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Research paper

Railway transition curves – optimization and assessment

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Abstract: This article relates to optimization and assessment of railway polynomial transition curves. The search for the optimum shape meant here the evaluation of the transition curve properties based on chosen dynamical quantity and generation of such a curve shape. In the study, 2-axle rail vehicle was used. The rail model represented 2-axle freight car of the average values of parameters. Mathematically understood optimization methods were also applied. As the transition curve, the authors used polynomials of 9$^{\text{th}}$ and 11$^{\text{th}}$ degrees. As the criterion of the assessment, the integral of change of lateral acceleration along the route was also used. Wide range of the circular arc radii was applied by the authors. The mentioned radii were: 600 m, 900 m, 1200 m, 2000 m and 3000 m. In the work the results of the optimization – types of the curvatures of the optimum transition curves, as well as the vehicle dynamics were presented.

Keywords: railway transition curves, optimization, railway dynamics, computer simulation

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1. Introduction

In the past, railway transition curves between straight track and circular arc weren't used. Nowadays, due to increasing velocity of passenger trains, it is impossible. This situation is mirrored in increasing number works published in recent 15–20 years, which deal with this topic.

As mentioned, the large number of works, which deal with topic of railway and road transition curves can be visible in recent years. Let the examples of such works can be: [3–5,7,9,10,12,16–23,26,27,27–33,35,36,38–40]. The resignation from the traditional transition curves properties assessment is also visible [40].

The influence of the transition shape on the dynamics of motion of the vehicle negotiating the transition is also strongly examinated in recent years. The examples of such works can be e.g.: [16, 20–22, 24, 26–29, 31, 32, 35–40]. In the light of high-speed trains is also visible e.g.: [8, 16, 20, 25, 34] and [35].

In many works, their authors suggested that the change of lateral acceleration is the most criterion influencing on the comfort of the passenger journey. Such mentioned works were: [11, 13, 14, 33].

In the light of opinion of the authors of the current work, the dynamics of the full vehicle-track system (complete description of the vehicle interactions with the track), is not still popular. The simplicity of the vehicle model makes a full examination of vehicle dynamics impossible. The use of advanced vehicle models makes a better assignment of dynamical properties of curves to freight or passenger trains possible. The other than the passenger comfort criteria of the transition curves assessment are taken into account in the mentioned method. For cargo trains, such basic criterion is wear in wheel-rail contact. For the high-speed trains comfort must be considered, combined with wear criteria.

The works [37, 38] have shown that the use of software for dynamical simulation of vehicle motion in the assessment of the shape of the railway transition curves can be useful. New knowledge not available using traditional approach, in which all key elements of the vehicle are treated as a point can be provided. The results of the simulations obtained in non-classical way are not the same as the results obtained with the classical approach. The mentioned works have stated that the simulation methods applying the advanced vehicle models can be a supplement to traditional approach. Simulation studies may be justified taking into account the fact that, there are many software packages for automatic generation of the equations of motion for railway vehicles. The programs by the lead author are also included.

2. Literature survey

The authors of the current work can see a visible division of work, which deal with the problem of railway transition curves optimization. In their opinion, four groups of the works can be distinct. These mentioned 4 groups can be represented by the works: [19, 26, 33], and [1].

Tari and Baykal in [33] examined the lateral change of acceleration (LCA) of vehicle negotiating transition curve as the most important criterion of comfort assessment. According to them, the continuity of LCA should be absolutely satisfied. They were searching only for the curves, which satisfied this criterion. Similar approach was also represented by Hasslinger [10]. He proposed the curve having LCA continuity. As the result, the curve had good dynamical properties. The works of: Eliou and Kaliabetsos [7], Kobryn [17], Koc [18], Li et al. [23], and Shen et al. [30] also belong to this group.

The work of Long et al. [26] is the example of the work, where the authors used the advanced model of a railway vehicle (simulation software) for railway transition curves dynamical properties of assessment. Six typical shapes of the transition curves were compared using some criteria: vehicle body lateral and vertical acceleration, wheel/rail lateral and vertical forces, derailment coefficient and reduction rate of wheel-load. In the work the function of curvature can not be the scaled function of superelevation ramp. Similar approach can also be represented by e.g. [3, 9]; [16, 20–22, 24, 26–40].

Kufver in [19] optimized the length of curves. The passenger ride comfort and track-rail vehicle dynamical interactions were taken into account. For this reason, he used the European standard published by CEN [6]. The percentage of passengers P_{CT} (both seating and standing) with discomfort feelings was describing in a formula proposed by the standard CEN. The P_{CT} values in the function of the transition curve length had a form of the second-degree parabola, so it was possible to find such a value of the length, for which P_{CT} had the minimum value.

