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The Pandemic-Type Demand Shocks in the Mean-Variance Newsvendor Problem

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Abstract
The paper considers the negative pandemic-type demand shocks in the mean-variance newsvendor problem. It extends the previous results to investigate the case when the actual additive demand may attain negative values due to high prices or considerable, negative demand shocks. The results indicate that the general optimal solution may differ to the solution corresponding exclusively to the non-negative realizations of demand.

Keywords
Production planning and control; Inventory; Newsvendor; Pricing; Stochastic additive demand.

Introduction

Demand shocks constitute surprise events which cause an increased or decreased demand for goods or services. The Covid-19 pandemic has stimulated a wave of negative impacts far beyond any recent global event. With the pandemic causing a major global recession, prices and demand for the majority of industrial commodities, except for the necessities, have been driven lower (Nikolopoulos *et al.*, 2020). The coronavirus pandemic exerts a significant impact upon demand in various sectors including the steel and iron ore industry, automotive industries, restaurants, beauty salon services and numerous others. The pandemic boosts unemployment and reduces consumers' capacity to purchase goods and services. As a consequence it can be considered as a demand shock (Collie *et al.*, 2020; Cecil, 2020; Guerrieri, 2020).

Various studies in Operations Research (OR) open with a specific stochastic function for demand. The random part is referred to as the demand shock. The price-dependent demand function with uncertainty constitutes a function of the price and demand shock. It is typically a random variable with a certain known distribution. Relatively frequently, the uncertainty is incorporated into deterministic demand functions by the additive demand shock being considered. In the event the assumption of non-negativity is absent, negative demand realizations may emerge in additive models for high prices or considerably negative demand shocks which, in this paper, will be referred as "*the negative pandemic-type demand shocks*" due to the recent dramatic economic events. When the set of possible parameters is restricted, an incomplete characterization of the optimal price occurs. If negative demand occurs at a given price, the non-negativity requirement on demand will considerably affect the price. In the additive case, provided that both the deterministic demand and demand shock are non-negative, an increase in the expected revenue may be anticipated without any upper bound if the company sets the price arbitrarily high. As a consequence, as far as the additive shock is concerned, the non-negativity cannot be imposed on the deterministic demand and demand shock separately. It must be imposed simultaneously. This can be achieved by, e.g. defining the random demand as the positive part of the demand function with a shock (Krishnan, 2010).

Obviously, the supply chain models with non-negative demand are far more finely tuned to actual conditions including the occurrence of unexpected events. The presence of negative realized demand in OR models has been recently investigated in (Krishnan, 2010; Kyparisis & Koulamas, 2018), and (Bieniek, 2021). Krishnan (2010) points out that the non-negativity assumption ought to be imposed on demand for the generality of the study to be ensured. If the non-negativity constraint is not employed, the so-

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lution may be suboptimal and the expected profit can be underestimated. Kyparisis and Koulamas (2018) consider a classical price-setting newsvendor problem with non-negative demand. They show that, in this case, the newsvendor problem invariably has an optimal solution, even in adverse market conditions. Bieniek (2021) examines a two-stage Vendor Managed Consignment Inventory contract with a similar constraint imposed on demand.

The newsvendor model has been under examination for approximately sixty years and lately, several relaxing assumptions concerning the elementary newsvendor problem have been offered in the OR literature (Qin, 2011; Khouja, 1999). One of the major generalizations of the classic newsvendor problem is the application of price as a decision variable. A very popular form of the price-setting newsvendor problem is the mean-variance analysis. It was first applied by Lau (1980) who studied the mean-standard deviation payoff criterion. Choi *et al.* (2008) introduced stockout costs in the mean-variance analysis. Subsequently, Wu *et al.* (2009) focused on the influence of stockout costs on the optimal ordering decisions while comparing the classic models with the mean-variance models. Agrawal & Seshadri (2000) presented the mean-variance analysis from the expected utility framework perspective. Rubio-Herrero *et al.* (2015), Rubio-Herrero & Baykal-Gursoy (2018), and Rubio-Herrero & Baykal-Gursoy (2019) examined the price-setting newsvendor problem under the mean-variance criteria as well. Based on the theory developed by (Zabel, 1970) Rubio-Herrero *et al.*, 2015) solved the problem sequentially as long as the level of risk-aversion is not overly high. They proved that the price is lower for a risk-averse newsvendor than for a risk-neutral one. Rubio-Herrero and Baykal-Gursoy (2018) extended the findings of (Rubio-Herrero *et al.*, 2015) and proved the unimodality or quasiconcavity of the objective function. The authors of the above-cited papers solved all risk-sensitive instances using the optimality conditions based on the lost sales rate uncertainty (Kocabiyikoglu & Popescu, 2011) and considered the additive demand uncertainty. A similar problem for the iso-elastic demand function was examined by (Rubio-Herrero & Baykal-Gursoy, 2019).

