**Management and Production Engineering Review**
Volume 13 • Number 2 • June 2022 • pp. 31–47
DOI: 10.24425/mper.2022.142053

# Heuristics for Dimensioning the Shelf Space on the Rack with Vertical and Horizontal Product Categorisation in the Distribution Centre with Zone Picking

Kateryna CZERNIACHOWSKA[1], Radosław WICHNIAREK[2], Krzysztof ŻYWICKI[2]

[1] *Wroclaw University of Economics and Business, Wroclaw, Poland*
[2] *Poznan University of Technology, Poznan, Poland*

Received:11 May 2021
Acepted: 26 May 2022

**Abstract**
Distribution centres are the important elements of modern supply chains. A distribution centre stores and ships products. In this paper, we investigate the model of the dimensioning of shelf space on the rack with vertical and horizontal product categorisation in a distribution centre, where the objective is to maximise the total product movement/profit from all shelves of the rack which is being managed by a packer who needs to complete orders selecting the products from the shelves and picking them to the container. We apply two newly developed heuristics to this problem and compare the results to the optimal solution found by the CPLEX solver. There are 8 steering parameters that allow for reducing the search space implemented in heuristics. Among them are parameters that decrease the number of products on the shelves, the category width range for assigning most space for the most profitable products within the category, two versions of steering parameters for the number of generated product allocations, the step parameters for the intensity of solution diversification, and the movement/profit below which the solutions are not generated. The computational results are presented and indicate that higher-quality solutions can be obtained using the new heuristics. In 10 from 15 tests, both heuristics can find optimal solutions without exploring the whole solution space. For the rest test sets, the solutions received by heuristics are not less than 92.58%.

**Keywords**
Distribution centre, Order picking, Decision making/process, Heuristics, Shelf space allocation.

## Introduction

A distribution centre is a spacious commercial building that stores items for merchants and wholesalers, importers and exporters ready to be redistributed to another site or sold directly to customers. In a distribution centre, an order picking strategy describes how pickers traverse the picking area to get products from storage locations (Parikh & Meller, 2008).

Better distribution centre picking systems are becoming more important as e-commerce grows. Order accuracy is highly dependent on distribution centre picking efficiency, which in turn has an impact on the company's business model and interaction with the customers (Bartholdi & Hackman, 1998).

Particularly for online merchants and e-commerce enterprises, distribution centres are an important aspect of the order fulfilment process. The standard shipping path is as follows: the manufacturer delivers the goods to the distribution centre, which then ships them to the client.

A customer order may be collected totally by one person, by a group of workers but only one at a time, or by a large number of workers all at once. Many factors influence the best strategy, but one of the most crucial is how rapidly orders must flow through the system (Bartholdi & Hackman, 1998).

Optimising the order picking operation, which includes any or all of the tasks listed below (Davarzani & Norrman, 2015; Bahrami et al., 2019)

- Picking method selection, which includes single order picking, batch picking, zone picking, and wave picking, is dealt with by the order picking policy.

***Corresponding author**: Krzysztof Żywicki – Poznan University of Technology, Faculty of Mechanical Engineering Piotrowo 3 Street, 61-138 Poznan, Poland, phone: (+48) 61 665-27-40, e-mail: krzysztof.zywicki@put.poznan.pl*

- Order batching refers to the procedure of gathering customer orders and picking them all up at the same time during a collection tour.
- Picker routing is concerned with the order in which the items are placed on the picklist in order to provide an effective path through the warehouse.
- Sorting/Order Accumulation: Operation of combining the selected products per client order or per shipping destination.

Because products come in and out at a rapid speed, they're commonly regarded to be demand-driven. Distribution centres are frequently built-in highly accessible regions, such as near major roads and highways, so that transport vehicles can drop off and pick up products more quickly. For example, if the distributors know all of the orders before they start choosing, they can develop efficient picking tactics ahead of time. If, on the other hand, orders arrive in real-time and must be picked in order to fulfil delivery deadlines, they have very little time to look for efficiency (Bartholdi & Hackman, 1998).

