avoided for simplicity.

Denoting by $\mathbf{y}^{sp}(k)$ and $\mathbf{y}^{pr}(k)$ composite vectors of set-points and predicted outputs on the prediction horizon, respectively,

$$\mathbf{y}^{sp}(k) = \left[y^{sp}(k+1|k)^T \quad \cdots \quad y^{sp}(k+N|k)^T \right]^T, \tag{6}$$

$$\mathbf{y}^{pr}(k) = \left[y(k+1|k)^T \quad \cdots \quad y(k+N|k)^T \right]^T, \tag{7}$$

we can formulate *the MPC optimization problem*, which calculates the optimal control trajectory at each sampling instant, as follows:

$$\begin{aligned} &\min_{\Delta \mathbf{u}(k)} \{ J(k) = \|\mathbf{y}^{sp}(k) - \mathbf{y}^{pr}(k)\|^2_{\underline{\boldsymbol{\Psi}}} + \|\Delta \mathbf{u}(k)\|^2_{\underline{\boldsymbol{\Lambda}}} \} \\ &\text{subject to : } (3), (4) \text{ and } (5), \end{aligned} \tag{8}$$

where

$$\underline{\boldsymbol{\Psi}} = \mathrm{diag}\{ \overbrace{\boldsymbol{\Psi}, \ \ldots, \ \boldsymbol{\Psi}}^{N \text{ times}} \}, \qquad \underline{\boldsymbol{\Lambda}} = \mathrm{diag}\{ \overbrace{\boldsymbol{\Lambda}, \ \ldots, \ \boldsymbol{\Lambda}}^{N_u \text{ times}} \} \tag{9}$$

and where the predicted trajectory $\mathbf{y}^{pr}(k)$ of the process outputs is calculated using the process model, in the form of unit step responses.

For MIMO processes, it is most convenient to formulate the overall process step response model in the matrix form, see [11]. Such model consists of D matrices $\mathbf{S}_l$, each matrix corresponding to one sampling instant l, covering the horizon of sampling periods needed for the outputs to stabilize after the step input change, i.e., the horizon of dynamics of length D of the process,

$$\mathbf{S}_l = \begin{bmatrix} s_l^{11} & s_l^{12} & s_l^{13} & \cdots & s_l^{1n_u} \\ s_l^{21} & s_l^{22} & s_l^{23} & \cdots & s_l^{2n_u} \\ s_l^{31} & s_l^{32} & s_l^{33} & \cdots & s_l^{3n_u} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ s_l^{n_y1} & s_l^{n_y2} & s_l^{n_y3} & \cdots & s_l^{n_yn_u} \end{bmatrix}_{n_y \times n_u}, \quad l = 1, 2, \ldots, D,$$

where s_l^{ij} denotes l-th element of the response of the i-th process output on the unit step change of the j-th process input, $i = 1, \ldots, n_y$, $j = 1, \ldots, n_u$, see [11]. Using this model, the following prediction formulae can be obtained (for the case without measured disturbances) [11]:

$$\mathbf{y}^{pr}(k) = \mathbf{M}\,\Delta\,\mathbf{u}(k) + \mathbf{y}(k) + \mathbf{M}^P \Delta\,\mathbf{u}^P(k), \tag{10}$$

where

$$\mathbf{M} = \begin{bmatrix} \mathbf{S}_1 & 0 & 0 & \cdots & 0 \\ \mathbf{S}_2 & \mathbf{S}_1 & 0 & \cdots & 0 \\ \mathbf{S}_3 & \mathbf{S}_2 & \mathbf{S}_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{S}_{N_u} & \mathbf{S}_{N_u-1} & \mathbf{S}_{N_u-2} & \cdots & \mathbf{S}_1 \\ \mathbf{S}_{N_u+1} & \mathbf{S}_{N_u} & \mathbf{S}_{N_u-1} & \cdots & \mathbf{S}_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{S}_N & \mathbf{S}_{N-1} & \mathbf{S}_{N-2} & \cdots & \mathbf{S}_{N-N_u+1} \end{bmatrix} \tag{11}$$

is the dynamic matrix,

$$\mathbf{M}^P = \begin{bmatrix} \mathbf{S}_2 - \mathbf{S}_1 & \mathbf{S}_3 - \mathbf{S}_2 & \mathbf{S}_4 - \mathbf{S}_3 & \cdots & \mathbf{S}_D - \mathbf{S}_{D-1} \\ \mathbf{S}_3 - \mathbf{S}_1 & \mathbf{S}_4 - \mathbf{S}_2 & \mathbf{S}_5 - \mathbf{S}_3 & \cdots & \mathbf{S}_{D+1} - \mathbf{S}_{D-1} \\ \mathbf{S}_4 - \mathbf{S}_1 & \mathbf{S}_5 - \mathbf{S}_2 & \mathbf{S}_6 - \mathbf{S}_3 & \cdots & \mathbf{S}_{D+2} - \mathbf{S}_{D-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{S}_{N+1} - \mathbf{S}_1 & \mathbf{S}_{N+2} - \mathbf{S}_2 & \mathbf{S}_{N+3} - \mathbf{S}_3 & \cdots & \mathbf{S}_{N+D-1} - \mathbf{S}_{D-1} \end{bmatrix}, \tag{12}$$

$$\mathbf{y}(k) = \begin{bmatrix} y(k) \\ y(k) \\ y(k) \\ \vdots \\ y(k) \end{bmatrix}, \qquad \Delta \mathbf{u}^P(k) = \begin{bmatrix} \Delta u(k-1) \\ \Delta u(k-2) \\ \Delta u(k-3) \\ \vdots \\ \Delta u(k-(D-1)) \end{bmatrix}, \tag{13}$$

and where $\mathbf{y}(k)$ consists of N repetitions of the vector $y(k)$.

Once the optimization problem (8) with the predictions (10) has been solved, only the first element $\Delta\hat{u}(k|k)$ of the optimal control trajectory is further taken as the current process input $u(k) = u(k-1) + \Delta\hat{u}(k|k)$. But, after the next measurement (at the next sampling instant) the whole DMC algorithm procedure is repeated (which is usually called a receding horizon strategy).