The last group of the work, in which transition curves are regarded only as the mathematical object can be represented by [1] and [2]. Their authors examinated the properties of the transition curves spanned between two straight lines or two circles. They used Bezier curves and the continuity of type G^2 in each point of the curve was satisfied. This meant the continuity of a function of curvature of the curve in a function of current length l. It also guaranteed no abrupt changes of a centrifugal force. The curve not possessing: discontinuities, inflexion points, and extrema of curvature was proposed by the authors.

According to the authors of the current work, the works, where the full dynamics of the vehicle-track system and optimization methods are considered jointly are still rare. In the majority of works, the classical approach to track-vehicle interactions is used. A simple vehicle model and the maximum unbalanced lateral acceleration a and its change, which should not be exceeded as the demand are of the interest to them. The approach in which in the TC shape' optimization the advanced rail vehicle model of the vehicle-track system and mathematically understood optimization methods are applied can be still treated the novelty.

3. General aim of the work

The main general aim of the current work was the optimization of railway polynomial transition curves of 9th and 11th degrees. These higher degrees were chosen regarding at their great flexibility during railway transition curve shape optimization. In the work one non-traditional optimization criterion was used. The authors of the work used 2-axle rail freight vehicle for dynamical simulation as well optimization method in mathematical understood meaning. Mentioned non-classical optimization (assessment) criterion (quality function QF) related to the lateral dynamics of a rail vehicle, and it was as follows:

$$QF = L_C^{-1} \int_0^{L_C} |\dddot{y}_b| \, \mathrm{d}l \tag{3.1}$$

The meaning of symbols used is as follows: L_C – length of whole TC and the adjacent 100 m of a circular arc, $\dddot{y}_b$ – change of lateral acceleration (jerk), l – current length of the road.

4. Transition curves adopted

The authors of the current work in their research, as the type of railway transition curve, assumed a polynomial of n-th degree. The assumption $x = l$ (x co-ordinate = length) was made, and consequently applied in the further part of the work. The shape of the curve is specified in the Cartesian coordinate system. The equation of the mentioned curve was as follows:

$$y = \frac{1}{R}\left(\frac{A_n l^n}{l_0^{n-2}} + \frac{A_{n-1} l^{n-1}}{l_0^{n-3}} + \frac{A_{n-2} l^{n-2}}{l_0^{n-4}} + \frac{A_{n-3} l^{n-3}}{l_0^{n-5}} + \ldots + \frac{A_4 l^4}{l_0^2} + \frac{A_3 l^3}{l_0^1}\right) \tag{4.1}$$

From equation (4.1), the curvature $k(l)$ (4.2) was determined by using an approximate formula.

$$k = \frac{\mathrm{d}^2 y}{\mathrm{d}l^2} = \frac{1}{R}\left[n(n-1)\frac{A_n l^{n-2}}{l_0^{n-2}} + (n-1)(n-2)\frac{A_{n-1} l^{n-3}}{l_0^{n-3}} + \ldots + 3 \times 2 \frac{A_3 l^1}{l_0^1}\right] \tag{4.2}$$

As the consequence of assumption $x = l$, equations (4.3) and (4.4) are also approximate.

$$h = H\left[n(n-1)\frac{A_n l^{n-2}}{l_0^{n-2}} + (n-1)(n-2)\frac{A_{n-1} l^{n-3}}{l_0^{n-3}} + \ldots + 3 \times 2 \frac{A_3 l^1}{l_0^1}\right] \tag{4.3}$$

$$\begin{aligned} i = \frac{\mathrm{d}h}{\mathrm{d}l} = H\Bigg[&n(n-1)(n-2)\frac{A_n l^{n-3}}{l_0^{n-2}} + (n-1)(n-2)(n-3)\frac{A_{n-1} l^{n-4}}{l_0^{n-3}} \\ &+ \ldots + 3 \times 2 \times 1 \frac{A_3 l^0}{l_0^1}\Bigg] \end{aligned} \tag{4.4}$$

Each symbol in Equations (4.1)–(4.4) has the following meaning: y – curve lateral coordinate (offset) [m], k – curvature [1/m], h – superelevation ramp [m], i – inclination of superelevation ramp [–], R – curve radius [m], H – cant [m], l_0 – total curve length [m], l – curve current length.

