Design optimization of the Petri net-based production process supported by additive manufacturing technologies

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Abstract. In the era of smart manufacturing and Industry 4.0, the rapid development of modelling in production processes results in the implementation of new techniques, such as additive manufacturing (AM) technologies. However, large investments in the devices in the field of AM technologies require prior analysis to identify the possibilities of improving the production process flow. This paper proposes a new approach to determine and optimize the production process flow with improvements made by the AM technologies through the application of the Petri net theory. The existing production process is specified by a Petri net model and optimized by AM technology. The modified version of the system is verified and validated by the set of analytic methods safeguarding against the formal errors, deadlocks, or unreachable states. The proposed idea is illustrated by an example of a real-life production process.

Key words: additive manufacturing technology; manufacturing processes; Petri nets; process modelling; verification; validation; optimization of production process supported by AM technologies.

1. INTRODUCTION

Changes brought about by the fourth industrial revolution (Industry 4.0) – such as the development of the Internet of things (IoT), cyber-physical systems (CPS), more and more frequent use of Big Data to do analysis or new production technologies such as additive manufacturing (AM) technologies affect the organization and nature of production processes, and contribute to a change in the approach to production planning and modelling [1, 2]. Manufacturing enterprises require modification and modelling production processes need remodelling in order to meet customer expectations [3]. AM technologies offer a number of possibilities with respect to producing products of complex geometry and manifold shape, as well as in the area of material utilization, waste reduction and the involvement of production resources [4]. AM technologies rely on layered product manufacturing based on a digital design. Computer-aided AM technologies allow for the manufacturing of precise geometric structures. Contrary to traditional technologies, where it is necessary to remove material (millling, machining), AM technologies allow for effective material management through the maximum use of materials and reduction of production waste. The implementation of AM technology in manufacturing can significantly contribute to the reduction of production costs [5, 6].

In this context, i.e., the need to implement AM technologies within manufacturing companies, the use of models to analyse and evaluate the production process both before and after the AM technology implementation is highly recommended, since inaccurate specification in this type of system could have economic consequences.

In literature there are a lot of studies that focus on modelling processes and phenomena in the field of production, a long list of the more prominent being: the design of machines and devices, the design of new technological solutions, the use of computational methods, mathematical models, electro-magnetic-thermal modelling [7], finite-element modelling (FEM) [8], integrated circuit modelling, the use of data models, analytical models [9], Monte Carlo analysis and simulation [10], performance models [11], etc. Presented in [12] are examples of analytical modelling in the area of mass production, using the finite element method. There are studies in which methods of modelling production processes are presented, based on comprehensive data on production process using the process specification language (PSL) method [13]. In the work of [14] a complex workflow model (CWM) was proposed in order to clearly present the production process in a graphic form. In the [15] study, an intelligent agent system was proposed for the analysis of production process models. The work of [16] presents the manufacturing process modelling approach based on Taguchi’s Design of Experiments, whose aim is to identify the optimum setting of NN parameters in a multilayer perceptron (MLP) network trained with the backpropagation algorithm. A. Hassan et al. makes a pro-
dactivity analysis taking into consideration various perspectives of the production process, and proposes an approach based on quality/cost-based conceptual process planning (QCCPP) by using quality function deployment (QFD), failure mode and effects analysis FMEA and activity-based costing methods (ABC) [17]. The work by [18] presents the concepts of a mathematical optimization model, developed to determine the optimal range of products in an integrated production and regeneration system.

There are also studies of modelling and analysis of challenges related to the adaptation of production to Industry 4.0, based on expert opinions and the interpretive structural modelling (ISM) method [19]. There are also models based on linear programming which facilitate decision-making in the area of investment in production [20, 21] presents a modelling approach based on data that can be utilized to characterize process streams and support the implementation of the circular economy principals, process resilience and waste valorisation. The work of [22] presents the assumptions for the hybrid model of enterprise self-assessment in the area of implemented lean manufacturing and Industry 4.0 components.