We shall assume $n_u = n_y$ in the paper, which in the linear case always yields a unique solution of the MPC optimization problem. However, by appropriate augmentation of the performance function, this can be easily generalized to the case $n_u > n_y$, not unusual in MPC applications, see, e.g., [11].

3. The DMC Algorithm with Laguerre Functions

The Laguerre functions are usually defined by their transfer functions – the transfer function of the Laguerre function of order n is

$$\mathcal{L}_n(z) = \frac{\sqrt{1-a^2}}{z-a}\left(\frac{1-az}{z-a}\right)^{(n-1)}, \tag{14}$$

where a is a scaling factor, $0 \leqslant a < 1$, see, e.g., [17, 19]. Thus the Laguerre function of order n is

$$l_n(k) = \mathcal{Z}^{-1}(\mathcal{L}_n(z)). \tag{15}$$

Define a set of n_L Laguerre functions, of increasing order

$$l(p) = [l_1(p) \cdots l_{n_L}(p)]^T, \quad p = 0, \ldots, N-1. \tag{16}$$

Taking into account the structure of these functions, it can be easily deduced that

$$l(p+1) = \mathbf{A}\, l(p), \tag{17}$$

where

$$\mathbf{A} = \begin{bmatrix} a & 0 & 0 & \cdots & 0 \\ \beta & a & 0 & \cdots & 0 \\ -a\beta & \beta & a & \cdots & 0 \\ a^2\beta & -a\beta & \beta & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -a^{n_L-2}\beta & -a^{n_L-3}\beta & \cdots & \beta & a \end{bmatrix}, \tag{18}$$

with the initial value

$$l(0) = \sqrt{1-a^2} \begin{bmatrix} 1 \\ -a \\ a^2 \\ -a^3 \\ \vdots \\ (-a)^{n_L-1} \end{bmatrix} \tag{19}$$

and where $\beta = 1 - a^2$, see, e.g., [19].

Let us assign a set of n_L Laguerre functions $l_1^j, l_2^j, \ldots, l_{n_L}^j$, $j = 1, \ldots, n_u$, to every component u_j of the process control input vector $u = [u_1 \cdots u_{n_u}]^T$. Generally, the numbers n_L can differ for components of u, but they are usually taken equal, for simplicity, we also apply this assumption. Next, components of the DMC decision control vector $\Delta\, \mathbf{u}(k)$ (2) are parametrised in the following way

$$\Delta\, u_j(k+p|k) = \sum_{m=1}^{n_L} l_m^j(p) c_m^j(k) = l^j(p)^T c^j(k),$$
$$j = 1, \ldots, n_u, \quad p = 0, \ldots, N-1, \tag{20}$$

where

$$l^j(p) = \left[l_1^j(p) \ \cdots \ l_{n_L}^j(p) \right]^T \tag{21}$$

is defined according to (16) and $c^j(k)$ is the vector of coefficients of the Laguerre functions,

$$c^j(k) = \left[c_1^j(k)\ c_2^j(k)\ \cdots\ c_{n_L}^j(k)\right]^T, \quad j = 1, \ldots, n_u. \tag{22}$$

Therefore, we have finally the following representation of the control vector

$$\Delta u(k+p|k) = \begin{bmatrix} l^1(p)^T c^1(k) \\ l^2(p)^T c^2(k) \\ \vdots \\ l^{n_u}(p)^T c^{n_u}(k) \end{bmatrix}, \quad p = 0, \ldots, N-1. \tag{23}$$

Further, defining the composite vector $\mathbf{c}(k)$ of all coefficients $c^j(k)$,

$$\mathbf{c}(k) = \left[c^1(k)^T\ c^2(k)^T\ \cdots\ c^{n_u}(k)^T\right]^T \tag{24}$$

and N matrices

$$\mathbf{L}(p) = \operatorname{diag}\left(l^1(p)^T, l^2(p)^T, \ldots, l^{n_u}(p)^T\right), \quad p = 0, \ldots, N-1, \tag{25}$$

we can transform (23) to the form

$$\Delta u(k+p|k) = \mathbf{L}(p)\,\mathbf{c}(k), \quad p = 0, \ldots, N-1. \tag{26}$$

Therefore, we get the final formula

$$\Delta \mathbf{u}(k) = \begin{bmatrix} \Delta u(k|k) \\ \Delta u(k+1|k) \\ \vdots \\ \Delta u(k+N-1|k) \end{bmatrix} = \begin{bmatrix} \mathbf{L}(0) \\ \mathbf{L}(1) \\ \vdots \\ \mathbf{L}(N-1) \end{bmatrix} \mathbf{c}(k) = \mathbf{L}\,\mathbf{c}(k). \tag{27}$$

Observe that the matrix $\mathbf{L}$ does not depend on time, it depends only on the value of the scaling factor a. Thus it is evaluated only once, off-line during the design phase. Generally, different scaling factors a^j can be used for different components u_j of the vector u, but this would only influence the off-line evaluation of the Laguerre functions l^j and the matrix $\mathbf{L}$.

Substituting (27) into the prediction equation (10), into the performance function (8) and into the inequality constraints (3),(4),(5) changes the decision variables of the DMC optimization problem. It becomes the problem of optimization with respect to the vector of the Laguerre coefficients $\mathbf{c}(k)$. The DMC algorithm with this optimization problem will be further denoted by the acronym DMCL (DMC with the control input parametrization using the Laguerre functions).

