Research paper

Proposal for application of risk analysis to assess robustness of floor slabs pre-stressed with unbonded tendons

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Abstract: The list of potential hazards related to concrete elements and structures prestressed with the use of unbonded tendons, including the flat slabs, is long and fairly well recognized. In addition to the standard accidental events this list includes: mishandling during construction, small fire, local corrosion, loss of bond at the anchorage, second order effects, brittle fracture of elements, etc. Despite of these hazards related to unbonded post-tensioning, this type of structures are extensively promoted and used in practice thanks to the possibility of the large span floors and innovative character of this technology. The paper presents a proposal for the application of risk analysis to assess the robustness of structures with flat slabs prestressed with unbonded tendons. The adoption of variables that determine risk and robustness as fuzzy numbers assigned to linguistic variables are proposed. Numerical example is presented to demonstrate risk and robustness assessment of building structure with unbonded post-tensioned slabs supported directly on columns.

Keywords: unbounded prestress, flat slabs, risk analysis, fuzzy logic, FEM method, robustness

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1. Introduction

In the USA and in Western Europe flat floor prestressed with unbonded tendons have been widely used for several years. The main advantages of these structures are as follows: concrete slabs with a complex contour, supported directly on columns and/or sections of bearing walls are easy to form, reduction of deflection and their cracking behavior makes possible reduction of the slab thickness and weight as well as to reduce the consumption of concrete and reinforcing steel.

Prestressing can increase the span of entire flat slabs to about 12 m, and the span of slab band constructions up to 20 m, which determines the free formation of functions in buildings with such floors. Small diameter of tendons, usually up to 20 mm, allows to obtain significant eccentricity of prestressing force. On the other hand, considerable flexibility of covers of unbonded tendons enables to route the course of tendons as well as affect the dissemination of this type of floors in modern designs. There are a lot of potential, although well recognized, risks associated with the use of prestressed concrete structures with unbonded tendons, including flat ceiling plate [1]. In addition to standard accidental actions and catastrophic events difficult to identify, there are as well: reactions caused by presstressing in the system statically redundant, brittle character of fracture, the second order effects, sensitivity to local damages of tendons due to errors of execution that are difficult to find and repair, local corrosion [2] of tendons, small fires, as well as corrosion and loss of bond of tendons at anchorages. Although recent publications indicate that there is not much fears of hazard to safety and low structural robustness of prestressed constructions with unbonded tendons after meeting appropriate standards of design and craftsmanship, their reasoning is rather questionable.

Current standards and publications recommend two strategies for safeguarding civil engineering works against identifiable and unidentifiable accidental actions. The first one is based on specific accidental actions and the other one on limitation of the extend of localised failure from an unspecified causes, taking into account structural robustness. For the construction of buildings with the highest consequences class it is recommended to conduct a systematic risk analysis, taking into account the predictable and unpredictable risks. Unfortunately, these studies show the procedures for analysis and risk assessment in an unclear way, hindering their use in practice. The proposal for application of risk analysis to assess the structural robustness of a flat plate ceiling prestressed with unbonded tendons on the example of such a ceiling is presented in this paper.

2. Measures of structural robustness

2.1. Traditional and probabilistic measures

Structural robustness is variously defined and assessed, usually in a descriptive and imprecise way. Today’s publications often describe it, referring to systems theory, as the property of a structural system that allows them to survive unforeseen or unusual
circumstances [1]. A quantitative estimation of robustness and evaluation of its acceptable level requires identification of the following elements: structure, function and constraints that the system should meet, the list of hazards as well as damage and disruption in the functioning of the system and their consequences. Systems theory shows at least a dozen definitions of robustness, including: system’s ability to perform the intended function in case of damage and/or exceptional actions and catastrophic events, measure of lack of sensitivity to contingency in the project, lack of sensitivity to small changes in the assumptions assumed in the project, the ability to adequately respond to emergency situations, proportionality of failure consequences to its causes and extent, the ability to minimize the damage in the most adverse conditions (Santa Fe Institute 2001) [2].