The polynomial coefficients A_i in general case are non-zero. The index i ranges from n to 3. The possibility of proper values of curvature and supereleveation ramp at terminal points of the curve is provided according to the following approach. It was equal to 0 at the initial point and $1/R$ and H at the end point of the curve, respectively. The values for both functions were always equal 0 for $l = 0$. To provide $1/R$ and H values for length $l = l_0$, normalization of the polynomial coefficients was be performed [39]. The normalized coefficients A_i' satisfied also additional constraints imposed on the mentioned functions – a tangency of the curvature and supereleveation ramp functions at their terminal points to these functions in straight track and circular arc.

In the current work, as mentioned, the polynomial transition curves only of degrees 9$^{\text{th}}$ and 11$^{\text{th}}$ were optimized. The possible number of non-zero polynomial terms were 7 and 9, respectively. In work during the optimization two initial transition curves of degree 9$^{\text{th}}$ and 11$^{\text{th}}$ were applied, respectively. These mentioned curves were as follows:

$$y = \frac{1}{R}\left(-\frac{5}{18}\frac{l^9}{l_0^7} + \frac{5}{4}\frac{l^8}{l_0^6} - 2\frac{l^7}{l_0^5} + \frac{7}{6}\frac{l^6}{l_0^4}\right) \tag{4.5}$$

$$y = \frac{1}{R}\left(\frac{7}{11}\frac{l^{11}}{l_0^9} - \frac{7}{2}\frac{l^{10}}{l_0^8} + \frac{15}{2}\frac{l^9}{l_0^7} - \frac{15}{2}\frac{l^8}{l_0^6} + 3\frac{l^7}{l_0^5}\right) \tag{4.6}$$

The curve (4.5) was presented by Long et al. in [26], whereas the curve (4.6) was presented for the first time in [39]. Both two curves have the inclination function i of superelevation ramp of a bell shape.

In the current work the problem of finding a local minimum is encountered. In order to solve this problem at least partially, the authors used additionally 3$^{\text{rd}}$ degree parabola – railway transition curve, which is traditionally applied in railway engineering practice. This curve was used, however only in two cases:

1. The first case was, when initial railway transition curve used turned out to be a local minimum and optimization procedure was not able to find another curve. In such a curve indicated solution was the optimum curve.
2. The second case was, when the lengths were not greater than 100 m, if the optimum transition curve found had the curvature different than linear.

Every railway transition curve has a minimum length l_0 according to the methods used in engineering practice [8]. This minimum curve length arised from 2 kinematic conditions, in which limits of two quantities:

- the velocity of the unbalanced lateral acceleration change,
- the velocity of wheel vertical rise along the superelevation ramp

during negotiating the curve should not be exceeded. Calculating the curves lengths in such a way, two values of the lengths were always obtained. The greater length was considered in further research.

5. Rail vehicle model and software used

In the work the model of a 2-axle rail freight car of average values of parameters was used. For the first time this model was described by Zboinski [37,38]. The earlier research of Zboinski and Woznica [39,40] showed, that this tool can be efficient in the optimization of the shape of railway transition curves. Relatively simple structure of the model allows reaching short computation times.

The structure of the rail vehicle model is shown in Fig. 1c. The vehicle model was supplemented with discrete models of vertically and laterally flexible railway track. The models of the track are shown in Figs. 1a and 1b, respectively. In the model also linearity of stiffness and damping elements was applied. Also the track models were linear. All key parameters of the whole vehicle-track system used in the research are presented in [40].

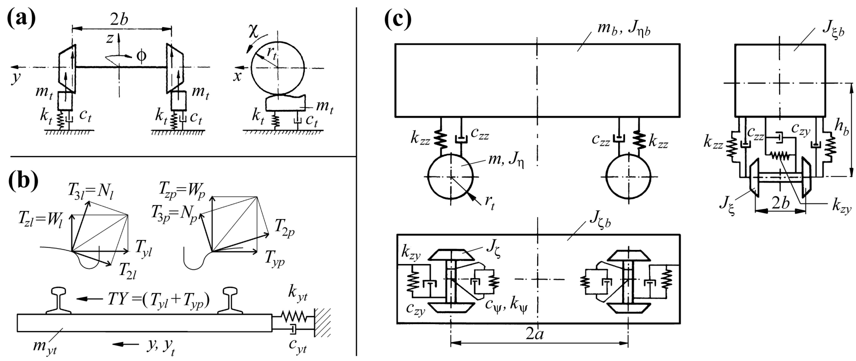

Fig. 1. System's nominal model: a) track vertically, b) track laterally, c) vehicle