In the case of the AM process modelling the work of [23] presents an approach to modelling AM processes based on key performance indicators (KPI), process parameters and an approach to modelling, taking into account analytical, numerical and empirical models. The work of [24] proposes an activity model for preliminary AM process planning. In the proposed approach, design for additive manufacturing (DfAM) was analysed with the optimization of the selection of the appropriate AM process and the production resources based on various evaluation criteria. Conventional modelling methods such as Markov Models [25], Bayesian Nets [26] and Petri Nets [27, 28] were used in the modelling of manufacturing systems. However, whether these models are still the best choice must now be addressed in respect of the need to link those models with procedures or systems related to the necessity of shortening the cycle for the introduction of new products and the time that customers have to wait for an order, and specifically in terms of the Industry 4.0 concept and AM technologies implementation.

Bearing in mind the above, we propose a new approach to modelling production processes in the context of the implementation of AM technology and its effectiveness with the use of stochastic Petri nets. The use of formal models for AM implementation analysis allows the verification and validation of system dynamics. It is shown that our solution in the form of applying Petri net to design the effects of AM technologies implementation within a production process can be used as an assistive tool for decision making within management boards in manufacturing companies in the process of investment in specialized machines and devices in the field of AM technologies.

This paper proposes a novel approach to design optimization of the production process supported by additive manufacturing technologies. The idea is based on the application of the Petri net theory to verify and validate the system at the early prototyping stage. The main contributions of the work can be summarized as follows:

- A new framework that permits the determination and optimization of the production process flow improved by the AM technologies in terms of the correctness of the structure and functionality is proposed in the paper;
- The presented technique involves Petri net theory for the purpose of verifying and validating the modelled system at the early specification stage;
- The overview of the related works is presented and discussed by pointing out the benefits and weak points of the other, similar techniques;
- The proposed idea is explained in detail by an example of the real-life production process.

2. PETRI NETS IN THE MODELLING OF MANUFACTURING PROCESSES

Petri nets are a formalism for the description of concurrency and synchronization inherent in modern distributed systems [29–31]. They also provide a convenient graphic representation of the modelled system [32, 33]. This section focuses on the design of the Petri net-based production processes. Firstly, an analysed production process is presented. Next, a brief overview related to the application of Petri nets to the modelling of production is presented and main notations and definitions are introduced. Finally, verification and validation of Petri net-based system aspects are discussed.

2.1. The real-life production process

The example of the real-life production process used is four machine tools on which the manufacturing processes of the constituent elements of the structure are performed. The material is transported to machine tools that are operated by four different employees at the same time. During the manufacturing process, material waste is generated and transported to the unit warehouse. The manufactured product is subject to quality control performed by four employees. The elements are then assembled in two stages and the finished structure is transported to the main warehouse. In the analysed production process, all production stages are based on material processing actions. All the processes run at the same time (concurrently).

2.2. Application of Petri nets to the modelling of manufacturing processes

Petri nets have been used in heuristic methods for production planning [34], modelling and simulation of smart manufacturing processes and decision support in production [28]. Their application can also be found in resource management in distribution networks, allowing for the minimization of delays in emergency interventions [35]. For modelling production processes, among others, hierarchical timed coloured Petri nets, continuous Petri nets and constant speed continuous Petri nets (CCPN) [36] are used. Timed Petri nets have been used to develop supervisory controller design for manufacturing systems [27]. Presented in the work of [37] is a method and framework to put together goal-oriented requirements and Petri nets as an alternative to the requirements for manufacturing systems.
capable of dealing with digital twins. The work of [38] proposes an approach to modelling in control of flexible manufacturing systems based on a Petri net representation of production process. There are studies where solutions for deadlock-free planning can be found for flexible production systems using indefinite Petri nets and model prediction control. Solutions of this type are useful in the event of production scheduling problems [39]. In the work of [25] an approach to modelling batch production systems based on generalized stochastic Petri nets (GSPN) was proposed. The solution considers machine failures and process defects.