“European Construction Product Directive” as a one of several documents, contains requirements for structural robustness. Structures must be designed and executed in a manner that protects them from damage, during construction and in use, disproportionately to the cause. Examples of traditional, deterministic and probabilistic measures of robustness are the following:

Relative Residual Resistance $RSR$ (ISO Standard 19902) [3]:

\[ RSR = \frac{R_c}{S_c} \]  

where: $R_c$ – the characteristic value of the load bearing capacity and $S_c$ – the design load corresponding to ultimate collapse of a structure.

Vulnerability index $V$ (Lind 1995) [4]:

\[ V = \frac{P(r_d, S)}{P(r_o, S)} \]

where: $P(r_d, S), P(r_o, S)$ – represents the probability of failure, $r_0$ is the resistance of the intact structure, $r_d$ is the resistance of the damaged structure and $S$ is the effect of actions.

Redundancy index $\beta_r$ (Frangopol, Curly 1987) [5]:

\[ \beta_r = \frac{\beta_i}{\beta_i - \beta_d} \]

where: $\beta_r$ is the redundancy index, $\beta_i$ is the reliability index of the intact structure and $\beta_d$ is the reliability index of the damaged structure [6].

### 2.2. Measures based on risk assessment

Risk may be referred to as a measure of the danger or hazard that undesired events represents for people, economy and environment, and is defined as a combination (usually a product) of the probability of occurrence and the consequence of a specified hazardous or undesired event. For a set of hazardous design situation $H_i$ the total risk $R$ can be calculated by the following formula [7,9,10]):

\[ R = \sum_{i=1}^{n_H} p(H_i) \sum_{j=k}^{n_D} p(D_j|H_i) p(S_k|D_j) C(S_k) \]
where: \( n_H \) – number of hazards that can cause damage to the construction in \( n_D \) different ways, \( n_S \) – number of adverse structure conditions \( S_k \), the consequences of which are \( C(S_k, H_i) \), \( p(H_i) \) – probability of occurring the \( i \) hazard in the reference time, \( p(D_j|H_i) \) – conditional probability of \( j \) damage condition of a construction causing the \( i \) hazard, \( p(S_k|D_j) \) – conditional probability of \( k \) adverse condition of the whole construction caused by \( j \) damage condition.

The value of risk calculated using the formula (2.4), expressed in monetary terms is inconvenient for evaluation and comparison of the robustness. The reasonable measure of robustness should be rather dimensionless.

I.W. Baker proposed to use a measure of robustness in the form of the robustness index, defined as the ratio of the direct to the total risk, being the sum of the direct \( R_{\text{dir}} \) and the indirect risk \( R_{\text{ind}} \) [7]:

\[
I_{\text{rob}} = \frac{\sum_i R_{\text{dir},i}}{\sum_i R_{\text{dir},i} + \sum_i R_{\text{ind},i}}
\]

Direct risk is related to the consequences of damage and destruction of elements on the safety of the construction. Indirect risk refers to the consequences of their effect on the structure and the surroundings. The risk \( R_D \) is associated with direct consequences due to exposure events and the risk \( R_{ID} \) to all indirect consequences of exposure events.

The \( I_{\text{rob}} \) – index takes values between zero and one \( 0 \leq I_{\text{rob}} < 1 \); \( I_{\text{rob}} = 1 \) if the system is completely robust and there is no risk due to indirect consequences, and \( I_{\text{rob}} = 0 \) if all risk is due to indirect consequences. If \( I_{\text{rob}} = 1 \), the construction is fully robust, if \( I_{\text{rob}} = 0 \), the construction has no robustness. Due to the fact that extreme cases generally do not occur in reality, the problem of calibration an acceptable value of the robustness index requires a comparison study of many a great number of practical examples. In addition, it is important to remember that, in practice, risk assessment is much more difficult and more uncertain.