The rail vehicle model applied possessed all elements of vehicle dynamical models identified in railway vehicle dynamics. These mentioned key mass elements were: wheelsets and vehicle body, suspension elements – stiffness and damping elements, and wheel and rail geometry. In the adopted model, tangential contact forces were calculated using the non-linear simplified contact theory invented by J.J. Kalker. The whole dynamical system, as mentioned, included the track models. The model was also equipped with a typical pair of wheel-rail (S1002/60E1) profiles. This pair was introduced in the model as the table of contact parameters. The mentioned table was built with the use of the RSGEO software [15].

The algorithm of the optimization procedure used in the work during the optimization of the shape of the transition curves is presented in [40]. Two iteration loops are present in this algorithm. The first loop was the loop of numerical integration. The loop stopped the simulation, when the model reached the assumed length of the route L_C. The second loop was the optimization process loop. This loop stopped, when the number of iterations reached the limit value i_{lim} earlier assumed. In the optimizations, $i_{lim} = 200$ was generally applied as a typical value. If the optimum solution was found earlier, then the optimization was automatically stopped.

The optimization problem of finding the global minimum of quality function (3.1) was a typical formulation of static constrained optimization. It was solved with the use of the library procedure that utilized a moving penalty function algorithm combined with Powell's method of conjugate directions. The quality function QF was a real-valued function. Mentioned method minimized a quadratic function of n variables. The library and optimization procedure was built in Warsaw University of Technology. It was applied with success in optimization problems in railway vehicle dynamics. Reliability and good efficiency of the procedure used so far encouraged the authors to use this procedure in transition curve optimization. Mentioned quality function is always calculated based on the result of dynamical simulations of a mechanical system.

6. The results of the optimizations

The authors assumed that every transition curve obtained in the optimization has the curvature (superelevation ramp), which can be qualified to one of 5 groups (types). These 5 mentioned groups were:

1. Type 1 – curvature was the curvature of standard transition curves of 9th [26] and 11th degree [39].
2. Type 2 – curvature was something between the curvature of standard transition curve of 9th or 11th degree and 3rd degree parabola.
3. Type 3 – linear curvature.
4. Type 4 – curvature had a convex character. it has slope subtype (4a) or g_1 continuity subtype (4b) at the beginning of tc, and it has a slope at the end.
5. Type 5 – curvature had a concave character.

All types of the mentioned curvatures (superelevation ramps) are shown in Fig. 2.

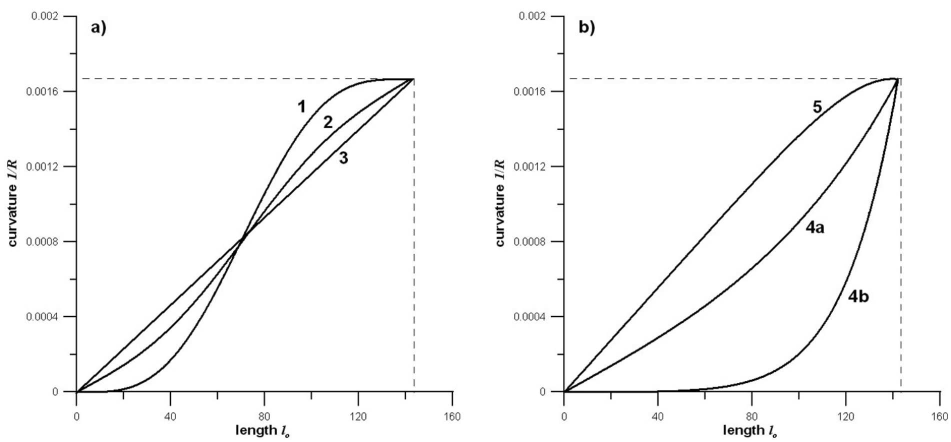

Fig. 2. Types of curvatures (superelevation ramps): a) 1, 2, 3, b) 4, 5

In Table 1 the authors shown the general results of the optimization of the transition curves – the types of TCs obtained in the research. The values of parameters assumed in the

