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Control of perishable inventory system with uncertain perishability process using neural networks and robust multicriteria optimization

Ewelina CHOŁODOWICZ* and **Przemysław ORŁOWSKI****

Faculty of Electrical Engineering, Department of Automation and Robotics, West Pomeranian University of Technology in Szczecin, al. Piastów 17, 70-310 Szczecin, Poland

Abstract. The inventory systems are highly variable and uncertain due to market demand instability, increased environmental impact, and perishability processes. The reduction of waste and minimization of holding and shortage costs are the main topics studied within the inventory management area. The main difficulty is the variability of perishability and other processes that occurred in inventory systems and the solution for a trade-off between sufficient inventory level and waste of products. In this paper, the approach for resolving this trade-off is proposed. The presented approach assumes the application of a state-feedback neural network controller to generate the optimal quantity of orders considering an uncertain deterioration process and the FIFO issuing policy. The development of the control system is based on state-space close loop control along with neural networks. For modelling the perishability process Weibull distribution and FIFO policy are applied. For the optimization of the designed control system, the evolutionary NSGA-II algorithm is used. The robustness of the proposed approach is provided using the minimax decision rule. The worst-case scenario of an uncertain perishability process is considered. For assessing the proposed approach, simulation research is conducted for different variants of controller structure and model parameters. We perform extensive numerical simulations in which the assessment process of obtained solutions is conducted using hypervolume indicator and average absolute deviation between results obtained for the learning and testing set. The results indicate that the proposed approach can significantly improve the performance of the perishable inventory system and provides robustness for the uncertain changes in the perishability process.

Key words: multiple objective programming; optimal control; genetic algorithm; perishable inventories; inventory control.

1. INTRODUCTION

Nowadays, rapid changes in inventory systems force the development of new control approaches which will be able to overcome the variety of uncertainties appearing in such systems. One of the important factors contributing to the uncertainty in storage systems is perishability. Perishable products have highly varying deterioration rates which means that an uncontrolled perishability process can cause high waste [1]. Due to the high loss ratio of products (about 30% in many countries), it is highly important to effectively and efficiently control the inventory flow, in particular the level of product perishability [2]. The process of product deterioration usually does not occur uniformly, hence, unsteadiness in quality decrease should also be considered during the control of perishable inventories [3]. New technologies provide the measurement and control of deterioration in perishable inventories, e.g. RFID tagging [4, 5], Internet of Things (IoT) [6]. Therefore, the latest technology advancements create the proper environment for the application of new control methods to perishable inventories. Novel approaches have to provide the adaptation to changes and robustness to uncertainties that nowadays often occur in supply chains [7].

*e-mail: ewelina.cholodowicz@zut.edu.pl
**e-mail: przemyslaw.orlowski@zut.edu.pl

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In inventory, there are a few types of uncertainties, the four most popular in the literature are: (a) uncertain demand, (b) uncertain lead-time, (c) uncertain supply (d) uncertain perishability. The most researched uncertainties are (a), (b), and (c). For example, demand and supply uncertainty is assumed in [8] and the methods of bi-objective integer programming are proposed to manage blood inventory – the supply uncertainty of blood is modelled from regional banks to hospitals. The main goal of blood banks is to maintain sufficient stock while minimizing wastage due to the expiration of blood. The next work which also considers demand and supply uncertainty is [9] where a mixed linear programming model (MILP) is used with Lagrangian relaxation to reduce memory usage and time. Demand uncertainty is also taken into consideration in [10] where the order quantity is calculated by popular order-up-to policy and logistic stability is examined. In [11] new variants of periodic review policy and continuous review policy are proposed to reduce the holding costs and shortages in inventory systems with demand uncertainty. Approach including lead-time uncertainty is presented in [12] where an optimal ordering decision model is developed using differential equations. The lead time tends to change due to capacity constraints, defects in products, delays in material supply, and changes in production processes [13]. For both demand and lead times uncertainty, the research is available in [14]. Dealing with these two uncertainties and NP-hard study is possible thanks to the hybrid solution approach based on Simulated Annealing and direct search method. On the contrary, there is a lack of research on the control of perishable inventory with an uncertain lifetime of products (d). The uncertain character of perishability is considered in [15] where a robust optimization model is designed for controlling uncertain parameters. The second work is the recent research [16] where a multi-objective mathematical programming model is developed to optimize the cost, energy consumption, and traffic congestion associated with such supply chain operations. What is more, in [16] uncertain lifetime of products is explicitly modelled as a Weibull random variable, and the perishability process depends on vehicle refrigerator utilization. According to recommendations in [15], it is more efficient to use multi-objective meta-heuristic algorithms such as the nondominated sorting genetic algorithm II (NSGA-II) for problem-solving due to an exponential increase in computation time.

From this viewpoint, to cope with uncertain perishability processes this study is devoted to the problem of developing a new approach for optimizing perishable inventory systems, especially with uncertain perishability. In a nutshell, this paper aims to develop a novel approach for control and optimization using neural networks, state-space models, and evolutionary algorithms. Overall, to identify our contributions, the literature review reveals that the perishable inventory control with perishability uncertainty has not been widely studied. Most previous studies on the control of inventory systems without perishability and uncertainty in the perishability process. The main contributions of this paper are:

1. Development of a novel solution for perishable inventory system with uncertain perishability and first-in-first-out (FIFO) issuing policy that combines neural networks, bi-criterial optimization, and robust optimization using the minimax method.
2. The numerical study shows that the proposed robust system can find about 18% better solutions than the non-robust approach in terms of a hypervolume indicator.

We conduct a series of simulations to investigate the performance of our approach. The achieved results show that the proposed approach allows the perishable inventory system to obtain good robustness on perishability uncertainty with a view to customer satisfaction, holding cost, and wastage reduction. The remainder of this paper is organized as follows. Section 2 reviews related literature. Section 3 presents the inventory model with perishable products. A neural controller for perishable inventory is introduced in Section 4. Section 5 is devoted to the learning process of a neural network using multicriteria optimization. Section 6 reports and analyses the computational results obtained through simulations in Matlab. Section 7 concludes the paper and provides perspectives for future study.

2. RELATED WORK

Our paper is mainly related to two streams of research in the literature: robustness of perishable inventory control methods and their optimization, artificial intelligence methods used for perishable inventory control problems (especially neural networks).