### 2.3. Measures based on fuzzy risk estimation

Risk estimation in cases of multiple dependance of consequences on event and when the available knowledge is imprecise, uncertain or vogue needs special methods. In these cases fuzzy set mathematics could be helpful. Qualitative information and uncertain data can be formally treated by linguistic variables which can be quantified using fuzzy numbers with the standard membership functions [11] for example triangular. The frequency – consequence diagrams can be used to present to present the risk in terms of probable consequences of catastrophic or undesired events. In Figure 1 consequences of structural failure expressed by means of the fuzzy target probability probabilities of these events and their relative costs defined in terms of the membership functions of fuzzy variables [12] are presented together with the corresponding frequency – failure diagram. Thanks to the adopted assumptions, the following three areas of risk can be extracted: acceptable,
controlled, unacceptable. Figure 1 shows a sample diagram with fuzzy cost resulting from the criterion of the unity distribution correlated with sharpened fuzzy probabilities of failure by [13]. It is practically impossible to precisely determinate both probabilities of occurrence of hazards and their consequences. Taking into account the impact of quality control in the design and implementation, the variables $p$ and $C$ can be presented by means of fuzzy numbers $\mu_c$, $\mu_{cf}$, $\tilde{p}$ and $\tilde{C}$ with membership functions $\mu_p$ and $\mu_c$ [13–15]:

$$
\tilde{p} = (1 - \tilde{n}) \tilde{p}_b + \tilde{n} \cdot \tilde{p}_k
$$

where: $\tilde{n}$ – fuzzy coefficient expressing degree of controle efficiency, taking a value between [0, 1], $\tilde{p}_b$ and $\tilde{p}_k$ – fuzzy probabilities with no control and with a different levels of control.

Measure of consequence $C$ can be defined as measurable but in principle impossible to control. Membership functions of fuzzy variables $\mu_p$ and $\mu_C$ can be considered, in a first approximation, as “triangular”, defined by three numbers representing the value of the dominant variable with a total membership of $m_X$ and two numbers defining the range of variation $[a_X, b_X]$: $\mu_X = (m_X, a_X, b_X)$.

The risk, with a consideration of the nature of fuzzy variables determining its size, is also a fuzzy variable $\tilde{R}$. The best measure of robustness will be defuzzifed value $\tilde{R}$, for example its dominant value $m_R$.

In order to facilitate the comparison of robustness of different constructions, it is best to take a dimensionless measure of robustness (risk index) as a ratio of the risk dominant value $m_R$. calculated for the dominant value of fuzzy damage cost or destruction of the
structure $m_{CR}$, to the dominant values of acceptable risk of destruction $m_D$.

\begin{equation}
_i R = \frac{m_R}{m_D + m_R}
\end{equation}

The acceptable risk value $m_D$ is a product of acceptable probability value of construction damage $p_{fd}(RCX, T_0)$ recommended in the standard EN 1990 [11] for a given reliability class $RCX = RC3$ or $RC2$ or $RC1$, the reference time $T_0$ and dominant value of fuzzy costs of the investments in the entire life cycle of the structure $m_{CD}$:

\begin{equation}
 m_D = p_{fd}(RCX, T_0) \times m_{CD}
\end{equation}

Risk index takes values from the range $0 \leq i_R < 1$. The virtual structure is completely robust if $i_R = 0$, (no risk), and fully sensitive to failure in accidental design situations if, $i_R \sim 1$.

Since it is extremely difficult to determine the quantitative assessment of the hazard occurrence $p(H_i)$, as well as conditional probabilities present in the formula (2.4), it is recommended to use rough estimates of the occurrence of hazards, for instance, by the following dependencies [16]:

- **often occurring:** $p(H_i) \geq 2.7 \times 10^{-2}$ (over 10/year);
- **frequent:** $2.7 \times 10^{-2} > p(H_i) \geq 2.7 \times 10^{-3}$ (from 10/year to 1/year);
- **occasional:** $2.7 \times 10^{-3} > p(H_i) \geq 2.7 \times 10^{-4}$ (from 1/year to 1/10 years);
- **unlikely:** $2.7 \times 10^{-4} > p(H_i) \geq 2.7 \times 10^{-5}$ (from 1/10 year to 1/100 years);
- **very rare:** $2.7 \times 10^{-5} > p(H_i) \geq 2.7 \times 10^{-6}$ (from 1/100 year to 1/1000 year);
- **improbable:** $2.7 \times 10^{-6} > p(H_i) \geq 2.7 \times 10^{-7}$ (from 1/1000 year to 1/10000 year).