2.3. Definitions and main notations of Petri net

A Petri net can be considered as a bipartite oriented graph with two types of nodes (places and transitions), connected by arcs. Formally [29–33]:

**Definition 1.** A Petri net $N$ is a 4-tuple: $N = (P, T, F, M_0)$, where: $P = \{p_1, p_2, \ldots, p_n\}$ is a finite set of places, $T = \{t_1, t_2, \ldots, t_m\}$ is a finite set of transitions, $F \subseteq (P \times T) \cup (T \times P)$ is a finite set of arcs, $M_0$ is an initial marking (state). Places and transitions are called the vertices of $N$.

A state of a Petri net is called marking:

**Definition 2.** A marking (state) of a Petri net $N$ is a subset of the net places: $M \subset P$. The value $M(p)$ corresponds to the number of tokens in place $p$. A place is marked if it contains at least one token: $M(p) > 0$.

Markings are changed by the firing of a transition. A transition can be fired if every one of its input places is marked: $t \subseteq M$. Firing of a transition adds a token to each of its output places and removes a token from all its input places.

**Definition 3.** A marking $M_n$ is reachable from marking $M_k$ if there exists a finite sequence of transition firings that transforms $M_k$ to $M_n$.

**Definition 4.** A Petri net is live if from any reachable marking $M_n$ it is possible to fire any transition by a sequence of firings of other transitions.

**Definition 5.** A place $p \in P$ of a Petri net $N$ is said to be $k$-bounded or simply bounded if the number of tokens in that place does not exceed a finite number of $k$ for any marking reachable from $M_0$. In other words, there is no marking (state) such that place $p$ contains more than $k$ tokens. A Petri net $N$ is bounded if each place in the net $\forall p \in P$ is bounded.

**Definition 6.** A place $p \in P$ of a Petri net $N$ is safe (1-bounded), if in every reachable marking place $p$ contains not more than one token. A Petri net $N$ is safe if each place in the net is safe.

Liveness, boundedness, and safeness of a Petri net are essential properties in the analysis of the modelled system. Live and bounded (or even safe) Petri net prevents the system from deadlocks and unreachable states.

2.4. Verification and validation of Petri net-based processes

One of the main benefits of the Petri net-based descriptions is the possibility of verification and validation of the system at the specification stage [40]. The system can be verified against the formal errors such as deadlocks or unreachable states [41]. The most useful methods include analysis of boundedness and liveness of the system [42]. Such methods permit modelling of the occurrences of deadlocks and modelling the process with limited resources.

Furthermore, its behaviour can be validated (simulated) with the use of one of the dedicated tools. Platform Independent Petri net Editor (PIPE) is a free available and open-source platform-independent Petri net tool. This platform enables an increase in modelling power through the introduction of inhibitor arcs, a new analysis module for generating siphons and traps, many interface features and various presentation improvements, etc. [43]. The IOPT-Tools (Input-Output Place-Transition Tools) derived from the place-transition low level Petri net class, employs the standard Petri net Places. This tool allows the user to tune some parts of the automatically generated code taking into consideration some characteristics of the implementation platform improving the performance of the generated code [44, 45].

3. THE PROPOSED MODELLING IDEA OF THE PETRI NET-BASED PRODUCTION PROCESS SUPPORTED BY AM TECHNOLOGIES

This section introduces the novel modelling technique of the Petri net-based production processes. The proposed idea includes four main steps:

1. Modelling of the currently realized production process in a manufacturing company with the application of Petri nets.
2. Verification and validation of the current production process. In the case of any errors or malfunctions, the model of the system should be corrected (repeated step 1).
3. Design of the optimization of the current production process supported by AM technology in terms of the correctness of the structure and functionality with the application of Petri nets.
4. Verification and validation of the production process supported by the AM technology. In case of any errors or malfunctions, the improved AM model of the system should be corrected (repeated step 3).

3.1. Modelling of the currently realized production process

In the analysed production process (II.A) all production stages are based on material processing actions. Figure 1 shows the Petri net-based description of the system. In general, places of the net refer to actions, while transitions are related to the conditions (inputs).

