Research paper

Modelling of the spatial structure of longwall buildings with large-scale quasi finite elements and applying the method in the interaction issues with subsoil – with regard to their implementation in stages

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Abstract: The paper presents the original concept of description and analysis of buildings (wall and floor structures), corresponding to the natural components of construction, quasi finite elements (QWSFS). This concept constitutes one of the component of the developed, interactive model of deep foundation buildings. The presented modelling method enables a significant reduction of the number of unknowns, which in the case of interaction building – subsoil, gives a possibility of including the factual geometry and building development stiffness into the FEM model. Therefore the true representation of static operation of the objects can be analysed. The paper gives basic assumptions to the construction of the QWSF-superelements as well as the results of numerical tests conducted. The potential of using the developed modelling concept in the analysis of the structural elements and deep foundation problems, in a three-dimensional system: subsoil – new building – potential neighbouring building development (at each stage of erection of investment, using a structural statics stage analysis) was presented.

Keywords: deep foundation buildings, interaction building – subsoil, large-scale quasi finite elements, superelements, three-dimensional schemes

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1. Introduction

The basic tool for analysing any kind of engineering problems is finite element method (FEM), now more and more often in implementation with the BIM methodology [1]. However, a reliable representation of the problem with the use of FEM requires a large number of unknowns to be entered to the system. Because of the limits of computer equipment (especially RAM operating memory [2, 3]), it is often impossible to carry out analysis or at least very difficult to conduct (excluding multiprocessor units – supercomputers).

Computing problems tend to occur most often during the analysis of large interaction issues: building – subsoil (Fig. 1a. Due to a large number of unknowns (up to several million), in the case of deep foundation buildings (e.g. in deep excavation), the numerical model is simplified to two-dimensional system (2D) or three-dimensional system (3D) – excluding the representation of the factual geometry and the stiffness of the analysed constructions (both of the new building and the neighbouring ones). The building development area is modelled in a simplified manner by including equivalent schemes to the model (foundation blocks in the case of the existing building development; floor slabs, walls and columns in the case of a new building) representing the building underground parts [4–6].

![Fig. 1. Components of the interactive and “tracking” model of deeply set buildings together with existing buildings in the neighborhood: a) 3D model view; b) subdivision of the building structure into constitutive elements](image)

Besides, the analysis of such problems should involve so called “tracking” procedures [7, 8] allowing for the analysis of investment in individual stages of its erection – from excavation, through erection phase, to occupancy phase.

Given the size of the model (in three-dimensional description) as well as the necessity of carrying out analyses with the use of the aforementioned “tracking” algorithms, such simplifications of the numerical model in the problems of interaction are – from the computation time-consumption point of view – rational. But such an approach, by not including the interaction between individual components of the model to the numerical simulation (among others by not including the factual geometry and the stiffness of the analysed buildings), can result, for
example, in incorrect assessment of the effort state of the new building being erected as well as
the existing ones in the impact area of the building investment [9–12].

The concept for using large-scale finite elements constitutes one of the components of
the developed model of deep foundation buildings in the system: subsoil – new building –
eighbouring building development (Fig. 1a) – in individual stages of erection of the building
investment [13,14].

The paper presents the developed, special (large-scale) quasi wall-slab finite superelements
(QWSFS – elements for wall structures description) together with methodology for their im-
plementation in the numerical model, allowing (with a significant reduction of the number of
unknowns – as compared with the traditional modelling) for real (including the geometry and
the stiffness of the objects being analysed) recognition of all – in a single computational system
(3D) – crucial components of the interaction system [13]: building – subsoil.

The QWSF – superelements were developed on the basis of some variant of finite element
method – quasi finite element method (QFEM) [14,15] – on the basis of which the models of new
and neighbouring buildings are described by elements corresponding to the natural components
of constructions, amongst which we can distinguish: wall strips, floor slabs, horizontal and
vertical joints (Fig. 1b).

In the presented paper the description of finite elements was significantly developed. The
plate state has been described and intermediate nodes in the finite elements were added.

Such practices – implementation of solutions aimed at a significant reduction of the number
of unknowns – can also be found in works of other researchers dealing with modelling of
building systems (problems): [16–19].