The dimension of the vector $\mathbf{c}(k)$ is $n_c = n_u{\cdot}n_L$ and is therefore not dependent on the control horizon N_u, whereas in the standard DMC algorithm the dimension of the decision vector $\Delta\,\mathbf{u}(k)$ is $n_u \cdot N_u$. Therefore, if $n_L < N_u$, dimensionality of the DMC optimization problem is diminished – what is important, as this problem is solved on-line at every sampling instant. As n_L is not dependent on N_u, we assume $N_u = N$ in the DMCL algorithm. Further, if $N_u = N$ and the prediction horizon N is sufficiently long, the Laguerre functions become orthonormal (see, e.g., [19]). Taking this into account, the second term in the performance function in (8) can be transformed to the following simple form:

$$\|\Delta\,\mathbf{u}(k)\|^2_{\underline{\mathbf{\Lambda}}} = \mathbf{c}(k)^T\mathbf{L}^T\underline{\mathbf{\Lambda}}\mathbf{L}\mathbf{c}(k) = \mathbf{c}(k)^T\underline{\mathbf{\Lambda}}_{\mathbf{L}}\mathbf{c}(k) = \|\mathbf{c}(k)\|^2_{\underline{\mathbf{\Lambda}}_L}, \tag{28}$$

where

$$\underline{\mathbf{\Lambda}}_L = \operatorname{diag}\left(\lambda_1\mathbf{I}_{n_L},\ \lambda_2\mathbf{I}_{n_L},\ \cdots,\ \lambda_{n_u}\mathbf{I}_{n_L}\right) \tag{29}$$

and where $\mathbf{I}_{n_L}$ is the identity matrix of dimension n_L and the weights λ_i are elements of the weighting matrix $\mathbf{\Lambda}$ in the initial performance function (1), $\mathbf{\Lambda} = \operatorname{diag}(\lambda_1, \lambda_2, \ldots, \lambda_{n_u})$. Therefore, the performance function takes the form

$$J(k) = \left\|\mathbf{y}^{sp}(k) - \left[\mathbf{M}\mathbf{L}\mathbf{c}(k) + \mathbf{y}(k) + \mathbf{M}^P\Delta\,\mathbf{u}^P(k)\right]\right\|^2_{\underline{\mathbf{\Psi}}} + \|\mathbf{c}(k)\|^2_{\underline{\mathbf{\Lambda}}_L}. \tag{30}$$

In the simplified case when there is one common weighting factor λ for all components of the control input vector, i.e., $\mathbf{\Lambda} = \lambda\mathbf{I}_{n_u}$, eq. (28) reduces to $\lambda\,\|\mathbf{c}(k)\|^2$.

4. Efficiency of the DMCL algorithm

As the dimensionality n_c of the decision vector $\mathbf{c}(k)$ does not depend on the control horizon, we can take both control and prediction horizons equal, $N_u = N$, without negative influence on the on-line computations. On the other hand, $n_c = n_u \cdot n_L$ and thus depends on the number of Laguerre functions n_L taken to parametrize the trajectory of each element u_i of the input vector u, over the prediction horizon. The practice of model approximations by the Laguerre functions has shown that taking relatively small number of these functions leads usually to satisfactory results. This fact will be also confirmed by simulation results analysed further in this section. Therefore, the DMCL algorithm should be a more effective solution in cases when longer control horizons are appropriate in the standard DMC algorithm. Taking this into account, we have selected two benchmark example processes corresponding to this case. The aim of our research will be to analyse properties of the DMCL algorithm in comparison with the results obtained using a standard formulation of the DMC alhorithm.

4.1. Example 1

First, we shall apply the DMCL algorithm to the composition control in the Wood-Berry (WB) methanol-water distillation column example, a well known benchmark for MIMO process control [20]. The 2×2 continuous time transfer function model in this column is as follows:

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-3s}}{21s+1} \\ \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix} + \begin{bmatrix} \dfrac{3.8e^{-8s}}{14.9s+1} \\ \dfrac{4.9e^{-3s}}{13.2s+1} \end{bmatrix} F(s). \tag{31}$$

A discrete-time model is needed, therefore the model (31) has been discretized, with different sampling periods T_p, as the sampling period can be an important design parameter. We have started with $T_p = 1$, the largest sampling period resulting in discrete-time model with accurate representation of delay times. Using for discretization the Matlab function `c2d` we got the following result:

$$\begin{bmatrix} Y_1(z) \\ Y_2(z) \end{bmatrix} = \begin{bmatrix} \dfrac{0.744z^{-1}}{z-0.9419} & \dfrac{-0.8789z^{-3}}{z-0.9535} \\ \dfrac{0.5786z^{-7}}{z-0.9123} & \dfrac{-1.302z^{-3}}{z-0.9329} \end{bmatrix} \begin{bmatrix} U_1(z) \\ U_2(z) \end{bmatrix} + \begin{bmatrix} \dfrac{0.2467z^{-8}}{z-0.9351} \\ \dfrac{0.3575z^{-3}}{z-0.927} \end{bmatrix} F(z). \tag{32}$$

Step responses of this model are shown in Fig. 1.