Based on our previous considerations, there exists a need for the non-negativity constraint being imposed on the demand in the mean-variance newsvendor model studied in (Rubio-Herrero *et al.*, 2015; Rubio-Herrero & Baykal-Gursoy, 2018). Such a transformation enables the results more general and complete. In addition, on the whole, solutions obtained in this mode will be optimal for the pandemic-type demand shocks as well. Compared with the studies mentioned hereinabove, the present paper aims to complement the non-negative actual demand case solved in (Rubio-Herrero & Baykal-Gursoy, 2018) with the prospect of a negative demand realization. To this end, the paper disregards the assumption that a price is not higher than a maximal price for which demand reaches zero value. However, in the event a possibly negative realized demand emerges, the general solution is complicated even for uniformly distributed demand. In such a case we solve this problem numerically. We indicate that if the considerations are restricted to the positive actual demand case, it may lead to the suboptimal solution. This outcome is illustrated in figures created with the Mathematica computing software.

Preliminary facts and notation

Let us begin with the examination of the non-negativity aspect of demand in the mean-variance newsvendor problem. The price-dependent demand function with uncertainty in the common form is denoted by $D(p,\varepsilon)$, where p is the price and ε is a demand shock, which is a random variable with a certain known distribution. In general, the additive demand model has the form $D(p,\varepsilon) = d(p) + \varepsilon$, where $d(p)$ is the deterministic demand, which is frequently used in the economic sciences. For the deterministic demand, being a linear function $d(p) = a - bp$, $a, b > 0$, the uncertain additive demand is given by

$$D(p,\varepsilon) = a - bp + \varepsilon. \tag{1}$$

The following assumptions will be used henceforth: ε constitutes the demand shock with the expectation $\mu = 0$ and the variance $\mathrm{Var}(\varepsilon)$, the cumulative distribution function F and the continuously differentiable probability density function f with the support $[A, B]$, where $A < 0$ and $B > 0$; $\bar{F}(z) = 1 - F(z)$ is a hazard rate and $h(z) = f(z)/\bar{F}(z)$ is a failure rate function; and finally $A + a - bc > 0$. WLOG it is assumed that $\mu = 0$ since if $\mu \neq 0$, its value can be added to a. This can be done if $a + \mu > 0$. If ε is defined on an open interval, we can take into account an efficient truncation capturing as much information as possible.

Under the assumptions made above, the actual demand realization $D(p^*,\varepsilon) = a - bp^* + \varepsilon$, where p^* is the optimal price, can be negative for $p^* > \dfrac{A+a}{b}$ and the pandemic-type demand shocks. This means that in these cases there is no demand. Furthermore, for $p^* > \dfrac{B+a}{b}$ the actual demand realization is always negative.This implies that there is no demand regardless of the conditions. For that reason, this case is disregarded from our considerations. Pursuant to all the previous statements, imposing the non-negativity forces the use of the following function for demand

$$D(p,\varepsilon) = (a - bp + \varepsilon)^+, \tag{2}$$

where $y^+ = \max(0, y)$, instead of (1). This transformation various mathematical problems in each supply chain model. The problems are frequently severely complicated or even intractable. This paper imposes the non-negativity constraint in the mean-variance newsvendor problem by implementing the demand function defined by (2). In addition, the optimization process is also conducted as if it is a new task. In this problem, the retailer aims at maximizing their expected profit while maintaining the variance of the profit under control. The decision-maker determines the quantity of the product to be purchased from the wholesaler at a given cost and fixes the price at which the product will be sold. The mean-variance risk-sensitive performance measure is formulated as

$$\begin{aligned}\Pi(p,q) = {} & pE\min(q, D(p,\varepsilon)) - cq \\ & - \lambda \mathrm{Var}(p\min(q, D(p,\varepsilon))),\end{aligned} \tag{3}$$

where q is the stock quantity. In this model, λ constitutes a risk parameter, which is positive for risk-averse, equal to 0 for risk-neutral, and negative for risk-seeking cases. The demand $D(p,\varepsilon)$, is defined by (1). Even though (3) lacks economic meaning, the optimization (3) is understood as profit being maximized while either minimizing variance for a risk-averse newsvendor or maximizing variance for a risk-seeking one. Furthermore, the risk parameter can be approached as a scaling factor that balances the expected profit and the variance of the profit (Rubio-Herrero *et al.*, 2015).

The performance measure maximization problem can be presented as

$$\max_{p,q} \Pi(p,q), \tag{4}$$

where $\Pi(p,q)$ is given by (3). When solving (4) the actual demand realization for the optimal price p^* may turn out to be negative. It implies that there is no demand. If the non-negativity constraint is imposed on the total demand, the maximization problem given by (4) becomes

$$\begin{aligned}\max_{p,q} \widetilde{\Pi}(p,q) = {} & pE(\min((a - bp + \epsilon)^+, q)) - cq \\ & - \lambda \mathrm{Var}\left[p\min((a - bp + \epsilon)^+, q)\right].\end{aligned} \tag{5}$$

Since $\varepsilon \in [A,B]$ then $\widetilde{\Pi}(p,q) = \Pi(p,q)$ for $p \le \dfrac{A+a}{b}$, $q \ge 0$. It ought to be noted that the demand realization $D(p,\varepsilon) = a - bp + \varepsilon \le B + a - bp$, which is always negative for $p > \dfrac{B+a}{b}$, thus the expected profit becomes negative. Therefore, our considerations are limited to the case $p \le \dfrac{B+a}{b}$.