2. The wall-slab finite superelement (QWSFS)
and methodology of its implementation

The methodology of building of the QWSF-superelement, describing plane-plate state in
wall structures, can be presented as follows:
– static condensation of wall panel to 9-node system:
  • plane state: discretization of the system with iQTDF-elements (with isoparametric,
    quasi two-dimensional, 9-node finite elements; depending on the number of areas with
    window and/or door openings) and condensation of the panel to the QWF-superelement
    (quasi wall strip finite superelement) – Fig. 2b, 2d;
  • plate state (adding plate stress to the plane state): discretization of system with
    9PF-elements (with 9-node, plate finite elements) and condensation of the panel to the
    QSF-superelement (quasi floor slab finite superelement) – Fig. 2c;
  – superposition of both plane and plate states (Fig. 2e).

Additionally – after Zienkiewicz [20] – in order to remove singularities from the equilibrium
equations a fictitious network of torsional stiffness coefficients in all elements of the aggregated
system was introduced.

The stiffness matrices of the iQTDF-element as well as the plate ones were developed in
the symbolic form (according to own software in Matlab language, using the Symbolic Math
Toolbox [21]).
Fig. 2. The developed finite elements: a) element of a wall strip (load diagram and stress state); b) the iQTDF-element (with/without rotational nodal displacements in the element axis); c–e) manner of construction of the wall-slab QWSF-superelements

### 2.1. Components of the QWSF-superelement

The stiffness matrix of the iQTDF-element (component of the plane state of the QWSF-superelement; taking, as theoretical foundation, the section deformation scheme as in Timoshenko type beam, completed with a vertical displacement component – \( w_j \)) can be determined on the basis of the following procedure.

The displacement field of element (where, index: \( X^{\text{iQTDFE},bT} \)); (iQTDFE,bT) refers to the iQTDF-element in version with rotational displacements in its axis – Fig. 2b) (2.1):

\[
(2.1) \quad d^{\text{iQTDFE},bT}_e = u^{bT} + u^{\text{iso}(w_j)} = \begin{cases} u \\ -x \cdot \varphi_y \\ \varphi_y \\ w - x \cdot \varphi_y \\ \varphi_y \end{cases}
\]

where: \( u^{bT} \) – vector of displacements of the “beam” element, Timoshenko type; \( u^{\text{iso}(w_i)} \) – vertical displacement vector; \( u, w, \varphi_y \) – displacement field components.

The strain field (2.2): 

\[
(2.2) \quad \varepsilon^{\text{iQTDFE},bT}_e = \begin{bmatrix} \varepsilon_x \\ \varepsilon_z \\ \gamma_{xz} \end{bmatrix} = L d^{\text{iQTDFE},bT}_e = L N d^{\text{iQTDFE},bT}_e
\]
where: \( L \) – matrix of the differential operators (2.3):

\[
L = \begin{bmatrix}
\frac{\partial}{\partial x} & 0 & 0 \\
0 & \frac{\partial}{\partial z} & -x \cdot \frac{\partial}{\partial z} \\
0 & \frac{\partial}{\partial z} & -1
\end{bmatrix}
\]

\( \mathbf{d}^{iQTDFE,bT}_e \) – vector of the unknown nodal displacements – Fig. 2b (2.4):

\[
\mathbf{d}^{iQTDFE,bT}_e = \begin{bmatrix}
w_1 \varphi_1 & w_2 \varphi_2 & u_3 w_3 & u_4 w_4 & \varphi_5 w_5 & u_6 w_6 & u_7 w_7 & u_8 w_8 & u_9 w_9
\end{bmatrix}^T
\]

\( \mathbf{N} \) – matrix of the shape functions (2.5):

\[
\mathbf{N} = \begin{bmatrix}
0 & 0 & 0 & 0 & N_{3u} & 0 & N_{4u} & 0 & 0 & 0 & N_{6u} & 0 & N_{7u} & 0 & N_{8u} & 0 & N_{9u} & 0 \\
N_{1w} & 0 & N_{2w} & 0 & 0 & N_{3w} & 0 & N_{4w} & N_{5w} & 0 & 0 & N_{6w} & 0 & N_{7w} & 0 & N_{8w} & 0 & N_{9w} \\
0 & N_{1\varphi} & 0 & N_{2\varphi} & 0 & 0 & 0 & 0 & 0 & N_{5\varphi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