All the processes run at the same time (concurrently). In the first step of the modelled production process, materials are collected (T0) from the warehouse (input) and prepared for transport to specific stations (T1). Secondly, materials (P2, P3, P4, P5) are transported to production machines (P6, P7, P8, P9).
Each machine is driven by one worker (P14, P15, P16, P17). The production process begins when the material is delivered to the position and an employee appears to operate the process (T2, T3, T4, T5). The material is processed on the machine by an employee, generating material waste. It is discharged (T10, T11, T12, T13) to the waste warehouse (P18, P19, P20, P21). Manufactured structural elements are subject to quality control (T18, T19, T20, T21) carried out by seconded employees (P30, P31, P32, P33). The model assumes that the element has been positively assessed during the quality control and has been qualified for assembly. The first step in assembly (T30, T31) is creating a component consisting of two parts (P38, P39). The next step is to assemble (T32) the entire structure (P40). The T33 transition illustrates the remaining post-production activities. Completed structure (P41) is transported (T34) to the warehouse (Input).

3.2. Verification and validation of currently realized process

Let us now examine the Petri net model. Firstly, we shall analyse the main properties of the system (safeness, boundedness, deadlock-freedom), then simulation (validation) of the model will be performed.

The net consists of 42 places and 36 transitions. In order to perform the verification of the system, it was initially described within PIPE tool (Platform Independent Petri net Editor, version 4.3). Such a tool includes several built-in analysis mechanisms that permit for verification and validation of the modelled Petri net. Unfortunately, the first attempt of the system verification finishes with an error. The PIPE tool results in information that the state-space tree for this net is too big. DNAMACA might be a more appropriate tool for this analysis. This means that built-in methods were unable to find the solution due to the complexity of the modelled system. Therefore, it becomes necessary to apply other, external computer aided verification programs for the verification procedure. For this task, the authors’ Hippo system [46] was involved. Such a tool includes several different effective and efficient algorithms oriented toward the analysis of Petri nets. Firstly, the Petri net was converted to the Hippo format (this operation is executed automatically when the conversion tool is applied within the Hippo system). Then, the system was examined by application of various methods, starting by boundedness and safeness verification. Hippo indicates that the system is unbounded, since places P18 and P19 accumulate an indefinite number of tokens. Such a mistake can be simply corrected by application of an additional arc that joins the unbounded place to the main flow (for example to transition T18 in the case of P18, and transition T19 in the case of P18, respectively).

Furthermore, deeper verification shows that the system is also unsafe, since it is not covered by the state machine components (SMCs) [47, 48]. Indeed, further validation of the system shows that several places (P6, P7, P8, P9, P25, P2, P27, P28 and P29) may collect more than one token. Incidentally, this is a common designer mistake in the modelled synchronization (join – in the fork/join model). Figure 2 shows such a state, obtained during validation of the system within PIPE tool. For example, place P6 can be marked by firing transition T2, but
also by firing transition T14, thus it contains two tokens instead of remaining as one. Moreover, places P18 and P19 may collect infinite number of tokens. After the above modifications, the system was verified once more.

This time, both tools (PIPE and Hippo) indicate that the system is bounded and safe. Moreover, PIPE reports that there are no deadlocks in the net. Finally, validation of the system was performed. This task was executed by simulation of the net within the PIPE tool. Validation of the system confirms proper functionality of the corrected system (Fig. 3).

3.3. Design of the optimization of the current production process supported by AM technology

Figure 4 shows a manufacturing process in which the device in the field of AM technologies is implemented. It is assumed that two material treatments can be replaced by one additive manufacturing process. As in the previous model, the process begins with downloading (T0) materials from the input, prepared for transport to specific stations (T1). Next, materials (P2, P3, P4, P5) are transported to manufacturing machines (P6, P7, P9), with P4 and P5 being transported to the AM machine (P9). The additive manufacturing process begins when the materials are delivered to the workstation and an employee (P17) appears in place to operate the process. The employee operating the AM machine (P17) supervises and conducts the production process, simultaneously carrying out quality control (T21) of the manufactured structure.