Let $q = a - bp + z \ge 0$, where $z \in [A,B]$ is the service level. In addition, let us define $\mu(z) = E(\min(\varepsilon, z)) = \int_z^B (z-u)f(u)\,\mathrm{d}u$ and $\sigma^2(z) = \mathrm{Var}(\min(\varepsilon,z)) = \mathrm{Var}(\varepsilon) + \int_z^B (z^2 - u^2)f(u)du - \mu^2(z)$. The following statements hold: $\dfrac{\mathrm{d}\mu(z)}{\mathrm{d}z} = \bar{F}(z)$, $\mu(.)$ is increasing, $\mu(A) = A < 0$ and $\mu(B) = \mu$. Furthermore, the function $z - \mu(z)$ is also increasing and $0 \le z - \mu(z) \le B$. Additionally, $\sigma^2(.)$ is non-negative and increasing and $\sigma^2(A) = 0$, $\sigma^2(B) = \mathrm{Var}(\varepsilon)$ and $\dfrac{\mathrm{d}\sigma^2}{\mathrm{d}z} = 2\bar{F}(z)(z - \mu(z))$. As a consequence, the decision maker's problem (4) can be presented as

$$\begin{aligned}\max_{p,z} \Pi(p,z) = {} & p(\mu(z) + a - bp) - c(z + a - bp) \\ & - p^2\lambda\sigma^2(z).\end{aligned} \tag{6}$$

Our results complement the findings of Rubio-Herrero & Baykal-Gursoy (2018), where the assumption $p \le \dfrac{A+a}{b}$ for any $z \in [A,B]$ is made. It holds if

$$2A + a - bc > 0. \tag{7}$$

Our study disregards the assumption (7) and resolves the mean-variance newsvendor problem. We supply a general and complete solution to the research problem.

The non-negative demand realization

Let us review the optimization problem given by (5), restricted to the feasible set $p \le \dfrac{A+a}{b}$. Then, $\widetilde{\Pi}(p,q) = \Pi(p,q)$. If we let $q = a - bp + z \ge 0$, than this restricted version of the problem can be presented as

$$\begin{aligned}\max_{p \le \frac{A+a}{b}, z} \widetilde{\Pi}_R(p,z) = {} & p(\mu(z) + a - bp) \\ & - c(z + a - bp) - p^2\lambda\sigma^2(z).\end{aligned} \tag{8}$$

The problem can be solved using the sequential optimization method proposed by (Zabel, 1970). The first step in our optimization is to establish the price that maximizes the performance measure for any given $z \in [A, B]$. Solving the first order condition $\delta\Pi(p,z)/\delta p = -2p(\lambda\sigma^2(z) + b) + \mu(z) + a + bc = 0$ we obtain

$$p^*(z) = \frac{\mu(z) + a + bc}{2(\lambda\sigma^2(z) + b)}. \tag{9}$$

Moreover, $\delta^2\Pi(p,z)/\delta p^2 = -2(\lambda\sigma^2(z) + b)$, and

$$\frac{\delta^2\Pi(p,z)}{\delta p^2} \begin{cases} < 0, & \lambda > 0 \text{ or } (\lambda < 0 \text{ and } \\ & \qquad \sigma^2(z) < -b/\lambda), \\ = 0, & \sigma^2(z) = -b/\lambda, \\ > 0, & \lambda < 0 \text{ and } \sigma^2(z) > -b/\lambda. \end{cases} \tag{10}$$

The first and the second derivatives of the optimal price are given by

$$p^{*'}(z) = \frac{\bar{F}(z)}{\sigma^2(z) + \dfrac{b}{\lambda}} \left(\frac{1}{2\lambda} - 2(z - \mu(z))p^*(z) \right)$$

and

$$p^{*''}(z) \mid_{p^{*'}(z)=0} = -\frac{2\bar{F}(z)F(z)}{\sigma^2(z) + \dfrac{b}{\lambda}} p^*(z).$$

The application of these formulas, the lemma with the shape of $p^*(.)$ to be proved.

Lemma 1. [Rubio-Herrero & Baykal-Gursoy (2018)] *If $\lambda \geq 0$ then $p^*(z)$ is a positive, increasing-decreasing or strictly increasing and concave function. If $\lambda < 0$ then $p^*(z)$ is first a positive, increasing and convex function with asymptote $\sigma^2(z) = -b/\lambda$ and then a negative function.*

Remark 1. The function $p^*(z)$ is increasing for all $z \in [A, B]$ if $0 < \lambda < \dfrac{1}{4Bp^*(A)}$. Moreover, the negative values of $p^*(z)$ correspond to a minimizer of the performance measure.