### 3. Risk and robustness assessment for floor slabs prestressed with unbonded tendons

The subject of the analysis is a three-storey building with the slab-column structure (Fig. 2), with flat slab floors, prestressed with unbonded tendons (Figs. 2 and 3). Floor slabs, of a thickness equal 350 mm were designed using concrete of the strength [27] class C45/55. Prestressing reinforcement was adopted in a form of tendons Ø 16 mm, made of wires with tensile strength $f_{pk} = 1860$ N/mm², in a number of 5 tendons (750 mm²), spacing of 120 mm in the column bands and 650 mm in the middle bands. In addition, a subsurface reinforcement was applied [22].

<table>
<thead>
<tr>
<th>Type of materials resistance</th>
<th>Mean value</th>
<th>Distribution</th>
<th>CoV</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete capacity</td>
<td>48 MPa</td>
<td>ND</td>
<td>0.128</td>
<td>EC</td>
</tr>
<tr>
<td>Reinforcement capacity</td>
<td>560 MPa</td>
<td>ND</td>
<td>0.038</td>
<td>JCSS</td>
</tr>
<tr>
<td>Post-tensioning (PT) steel capacity</td>
<td>1630</td>
<td>ND</td>
<td>0.025</td>
<td>JCSS</td>
</tr>
</tbody>
</table>
Two different hazards \( n_H = 2 \) to the safety of structure were taken into account in the analysis: (a) collapse of one column that can (cause failure of 1, 2 or 3 panels of the floor slab \( n_D = 3 \), \( n_S = 3 \) in different ways (Figs. 4, 5), damage caused by corrosion of at least two adjacent tendons that in a period of \( T_0 = 50 \) years [23, 24] can results in failure of 3 panels of the floor slab (Fig. 6).
Fig. 4. Collapse of one column ($\Delta h$ the vertical movement in millimetres and crack patterns at the surface of the slab): a) collapse intermediate supports monolithic reinforced concrete columns. wk(d) – cracking pattern, crack patterns at the lower surface of the slab wk(g) – cracking pattern, crack patterns at the upper surface of the slab $\varepsilon_Z$ – displacement of the flat slabs

Unconditional and conditional fuzzy probabilities expressed in terms of fuzzy numbers with membership functions $\mu_X = (m_X, a_X, b_X)$ assigned to linguistic variables defined in point 2 for hazard $p(H_1)$ as “very rare” adjusted depending on the number of slab fields damaged by destruction of the column, for $H_2$ and $p(D|H_2)$ after 50 years of use as “frequent” and $p(S_1|D)$ as depending on the number of slab panels damaged due to corrosion of 2 or 3 tendons [25]. The acceptable risk value $m_D = (7.23E - 5)C$ was
Fig. 5. Collapse of one column ($\Delta h$ the vertical movement in millimetres and crack patterns at the surface of the slab): collapse extreme supports monolithic reinforced concrete columns. wk(d) – cracking pattern, crack patterns at the lower surface of the slab wk(g) – cracking pattern, crack patterns at the upper surface of the slab $\varepsilon_Z$ – displacement of the flat slabs

calculated for the reliability class of the structure RC2, the reference time $T_0 = 50$ years and the total costs $C$. The mean value of the total life costs $C_D$ of the analyzed structure was assumed as 100%: $m_D = (7.23E - 3)C$. The mean cost values due to failure of one column were estimated at 10% of the total life cost $C_D$: $m_{D1} = 7.23E - 4$, two adjacent tendons at 15% $C_D$: $m_{D2} = 10.845E - 4$ and the simultaneous failure of a column and 2 tendons at 40% $C_D$: $m_{D3} = 28.92E - 4$. In turn, the mean values of the risks
Fig. 6. Damage due to cut off 2 adjacent tendons owing to corrosion (numeral 1 in figure) – wk2(d) crack patterns at the upper surface of the slab