There is a rich body of literature on the control of perishable inventory systems. The types of products in perishable inventories are mainly food (mostly dairy, fruits, vegetables) and blood. Major challenges in the blood supply chains are connected to the shortage and wastage of blood products [8]. The main challenge of supply chains is keeping enough stock level to provide full product availability while minimizing the losses due to outdating [17, 18]. There are also different approaches applied in perishability simulation. For example, the inventory models presented in [19–21] contain the constant perishability rate, which means that for example in each review period 5% of unsold stock decays and 95% remains. On the other hand, in [22] it is assumed that the freshness of the product is a linearly decreasing function of the age of the perishable goods. Another way of perishability modelling is presented in [1] where the deterioration rate is affected not only by the storage environment and the preservation effort but also by different perishability characteristics of different agri-fresh products (for example, the deterioration rate for bananas is different from the deterioration rate for apples). The variable character of perishability is caused by handling and transportation equipment, product temperature and air-conditioning, etc. In order to resemble this variability more, the Weibull distribution is also used to describe a perishability rate [16, 23, 24]. But still, there is a lack of research in which the uncertain character of process perishability is included in simulation research. To the best of our knowledge, none of these studies has discussed the effects of the uncertainty of perishability rate on the performance of the inventory system.

2.1. Robustness of perishable inventory control methods and their optimization

Increasing uncertainty of logistic and production processes initiated the rapid development of approaches to ensure system robustness. Authors from [25] develop a robust proportional-integral-derivative (PID) tuning model based on simulation-optimization and computational intelligence methods which provide insensitivity to variability (e.g. demand variability). They combine surrogate techniques and evolutionary algorithms to decrease the level of computational complexity in the tuning of PID controller. In turn, a new optimal model-based sliding mode controller dedicated to the perishable inventory system is presented in [26] which provides a fast reaction to the unknown disturbance, e.g. customer's demand, ensures limited and smooth orders, and reduces the holding and operating costs.

In [27] new robust approach is proposed which is based on classical order-up-to policy including various demand uncertainty using robust dynamic programming approaches. What is more, the solution approach provides optimality of the results and stable changes in costs in case of demand variability. The problem of uncertainty in the decay of raw material is addressed in [15], where methods of robust optimization are applied. Moreover, multi-objective algorithms are recommended for more efficient optimization performance (shorter time of computations), which is utilized in this research.

Optimization of the inventory systems is presented in e.g. [28–31], which are used only for control parameters tuning, regardless of possible uncertainties inclusion in the optimization process, and robust optimization. The robust approach facilitates minimizing the possible loss for a worst-case scenario approach that provides the feasibility of the solutions for all possible values of the uncertain parameters included in the uncertainty set [32, 33]. The research in [34]] consists of bi-criterial optimization using two criteria: customer service level (SL) and overall holding cost (HC) and evolutionary algorithm: NSGA-II to improve the distribution of the product in a specific type of logistic systems network (mesh-type).

2.2. Artificial intelligence methods used for the perishable inventory control problem

In the recent decade, there is a significant increase in the popularity of neural network techniques applied to inventory control [35]. The most popular application of neural networks in the inventory control system is demand prediction as in [36,37]. The most popular model of neural network used for inventory control is the backpropagation neural network. In [38] an online neural network controller that optimized a three-stage supply chain is developed which means that the weight tuning process is a continuous optimization problem in which a backpropagation (BP) algorithm is applied. However, the standard BP algorithm has inherent disadvantages such as slow convergence, the problem of converging to a local minimum, a complication of the system, and random network structure selection. Trying to overcome these difficulties, researchers develop different improved BP neural network models [35]. In [35] improved BP algorithms have an advantage both in convergence and prediction accuracy in comparison to the existing approaches. To improve neural network control, the application of evolutionary algorithms becomes more popular. One of the solutions existing in the literature is genetic algorithm-based learning. In [39] is shown that artificial neural networks combined with genetics for a variety of complex functions can achieve superior optimization solutions when compared with BP. Comparing the BP and genetic algorithm (GA), it can be observed that BP moves from one point to another whereas the GA searches the weight space from one set of weights to another set in many directions simultaneously which increases the chance of reaching the global minimum. A more effective process of weights tuning is achieved in [40] where a method that combines BP and GA for the learning process is presented. The combination of neural network and fuzzy logic is also a presence in the literature on inventory control [41] where a two-layered feed-forward with a backpropagation learning algorithm is used. In the research [40] the genetic algorithm is used to train the weights of the neural network which significantly improves the performance of the system. However, there is still a gap in the application of these methods to uncertain inventory control problems with perishable products. There are still few studies on control systems and optimization approaches to deal with uncertain perishable inventory systems.

2.3. Contribution of this study

The review of the pertinent literature implies that there is a lot of research on designing control policies with the application to inventory systems. Most of the work focus on investigating obtaining an optimal performance with a view to possible demand deviations. In addition, a few pieces of research devote study to developing new approaches for handling the uncertainty of perishability processes. Through the review of these papers, it was found that there is no comprehensive solution for the determination of the optimal quantity of orders in perishable inventory systems affected by uncertain perishability rates. Therefore, designing the approach to resolving this matter is a contribution to the current state of the art. The main contribution of this study is the application of the neural network controller and robust multi-objective optimization to the problem of uncertain perishable inventory system control. The main task of the designed controller is to calculate order quantity such that the customer's demand is satisfied without holding the excess stock.

According to the literature, a lot of new approaches, and methods are designed for inventory systems without perishability and there is a research gap in the research of new methods which have application in perishable inventory systems. As it can be noted, there is a lack of work that considers process perishability. Hence, what differentiates this work from the previous studies is the development of an approach for optimal control of perishable inventory systems with uncertain perishability processes, which is based on Weibull distribution and neural network controllers. In contrast to earlier results devoted exclusively to the inventory control systems, in this paper control approach for perishable inventory with uncertainty is developed. In order to verify our approach usability, simulation research with a wide range of initial conditions is conducted. To the best of our knowledge, works [15, 16] as the only two of the few which have included uncertainty of perishability in the perishable inventory model. According to the conclusion in [15] in this work it is decided to use NSGA-II for optimization in order to shorten the computational time. We also assume a random lifetime of perishable products, which is still less studied than fixed-lifetime perishability and non-perishable products. Our approach provides robustness for the uncertain character of the perishability process and the generated order signal is finite and stable.

3. AN INVENTORY MODEL WITH PERISHABLE PRODUCTS

3.1. Preliminaries

Let us consider the nonlinear, discrete-time perishable inventory with random lifetime products proposed in [24]. The considered class of inventory system assumes that stored products have a limited shelf-life. The main purpose of the inventory system is to satisfy customer demand and optimize the stock levels and losses due to perishability. The model fundamentals are based on the following assumptions:

1. The inventory system considers a single item only.
2. A review period is constant and equals one day.
3. Lead time is deterministic and positive and equals s days ($s > 0$).
4. Shortages are allowed but are not backlogged. Excess demand is lost.