For example, the functions \( N_{3u}, N_{6w}, N_{2\varphi} \) can be expressed by formulas (2.6, 2.7, 2.8):

\[
N_{3u} = -\frac{1}{2} (\xi - 1) \left(1 - \zeta^2\right)
\]

\[
N_{6w} = \frac{1}{4} \xi \zeta (1 - \xi) (1 - \zeta)
\]

\[
N_{2\varphi} = \frac{1}{2} (\zeta + 1) \quad \text{where} \quad \xi = \frac{2x}{L}, \zeta = \frac{2z}{H}
\]

The stress field of the wall strip (Fig. 2a) (2.9):

\[
\sigma^{iQTDFE,bT}_e = \begin{bmatrix}
\sigma_x \\
\sigma_z \\
\tau_{xz}
\end{bmatrix}
\]

\[
= \mathbf{D} \varepsilon^{iQTDFE,bT}_e
\]

where: \( \mathbf{D} \) – constitutive matrix of the wall strip (2.10):

\[
\mathbf{D} = \frac{E \cdot b}{1 - \nu^2} \cdot \begin{bmatrix}
1 & 0 & 0 \\
\nu & 1 - \nu & 0 \\
0 & \frac{1 - \nu}{k} & \frac{1}{k}
\end{bmatrix}
\]

where: \( k \) – shear factor (\( k = 1, 2 \) – for rectangular cross-sections) – [14], \( b \) – thickness of planar elements.

Stiffness matrix of the \( iQTDFE,bT \)-element – based on the equation (2.11) of virtual work – can be determined by dependence (2.12):

\[
\int_A (\delta \varepsilon)^T \sigma \, dA - \int_A (\delta \mathbf{u})^T \mathbf{p} \, dA = 0
\]
where: \( \mathbf{p} \) – unit external load vector; \( \mathbf{B} = \mathbf{L N} \) – matrix of relations between the strain and node displacements.

Finally (transformed into replacement system – without rotations) the iQTDFE stiffness matrix can be written as (2.13):

\[
K_{e}^{iQTDFE,bT} = A^{T}K_{e}^{iQTDFE,bT}A
\]

And the new vector of the unknown nodal displacements (Fig. 2b) (2.14, 2.15):

\[
\overrightarrow{\mathbf{d}}_{e}^{iQTDFE,bT} = A^{T}\overrightarrow{\mathbf{d}}_{e}^{iQTDFE,bT}
\]

\[
\overrightarrow{\mathbf{d}}_{e}^{iQTDFE} = \{ w_{1}, w_{2}, u_{3}, w_{3}, u_{4}, w_{4}, u_{5}, w_{5}, u_{6}, w_{6}, u_{7}, w_{7}, u_{8}, w_{8}, u_{9}, w_{9} \}^{T}
\]

where: \( A \) – transformation matrix from the system with rotational components in the element axis (iQTDFE,bT) to the system without rotations (iQTDFE) [14,15].

The methodology of building of the 9PF-element – taking shape functions in the form of the Hermit polynomials as for the 3-node beam [22] (Fig. 3) – was based on Kirchhoff–Love’s thin plate theory.

Fig. 3. Graphs of the shape functions for the selected plate (9PFE) element nodes: a) graph of \( N_{7v} \) function; b) graph of \( N_{7\varphi x} \) function; c) graph of \( N_{7\varphi z} \) function

Deflection of the plate can be written using the equation (2.16):

\[
w(X,Z) = \overrightarrow{\mathbf{N}}\overrightarrow{\mathbf{d}}_{e}^{9PFE}
\]

where: \( \overrightarrow{\mathbf{d}}_{e}^{9PFE} \) – vector of the unknown nodal displacements (2.17):

\[
\overrightarrow{\mathbf{d}}_{e}^{9PFE} = \{ v_{1}, \varphi_{1x}, \varphi_{1z}, \ldots, v_{9}, \varphi_{9x}, \varphi_{9z} \}^{T},
\]