A retailer's competitiveness is heavily reliant on a well-functioning distribution centre (Ostermeier et al., 2020). The distributor should manage the layout of the distribution centre and storage to improve labour efficiency. High-volume items should be stored near the front of the distribution centre to avoid unnecessary transit time. Items that are regularly sold together should be stored close to one another's zones for the convenience of the pickers.

Businesses should treat their order picking solutions wisely since distribution centres that perform order picking activities accurately and quickly are usually the most effective. This is particularly true when it comes to storing supplies from distribution centres. When it comes to supply, stores demand a lot of flexibility. As a result, quick order processing is critical. At the distribution centre, order processing affects various subsystems: orders are picked in multiple picking zones, transported to intermediate storage, and delivered by dedicated tours. These steps in the procedure are extremely intertwined (Ostermeier et al., 2020).

In most of the works, researchers have been looking for methods for optimising order-picking routes for solutions to minimise the order-picking travel distance in distribution centres. This study considers the problem of dimensioning the shelf space on the rack with vertical and horizontal product categorisation, which is being utilised by a packer who needs to quickly find the product on the rack and add it to the order container in their picking zone.

Rack space distribution approaches are important in the warehouse and distribution centre industry. Understanding these methods provides ideas for optimising the distribution centre shelf space allocation and later packing methods in day-to-day operations.

The research is conducted according to the following steps.

- Precisely define the problem.
- Design a heuristic solution and implement it using computer software.
- Conduct the computational experiments in different problem data sets.
- Compare the results with the optimal solution found by a commercial solver.

The contributions of the research are the following:

- Finding similarities of the distribution of shelf space on the racks in a distribution centre with the more advanced shelf space allocation on planograms problem.
- Problem definition specifically to zone-picking in a distribution centre.
- Developing two heuristics which allow achieving an optimal or near-optimal solution.
- Proposing eight steering parameters which allow to reduce the search space of heuristics and significantly decrease the solution time.

In the next section, a problem definition and mathematical model of the problem are given. We then present the heuristics explanation and steering parameters which allow for reducing the search space. This is followed by computational results. We close with concluding remarks.

## Literature review

### Order-picking

The process of extracting products from storage in response to a specific client request is known as order picking (de Koster et al., 2007). Order picking, which is the most labour-intensive process in warehouses using manual systems, is projected to account for up to 55 per cent of overall warehouse operating expenses (Manzini, 2012). As a result, order choosing planning should be considered the most pressing issue in terms of increasing productivity (Chen et al., 2022).

Order-picking is one of the most important functions of a warehouse. It is a subprocess of the superior warehouse process that involves extracting (block-stacked or racked) items from inventory to fulfil a client's request (qualitatively and quantitatively) (Parikh & Meller, 2008; Emmett, 2005)

The most frequent picking method is single-order/piece/discrete picking, in which one warehouse picker retrieves goods (line by line in order) to complete a single order at a time. Among other picking methods are multi-order/batch picking, cluster picking, parallel zone/wave/consolidation picking, and sequential zone picking (Parikh & Meller, 2008; Emmett, 2005; Redmer, 2020).

Numerous warehouse pickers retrieve items in specific storage zones sequentially and merge them simultaneously to accomplish a single order. The time of merging obtained items is critical (Redmer, 2020).

Specific order picking tactics can be adopted (Emmett, 2005; Redmer, 2020; Garbacz & Łopuszyński, 2015; Kostrzewski & Kostrzewski, 2014):

- in warehouse storage or picking regions, or both
- in picking by item or by order,
- in person-to-goods or person-to-goods picking.

Redmer's (2020) study's main goal was to discover a link between the length of picking paths calculated using the S-shape method and variables like storage strategy, distribution centre location, order size, ABC-storage class size, and the probability of retrieving items from specific ABC-storage classes (the last three of which typify a demand pattern).

Order-picking may be structured like an assembly line in warehouses or distribution centres that move a lot of small items for a lot of clients, such as those serving retail stores. The warehouse is divided into zones that correspond to workstations, pickers are assigned to zones, and employees prepare each order one at a time, passing it from zone to zone (Bartholdi III & Hackman, 1998). Zone picking entails assigning each picker to a specific region of the storage space and ensuring that only the items in that area are picked (Parikh & Meller, 2008).