cauised by these accidental events were calculated from the formula (2.4) are as follows: $m_{R1} = 1.177E-4$, $m_{R2} = 5.044E-4$ and $m_{R3} = 19.195E-4$. As a result, the values of risk index calculated according to the formula (2.7) in case of a hazard due to damage of the column, situated anywhere, is $i_R = 0.14$. For hazards of corrosion of tendons in a period of 50 years the risk index value is $i_R = 0.40$, and having taken into account both hazards is $i_R = 0.49$. These results confirm the fairly widespread opinion about the critical impact of hazard caused by the loss of bearing capacity of the tendons due to local damage.
caused by corrosion of tendons, tendon bond loss at anchorages [28], small fires, etc. Mostly considered accidental events, involving the destruction of the floor support [18, 20], turn out to be less dangerous [17, 20]. It can also be noted that with increasing numbers of catastrophic events [25, 26], risks associated with the failure of the structure do not increase proportionally to their numbers, even without taking into account the correlation of hazards and their consequences.

4. Conclusions

The paper presents a proposal for the application of risk analysis to assess the robustness of flat slab floors prestressed with unbonded tendons. Basic constraints for risk assessment procedures and resistance of construction in accidental situations are recommended in the EN 1991-1-7 and in the literature and they are based on the values, especially conditional probabilities and those associated with the indirect risk. Therefore, the adoption of variables that determine risk and robustness as fuzzy numbers assigned to linguistic variables are proposed [19–21]. They directed primarily expertise as well as subject to updates as broaden its base of experimental data. The expertise was primarily taken into consideration as well as updates while broadening the base of experimental data [18, 19]. A modified, dimensionless measure of robustness, called risk indicator was defined. It is related to estimated and acceptable damage risk and it is independent of the consequences of structural damage expressed in monetary units. The example of risk and robustness assessment of building structure with unbonded post-tensioned slabs supported directly on columns illustrates functionality and relative simplicity of the proposed measures and procedures to evaluate the impact of exceptional events on a safety hazard of complex building structures.

References


Propozycja procedury oceny ryzyka i odporności poawaryjnej płaskich stropów płytowych dwukierunkowo sprężonych cięgami bez przyczepności

Słowa kluczowe: stropy dwukierunkowo sprężone, cięgna bez przyczepności, analiza ryzyka, funkcje rozmyte, analiza MES, odporność poawaryjna

Streszczenie:

Wprowadzenie

Lista potencjalnych zagrożeń związanych z elementami i konstrukcjami betonowymi sprężonymi za pomocą cięgien bez przyczepności, w tym płyt płaskich, jest długa i dość dobrze rozpoznana. Chociaż w nowszych publikacjach formułowane są opinie, że obawy związane z zagrożeniami bezpieczeństwa i małą odpornością poawaryjną konstrukcji sprężonych cięgami bez przyczepności po spełnieniu odpowiednich standardów projektowych i starannym wykonaniu są bezpodstawne, to ich uzasadnienie jest problematiczne. Poza standardowymi zdarzeniami losowymi lista ta obejmuje: błędy wykonawstwa, niewielki pożar, miejscową korozję cięgien sprężających, utratę przyczepności cięgien w zakotwieniach, efekty drugiego rzędu, kruche pękanie elementów itp. Tego rodzaju konstrukcje są szeroko promowane i wykorzystywane w praktyce dzięki możliwości konstruowania stropów o dużej rozpiętości oraz innowacyjnemu charakterowi tej technologii. W pracy przedstawiono propozycję zastosowania analizy ryzyka do oceny bezpieczeństwa konstrukcji stropów z płyt płaskich sprężonych cięgami bez przyczepności. Proponuje się przyjęcie zdefiniowanych implicit wymagań wpływających na poziom ryzyka i odporność jako liczby rozmytych przypisanych odpowiednim zmiennym lingwistycznym. Przedstawiono przykład liczbowy oceny ryzyka i nośności ustroju słupowo-płytnego z płaskimi płytami sprężonymi płaskimi cięgami bez przyczepności.

Received: 5.05.2021, Revised: 28.05.2021