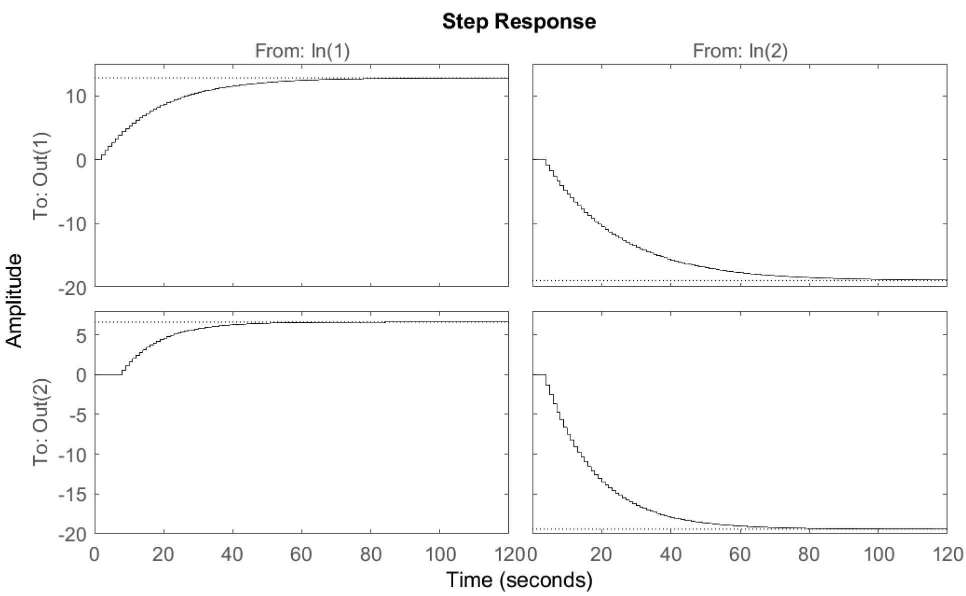

Figure 1: Step responses of discrete-time Wood-Berry column model, $T_p = 1$

The number n_L of the Laguerre functions modeling every control input trajectory is a key factor influencing efficiency of the DMCL algorithm. We assumed this number the same for each component of the control vector, for simplicity and without loss of generality. The influence of n_L on the results of a single optimization of the DMCL performance function (30) was analysed first. The scaling factor $a = 0.55$ was used, values of a around this value were found to be appropriate. Sample results are presented in Fig. 2 and Fig. 3, for a case with unit step changes of both output reference values (set-points) at the current sample time (0 in the figures).

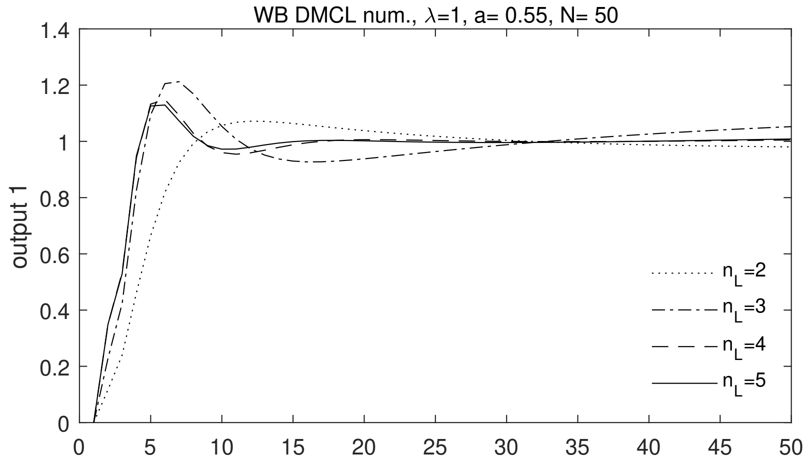

Figure 2: Optimized trajectories of the first output, after unit step changes of both set-points, for different numbers of the Laguerre functions n_L, $T_p = 1$

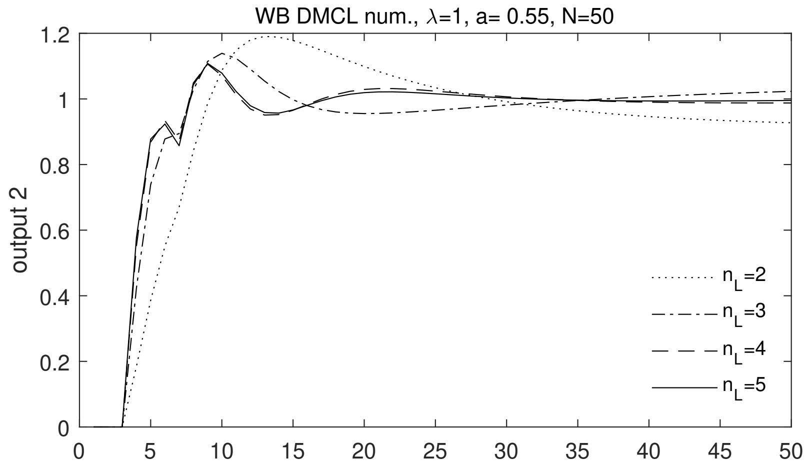

Figure 3: Optimized trajectories of the second output, after unit step changes of both set-points, for different numbers of the Laguerre functions n_L, $T_p = 1$

It can be easily seen that shapes of the trajectories stabilize with the increase of the value of n_L, differences between the trajectories for $n_L = 4$ and $n_L = 5$ are small. This indicates that the choice $n_L = 5$ should be safely satisfactory, even taking $n_L = 4$ should suffice. Let us notice that $n_L = 5$ corresponds to the control horizon $N_u = 5$ in the standard DMC algorithm, assuming the same dimensionality of optimization problems in both algorithms.

Output and input trajectories of a sample simulation of the feedback control system with the DMCL algorithm with $T_p = 1$ are depicted in Fig. 4. The applied scenario of external influences consists of unit step changes of the reference values, at sampling instants $k = 2$, $k = 60$ and $k = 120$, and of a step change of the disturbance F (see (32)) at the samppling instant $k = 200$. Following the inspection of Figs. 2 and 3, the prediction horizon was chosen equal to $N = 20$ for the sample simulation, which seems to be the shortest value assuring that the controlled outputs stabilize over the prediction horizon. Following the previous discussion, $N_u = N = 20$ was selected, as also the control inputs should stabilize with stabilizing outputs. The horizon of dynamics of the model used for the prediction in the DMC controller was assumed $D = 80$, shorter values were decreasing the quality of the control. For the process simulation in the feedback control loop the horizon of dynamics $D_{pr} = 120$ was taken – compare the step responses presented in Fig. 1.