## Batching and zoning

Batching and zoning are two critical aspects that influence order picking efficiency and are discussed in (Chen et al., 2022). Order batching is the practice of combining orders together and releasing them all at the same time for picking in order to save travel time and distance. The front warehouse's picking area is divided into many zones housing various merchandise. Pickers are assigned to distinct zones and are only allowed to pick objects within that zone. Each zone has an order processing desk in the bottom right corner where information is processed and picking jobs are prepared. Each order consists of many things, and orders will be divided into batches (Chen et al., 2022).

Parikh and Meller (2008) studied whether batch or zone picking is more appropriate for an existing distribution centre, and a cost model was provided to help choose between the two picking procedures. Van Nieuwenhuyse and de Koster (2009) presented a paradigm for batching in specified time windows.

Picking is the most time-consuming and costly part of distribution warehouse operations. As a result, research into how to make distribution warehouse picking more efficient has become critical (Tanaka et al., 2019).

Workers in a zone picking method work together as a team (de Vries et al., 2016). Workers can be assigned to zones containing work in amounts according to their work speed to improve zone picking (Bartholdi III & Hackman, 1998).

Tanaka et al. (2019) presented an asynchronous parallel processing system for distribution warehouses with numerous zones and utilised real-world data to demonstrate the efficiency gains. This has been referred to as an order batching problem because it involves combining orders for multiple shipping locations. Each divided zone is assigned a zone picking method in this manner (Tanaka et al., 2019).

Parikh and Meller (2009) constructed analytical models for pick-column blockage in wide-aisle warehouses in a follow-up study, demonstrating that blocking grows monotonically as the number of stock-keeping units (SKUs) at the same pick column increases. Parikh and Meller (2010a) extended the recent paper to the case of narrow-aisle warehouses. The authors demonstrated that underestimating picker blockage can jeopardise the process' efficiency (measured in order picking time per order). When pick density grows, scientists found that blocking becomes more important. Picker blocking becomes more common when fast-moving SKUs are placed near the depot, according to Parikh and Meller (2010b).

Because each picker only completes a portion of an order, the order's throughput time is influenced by the performance of each individual picker. Furthermore, in a situation where buffers are restricted, a worker's maximal work pace in a later zone is serially reliant on the speed of workers in previous zones (Schultz et al., 1999). As a result, zone choosing is linked to a high level of task interdependency (de Vries et al., 2016).

Pickers will likely be more motivated at work if the incentive system is group-oriented to some extent, as large levels of task interdependency facilitate the motivating effects of a group incentive structure (Ho & Liu, 2005).

Parikh and Meller (2008) investigated the difficulty of deciding between batch picking and zone picking. They present a cost model to quantify the cost of each sort of picking technique for this task. Therefore they take into account the consequences of pick-rate, picker blockage, workload imbalance, and the sorting system required in their cost model. They demonstrate how system throughput, order sizes, item placement in orders, and wavelength affect the picking technique selection decision using an example case.

In a rectangular manual order picking warehouse, Franzke et al. (2017) looked into the impact of picker blockage on mean order throughput times. The study by Ho et al. (2007) looked at a distinct problem: transforming a normal warehouse into several forms of geometric zone-picking warehouses. They looked at alternative order-batching strategies and route-planning approaches in the trials (as they assumed order batching was used in the warehouse). The first part of their research was to identify the products in each zone, and the second stage was to identify the storage sites for all of the products in each zone (Ho et al., 2007).

The main advantage of the zone picking method is that it is most suitable for large warehouses that deal with a large number of SKUs that have distinctive features or picking standards. It significantly reduces travel time because inventory pickers stay in their assigned areas rather than moving throughout the warehouse, and cartons containing the items needed for each order are delivered to them. Working in a smaller zone allows pickers to become more acquainted with the SKUs in their zone and their pick locations lead to faster, more correct picking.