5. There is only one stocking point in each period k.
6. Demand is a time-varying function.
7. The maximum shelf-life l is fixed and known a priori. Lost units are not replaced.
8. Items deteriorate according to a variable rate. The Weibull distribution is used to represent the distribution of the time to deterioration. It is assumed that after a period of l days distribution function is almost equal to 1. It means that the error resulting from the assumption of a finite time horizon is negligible.
9. The part of a new batch of products entering the inventory could spoil during the transport.
10. The products are sold according to FIFO policy.

3.2. Notation

The variables used in the model are presented in Table 1.

Table 1
Applied notation

Symbol	Definition
N	The length of the simulation horizon
$k \in \{1,2,\ldots,N\}$	Discrete-time
l	The maximal lifetime of the item
$i \in \{1,2,\ldots,l\}$	The index of state variables
s	Deterministic lead time
$d_{\max}$	The maximum demand in one period k
w_i	Perishability coefficients
$\mathbf{x}(k)$	The vector of state variables
$y(k)$	Inventory level (on-hand stock)
$u(k)$	Order quantity
$F(p,\gamma,\lambda,\beta)$	The cumulative distribution function of the Weibull distribution
λ_n	The nominal value of scale parameter λ from the Weibull distribution
λ_Δ	The perturbated value of scale parameter λ from the Weibull distribution
Δ	Perturbation
$\mathbf{z}(k)$	The aggregated amount of perished product
$z_i(k)$	The sum of perished product
$d(k)$	Aggregated demand
$d_i(k)$	Demand for a product of age i
$h(k)$	The aggregated amount of sold product
$h_i(k)$	The sold product of age i
n	Number of neurons in the hidden layer
$\mathbf{v}$	The vector of network weights
a_j	The activation function in the first layer
e	The activation function in the second layer
c_j	The transformation function in the second layer

3.3. The applied model – in brief

The model reflects the real processes in the inventory system with perishable products. The main goal of every inventory system is to satisfy customer demand. In order to maintain high customer service and minimize the on-hand stock, the replenishment orders must be just in time delivered. It is important to note that the orders arrive in the inventory after lead-time denoted as s. It means, that applied control input makes an impact on the system after the period s. It makes these systems much more different from the classical systems. The dynamics of the system are influenced by the on-hand stock and work-in-progress orders. However, the demand does not affect the dynamics and is modelled as an unknown a priori, bounded function of discrete time $0 \leq h(k) \leq d(k) \leq d_{\max}$. There is full demand satisfaction when the number of sold products $h(k) \in R_{\geq 0}$ is equal to the current demand $d(k) \in R_{\geq 0}$, $h(k) = d(k)$ The maximum value of imposed demand for products per k period is constrained by $d_{\max} \in R_{\geq 0}$. The general description of the assumed inventory system is that: (a) the orders are generated in regular intervals on the basis of the on-hand stock quantity $y(k)$, work-in-progress deliveries $x_i(k)$, where $i = \{1,2,\ldots,s-1\}$ and expected demand $d(k)$, (b) the products are sold according to FIFO issuing policy, (c) products ages according to Weibull distribution. Total demand consists of a sum of demand for products characterized by age i: $d(k) = \sum_{i=1}^{l} d_i(k)$. In this model, i represents the age of products, e.g. $i = 1$ means that $d_1(k) \in R_{\geq 0}$ is the demand for the freshest products available in the inventory. The total number of the sold product is given by $h(k) = \sum_{i=1}^{l} h_i(k)$, where $h_i(k) \in R_{\geq 0}$ – sold products of age i. As inventory systems become more complex, representing them with differential equations or transfer functions becomes highly advanced. Considering that, for efficient implementation in Matlab and on computing server, the model is formulated using a state-space approach. State-space representation of this system is given by l equations

$$\begin{cases} x_1(k+1) = (1-w_1)\,u(k), \\ x_2(k+1) = (1-w_2)\,(x_1(k) - h_1(k)), \\ \quad\vdots \\ x_l(k+1) = (1-w_l)\,(x_{l-1}(k) - h_{l-1}(k)). \end{cases} \tag{1}$$

State variable $x_i(k) \in R_{\geq 0}$ keeps the information about products quantity of age i. Items start to deteriorate during transport to the inventory. Order quantity $u(k)$ is a positive and real number. Perishability coefficients $w_i \in \langle 0, 1\rangle$ of product of age i are provided by the Weibull distribution function

$$w_i = \begin{cases} F_1 & i = 1, \\ \dfrac{F_i - F_{i-1}}{1 - F_i} & i = 2,3,\ldots,l, \\ 1 & i = l+1. \end{cases} \tag{2}$$

Assuming that the inventory deterioration rate follows a Weibull distribution, its cumulative distribution function can be presented in the following form

$$F_p = F(p,\gamma,\lambda,\beta) = \begin{cases} 0 & p \in \langle 1,\gamma\rangle, \\ 1-e^{\left(\frac{p-\gamma}{\lambda}\right)^{\beta}} & p \in (\gamma, l\rangle, \end{cases} \tag{3}$$

where $\beta > 0$ is the shape parameter, $\lambda > 0$ is the scale parameter, $\gamma > 0$ is the location parameter defining the beginning of deterioration, p is the time of deterioration. On-hand stock is a sum of the products stored to the inventory with different age i which can be described as $y(k) = \sum_{i=s}^{l} x_i(k)$. The products are sold according to FIFO policy, e.g. the oldest products, that is, the quantity with the highest i, is consumed first. In order to preserve the inventory processes, the following inequality must be met $y(k) - h(k) \geq 0$. It means that the system cannot sell more units than available on-hand stock. Perishability process of products for each moment k and age i is given by $l+1$ equations

$$\begin{cases} z_1(k) = w_1 u(k), \\ z_2(k) = w_2 (x_1(k) - h_1(k)), \\ \vdots \\ z_{l+1}(k) = w_{l+1} (x_l(k) - h_l(k)). \end{cases} \tag{4}$$

Products perish according to Weibull distribution what means that perishability can occur for every group of products (according to age i). For a general overview of the quantity of products losses, the sum of perished quantities of ages i is given as $z(k) = \sum_{i=1}^{l+1} z_i(k)$.

4. NEURAL CONTROLLER FOR PERISHABLE INVENTORY

Artificial neural networks consist of the number of connected neuron cells with weights imitating the real processes which occur in brains. The main goal of this work is to develop a robust neural network controller for uncertain perishable inventory systems in order to optimize the performance of this system using multicriteria optimization. The developed neural network controller consists of three layers: input, hidden, and output layer. It is assumed that the developed network has one hidden layer which has n neurons. The hidden layer has the saturating linear transfer function (satlin) whereas on the output layer is a positive linear transfer function (poslin). The applied structure of the neural network is depicted in Fig. 1.