Table 1. The results of the optimizations – the types of curvatures

R [m]	a_{lim} [m/s^2]	v [km/h]	H [mm]	n	l_0 [m]	Type of curv.
600	–0.3	72.72	150	9	118.35	1
				11	133.10	1
600	0	87.33	150	9	142.15	2
				11	159.86	1
600	0.3	97.95	150	9	162.36	3
				11	182.58	1
600	0.6	110.84	150	9	180.46	2
				11	202.94	1
900	–0.3	89.06	150	9	144.96	1
				11	163.01	1
900	0	106.92	150	9	174.02	3
				11	195.70	2
900	0.3	109.08	110	9	129.93	3
				11	146.31	3
900	0.6	109.33	65	9	77.11	4a
				11	86.71	4a
1200	–0.3	102.85	150	9	167.40	2
				11	188.25	1
1200	0	87.33	75	9	71.07	3
				11	79.93	3
1200	0.3	108.54	70	9	82.41	3
				11	92.68	4a
1200	0.6	130.21	75	9	105.98	4a
				11	119.18	4a
2000	–0.3	108.18	115	9	134.99	2
				11	151.80	3
2000	0	87.33	45	9	42.64	3
				11	47.95	3
2000	0.15	113.13	50	9	60.29	3
				11	67.80	3
2000	0.3	124.09	45	9	60.60	3
				11	68.15	3
3000	–0.6	104.76	135	9	153.45	2
				11	172.57	1
3000	–0.3	105.84	90	9	103.35	3
				11	116.23	3
3000	0	112.68	50	9	61.13	3
				11	68.74	3
3000	0.15	110.37	25	9	29.94	3
				11	33.67	3

optimization were: curve radius R, lateral unbalanced acceleration limit in circular arc a, vehicle velocity v, cant H (ranged between 20 mm and 150 mm), degree of polynomial n, and transition curve length l_0. In some cases, the maximum cant value H = 150 mm was assumed in a situation, where there was a large reserve in relation to the permissible value of unbalanced acceleration. It is caused by the fact that a smaller value of cant increases vehicle velocity and this velocity approaches a critical velocity of vehicle. Due to the same fact, a relatively small velocity was assumed for arc radii 2000 m and 3000 m.

If we make the analysis of the curvature types shown in Table 1, we may conclude different optimum shapes of TCs had been found by the software. It is visible, however, that linear type of the curvature had been found the most often. It is obvious that such curvature does not possess the inflexion point in the middle part. In the authors' opinion, such point has greater importance on rail vehicle dynamics, than the existence of G^0 continuity in beginning and the end of the curvature of the transition curve.

Analysing Table 1, second important observation for the practice can be made. We may observe a tendency to change of type of the curvature in accordance with the rule 4, 3 → 2, 1 in a function of the increasing TC length. Fig. 3 shows types of curvatures in the function of curve length. We see that the lengths smaller than 100 m, mainly the type no. 3 was obtained. For the greatest lengths (greater than 180 m), only transition curves with the type of curvature no. 1 and 2 (curvatures having the inflexion point in the middle part) were found.

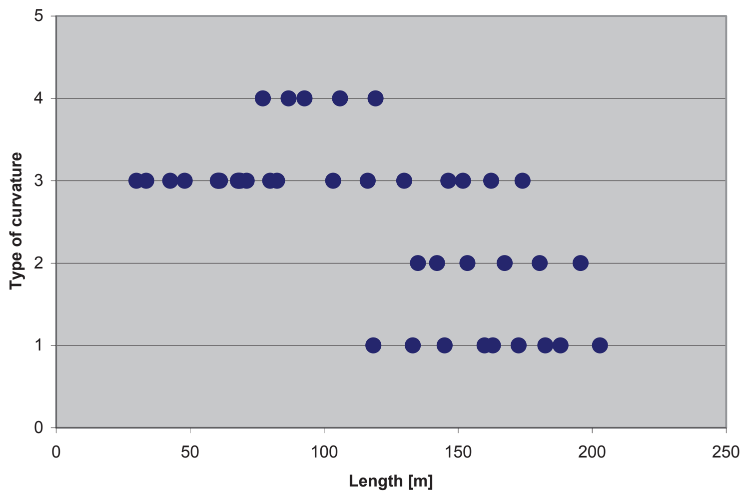

Fig. 3. Types of curvatures in the function of curve length

7. The results of the optimization – vehicle dynamics

In this section, the authors presented selected results of the research in the form of both curve and dynamical characteristics for one optimization process for transition curve of the 9th degree (Table 1, v = 109.33 m/s, l_0 = 77.11 m). The mentioned results were:

– lateral coordinate of the curve (Fig. 4a),
– curvature of the optimum TC (Fig. 4b),
– superelevation ramp slope (Fig. 5a),
– change of superelevation ramp slope with respect to x co-ordinate (Fig. 5b),
– lateral displacements and accelerations of the centre of mass of the body (Fig. 6),
– vertical displacements and accelerations of the centre of mass of the body (Fig. 7),
– angular displacements and accelerations of the body around the x-axis (Fig. 8).