It should be established whether the function $p^*(z)$ is hedged in the interval $\left(c, \dfrac{A+a}{b}\right]$. Let the hedged optimal price function be constructed as the following piecewise function:

$$\pi^*(z) = \begin{cases} p^*(z), & z \in [A, z_1) \cup [z_2, z_c); \\ \dfrac{A+a}{b}, & z \in (z_1, z_2]; \\ c, & z \in [z_c, B], \end{cases}$$

where

$$z_1 = \min\left\{ \min\left\{ z \colon p^*(z) = \frac{A+a}{b} \right\}, B \right\},$$

$$z_2 = \min\left\{ \max\left\{ z \colon p^*(z) = \frac{A+a}{b} \right\}, B \right\}$$

and

$$z_c = \min\left\{ \{z \colon p^*(z) = c\}, B \right\}.$$

If any of the equations $p^*(z) = x$ has no solution, we assume that $z_x = B$. It ought to be noted that the formula for the hedged price is valid for any risk behaviour. However, for $\lambda < 0$ it consists of one or two pieces only. However, for $\lambda \geq 0$, it may consist of up to four pieces, which differs from (Rubio-Herrero & Baykal-Gursoy, 2018), where the function $p^*(z)$ may include two pieces at most (Figure 1).

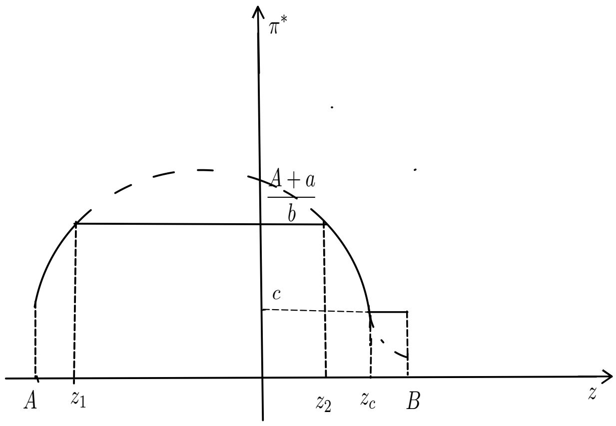

Fig. 1. Possible optimal price function in risk-averse cases

Consequently, let us also define the following useful functions of $z \in [A, B]$:

$$\Pi_1(z) = \Pi(p^*(z), z) = \frac{1}{2}p^*(z)(\mu(z)+a+bc) - c(z+a),$$

$$\Pi_2(z) = \Pi\left(\frac{A+a}{b}, z\right) = -\left(\frac{A+a}{b}\right)^2 (\lambda\sigma^2(z) + b) + \frac{A+a}{b}(\mu(z) + a + bc) - c(z+a)$$

and

$$\Pi_3(z) = \Pi(c, z) = -c^2(\lambda\sigma^2(z) + b) + c(\mu(z) + bc - z).$$

Then (8) can be transformed into $\max_{z \in [A,B]} \Pi^*(z)$, where

$$\Pi^*(z) = \begin{cases} \Pi_1(z), & z \in [A, z_1] \cup (z_2, z_c]; \\ \Pi_2(z), & z \in (z_1, z_2]; \\ \Pi_3(z), & z \in (z_c, B]. \end{cases}$$

It is noteworthy that for $\lambda < 0$ the function $\Pi^*(.)$ may consist of two pieces at most, while for $\lambda > 0$ it may consist of one, up to a maximum of four pieces. If our considerations are limited to the set of parameters satisfying (7) as it was done in (Rubio-Herrero *et al.*, 2015; Rubio-Herrero & Baykal-Gursoy, 2018), the performance measure will consist of max. two pieces for $\lambda > 0$. Therefore, our approach considers the research problem more broadly.

The function $\Pi^*(.)$ is continuous and its first derivative is equal to

$$\Pi^{*\prime}(z)=\begin{cases} p^*(z)\bar{F}(z)(1-2\lambda(z-\mu(z))p^*(z))-c, \\ \qquad z\in[A,z_1]\cup(z_2,z_c]; \\ -2\lambda\left(\frac{A+a}{b}\right)^2(z-\mu(z))\bar{F}(z) \\ \qquad +\frac{A+a}{b}\bar{F}(z)-c, \\ \qquad z\in(z_1,z_2]; \\ 2c^2\lambda(z-\mu(z))\bar{F}(z)-c(1-\bar{F}(z)), \\ \qquad z\in(z_c,B]. \end{cases} \tag{11}$$

Since $\Pi_3'(z)<0$ for any $z\in[z_c,B]$, $\Pi_3(.)$ is decreasing and the optimal solution, if it exists, falls into the interval $[A,z_c)$. We only need the second derivatives of $\Pi_1(.)$ and $\Pi_2(.)$ given by

$$\begin{aligned}\Pi_1''(z)=\bar{F}(z)p^*(z)\Bigg[\left(\frac{p^{*'}(z)}{p^*(z)}-h(z)\right)\big(1 \\ -2\lambda(z-\mu(z))p^*(z)\big) \\ -2\lambda(F(z)p^*(z)+(z-\mu(z))p^{*'}(z))\Bigg],\end{aligned} \tag{12}$$

and

$$\begin{aligned}\Pi_2''(z)=\bar{F}(z)\frac{a+A}{b}\left[-h(z)\left(1-2\lambda(z-\mu(z))\frac{a+A}{b}\right)\right. \\ \left.-2\lambda F(z)\frac{a+A}{b}\right],\end{aligned} \tag{13}$$

respectively. The following lemma proves the concavity of Π_1 and Π_2 under the conditions in terms of mathematical functions. We believe the formulas in a simple mathematical form are elegant, thus ready to be used in numerical computations. Let
$z_\psi=\min\{z\colon p'(z)=0\}$,
$z_\alpha=\min\{z\colon 1-2\lambda(z-\mu(z))p(z)=0,z_c\}$ and
$z_\beta=\min\left\{z\colon z-\mu(z)=\frac{b}{2\lambda(a+A)},B\right\}$.