The issue with zone-picking is that it involves all of the labour that goes into balancing an assembly line: a work-content model and task partitioning. An experienced professional is mainly responsible for this (Bartholdi III & Hackman, 1998). In a cost comparison of batch and zone picking systems, Parikh and Meller (2008) found that workload imbalance is greater in zone picking systems, despite the fact that such systems eliminate picker blocking. Picker-to-parts warehouses are higher in zone picking systems, despite the fact that such systems reduce picker blocking (Franzke et al., 2017).

Some zone-picking disadvantages also exist. Working with a fixed scheduling period per shift enables warehouse managers to plan ahead, but it also means that orders queued for picking have a deadline. Orders received after the cutoff will be processed during the following shift. Furthermore, zone load balancing for proper labour management can be difficult. If orders queued for picking during a specific shift only contain SKUs from one or two zones, warehouse employees in other zones could use downtime to refill forward pick areas.

## The significance of heuristics in shelf space planning

The problem of efficiently arranging retail products on shelves in order to maximise profits, enhance stock control, and improve customer experience, among other things, is known as shelf space allocation. The majority of research on this topic in the literature has focused on large stores, such as supermarkets (Landa-Silva et al., 2009). There is a lack of research in the distribution centre for the convenience of picking up products, and there are no relevant algorithms.

Choosing an assortment and allocating it to shelves is one of the most important strategic decisions that grocery stores must make. Retailers must balance diversity (number of products) and shelf levels (number of items of a product) to match consumers' needs with shelf supply. Because shelf space is limited, having a larger assortment reduces the proper service levels and vice versa. As a result, shops must make decisions about which products to provide and how much space to assign for each product at the same time (Hübner & Kuhn, 2011).

The shelf space allocation on planograms in retail stores corresponds to the distribution of shelf space on the racks in a distribution centre. In supply chain management, shelf space allocation approaches help distribution centre managers to perform their functions.

There are a lot of companies which specialise in logistic processes, such as sorting and packaging, among others electronic and medical equipment, cosmetics or other products. Warehouse or distribution centre staff unload and load product deliveries of materials to the production line taking care of the level of their inventory. Each distribution centre creates sequenced instructions for the picker to be executed, and every distribution centre requires different instructions for different tasks to be completed.

Shelf space allocation is utilised in retail stores as a decision problem to achieve the best possible goal while working within operational restrictions. In general, commercial space management systems build operational procedures based on very simple intuitive guidelines that make it easy to make decisions about shelf space distribution in practice (Yang & Chen, 1999). In reality, the shelf space allocation approach is just a way of solving problems really quickly.

The effectiveness of the space allocation decisions is the focus of the strategies' concern for practicability and simplicity. The development of optimisation methodologies to tackle the shelf space allocation has reached a practical solution to the space management systems stage due to the technology advancements (Yang, 2001).

Commercial space management systems utilise a variety of heuristics for allocating shelf space. A heuristic is a method for finding near-optimal solutions at a low computing cost without guaranteeing feasibility or optimality (Reeves, 1993). Heuristics include greedy algorithms and hill-climbing methods. They are simple, but they have the disadvantage of being prone to become caught in a local optimum (Kumar & Kulkarni, 2019).

Shelf space allocation algorithms are often associated with modern technology. Some heuristics are used in the shelf space allocation problem, such as those described in (Landa-Silva et al., 2009; Yang, 2001; Hansen et al., 2010; Erol et al., 2015), a greedy heuristic described in (Urban, 1998), the genetic algorithm proposed in (Urban, 1998; Czerniachowska et al., 2021; Czerniachowska, 2022; Hwang et al., 2009), simulated annealing described in (Borin et al., 1994; Czerniachowska & Hernes, 2021), hybrid heuristics and metaheuristics as described in (Castelli & Vanneschi, 2014), metaheuristics with the local search are presented (Lim et al., 2004).

Erol el al. (2015) proposed a new heuristic for obtaining a good allocation of shelf space for various products in order to increase profitability under a variety of constraints, including limited shelf space and elasticity factors. To solve retail problems, some heuristic approaches such as the first-fit algorithm, tabu search algorithm, and genetic algorithm were developed.