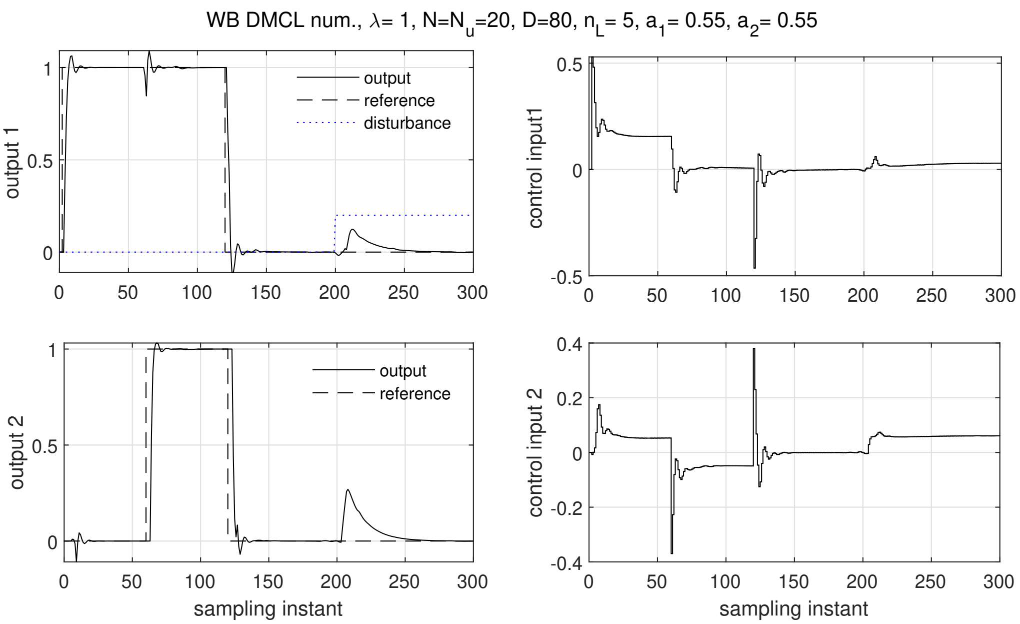

Figure 4: Trajectories of the process outputs and inputs in a simulation of the DMCL feedback control system under step changes of the reference values and the disturbance

To compare efficiency of the DMCL and standard DMC algorithms, a series of simulations were performed, for different values of n_L and N_u, also changing other main parameters of the DMCL and DMC algorithms, the horizons and, first of all, the sampling period Tp. Recall that the number of computations and thus the time of computations, measured roughly by dimensionality of the optimization problem, is equal for both algorithms for $n_L = N_u$. Each simulation was performed for the reference and disturbance scenario as applied to the simulation presented in Fig. 4. To measure the control quality, the associated value of the commonly used integrated squared control error (ISE) was calculated, over the simulation horizon.

Numerous simulation runs were performed, for different parameters of both DMCL and DMC algorithms. Most relevant aggregated results are presented in Figs. 5 to 8. To compare, simultaneously, the efficiency of both algorithms, the results are shown in terms of variable values $n_L = N_u$, for different sampling

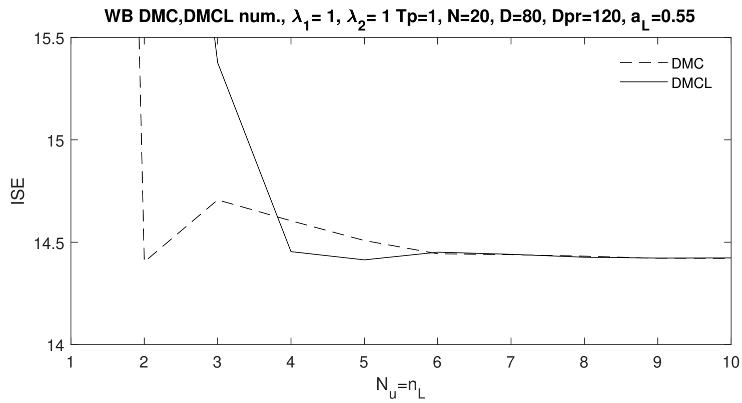

Figure 5: Comparison of control quality of DMCL and DMC algorithms, $T_p = 1$

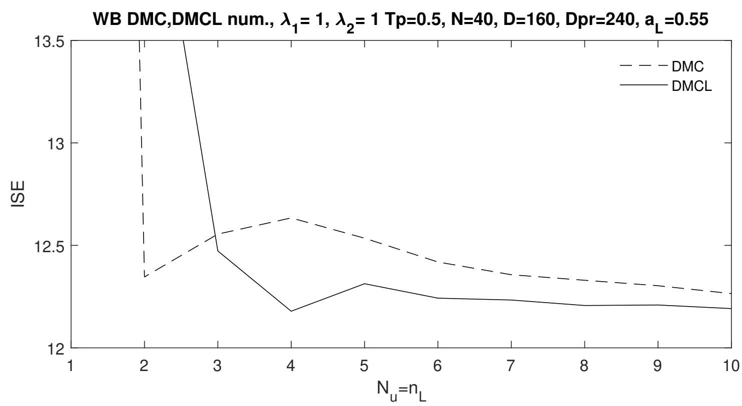

Figure 6: Comparison of control quality of DMCL and DMC algorithms, $T_p = 0.5$

periods T_p from 1 to 0.125. The control horizons are recalculated to maintain the same length in terms of time, for different sampling periods.

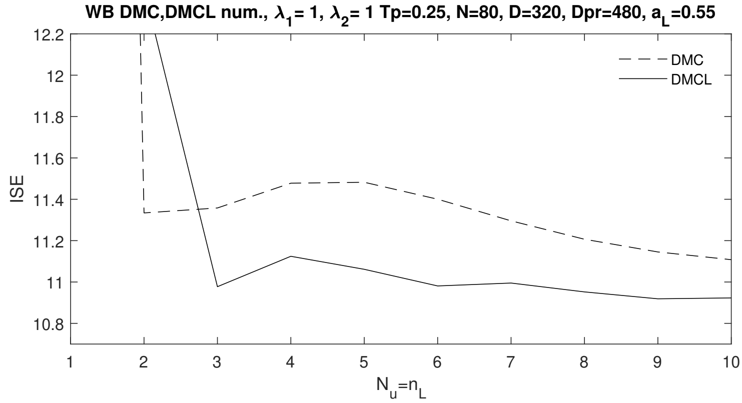

Figure 7: Comparison of the control quality of DMCL and DMC algorithms, $T_p = 0.25$