\( \mathbf{N} \) – matrix of the shape functions (2.18):

\[
\mathbf{N} = [ N_{1}, N_{2}, N_{3}, \ldots, N_{9} ]
\]
For example, submatrix for node 7 (2.19) has the form as follows (function graphs are shown in Fig. 3a—3c):

\[
N_7 = \begin{bmatrix}
N_{7v} & N_{7\varphi x} & N_{7\varphi z}
\end{bmatrix}
\]

where:

\[
N_{7v} = -\frac{1}{16}\xi^2\zeta^2 (3\xi - 4) (3\zeta + 4) (\xi + 1)^2 (\zeta - 1)^2
\]

\[
N_{7\varphi x} = -\frac{1}{16}H\xi^2\zeta^2 (3\xi - 4) (\xi + 1)^2 (\zeta - 1)^2 (\zeta + 1)
\]

\[
N_{7\varphi z} = -\frac{1}{16}L\xi^2(3\zeta + 4) (\xi + 1)^2 (\zeta - 1)^2 (\xi - 1)
\]

The state of deformation can be represented by curvatures [20] (2.20, 2.21):

\[
\epsilon_{9PFE} = \begin{bmatrix}
\kappa_x & \kappa_z & \kappa_{xz}
\end{bmatrix}^T
\]

\[
\epsilon_{9PFE} = \begin{bmatrix}
-\frac{\partial^2}{\partial x^2} \\
-\frac{\partial^2}{\partial z^2} \\
2\frac{\partial^2}{\partial x \partial z}
\end{bmatrix} \cdot w(X, Z) = LN\tilde{d}_{e, 9PFE}.
\]

The stress field, in form of the internal forces (bending moments) per unit length, can be written as below (2.22):

\[
\sigma_{9PFE} = \begin{bmatrix}
m_x & m_z & m_{xz}
\end{bmatrix}^T = D\epsilon_{9PFE} = \frac{E}{12 (1 - \nu^2)} \begin{bmatrix}
1 & \nu & 0 \\
\nu & 1 & 0 \\
0 & 0 & \frac{1 - \nu}{2}
\end{bmatrix} \epsilon_{9PFE}
\]

Finally, the plate element stiffness matrix can be determined based on the relationship (2.23):

\[
K_{9PFE} = \int_A (B^T DB) \, dx \, dz
\]

Plane-plate element (QWSF-superelement) was obtained by superposition states: plane (iQTDFelement) and plate (9PF-element), after static condensation of the panel to 9-node system – Fig. 2e.

In the general form, vector of unknown displacements at the "i-th" node can be saved in the form of (2.24):

\[
d_{e,i, QWSFS}^T = \begin{bmatrix}
u & w & \varphi_x & \varphi_y & \varphi_z
\end{bmatrix}
\]

Walls with openings are modeled according to the methodology provided in [15, 23], i.e., by replacing the area with openings with the iQDES element, evenly distributing the lintel stiffness over the storey height.
2.2. Modelling of flexible connections

Many building structures are constructed using coupled shear walls or shear wall-frame systems. The transition region, in which beam and shear walls or frames are interconnected, is often the weakest area [15, 24]. Description of the three-dimensional joint element between wall elements or between wall and floor elements will be presented in this chapter [24], (Fig. 1b).

The distribution of the unknown displacements in 12-node spatial joint is shown in Fig. 4.

Displacement field and strain field $\varepsilon_{e}^{\text{joint}}$ is assumed as in the three-dimensional state of stress. The stress field can be expressed by (2.25):

$$\sigma_{e}^{\text{joint}} = D \varepsilon_{e}^{\text{joint}} = D L d_{e}^{\text{joint}}$$

where: $D$ – constitutive matrix of the spatial joint:

$$D = \frac{E_{\text{joint}}}{(1 + \nu)(1 - 2\nu)} (D_{\text{diag}} + D_{12})$$

$$D_{\text{diag}} = \text{diag} \left[ 1 - \nu, 1 - \nu, 1 - \nu, \frac{1 - \nu}{2}, \frac{1 - \nu}{2}, \frac{1 - \nu}{2} \right]$$

$D_{12}$ – matrix $6 \times 6$ in which: $d_{12} = d_{13} = d_{21} = d_{23} = d_{31} = d_{32} = \nu$ and the other elements are equal 0.