Support and control of AM processes carried out by one employee allows reduction of the number of people involved in the manufacturing process. The additive manufacturing process allows for production of a finished structure, reducing the need for assembly steps. The structure produced during the
AM (P39) process goes to the second stage of assembly (T32), where the final structure is assembled. The finished structure (P40) is subjected to post-production activities (T33). The completed structure (P41) is transported (T34) to the warehouse (Input). Considering the need of implementing AM technologies within manufacturing companies, the use of models to analyse and evaluate the production process before and after the implementation of AM technologies is highly recommended, since inaccurate specification in this type of system could have economic consequences. Furthermore, the use of formal models for AM implementation analysis allows the verification and validation of system dynamics.

3.4. Verification and validation of the production process supported by the AM technology

Finally, the model of the production process supported by the AM technology was verified and validated. The common part of the description with the earlier manufacturing process is conform to what has been described above. The analysis of the system was divided into two parts. Firstly, verification of the model was performed. This step was performed using PIPE and Hippo tools.

The first one resulted in the information that the net is purged of deadlocks. Furthermore, both tools indicated that the net is bounded, but unsafe. Therefore, validation of the system was executed to check the safeness of the net. Indeed, simulation of the net confirmed problems with the AM process that was added to the system, as is shown in Fig. 5.

![Fig. 5. Validation of model with AM process (part of system is shown)](image)

It can be noticed that place P9 is unsafe. Moreover, place P25 may also contain more than one token (e.g., after firing of transition T9 twice). To solve this problem, a proper synchronization ought to be introduced. This can be resolved by connecting place P17 to transition T5, and transition T21 to place P17. Such modifications assure the safeness of places P9 and P25 (and thus, the whole system).

The improved specification is presented in Fig. 6. The corrected model was verified once within PIPE and Hippo. This time, both tools returned the result that the net is bounded, safe and has no deadlocks. Finally, further validation of the net confirmed proper functionality of the process.

![Fig. 6. Corrected model of the manufacturing process with implementing AM](image)

4. RESEARCH RESULTS AND DISCUSSION

Table 1 summarizes the main features that distinguish the proposed work from existing related works. It enriches the discussion with a rapid overview of the main outcome of the presented paper with respect to the analysed state of the art. Going into more detail, Table 1 reports information on the related works about: a) approach to modelling production taking into account the tools used, b) the exploitation of the modelling approach, c) the adoption of a model, d) the proposal of modifications and finally, e) the effectiveness of the model. To the best of our knowledge, and as already highlighted in the state-of-the-art analysis, none of the existing works supports all the presented features.

The use of Petri nets for the graphic presentation of production processes can help managers in the effective management and control of manufacturing processes.

Formalism and the structure of the Petri nets allow for the presentation of the stages of the production process and the identification of fragments in which changes are possible and the implementation of additive technology solutions. Modelling production processes with the use of Petri nets also allows for the analysis of the manufacturing process and the support of
Table 1
Comprising the proposed approach from existing related works