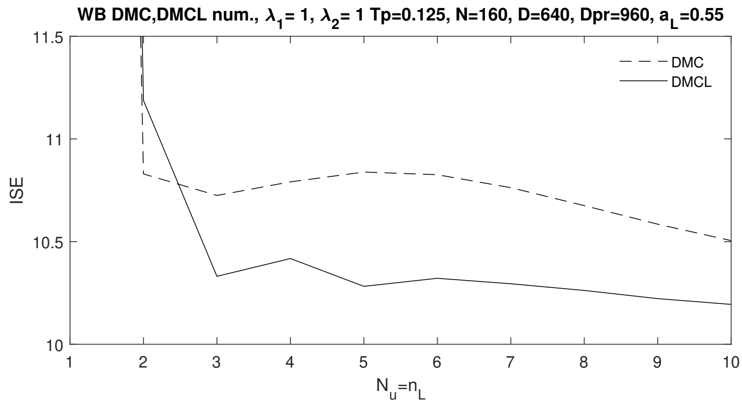

Figure 8: Comparison of the control quality of DMCL and DMC algorithms, $T_p = 0.125$

It can be easily seen from this comparison that the DMCL algorithm is a sound alternative to the classical DMC algorithm with the process control inputs as the decision variables, the more the smaller the sampling period (the ISE axes are comparable in all figures). It achieves good control quality for $n_L = 4$ Laguerre functions and, certainly, keeps it with increasing n_L (for each control input), whereas the standard DMC algorithm achieves the same level of control quality for larger values of the control horizon N_u, much larger for smaller sampling periods. This can be easily explained, it follows from the fact that with shorter sampling intervals longer horizons are needed in the standard DMC algorithm to achieve best control performance, for the same physical dynamical properties of the process. Therefore, the smaller sampling period, the longer control horizon is needed in DMC to attain the control performance of the DMCL algorithm, which can be easily seen in Figs. 5 to 8. The DMCL algorithm operates all the time with the full length control horizon equal to the prediction horizon, all the time with the same small number of Laguerre functions independent on the length of horizons. For $T_p = 1$ DMC arrives at asymptotically optimal performance with $N_u = 6$, obtained in DMCL from $n_L = 4$ and upwards, see Fig. 5. For smaller T_p DMC needs significantly longer control horizons N_u, whereas in DMCL the sufficient number of Laguerre functions n_L is about 5, independently of T_p. Observe that smaller sampling periods lead also to smaller optimal (asymptotic) values of ISE, due to smaller additional delay added in the feedback control loop by the sampling process itself. The presented results are important, as implementation of faster sampling, even for computationally more demanding control algorithms, is possible due to enormous potential computational power of modern process controllers.

Other parameters important for tuning the MPC, including DMCL and DMC algorithms, to achieve appropriate control properties, in particular robust stability, are the weighting factors in the performance function. Influence of these parameters was also thoroughly investigated for the considered example. All obtained results confirmed the conclusions stated earlier for the results presented in Figs. 5 to 8. To shorten the length of the paper only one, representative sample example is shown in Fig. 9.

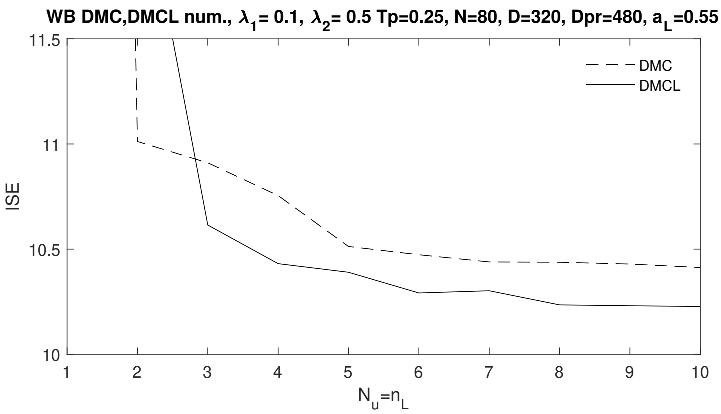

Figure 9: Comparison of control quality of DMCL and DMC algorithms, $T_p = 0.25$, different values of weighting coefficients λ_1 and λ_2

It should be noticed that the dynamics of the considered WB column seems to be representative, with different delays and time constants leading to long horizons in the DMC algorithms. However, transfer functions of this column model are without zeros in the numerators, in particular without unstable zeros. This will be the case in the second process example considered.

4.2. Example 2

We shall apply the DMCL algorithm to the MIMO 2×2 process examined in [18], where an MPC controller with state-space process model was investigated. The continuous time transfer model of this process is as follows

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} \dfrac{12.8(-s+4)^2}{(16.7s+1)(s+4)^2} & \dfrac{-18.9(-3s+4)^2}{(21s+1)(3s+4)^2} \\ \dfrac{12.8(-7s+4)^2}{(10.9s+1)(7s+4)^2} & \dfrac{-19.4(-3s+4)^2}{(14.4s+1)(3s+4)^2} \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}. \tag{33}$$

We have investigated this process with the DMCL controller for different sampling intervals, from $T_p = 1$ to $T_p = 0.125$, as for the WB column example (in [18] only $T_p = 0.1$ was used).

Discrete-time step responses corresponding to transfer functions (33) for the sampling interval $T_p = 1$ are shown in Fig. 10. Notice the non-minimum phase behaviour of the responses corresponding to unstable zeroes of the transfer functions.