The input of the neural network controller is the state vector $\mathbf{x}(k) \in R_{\geq 0}$, which is the number of products on every shelf – the shelf represents the age of the product. The output of the neural network is the control signal $u(k) \in R_{\geq 0}$, which is the order quantity generated in order to satisfy the demand $d(k) \in R_{\geq 0}$. The applied structure is a feed-forward network, in which the activation functions a_j, e, and transformations c_j and u occur. Based on the current stock age and work-in-progress deliveries, the controller can generate the optimal order quantity for each day k. The weights are the elements of vector $\mathbf{v}$. Neural network learning is conducted for constant demand value $d(k)$, which is 1 unit per day. However, for simulation purposes, the demand scaling is proposed, which provide the proper controller behaviour for different value of demand than 1.

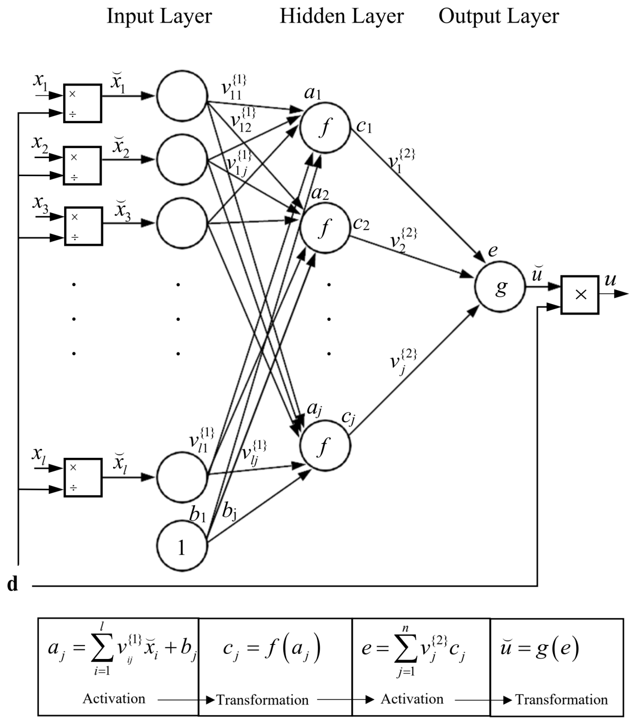

Fig. 1. The applied structure of the neural network controller

5. LEARNING USING MULTICRITERIA OPTIMIZATION

For the tuning of the neural network weights, multicriteria optimization is applied. First, the proper optimization criteria have been formulated. The first criterion is describing the number of lost sales due to stock shortages

$$J_h = \sum_{k=s+1}^{N} (d(k) - h(k)). \tag{5}$$

As a second criterion for optimization, the surplus of stock over demand is considered

$$J_y = \sum_{k=s+1}^{N} m(k), \tag{6}$$

where

$$m(k) = \begin{cases} y(k) - d(k) & \text{for } y(k) \geq d(k) \wedge \widehat{y}(k) \leq d(k), \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

The inequalities in the above relationship (7) eliminate the penalty for the stock which results only from the initial conditions $\mathbf{x}_0$. In other words, the penalty begins to be counted when the quantity of products from the initial vector is consumed.

To be more precise, if a free response (response for initial conditions without any orders) $\widehat{y}(k)$ of the system is equal to or smaller than the current demand and there is a surplus in stock, the cost criterion is nonzero. Otherwise, the shortages caused by initial conditions do not increase the criterion (6). It is important to note, that the free response of a system is when the input is zero – when the controller does not generate non-zero results.

Formulated criteria can be written as the following vector:

$$\mathbf{J} = \left[J_h, J_y\right]. \tag{8}$$

For a given nonlinear model of the uncertain, perishable inventory system (1)–(4) and the formulated cost vector (8), the optimization task can be defined as follows:

$$\begin{aligned} &\min_{\mathbf{v}} \max_{\Delta} J\left(\mathbf{v}, \mathbf{x}_0, \Delta\right) \\ &\text{s.t.} \quad -\delta \leq \Delta \leq \delta, \end{aligned} \tag{9}$$

where $\mathbf{v}$ is the vector of network weights.

The optimization process is depicted in Fig. 2.

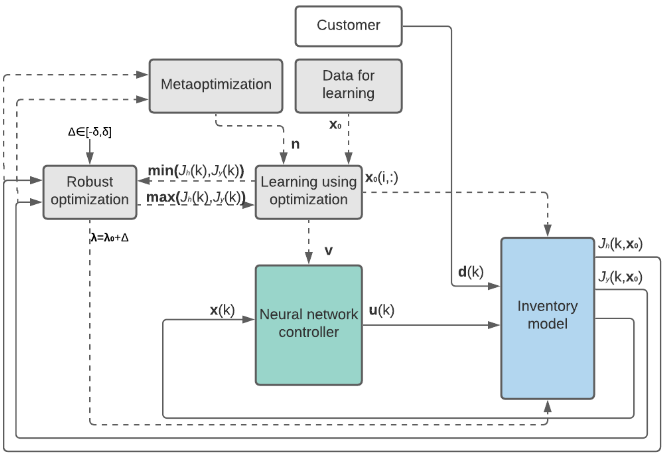

Fig. 2. The structure of conducted optimization process

The process of system optimization starts with the meta optimization task. Meta-optimization is used to choose an optimal number of neurons in the hidden layer. It is assumed that for meta optimization the 5 different numbers of neurons are considered $n = \{1, 2, 3, 4, 5\}$. Next, after choosing the number of neurons n, the learning process of the neural network controller begins. For the learning process, the learning set of the initial state vectors for the inventory model is generated using a set of random numbers. The learning set consists of different inventory states. Taking into consideration the whole range of assumed initial conditions, the sum of the inventory costs (8) is calculated. In this way, the performance of the controller is evaluated not only based on one single case but on a variety of cases. The main goal of the optimization is to minimize the shortage and holding costs. Due to uncertainty in real processes, the next step is highly necessary. In order to make the system more robust for the uncertainty, the robust optimization process is included. In this study, we apply the minimax decision rule which the main goal is to minimize the possible loss for a worst-case scenario. In this research, a worst-case scenario is the maximum value of criteria (8) obtained for a system with uncertainty. For robust optimization, the uncertainty of process perishability is considered, more specifically the scale parameter λ of Weibull distribution is perturbated. The value of λ determinates the speed of perishability, for example, decreasing value of lambda means that products deteriorate faster. Based on the minimax decision rule, the weights of the designed controller are optimized NSGA-II. The optimization is conducted with the use of parallel calculation mode in Matlab. The stopping criterion is the maximal number of generations which equals 4000. The population size is 2000 individuals.