Lemma 2.

(1) $\Pi_1(.)$ *is concave*

- *(a) in* $[A,z_c]$ *if* $\lambda>0$ *and* $\bar{F}(z)p^*(z)+(z-\mu(z))p^{*'}z>0$, *and* $h(z)>\frac{p^{'*}(z)}{p^*(z)}$ *in* $[A,z_\alpha]$, *and* $h(z)<\frac{2p^{'*}(z)}{p^*(z)}+\frac{F(z)}{z-\mu(z)}$ *in* $(z_\alpha,z_c]$
- *(b) in* $[A,z_1]$ *if* $\lambda<0$ *and* $h(z)>\frac{2p^{'*}(z)}{p^*(z)}-2\lambda F(z)p^*(z)$.

(2) $\Pi_2(.)$ *is concave in* $[A,B]$ *if* $\lambda>0$ *and* $h(z)<\frac{F(z)}{z-\mu(z)}$ *in* $(z_\beta,B]$, *and also if* $\lambda<0$ *and* $h(z)>-2\lambda F(z)\frac{a+A}{b}$.

(3) Π_3 *is decreasing in* (z_c,B).

Proof. (1) First, let us analyze Π_1 in $[A,z_c]$. Let us make the assumption

$$\bar{F}(z)p^*(z)+(z-\mu(z))p^{*'}(z)>0. \tag{14}$$

For $\lambda>0$, the optimal price $p(z)$ is either an increasing or unimodal function and we consider three subintervals: $[A,z_\psi]$ where $p'(z)\geq 0$, $(z_\psi,z_\alpha]$, where $p'(z)<0$ and $1-2\lambda(z-\mu(z))p(z)\geq 0$, and $(z_\alpha,z_c]$ where $p'(z)<0$ and $1-2\lambda(z-\mu(z))p(z)<0$ since by (14) $1-2\lambda(z-\mu(z))p(z)$ is a nonincreasing function. Using (12) we get that $\Pi_1''<0$ in $[A,z_\psi]$ if $h(z)>\frac{p'(z)}{p(z)}$ and also in $(z_\psi,z_\alpha]$ if (14) holds. In $(z_\alpha,z_c]$ we get that Π_1 is concave if $h(z)<\frac{p'(z)}{p(z)}+\frac{2\lambda(F(z)p(z)+(z-\mu(z))p'(z))}{2\lambda(z-\mu(z))p(z)-1}>\frac{2p'(z)}{p(z)}+\frac{F(z)}{z-\mu(z)}$. The proof of (1) is complete.

(2) The constraints on concavity stem from (13). If $\lambda>0$ then $\Pi_2''(z)<0$ for $z>z_1$ if $h(z)<\frac{2\lambda F(z)\frac{A+a}{b}}{2\lambda(z-\mu(z))\frac{A+a}{b}-1}>\frac{F(z)}{z-\mu(z)}$. If $\lambda<0$, we get the concavity of Π_2 if $h(z)>\frac{2\lambda F(z)\frac{A+a}{b}}{2\lambda(z-\mu(z))\frac{A+a}{b}-1}<-2\lambda F(z)\frac{A+a}{b}$, which completes the proof of (2).

(3) is obvious. □

In light of the previous facts we arrive at the unified and complex mathematical solution to the problem (8), which is valid for any risk behaviour and any set of the model parameters. This theorem generalizes (in the sense that we remove the restriction on the optimal price (7)), organizes and simplifies the results presented in (Rubio-Herrero & Baykal-Gursoy, 2018).

Theorem 1. *On the set* $c<p<\frac{A+a}{b}$, $z\in[A,B]$, *the equilibrium decisions of the decision maker of the problem* (6) *are as follows.*

(1) $p^*=p^*(z^*)$ *and* z^* *is the unique solution to* $\Pi_1'(z)=0$ *if*

- *(a)* $\lambda>0$ *and* $\bar{F}(z)p^*(z)+(z-\mu(z))p^{*'}(z)>0$, *and* $h(z)>\frac{p^{'*}(z)}{p^*(z)}$ *in* $[A,z_\alpha]$, *and*

$$h(z) < \frac{2p^{'*}(z)}{p^*(z)} + \frac{F(z)}{z-\mu(z)} \text{ in } (z_\alpha, z_c], \text{ and}$$
$$h(z) < \frac{F(z)}{z-\mu(z)} \text{ in } (z_\beta, B], \text{ and}$$