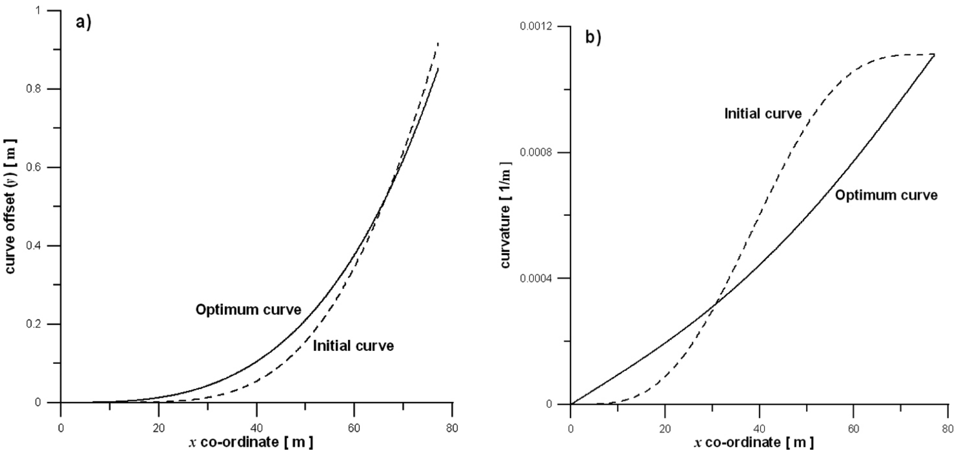

Fig. 4. a) curve offset y, b) curvature of the initial and optimum TC

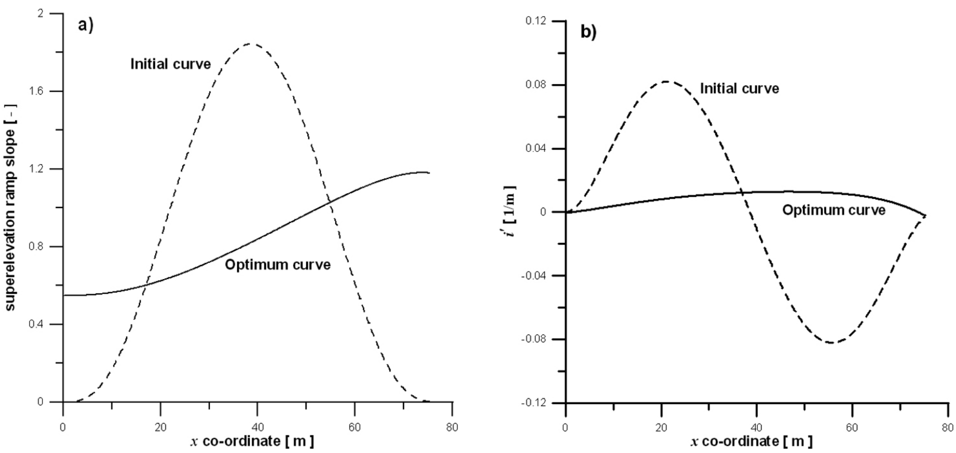

Fig. 5. a) superelevation ramp slope, b) change of superelevation ramp slope of the initial and optimum TC

Making the analysis of the dynamical characteristics from Figs. 6– 8, we may observe that the optimization of the shape of the transition curves for large circular arc radii brought