6. SIMULATION STUDY

The simulation research consists of three parts. The first one compares the optimization performance of controllers for the selected number of neurons in the hidden layer and the uncertain character of the perishability process. In the second stage, the testing of learning is conducted – the solution front is generated for different initial conditions of the inventory model. The learning set consists of 180 different inventory states. The third research is focused on the comparison between non-robust and proposed robust systems.

The system parameters are set in the following way: review period 1 day, delivery delay 3 days, perishability horizon 7 days, simulation horizon 8 days, adopted issuing policy is FIFO. The selected type of perishability is random lifetime perishability and is modelled by three-parameter Weibull distribution. For simplification purposes, after l days the whole batch of products perish, but not all at once. In more detail, during the storing period, the product quality worsens on daily basis according to Weibull distribution. It is assumed that the process of goods perishability is affected by unknown perturbation, bounded by Δ, such that $|\Delta| \leq \delta$. The scale parameter λ_Δ of the Weibull distribution is uncertain which is a sum of a nominal value of the scale parameter $\lambda_n = 5$ and perturbation Δ. It is assumed that the demand scenario is constant and there is uncertainty about the perishability process in the inventory. The initial conditions of the state vector $\mathbf{x}_0$ are generated using random numbers in the range (0, 2) containing 180 different inventory states in total.

6.1. Result of the learning process for neural network controllers

In this subsection, the results of the learning process are presented. The learning process is conducted with the use of the selected structure of a neural network, evolutionary algorithm, and bicriteria optimization. The used structure of a neural network is presented in Fig. 1 (in the previous section). For learning the optimization criteria are defined (5)–(6) and NSGA-II is used.

In Fig. 3, there is obtained Pareto front for the different number of neurons in the hidden layer $n \in \{1, 2, 3\}$ and different values of perturbation $\Delta \in \{0, 0.2, 0.5, 1\}$.

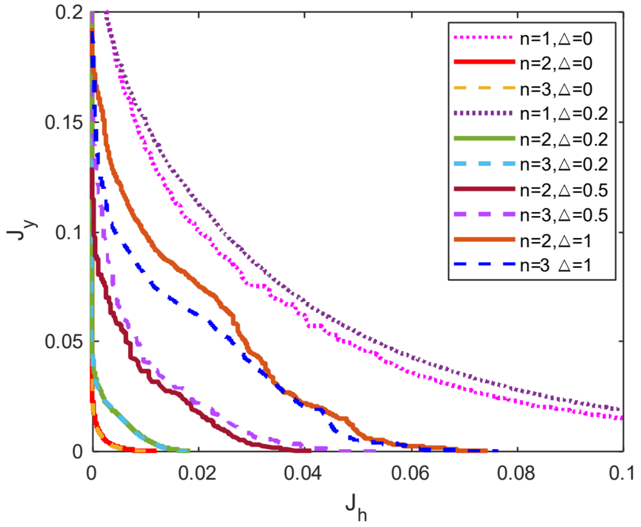

Fig. 3. Pareto fronts of results obtained for NN controllers and uncertain inventory model for selected values of perturbation – learning stage

It can be seen that the controller with 1 neuron in the hidden layer is dominated by solutions for $n \in \{2,3\}$ all considered perturbations Δ. Secondly, the results for $n \in \{2,3\}$ are similar for no perturbation and small perturbation range $\Delta \in \{0, 0.2\}$. The advantage of structure with three neurons is visible for the highest perturbations values, in particular, the difference is the most significant for $\Delta = 1$. For detailed analysis, in Table 1 the value of the hypervolume indicator (HV) for the obtained Pareto front is shown – the reference point for HV calculation is $(0.1, 0.2)$.

Table 2 shows that without perturbation there is no difference between $n = 2$ and $n = 3$, which is consistent with the Pareto front analysis. Moreover, it can be noted, that the best value of HV is achieved for $n = 3$ for the highest value of perturbation $\Delta = 1$. It is important to note that the higher the HV, the closer the solution to the optimal point.

Table 2

HV values for the selected number of neurons and perturbation for a reference point (0.1, 0.2) – learning stage

Number of neurons	$\Delta = 0$	$\Delta = 0.2$	$\Delta = 0.5$	$\Delta = 1$
$n = 1$	0.679	0.645	–	–
$n = 2$	0.998	0.991	0.951	0.834
$n = 3$	0.998	0.991	0.935	0.862

Furthermore, we compare the HV calculated for the Pareto fronts achieved by NN based controller and one of the classical ordering policies known as the order-up-to policy. Based on the papers [42, 43], it is assumed that the OUT controller can be described by the following equation

$$u(k) = \begin{cases} (y_{\mathrm{OUT}} - y(k) - \mathrm{WIP}(k))\, d(k), & \\ \qquad\qquad \text{for } y_{\mathrm{OUT}} > y(k) + \mathrm{WIP}(k), \\ 0 \qquad\qquad \text{otherwise}, \end{cases} \tag{10}$$

where y_{OUT} is the order-up-to level, and $\mathrm{WIP}(k)$ represents the placed but not yet completed orders due to the occuring delays.

For comparison of purpose parameters, the OUT controller has been computed using multiobjective optimization as for the NN-based controller. The assumed reference point for HV calculation is $(0.1, 3)$. The obtained results for the zero perturbation case ($\Delta = 0$) show that HV is about 42% smaller than for the NN-based controller with two neurons. It means that the NN-based controller significantly improves quality indicators both in terms of shortages and holding costs in comparison to the OUT controller.

6.2. Testing of optimized control system structures

In the next stage, the testing for the different seeds of random initial conditions that are used for learning is performed. As quality indicators, HV and average absolute deviation (AAD) between solutions obtained for the learning and testing stage, is used. Firstly, the HV values for one reference point $(0.1, 0.2)$ for all considered controllers are presented in Table 3.

Table 3

HV values for the selected number of neurons and perturbation for a reference point (0.1, 0.2) – testing stage

Number of neurons	$\Delta = 0$	$\Delta = 0.2$	$\Delta = 0.5$	$\Delta = 1$
$n = 1$	0.694	0.631	–	–
$n = 2$	0.997	0.989	0.931	0.780
$n = 3$	0.996	0.989	0.911	0.825

Judging by the HV values, the same relationship as in the learning stage is observed. For smaller perturbations $\Delta = \{0, 0.2\}$ the difference between $n = 2$ and $n = 3$ is merely visible. The only increasing value of perturbation $\Delta = \{0.5, 1\}$ resulted in gaining a significant advantage of $n = 3$. In order to see the individual differences between obtained results, Fig. 4 presents the summarized comparison of HV for the learning and testing stages.