<table>
<thead>
<tr>
<th>Model approach/tools</th>
<th>Model exploitation and adaptation</th>
<th>Modification proposal</th>
<th>The effectiveness of the model</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finite element method (FEM)</td>
<td>Modelling of phase transformations, predicting the influence of process parameters on the microstructure.</td>
<td>Improving the accuracy of simulating thermal processes, improving the identification of thermal history.</td>
<td>Facilitating the adaptation of the production of materials and components to improve the mechanics of performance.</td>
<td>[51]</td>
</tr>
<tr>
<td>Process specification language (PSL)</td>
<td>Standard, complete, and accurate definition of complex information about the production process.</td>
<td>Extending the concept of the production process, types of relationships to the description of resources, factors, and materials.</td>
<td>Standardization of the representation of data about the production process.</td>
<td>[13]</td>
</tr>
<tr>
<td>Complex workflow node (CWM)</td>
<td>Study of the impact of planning and performance analysis of the production process in terms of graphics.</td>
<td>Two-stage simulation analysis identifying the relationship and areas for improvement.</td>
<td>Easy-to-understand graphical representation of the production process.</td>
<td>[14, 15]</td>
</tr>
<tr>
<td>Production planning model</td>
<td>Assessing the manufacturability and estimating the cost of conceptual design in the early product design stage.</td>
<td>Quality/cost-based conceptual process planning (QCCPP),</td>
<td>Facilitating the determination of key process resources along with the estimation of the manufacturing cost, considering the risk cost associated with the process plan.</td>
<td>[17]</td>
</tr>
<tr>
<td>Mathematical optimization model</td>
<td>The model makes it possible to determine the optimal range of products for an integrated production and remanufacturing system.</td>
<td>The model can be used in Industry 4.0-enabled network to facilitate the transition to circular economy manufacturing systems.</td>
<td>Improving the value recovery through tracking, monitoring, and processing used products.</td>
<td>[18]</td>
</tr>
<tr>
<td>Interpretive Structural Modelling (ISM)</td>
<td>Searching for management analysis by analysing relations and challenges with Industry 4.0.</td>
<td>Analysis based on expert opinions and Interpretive Structural Modelling (ISM).</td>
<td>Identification of barriers to the implementation of new industrial solutions.</td>
<td>[19]</td>
</tr>
<tr>
<td>Modelling based on key performance indicator (KPI)</td>
<td>Indicative research for each group of KPIs distinguished from a given technology.</td>
<td>Classifying and modelling additive manufacturing processes based on KPIs group.</td>
<td>Practical methods for the solution of a specific problem, but they are unsuitable for the extraction of generalized conclusions concerning the identification and optimization of the problems faced by AM technology.</td>
<td>[20]</td>
</tr>
<tr>
<td>Supervisory controller design for manufactur-</td>
<td>Use of timed Petri nets (TPNs) to present the states of the production system.</td>
<td>Implication of the stretching method for TPNs.</td>
<td>Avoiding deadlock and designing supervisory controllers for other purposes, such as liveness, boundedness, and/or reversibility may also be considered.</td>
<td>[27]</td>
</tr>
<tr>
<td>ing systems</td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>Modelling with artificial intelligence (AI)</td>
<td>Modelling of workflow diagrams and conversion to Petri nets.</td>
<td>Correlation between goal-oriented systems requirements and Petri Nets.</td>
<td>Can be adapted to the transition phase and also be used to model full service-oriented digital manufacturing.</td>
<td>[37]</td>
</tr>
<tr>
<td>Flexible Manufacturing System modelling</td>
<td>Extracting a linear discrete time system from a rough description the production problem; useful in deadlock-free scheduling problems.</td>
<td>The use of petri network formalism to identify production problems.</td>
<td>The model can be used to apply methods developed within control systems and theories, which can open up new opportunities by introducing online feedback and optimization.</td>
<td>[38, 39]</td>
</tr>
</tbody>
</table>
the decision-making process in the area of AM technology introduction. However, there is a continuous need to conduct the research of an intelligent planning system for technological processes towards additive processing due to the huge variability of 3D printing processes and towards a broader use within the Industry 4.0 paradigm [49, 50].

5. CONCLUSION

In this paper, an approach to model changes and benefits in the production process by implementing AM technologies within a manufacturing company using stochastic Petri nets is proposed. In implementation of the procedure in which the AM process is planned a structure consisting of four components manufactured from four types of materials is produced. Machine operators and quality control workers are involved in the production of each of them. The assembly is performed in two stages. It was assumed that the implementation of AM technology includes the replacement of two processes with one in which a ready semi-structure is produced and transferred immediately to the second stage of assembly. The main innovation introduced in the paper relies on the analysis of the system in the terms of the correctness of its structure and functionality. In particular, liveness and boundedness properties are examined at the early specification stage in order to avoid deadlocks and unreachable states. Moreover, the validation (simulation) of the system is performed. In the further works, it is planned to involve Petri net theory for the purpose of assessing the efficiency of modelled manufacturing processes. Moreover, a comparison of the conventional process parameters (material processing) to the systems supported by Additive Manufacturing Technologies will be considered.

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