$$d_{e}^{\text{joint}} = N \bar{d}_{e}^{\text{joint}}$$

where: $d_{e}^{\text{joint}}$ – vector of unknown displacements has the form (Fig. 5), $N$ – matrix of the shape functions:

$$\bar{d}_{e}^{\text{joint}} = [u_1 \ w_1 \ v_1 \ \ldots \ u_{12} \ w_{12} \ v_{12}]^T$$

$$N = [N_1 \ I \ N_2 \ I \ N_3 \ I \ \ldots \ N_{12} \ I]$$

where: $I$ – $3 \times 3$ unit matrix.
The shape (for nodes: 1, 5 and 9) function of 12-node joint element has the form [24] (2.27, 2.28, 2.29):

\[ N_1 = \frac{1}{4} \left( 1 + 2\xi + \xi^2 - \eta^2 \right) \left( \zeta^2 - \zeta \right) \]

\[ N_5 = \frac{1}{4} \left( 1 + 2\xi + \xi^2 - \eta^2 \right) \left( 1 - \zeta^2 \right) \]

\[ N_9 = \frac{1}{4} \left( 1 + 2\xi + \xi^2 - \eta^2 \right) \left( \zeta^2 + \zeta \right) \]

The stress field vector has now the form (2.30):

\[ \sigma_{\text{joint}}^e = D L N_{\text{joint}}^e = D B \bar{d}_{\text{joint}}^e \]

Stiffness matrix is determined from the well known FEM formula (2.31):

\[ K_{\text{joint}}^e = \int_{V} (B^T D B) \, dV \]

3. Computational example – potential of using the QWSF-superelements

The computational example, selected from many realized, presents the potential of using the developed QWSF-superelements in the analysis of the interaction problems: subsoil – new building – neighbouring building development (in the analysis of deep foundation buildings) – in a three-dimensional system (3D), in individual stages of the newly erected building) [8–10, 13].

The land development plan is shown in Fig. 5. The 3D view of system is shown in Fig. 1a.

Assuming that the new building was constructed in monolithic technology of the C25/30 concrete – with at the wall thickness and storey height, equal: 0.25 m and 2.7 m.

Fig. 6 shows – schematically, together with the distribution of the “reduced” iTQDF-elements on the repetitive floor – an algorithm of discretization of a new building with the
QWSF-superelements. In the case of buildings in the neighbourhood, the discretization procedure may look analogically.

As a method of the new structure realization was assumed the Milan method with a large, central technological hole.

In the analysis of the system, the developed, block-incremental “tracking” model was used [8, 25] – in which it is possible to distinguish, for example, the stages of erection of the construction (new building) as shown in Fig. 7.

The “tracking” model, due to the block-incremental numerical structure at stage “t” (Fig. 8 – scheme of adding computational block to the global FE system), can be represented by the equation (3.1):

\[
\begin{bmatrix}
K_{ex} & K_{en} \\
K_{ne} & K_n
\end{bmatrix}
\begin{bmatrix}
d_{ex} \\
d_n
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
P_n
\end{bmatrix}
+ 
\begin{bmatrix}
R_{ex} \\
R_n
\end{bmatrix}
\]

where: \(K_n\) and \(K_{ex}\) – global stiffness matrices, formulated on the basis of construction schedule (indexes: \(X_n/(n)\) – newly added substructure (placement area) at stage “\(t\)”; \(X_{ex}/(ex)\) – existing part of the model at stage “\(t−1\)”; \(d\) – vector of the unknown displacements; \(P\) – vector of external
Fig. 7. Exemplary stages of deep foundation building erection (descriptions in the figure)

Fig. 8. Scheme of adding computational block (placement area) from the global FE system according to the developed “tracking” model

loads; $\mathbf{R}$ – vector of internal forces (forces equivalents – “corrective” forces), constructed on the basis of the current (at stage “t”) stiffness matrix of the system and the displacement vector from step “$t-1$” (from the “preceding” step).

The computations were carried out with the use of own algorithms in Matlab language [26]. For discretization of the system Robot Structural Analysis software was used, which through appropriate data processing was used as a kind of Preprocessor of the developed QFEM software.