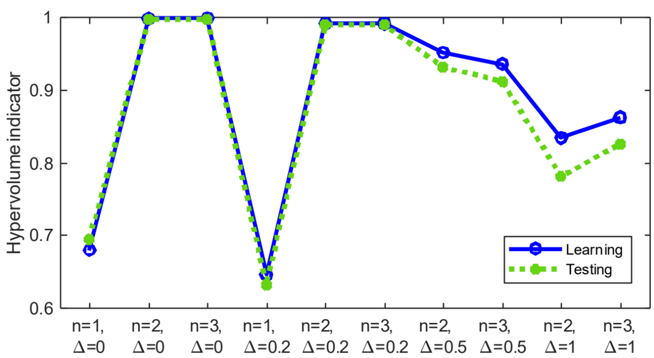

Fig. 4. The value of HV – comparison of the results for learning and testing stages

In Fig. 4, there are HV values for the different number of neurons in the hidden layer and different values of perturbation in the perishability process. First, the worst performance is for

$n = 1$, because it achieves the lowest value of HV for the selected two perturbation values $\Delta = 0$ and $\Delta = 0.2$. It is assumed that for $n = 1$ highest values of perturbation are not taken into consideration. This is because the controller with one neuron in the hidden layer is not able to find satisfactory results for this system. The best value of HV is achieved assuming $n = 2$ and $n = 3$ for the system with no perturbation ($\Delta = 0$), for $n = 2$, $n = 3$ for small perturbation ($\Delta = 0.2$). The significant advantage of $n = 3$ begins to be visible for high perturbation values ($\Delta = 1$). Secondly, the achievements of the learning process are analyzed using the testing process. Therefore, the average ratio of the HV for testing to the HV for learning is equal to 98.6 % for all selected neural network structures and perturbations. Furthermore, it can be seen that the higher the perturbation value Δ, the more significant differences between the HV for the learning and testing stages. The worst learning effectiveness among considered cases is for $n = 2$ and the perturbation $\Delta = 1 - 93.8\%$.

Secondly, let us look at the sets of solutions obtained in the testing stage. Figures 4(a)–(f) and 5(a)–(d) show the comparison plots for low perturbations $\Delta = \{0, 0.2\}$ and high perturbations $\Delta = \{0.5, 1\}$.

Judging by the results depicted in Fig. 6, the set of solutions obtained in the testing phase is similar to the one achieved in the learning stage, but some deviations occur. In order to quantify these deviations, two indicators are used – the hypervolume percentage difference (HVD) between HV values for the learning and testing stage and AAD. In Table 4 a simple summary of the hypervolume percentage differences between learning and testing sets is provided. The HVD and AAD values are calculated taking into account marked reference points in Figs. 5 and 6.

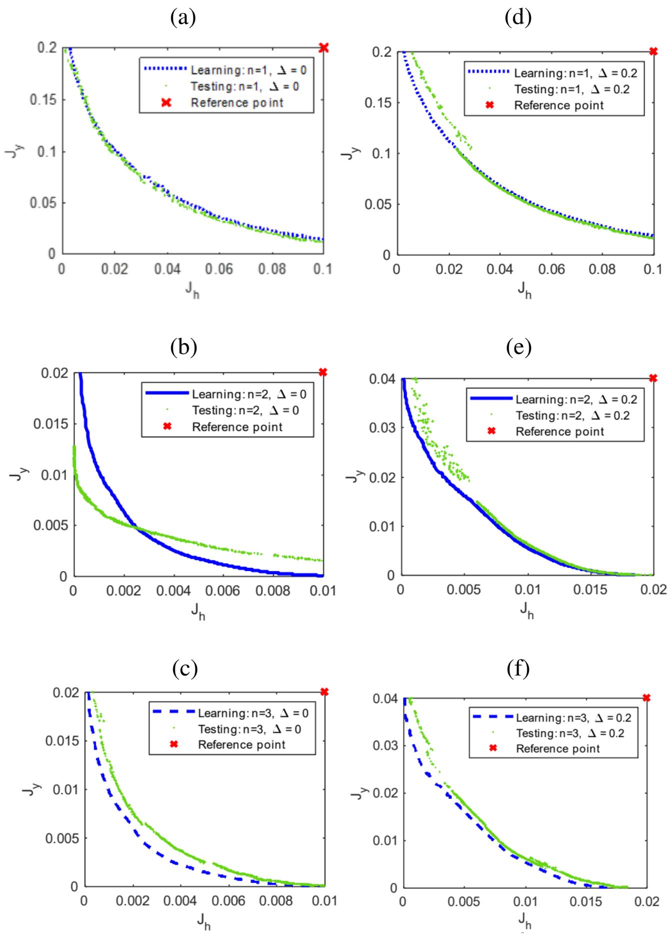

Fig. 5. Set of solutions obtained for NN controllers and uncertain inventory model for low values of perturbation $\{0.5, 1\}$

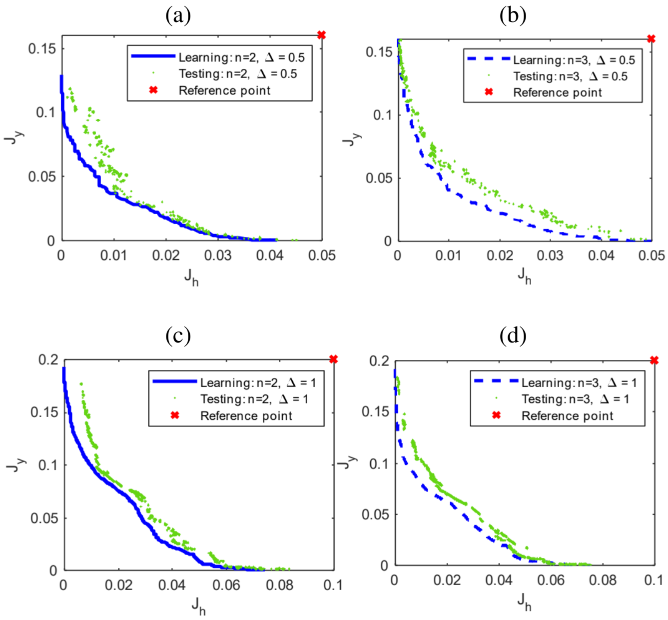

Fig. 6. Set of solutions obtained for NN controllers and uncertain inventory model for high values of perturbation $\{0.5, 1\}$

Table 4
HVD between solutions sets for learning and testing for the selected number of neurons and perturbation

Number of neurons	Uncertainty			
	$\Delta = 0$	$\Delta = 0.2$	$\Delta = 0.5$	$\Delta = 1$
$n = 2$	0.1%	4.6%	5.0%	6.3%
$n = 3$	7.1%	5.4%	7.1%	4.4%

Taking into consideration only the cases with uncertainty in the inventory system, the lowest value of the HVD – is for $n = 3$, $\Delta = 1$ – the advantage is equal to 1.9% in comparison to the result for $n = 2$. It means that solutions obtained for $n = 3$ provide the best set of solutions in the testing stage relative to the learning set. For presented perishable inventory system with uncertainty, three neurons in the controller structure cause that system can learn more effectively than the system with two neurons.