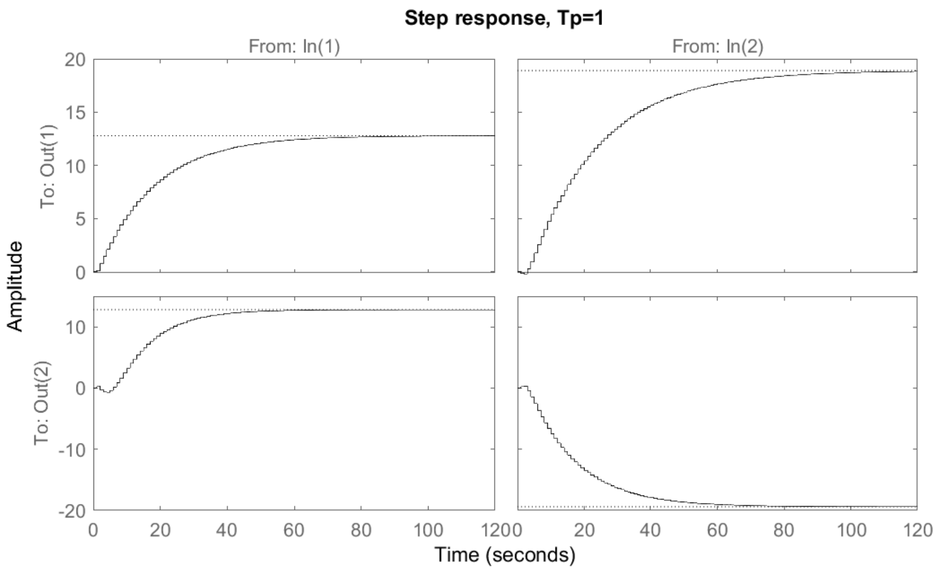

Figure 10: Step responses of discrete-time model corresponding to (33), for $T_p = 1$

Because the range of values of time constants in the transfer functions in (33), and thus thus the lengths of the step responses, are similar as in the case of the WB column model in Example 1, we have taken the same values of prediction, control and dynamics horizons in the design of the DMCL and DMC controllers. Further, the same time interval of simulation and similar scenario of external influences was assumed, the only difference being the replacement of the step change of the disturbance F of the WB column at the samppling instant $k = 200$ by two step changes of external disturbances acting directly on the process inputs. Output and input trajectories of a sample simulation of the feedback control system with the DMCL algorithm with $T_p = 1$ are depicted in Fig. 11 (notice the input disturbances denoted by z_{inp1} and z_{inp2}).

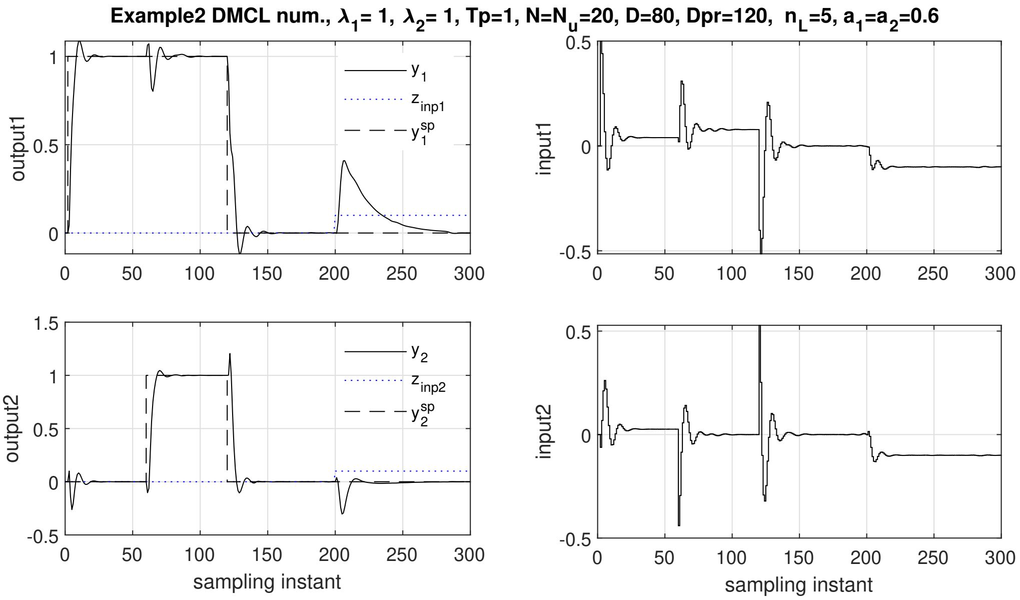

Figure 11: Trajectories of process outputs and inputs in a simulation of the DMCL feedback control system under step changes of reference values and input disturbances

To compare efficiency of the DMCL and standard DMC algorithms, a series of simulations was performed, for different values of n_L and N_u, also changing other main parameters of the DMCL and DMC algorithms, the horizons and, first of all, the sampling period Tp – in a similar way as for Example 1 in the previous section. Each simulation was performed for the reference and disturbance scenario as applied to the simulation presented in Fig. 11. To measure the control quality, the associated value of the integrated squared control error (ISE) was calculated, over the simulation horizon. The simulation runs were performed for both DMCL and DMC algorithms. Most relevant aggregated results are presented in Figs. 12 to 15. To compare, simultaneously, the efficiency of both algorithms, the results are shown in terms of variable values $n_L = N_u$, for different sampling periods T_p from 1 to 0.125. The control horizons are recalculated to maintain the same length in terms of time, for different sampling periods.

It can be easily seen from this figures that the comparison of the control quality between DMCL and DMC algorithms is similar as it was in Example 1 for the

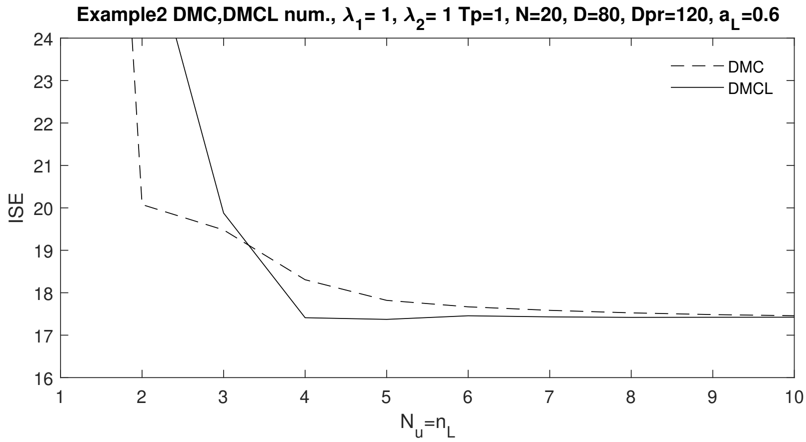

Figure 12: Comparison of control quality of the DMCL and DMC algorithms, $T_p = 1$