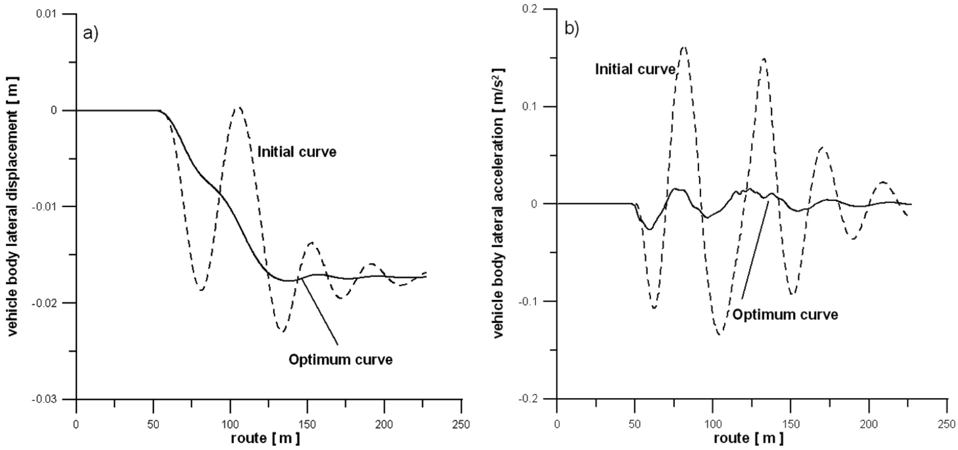

Fig. 6. Vehicle body lateral: a) displacement, b) acceleration

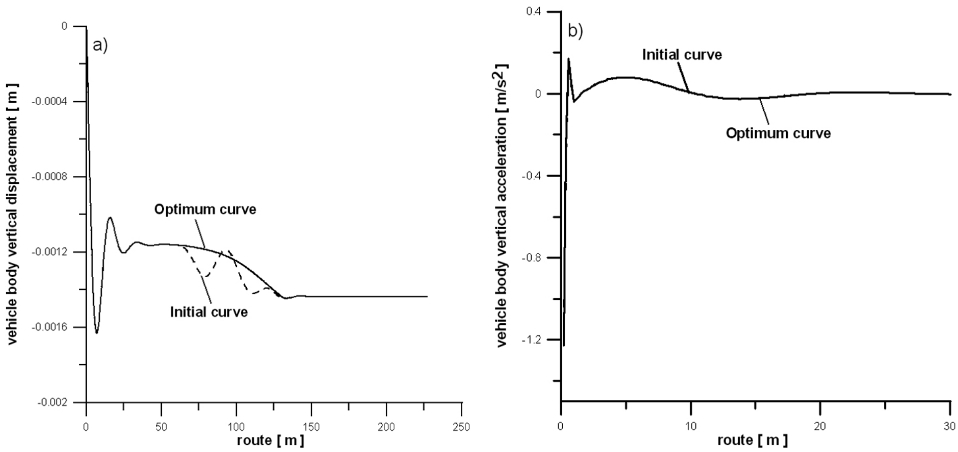

Fig. 7. Vehicle body vertical: a) displacement, b) acceleration

valuable results. This is especially visible for milder displacements and accelerations – both lateral and angular one of the body (Figs. 6 and 8). Transition curves with curvature bends at the end of the curve were preferred by the optimization procedure. Mentioned bend did not adversely influenced the dynamics of the vehicle. The optimum transition curve, found by the software in the optimization process, is a curve with the type no. 2. This curve was found in the 813$^{\text{th}}$ step. The ratio of the value of the objective function – integrals from the change of the lateral acceleration of the centre of mass of the vehicle body – for the optimum curve to the value of the objective function for the initial curve was $0.12 = (0.16649E + 00\ [\mathrm{m/s^3}]/0.13242E + 01\ [\mathrm{m/s^3}])$.

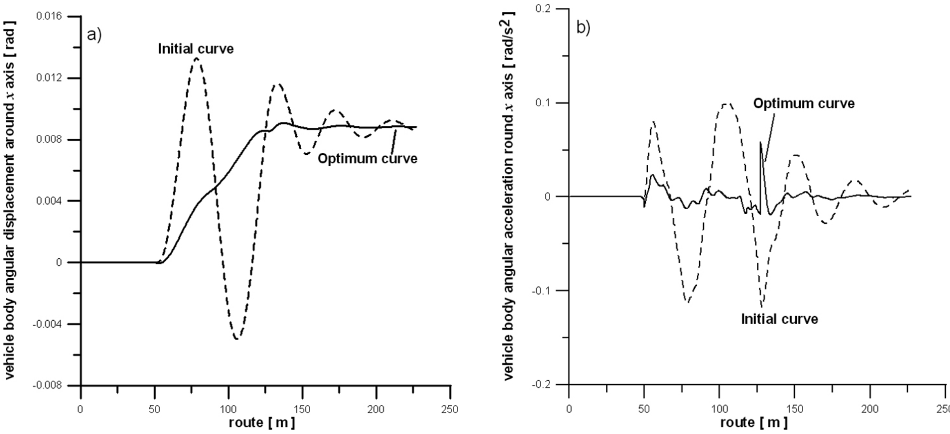

Fig. 8. Vehicle body angular: a) displacement, b) acceleration around x-axis

8. Conclusions

In this work the authors presented the original method of railway transition curves pioneered by one of them. The 2-axle rail vehicle model and mathematically understood optimization method was used to optimize railway polynomial transition curve shape. The authors of the work presented that criterion of a minimum integral of change of lateral acceleration and different circular arc radii, gave in general different shapes of the transition curves.