Moreover, the AAD values (Table 5) indicate that for the small perturbations there is no significant difference between obtained results for $n = 2$ and $n = 3$.

The following relationship is visible: the highest the uncertainty, the more significant the difference between AAD for $n = 2$ and $n = 3$. For the highest perturbation $\Delta = 1$, the $n = 3$ controller provides a smaller AAD than for $n = 2$. On the other

Table 5
The average absolute deviation between solutions obtained after learning and testing for the selected number of neurons and perturbation

Number of neurons	Uncertainty			
	$\Delta = 0$	$\Delta = 0.2$	$\Delta = 0.5$	$\Delta = 1$
$n = 2$	0.009	0.011	0.041	0.094
$n = 3$	0.007	0.013	0.053	0.068

hand, the controller with two neurons gives better results for $\Delta = \{0.2, 0.5\}$ which are lower than a three-neuron case by about 20%. However, the controller with 3 neurons achieves the best effectiveness of the learning process for $\Delta = 1$ because the testing set deviates about 30% less from the learning set than for $n = 2$.

6.3. Comparison of the robust and non-robust neural network controller

In order to verify the proposed robust controller, numerical simulations of the perishable inventory flow process have been performed. In Fig. 7, the specific points are selected to analyze the time responses of the designed controllers.

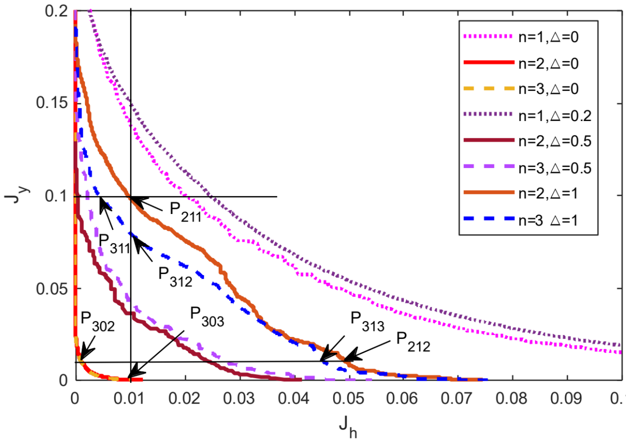

Fig. 7. Selected points from the Pareto fronts of results obtained for NN controllers and uncertain inventory model for selected values of perturbation at the learning stage

The points are selected using the following criteria: (a) 1% average of shortages, (b) 1% average of excess stock, (c) 10% average of excess stock – in the whole range of initial conditions $\mathbf{x}_0$. It is important to note that J_h and J_y are calculated in these cases on which the controller has an impact. In detail, the controller does not have an impact on the initial conditions $\mathbf{x}_0$ so the criteria are calculated apart from the impact of the initial conditions $\mathbf{x}_0$ which is reflected by equations (5)–(7). In Table 6, there are the symbols of marked points from Fig. 7 and the average of optimization criteria (5)–(6) obtained in the learning process for a certain value of perturbation Δ.

As we can see, the values of cost functions are the smallest for the controller $n = 3$. The difference in shortage costs J_h between robust systems increases about 20% for (c) than for (b). For the purpose of an analysis of the controller performance,

Table 6
Selected points and cost function values for assumed cases

Case	Point	J_1	J_2
(a) $J_h = 0.01$	P_{211}	0.010	0.100
	P_{312}	0.010	0.080
	P_{303}	0.010	0.000
(b) $J_h = 0.01$	P_{302}	0.000	0.010
	P_{313}	0.045	0.010
	P_{212}	0.050	0.010
(c) $J_y = 0.1$	P_{311}	0.004	0.100
	P_{211}	0.010	0.100

one case of initial condition $\mathbf{x}_0$ is selected based on the maximal difference between values of criterion J_h for a non-robust (NC) and robust system (RNC). In order to compare the performance of the selected case, the time plots for case (a) are presented (Figs. 8 and 9).

In Fig. 8, it can be seen that the non-robust system (learned without perturbation $\delta = 0$ – marked as NC) orders the smallest

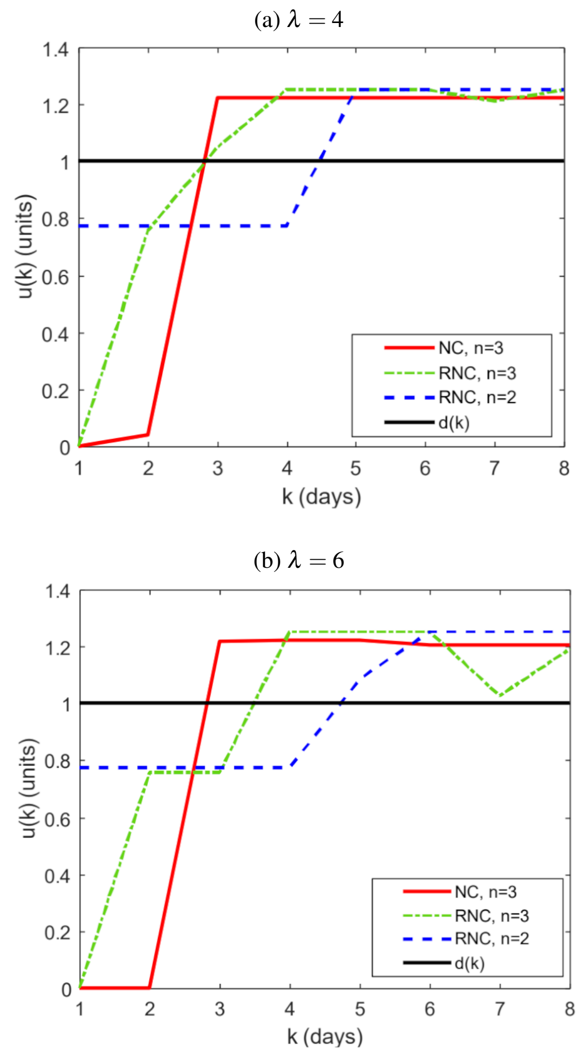

Fig. 8. Order quantity of three systems for chosen initial conditions $\mathbf{x}_0$, constant customer service level ($J_h = 0.01$), perishability parameter $\lambda = \{4, 6\}$, and two or three neurons in the neural network $n = \{2, 3\}$