4. Basic results for the newly built building

The results of analyses – obtained according to “QFEM + tracking model” in the spatial system – were given as deformation maps of the system for the selected phases of the erection of the new investment [13] – Fig. 9. Based on Fig. 9, for selected longitudinal walls, charts of the settlement of the building in the foundation level were made – Fig. 10. Fig. 10a shows the settlement of the building in the erection phase, while Fig. 10b shows the stage of occupancy.
Fig. 9. State of deformation of the new building in selected stages of erection (deformation maps of the system based on the transformed kinematic constraints – obtained according to the “QFEM + tracking model” method): a) erection phase (one of the construction stage); b) occupancy phase (last stage of construction).

Fig. 10. Building deformation for selected longitudinal walls (numeration of walls given in the picture): a) erection phase; b) occupancy phase.

5. Conclusions from numerical analyses. Summary

The presented method of modelling interaction systems: building – subsoil allows for analysing the problem in spatial (3D) terms on an average (with limited computing power) computer equipment. The correctness of the method was verified in many numerical tests. In the case of the analyzed computational example from dissertation [13], it should be pointed out that the example has been selected to obtained unequivocally assessed results from the QFEM analysis (so that the building’s behavior in its individual stages of erection is clearly determined). The obtained results (Fig. 9 and Fig. 10) confirm the assumed deformation of the building during construction. The developed method of discretization of wall structures
with quasi finite superelements – QWSFS – enables a significant reduction of the number of unknowns in the global FEM model.

Approach in spatial (3D) numerical model taking into account both the building being erected and the objects in the neighbourhood (in accordance with area urbanization) allows for:

- analysing buildings with taking into account the factual interaction between the construction and ground and the existing building – in the individual erecting stages (from excavation phase to the use of the new building stage),
- determining the degree of the effort state of constructions located in the impact zone of the new deep foundation building.

By comparing the developed (QFEM) modelling method with the classic FE-method, we can distinguish the main advantage of using the QFE-method, i.e. a possibility to analyse large, 3D computational models. The numerical model of deep foundation buildings, which use of QFE-method, due to the reduced (in relation to the classic approach) number of unknowns and quick access to “new” results gives additional options for using it in the following cases:

- to make changes to the model, for example for design, technological or computational reasons (e.g. “reverse” analysis – calibration of the model);
- to optimise of wall support (stiffness) system of excavation, paying particular attention to its impact on the effort state of the neighbouring constructions.

Through application of special transition elements (e.g. frame-wall), there is a simple manner for adapting the developed concept of modelling of buildings to the description of other construction structures, such as frame-wall structures – which significantly extends the range of applications of the proposed interactive model. The interactive three-dimensional model should be used by designers to more reliably and safely determine the values of internal forces in building structures.

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References

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Modelowanie przestrzennej konstrukcji budynków ścianowych – w zagadnieniach interakcji z podłożem gruntowym, w poszczególnych etapach wznoszenia – wielkowymiarowymi quasi elementami skończonymi

Słowa kluczowe: budowle z głębokimi fundamentami, budowanie interakcji – podłoże, wielkogabarytowe elementy quasi-skonczone, superelementy, schematy trójwymiarowe

Streszczenie:

W artykule przedstawiono oryginalną koncepcję opisu i analizy budynków (konstrukcji ścian i stropów), odpowiadającą naturalnym komponentom konstrukcji opisanych elementami quasi skończonymi
(QWSFS). Koncepcja ta stanowi jeden z elementów składowych opracowanego interakcyjnego modelu budynków o głęboko posadowionych fundamentach. Przedstawiona metoda modelowania pozwala na znaczne zmniejszenie liczby niewiadomych, co w przypadku modelu interakcji budowla–podłoże daje możliwość włączenia faktycznej geometrii i sztywności zabudowy do modelu MES. Dzięki temu można przeanalizować rzeczywiste odwzorowanie statycznej pracy obiektów.

W artykule przedstawiono podstawowe założenia konstruowania superelementów QWSF oraz wyniki przeprowadzonych testów numerycznych. Przedstawiono możliwości wykorzystania opracowanej koncepcji modelowania w analizie problemów konstrukcji oraz głębokich fundamentów w układzie trójwymiarowym: podłoże–nowy budynek i ewentualnie zabudowa sąsiednia (na każdym etapie realizacji inwestycji, tzn. stosując analizę etapową statyki konstrukcji).

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