(i) $\Pi_1'(z_1) < 0$, *or*

(ii) $\Pi_2'(z_2) > 0$,

(b) $\lambda < 0$ *and* $h(z) > \dfrac{2p^{'*}(z)}{p^*(z)} - 2\lambda F(z)p^*(z)$ *in* $[A, z_1]$ *and* $\Pi_1'(z_1) < 0$;

(2) $p^* = \dfrac{A+a}{b}$ *and* z^* *is the unique solution to* $\Pi_2'(z) = 0$ *if*

(a) $\lambda > 0$ *and* $\bar{F}(z)p^*(z) + (z-\mu(z))p^{*'}(z) > 0$, *and* $h(z) > \dfrac{p^{'*}(z)}{p^*(z)}$ *in* $[A, z_\alpha]$, $\Pi_2'(z_1) > 0$ *and* $\Pi_2'(z_2) < 0$;

(b) $\lambda < 0$ *and* $h(z) > -2\lambda F(z)\dfrac{a+A}{b}$ *and* $\Pi_2'(z_1) > 0$.

Proof. Let us note that Π is smooth. Moreover, Π_1 is continuous, smooth and concave in $[A, z_c]$ with $\Pi_1'(A) > 0$ and $\Pi_2'(z_c) < 0$. Therefore, by Lemma 2 under similar conditions there exists a unique maximum z^* in $[A, z_1]$ if $\Pi_1'(z_1) < 0$, or in $[z_2, z_c]$ if $\Pi_2'(z_2) > 0$, and we arrive at the solution of the theorem. Moreover, Π_2 is continuous, smooth and concave in $[A, B]$ with $\Pi_2'(A) > 0$ and $\Pi_2'(B) < 0$. By Lemma 2 under the similar conditions there exists a unique maximum z^* in $[z_1, z_2]$ if $\Pi_2'(z_1) > 0$ and $\Pi_2'(z_2) < 0$. The proof of the theorem is complete. □

The negative actual demand realization

Let us examine the optimization problem given by (5) limited to the feasible set $p \in \left[\dfrac{A+a}{b}, \dfrac{B+a}{b}\right]$. The restricted version of the mean-variance newsvendor problem can be presented as

$$\max_{\substack{\frac{A+a}{b} \le p \le \frac{B+a}{b},\\ B \ge z \ge bp-a}} \widetilde{\Pi}(p,z)$$
$$= pE(\min((a-bp+\epsilon)^+, a-bp+z)) - c(z+a-bp)$$
$$-\lambda p^2 \mathrm{Var}[\min((a-bp+\epsilon)^+, a-bp+z)]. \qquad (15)$$

The application of the standard algebraic arguments leads to the next lemma.

Lemma 3. For $\dfrac{A+a}{b} < p \le \dfrac{B+a}{b}$ and $B \ge z \ge bp-a$ we have

$$E(\min((a-bp+\epsilon)^+, a-bp+z)) = \mu(z) - \mu(bp-a),$$
$$\mathrm{Var}\left[\min((a-bp+\epsilon)^+, a-bp+z)\right] = \sigma^2(z) - \sigma^2(bp-a) - 2\left(\mu(z) - \mu(bp-a)\right)\left(bp-a-\mu(bp-a)\right).$$

By Lemma 3 we can write the performance measure (15) as

$$\max_{\substack{\frac{A+a}{b} < p \le \frac{B+a}{b},\\ B \ge z \ge bp-a}} \widetilde{\Pi}(p,z) = p(\mu(z) - \mu(bp-a))$$
$$- c(z+a-bp) - \lambda p^2\left[\sigma^2(z) - \sigma^2(bp-a)\right.$$
$$\left. -2(\mu(z)-\mu(bp-a))(bp-a-\mu(bp-a))\right]. \qquad (16)$$

Theorem 2. *The problem defined by* (16) *has a possibly non-unique optimal solution.*

Proof. By the Extreme Value Theorem, the continuous function $\widetilde{\Pi}(p,z)$ in (16) attains at least one maximum value on the convex set
$$\left\{(p,z)\colon \frac{A+a}{b} \le p \le \frac{B+a}{b},\quad B \ge z \ge bp-a\right\}.$$
The proof is complete. □

The considered problem is mathematically complex even for a uniform distribution. As a consequence, we solve it numerically. If the demand shock is uniformly distributed on the interval $[A, -A]$ then the performance measure is a polynomial of sixth degree of p. By solving the first-order condition $\dfrac{\delta\widetilde{\Pi}(p,z)}{\delta p} = 0$ we come up with the fifth degree polynomial equation. Then the optimal solution can be derived numerically for any given parameters satisfying the general assumptions.