The work indicated the novelty in railway transition curve shape formation. For relatively short lengths (not greater than 100 m) only two types of curvatures were found by the software. These types were 3 and 4. Above the length of 100 m additionally the types 1 and 2 were observed as the optimum ones. For the greatest length of 202.94 m the type was 1. So the certain transition from linear type (type no. 3) to type no. 1 versus the length can be the fact.

Proposed optimum curves of 9^{th} and 11^{th} degrees can be considered to use, taking the length of the curve primarily. Mentioned curves can constitute a novelty taking into account non-classical method of optimization of railway polynomial transition curves shape. New curves can be treated as the curves with dynamical properties not found in the world literature. It is especially important for the engineering practice.

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Kolejowe krzywe przejściowe – optymalizacja i ocena

Słowa kluczowe: kolejowe krzywe przejściowe, optymalizacja, dynamika kolejowa, symulacja komputerowa

Streszczenie:

Celem pracy była optymalizacja kolejowych wielomianowych krzywych przejściowych 9. i 11. stopnia z wykorzystaniem nieklasycznego kryterium oceny oraz modelu pojazdu szynowego. Jako wspomniane nieklasyczne kryterium oceny autorzy zastosowali tu minimalizację wartości całki ze zmiany przyspieszenia poprzecznego nadwozia pojazdu po długości drogi.

W pracy tej użyto jeden model pojazdu kolejowego. To model 2-osiowego wagonu towarowego o uśrednionych wartościach parametrów, który jest rozważany w stanie ładownym. Prosta konstrukcja pojazdu skutkuje akceptowalnymi czasami obliczeń, co jest korzystne w dużej liczbie optymalizacji. Jego strukturę pokazano na rys. 1c. Jest on uzupełniony o dyskretny pionowo i poprzecznie model toru pokazany na rys. 1a i 1b. W modelu przyjęto liniowość zawieszenia pojazdu – liniową sztywność i tłumienie elementów zawieszenia pojazdu. To samo zastosowano w modelu toru.

Wykorzystany model zawiera wszystkie kluczowe elementy modeli dynamicznych pojazdów szynowych, takie jak: kluczowe elementy masowe (zestawy kołowe i nadwozie pojazdu), elementy

zawieszenia (elementy sprężyste i tłumiące), koła i geometrię szyn opisaną przez rzeczywisty, nieliniowy kształt ich profili. Poza tym, styczne siły kontaktowe są obliczane przy użyciu uproszczonej nieliniowej teorii kontaktu J.J. Kalkera. Ponadto, model pojazdu jest uzupełniony modelem toru, co oznacza, że w rzeczywistości rozważany jest układ dynamiczny pojazd-tor. Może on być traktowany jako zaawansowany model dynamiczny, zwłaszcza gdy porówna się go do punktu materialnego reprezentującego pojazd w tradycyjnych metodach oceny i kształtowania krzywych przejściowych.

W pracy przyjęto następujące wartości promienia łuku kołowego R równe: 600 m, 900 m, 1200 m, 2000 m oraz 3000 m. Dla konkretnych wartości R i przechyłki H, autorzy zawsze obliczali prędkość pojazdu, zgodnie ze wzorami tradycyjnie przyjętymi w praktyce inżynierskiej.

Przyjęto, że każda krzywa otrzymana w pracy ma krzywiznę oraz rampę przechyłkową, która zostanie zakwalifikowana do jednej z 5 grup. Wspomniane 5 grup (typów) to:

- typ 1 – krzywizna jest w praktyce zbliżona do krzywizny wzorcowej 9. i 11. stopnia,
- typ 2 – krzywizna ma kształt pośredni pomiędzy krzywizną wzorcową 9. i 11. stopnia, a parabolą 3. stopnia, krzywizna ta ma styczność typu G^1 w skrajnych punktach,
- typ 3 – krzywizna quasi-liniowa, bardzo zbliżona do krzywizny paraboli 3. stopnia,
- typ 4 – krzywizna ma wklęsły charakter, jest ostra (4a) lub ma ciągłość typu G^1 (4b) na początku KP i zawsze ostra na końcu KP,
- typ 5 – krzywizna ma wypukły charakter i styczność typu G^0 na początku i końcu krzywej.

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