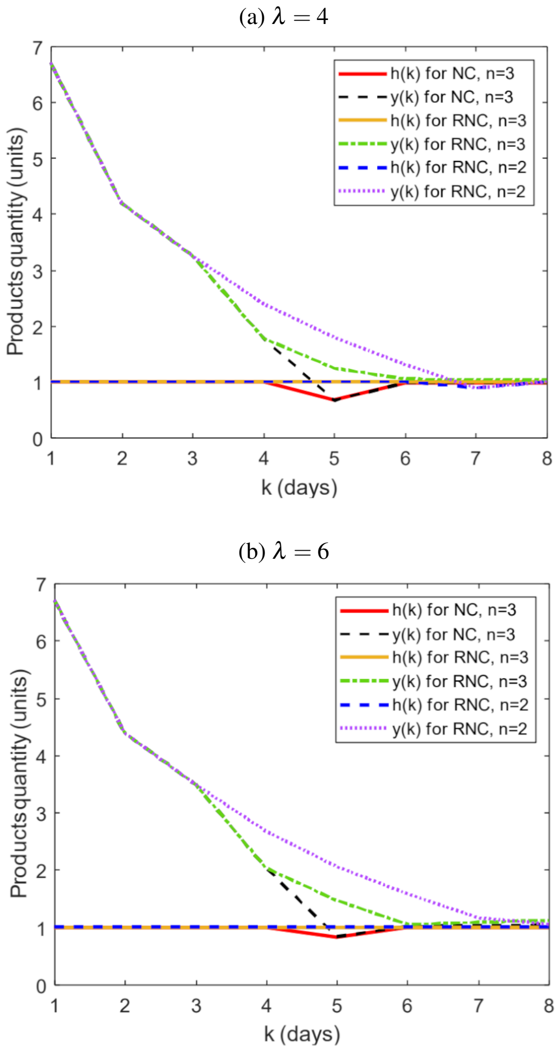

Fig. 9. Time plots ($h(k)$ – sold goods, $y(k)$ – inventory level) of three systems for chosen initial conditions $\mathbf{x}_0$ (maximal difference on J_h), constant customer service level ($J_h = 0.01$), perishability parameter $\lambda = \{4,6\}$, and two or three neurons in the neural network $n = \{2,3\}$

amounts of products in the two first-time moments $k = \{1,2\}$. From the third day, the non-robust system orders the same quantity on the whole simulation horizon for $\lambda = 4$, for $\lambda = 6$ it decreases slightly after $k = 5$. Moreover, the controller with two neurons ($n = 2$) starts with the highest order quantity and after $k = 5$ maintains high orders. Figure 8a shows that the robust network controller (learned including perturbation $\delta = 1$ – marked as RNC) for $n = 3$ provides total demand fulfilment (without shortages), whereas the NC, $n = 3$ system achieves about 5% loss in sales. The RNC, $n = 3$ thanks to maintaining about 4% higher stock can flexible response to the uncertain character of the uncertain process. The robust controller with two neurons keeps about 6% higher stock than the controller of $n = 3$ (Fig. 9). What is more, two considered closed-loop systems states converge in finite time (about 8 steps). These results suggest that the proposed robust optimization approach with the uncertainty set is effective in dealing with perishability distribution uncertainty. What is important, the robust system keeps the higher stock only for one period, and when demand is stable, the system keeps the smallest possible stock, which can be seen starting from $k = 6$.

Finally, in order to show the superiority of the robust controller for $n = 3$, the simulations for the test set of initial conditions $\mathbf{x}_0$ are performed. In detail, the solutions in the objective space are generated using obtained weights from the learning process from section A. Two types of controllers: NC and RNC are simulated for the same value of perturbation. It is assumed that the perturbation of the perishability process is equal to $\Delta = 1$, where the nominal value of the scale parameter is $\lambda_n = 5$. Instead of picking a single scenario of initial conditions like in section C, the test set of 180 initial conditions is taken into consideration. For the purpose of the testing phase, the new set of initial conditions $\mathbf{x}_0$ is generated with the same distribution as in the learning phase but with a different seed. The obtained solutions sets are depicted in Fig. 10.

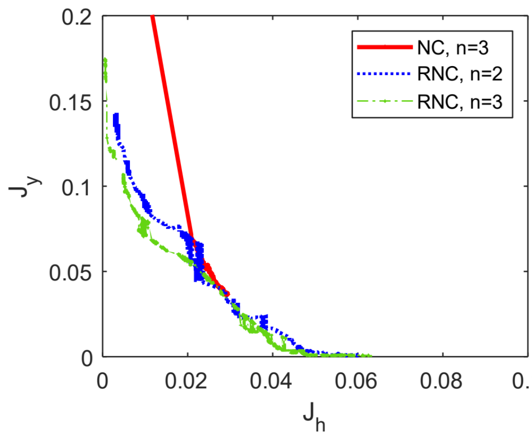

Fig. 10. Solution space for the neural controller and robust neural controllers for the test of the set initial conditions $\mathbf{x}_0$

As can be seen from Fig. 10, the best results are for robust neural controllers in the majority of the time horizon. The RNC, $n = 3$ provides about 18% higher quality of solution space in terms of HV than for NC, $n = 3$. It means that RNC, $n = 3$ can minimize the uncertainty of perishability more effectively, especially for the solutions of the smallest value of shortage cost, i.e. $J_h < 0.02$. Among robust controllers, the RNC, $n = 3$ achieves about 5% higher HV than RNC, $n = 2$ which shows that the controller with a three-neuron system is more robust than the two-neuron controller. Consequently, in the case of significant perturbation, i.e. $\Delta = 1$ RNC, $n = 3$ can crucially maintain less stock providing higher demand satisfaction at the same time. This also means that for robust systems there are smaller losses due to product perishability.

7. CONCLUSION

Improving the approaches of goods flow in the production-logistic systems is highly desirable in nowadays industry, which makes the research on inventory control approaches of utmost importance. Not only does the proposed solution reduce the costs but also provides robustness for the uncertain character of perishability. We show that a robust neural network controller, especially for $n = 3$, exhibits improved performance compared with the classical neural controller and robust neural controller with a lower number of neurons in the hidden layer. This work also shows that multicriteria optimization can be used for neural network optimization in the problem of perishable inventory control. What is important, this is evident that the controller with three neurons limits the stock and provides full demand satisfaction at the same time. The research shows that the cost performance obtained by applying the robust state-space controller to the model with uncertain parameter λ is superior to the non-robust controller. It is found that a robust system can achieve about 18% better solutions than the non-robust approach in terms of hypervolume indicators. Our proposed solution approach provides formal ground for real-life inventory optimization and can improve warehouse management systems to make ordering decisions under perishability uncertainty. Furthermore, the approach can be also adopted for inventory systems with multiple items.

Further research should focus on a perishable inventory model with two uncertainties such as lead-time, demand, and development of the proposed approach to perishable inventory system optimization.

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