The total solution to the mean-variance newsvendor problem

Based on the results obtained in the previous sections, it can be concluded that the problem given by (5) can be presented as

$$\max\left(\max_{\substack{p \le \frac{A+a}{b},\\ B \ge z \ge A}} \Pi(p,z), \max_{\substack{\frac{A+a}{b} \le p \le \frac{B+a}{b},\\ B \ge z \ge bp-a}} \widetilde{\Pi}(p,z)\right). \qquad (17)$$

The following examples vividly illustrate the obtained results. First, let us consider the risk-seeking newsvendor problem with $\varepsilon \sim U[-20,20]$, $\lambda = -0.001$, $a = 35$, $b = 1$ and $c = 1$. Then, the assumptions of the Theorem 1 and 2 are satisfied and, consequently, we infer that the subproblem given by (17) has a unique optimal solution $\hat{p}^* = 21.43$ and $\hat{z}^* = 18.96$ with $\widetilde{\Pi}^*(\hat{z}^*) = 322.48$. The results are specified in Table 1 and illustrated in Figure 2. We can state that, in this case, the optimal values correspond to the problem with the possibly negative actual demand.

Table 1
The risk-seeking newsvendor

$c \le p \le \frac{A+a}{b}$	$\frac{A+a}{b} \le p \le \frac{B+a}{b}$
$p^* = 15$	$\hat{p}^* = 21.43$
$z^* = 18.28$	$\hat{z}^* = 18.96$
$\Pi(z^*) = 290.85$	$\widetilde{\Pi}(\hat{z}^*) = 322.48$
$\varepsilon \sim U[-20,20]$, $a = 35$, $b = 1$,	$\lambda = -0.001$, $c = 1$

Table 2
The risk-averse newsvendor

$c \le p \le \frac{A+a}{b}$	$\frac{A+a}{b} \le p \le \frac{B+a}{b}$
$p^* = 20.925$	$\hat{p}^* = 21$
$z^* = -9.131$	$\hat{z}^* = -9.065$
$\Pi(z^*) = 0.4735$	$\widetilde{\Pi}(\hat{z}^*) = 0.4702$
$\varepsilon \sim U[-10,10]$, $a = 31$, $b = 1$,	$\lambda = 0.001$, $c = 20$

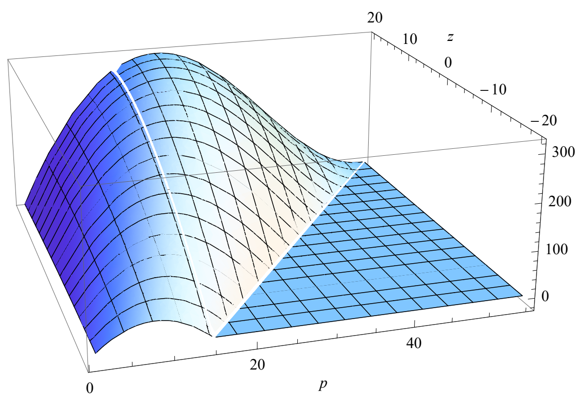

Fig. 2. Performance measure, $U[-20,20]$, $\lambda = -0.001$, $a = 35$, $b = 1$, $c = 1$

At this point, let us examine the risk-averse newsvendor problem with $\varepsilon \sim U[-10,10]$, $\lambda = 0.001$,

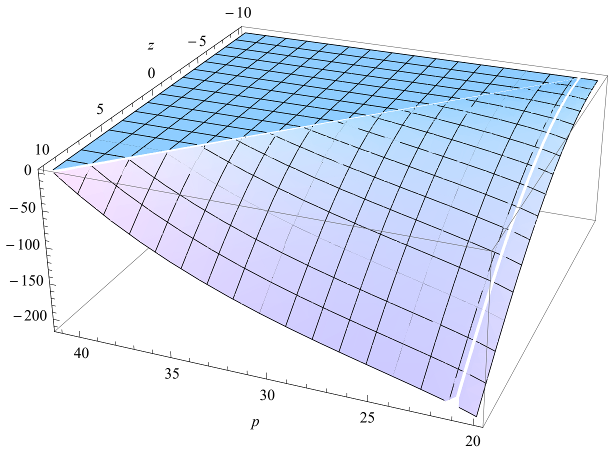

Fig. 3. Performance measure, $U[-10,10]$, $\lambda = 0.001$, $a = 31$, $b = 1$, $c = 20$

$a = 31$, $b = 1$ and $c = 20$. In this case, the assumptions of the Theorem 1 and 2 are satisfied. It can be concluded that the problem given by (17) has an optimal solution $p^* = 20.925$ and $z^* = -9.131$ with $\Pi^*(z^*) = 0.4735$ specified in Table 2 and shown in Figure 3. It ought to be noted the optimal values correspond to the problem with the non-negative realizations of demand.