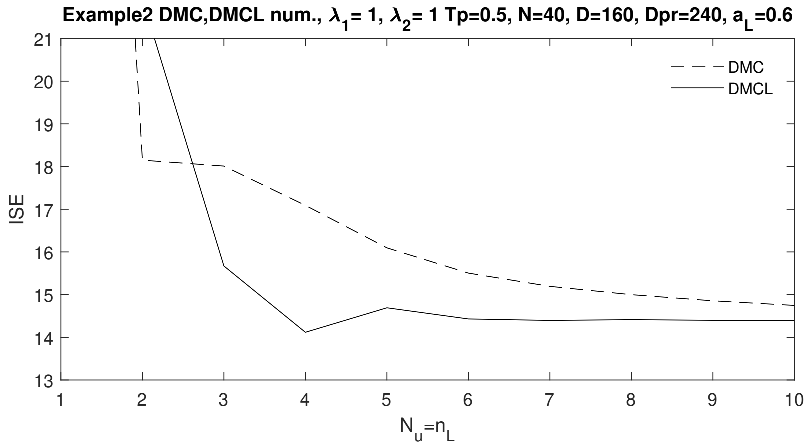

Figure 13: Comparison of control quality of the DMCL and DMC algorithms, $T_p = 0.5$

Wood-Berry column, in this case even more in favor of the DMCL algorithm – the differences are larger. The DMCL algorithm is again a sound alternative to the classical DMC algorithm with the process control inputs as the decision variables, the more the smaller the sampling period (the scale of ISE axes are comparable in all figures). It achieves good control quality for about $n_L = 5$ Laguerre functions and, certainly, keeps it with increasing n_L, only for the smallest sampling period $T_p = 0.125$ slightly more Laguerre functions are needed. Whereas the standard DMC algorithm achieves the same level of control quality for larger values of the control horizon N_u, the larger the smaller sampling periods. Observe that smaller

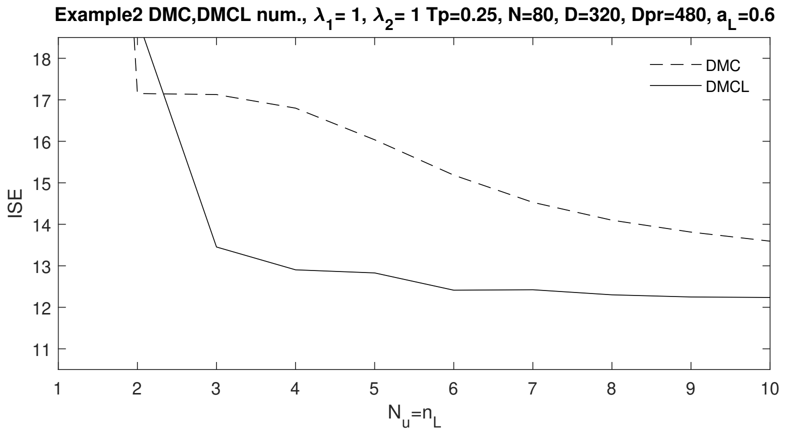

Figure 14: Comparison of control quality of the DMCL and DMC algorithms, $T_p = 0.25$

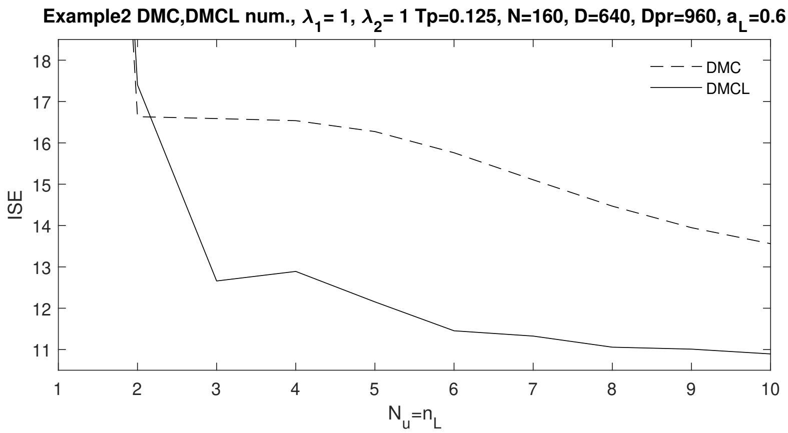

Figure 15: Comparison of control quality of the DMCL and DMC algorithms, $T_p = 0.125$

sampling periods lead also in this example to smaller optimal (asymptotic) values of ISE, due to smaller additional delay added in the feedback control loop by the sampling process itself.

5. Conclusions

Development and analysis of the DMCL (DMC with Laguerre functions) algorithm, the DMC model predictive control algorithm with parametrisation of the control input trajectories by sets of the Laguerre functions, was the aim of the paper. The appropriate formulation of the algorithm was developed, where coefficients of the approximation by the Laguerre functions instead of process control input values are the decision variables of the MPC optimization problem. Then the developed DMCL algorithm was applied to two MIMO benchmark processes with difficult dynamics. Aggregate results of extensive simulation studies were presented showing that the DMCL algorithm is a sound alternative to the standard DMC algorithm. It offers the possibility to deliver equivalent results more effectively, with lower computational effort and thus shorter computational time, especially for problems with long prediction and control horizons. It was shown that this happens, in particular for faster sampling (smaller samppling intervals). These results are important, as implementation of faster sampling, even for computationally more demanding controllers, is now possible and is applied due to enormous potential power of modern process controllers. This enables also to replace PID control loops and structures by more effective SISO or MIMO predictive controllers when PID performance is not satisfactory due to complex process dynamics, including stronger interactions in MIMO processes. Application of predictive controllers in embedded systems needs also effective implementations.

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