Discussion and conclusions

The investigation of additive uncertainty is especially interesting due to a special feature, i.e. the fact that models with such uncertainty allow negative demand realizations. These realizations may emerge, in particular, due to considerably negative pandemic-type demand shocks. The negativity of the demand has often been neglected in numerous current OR problems, including the newsvendor problem, which implies the loss of generality, and incompleteness of results. In its elementary formulation, the newsvendor problem aims at finding an optimal replenishment policy for a perishable product in the face of uncertain demand. One major modification of the newsvendor problem is the extension of decision variables to include a price as well as an order quantity. In further modifications of this problem, the objective function includes not only the expectation but also the variance of the profit. The present article supplements the mean-variance newsvendor problem with the non-negativity constraint. The investigation is designed to assess the hypothesis that the non-negativity assumption may substantially change the optimal solution. The present article proves that restricting the model parameters to those assuring the non-negativity of the demand violates the generality of considerations. The hypothesis is illustrated with the example in which the optimal solution corresponds to the pandemic-type demand shock. In the newsvendor model, instead of the risk-neutral or mean-variance approach, researchers frequently adopt the conditional value-at-risk (CVaR) ((Rockafellar & Uryasev, 2000; Jammernegg & Kischka, 2007) and references therein), which is a risk measure commonly used in finance. One of the early works with the CVaR adopted to the newsvendor problem is (Chen *et al.*, 2009). In that article, the CVaR risk measure constituted the decision criterion in a risk-averse newsvendor with the stochastic price-dependent demand. The aim of the study was to investigate optimal pricing and ordering decisions in such a setting for both the additive and multiplicative demand. The results were compared with those of the newsvendor with a risk-neutral attitude and a general utility function.

It should be noted that there exists a correspondence between CVaR and uncertainty sets in robust optimization, which can be used to generalize the concepts of risk measures. Using properly defined uncertainty sets in robust optimization models, one can construct coherent risk measures (Natarajan *et al.*, 2009). If the exact distribution of the uncertain demand in the newsvendor problem is unknown, it is necessary for the decision maker to find robust solutions. Scarf (1958) and Gallego and Moon (1993) introduced robust optimization as a practical extension of the classical newsvendor problem. The robust approach allows the optimal order quantity for the worst case scenario to be determined only if he mean and variance of demand are known. In Scarf (1958), a closed form formula for the optimal ordering rule was obtained. The formula maximizes the expected profit against the worst possible distribution of the demand. In (Gallego & Moon, 1993), a simpler proof than in (Scarf, 1958) was given and Scarf's ideas were extended. Since then, numerous articles related to Scarf's theory have been published, i.e. (Jiang et al., 2011; Zhu *et al.*, 2013; Xiao & Chen, 2017; Carrizosa et al., 2016). In (Jiang et al., 2011), the authors generalized the analysis of competition among newsvendors to a setting in which competitors possess asymmetric information about future demand realizations. In that case, traditional expectations based upon optimization criteria were not adequate. Additionally, they focused on the alternative criterion used in the robust optimization literature, namely the absolute regret minimization. In Zhu *et al.* (2013), the newsvendor problem in which the distribution of the random demand is specified by its mean and standard deviation or support was considered. A robust model which minimizes the regret was created where the regret was defined as the ratio of the expected cost based on limited information to that based on complete information. In Xiao and Chen (2017), the newsvendor problem with uncertain market demand with unknown probability distribution was investigated. The robust optimal decision was based on minimizing the expected legacy loss and CVaR concerning the legacy loss. Three robust mean-CVaR models were built when the distribution varies in a box uncertainty set. Moreover, the equivalent forms of the robust models were derived. In (Carrizosa et al., 2016), the newsvendor problem under a setting which combines temporal dependence and tractable robust optimization was explored. The demand was modeled as a time series which follows an autoregressive process. Additionally, a robust approach was introduced to maximize the worst case revenue.

Uncertainties due to unexpected events such as the coronavirus outbreak can be modeled in a different way than in this article. To this aim, the regime switching (Savku & AU Weber, 2018) was applied recently in (Ahmed & Sarkodie, 2021). In that work, the switching effect of the Covid-19 pandemic, and the impact of economic policy uncertainty on commodity prices were considered. The Markov regime switching dynamic model was employed to explore price regime dynamics of a few widely traded commodities. Two Markov switching regimes were fitted to allow parameters to respond to low and high volatilities. It was shown that most commodities were responsive to the historical price in terms of demand and supply in both volatility regimes. Moreover, there was a high probability that commodity prices will remain in low volatility regime rather than in high volatility regime owing to market uncertainties caused by Covid-19. Financial markets with high uncertainties were the subject of (Kara *et al.*, 2019) which highlights that the trade-off between maximizing the expected return and minimizing the risk is one of the main challenges in modeling and decision making. In their study, they considered uncertainty in parameters based on uncertainty in prices, and a risk-return analysis. They modelled a robust optimization problem based on data and found a robust optimal solution to the portfolio optimization problem by using robust CVaR under parallelepiped uncertainty.

In view of the foregoing, the following issues may become a hot topic and an interesting avenue for prospective research. First of all, one can investigate the CVaR criterion instead of the mean-variance one in the newsvendor problem with the non-negative demand. Secondly, it should be noted that the additive uncertainty investigated in this article is frequently encountered in the electricity industry where the presence of the negative demand is possible. It may occur in relation to the energy prosumer defined as a customer who both produces and consumes energy. When the prosumer sells excess electricity to the electric energy provider, in a sense, their demand may be regarded as the negative demand. Intensive work on the electric energy prosumer law is underway in several countries and by numerous institutions including the Polish Ministry of Entrepreneurship and Technology (Chojnowski; 2020).

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