Landa-Silva et al. (2009) presented a heuristic approach to automated shelf space allocation. This method was created in collaboration with a retailer who had a thorough understanding of the problem and the criteria for a computer solution. The suggested method incorporated several initialisation heuristics and local search steps to provide high-quality, practical layouts.

Gajjar and Adil (2010) developed local search heuristics for solving the retail shelf space allocation problem with a linear profit function, which provides initial arrangements and then iteratively improves the profit of the candidate solutions via adjustment moves. For this two-dimensional shelf space allocation problem, a simulated-annealing multiple-neighbourhood hyper-heuristic technique was developed (Gajjar & Adil, 2010).

In parallel-aisle picking systems, Hong et al. (2012) presented an order batching formulation and heuristic solution approach suitable for large-scale order picking. Their construction-based routing method guided the search procedure by narrowing order-to-batch assignments to find batches with potentially shorter routes (Hong et al., 2012).

In an automated warehouse with capacity limits and numerous objectives, Chang et al. (2007) developed a new mathematical model for the warehouse order picking optimisation problem. They utilised a genetic algorithm with an enhanced initialising population method.

Calzavara et al. (2019) suggested a heuristic storage assignment process to aid in the determination of which items should be stored on which pallet.

Al-Araidah et al. (2017) created a heuristic that groups things into clusters before assigning them to storage places. The goal is to ensure safe selection while avoiding putting the order picker into unnatural physical postures. The proposed heuristic generated order selection tours with the shortest cycle time (Al-Araidah et al., 2017).

Wu et al. (2017) developed a greedy heuristic-based solution approach in which they paid attention to the following topics in particular; (1) determining whether merging sequence control can be used to reduce idle and order fulfilment periods; (2) creating a mathematical model and developing a useful heuristic algorithm.

Önüt et al. (2008) examined a distribution-type warehouse and tried to come up with a multi-level warehouse layout that would reduce annual carrying costs. As a novel heuristic approach for determining the ideal layout, they devised a particle swarm optimisation algorithm. Nevertheless, both annual and daily operational costs are examined in this study.

Warehouse management solutions must be scalable and adaptable as the business evolves. It's a good idea to be able to incorporate a variety of order management and picking approaches, including zone picking, pick and pass, wave picking, batch and cluster picking, and hybrids of these.

While distributing shelf space of the rack between products, a distribution centre leader requires the development of effective heuristics, especially for larger problem instances. As for the limitations, only near-optimal solutions could be found by using heuristics.

In the literature, there are lots of heuristics proposed for routing determining, minimising travel distance between input and output locations and aisle spots, and scheduling for processing order batches. But there is a lack of heuristics to order picking. For real-life distribution centres, the number of items per order can be large, but the number of zones from which these items should be picked can be rather few. Nevertheless, a heuristic approach may be needed for solving large instances. Therefore in this research, we develop heuristics for dimensioning the shelf space on the rack in the distribution centre with zone picking.

## Problem definition

Parameters and indices used in a model:

$S$ total number of shelves;
$P$ total number of products;
$K$ total number of categories;
$T$ total number of tags.
$i$ shelf index, $i = 1, ..., S$;
$j$ product index, $j = 1, ..., P$;
$k$ category index, $k = 1, ..., K$;
$t$ tag index, $t = 1, ..., T$;
$r$ orientation index, $r \in \{0, 1\}$;

$$r = \begin{cases} 0, & \text{for front orientation,} \\ 1, & \text{for side orientation.} \end{cases}$$

Shelf parameters:

$s_i^l$ length of the shelf $i$;
$s_i^d$ depth of the shelf $i$;
$s_{ti}^g$ binary tag $t$ of the shelf $i$;

$$s_{ti}^g = \begin{cases} 1, & \text{if shelf } i \text{ is tagged,} \\ 0, & \text{otherwise.} \end{cases}$$

Product parameters:

$p_j^w$ width of the product $j$;
$p_j^d$ depth of the product $j$;
$p_j^u$ unit movement/profit of the product $j$;
$p_j^l$ cluster of the product $j$;
$p_{tj}^t$ tag $t$ of the product $j$;
$p_j^k$ category of the product $j$;
$p_j^s$ supply limit of the product $j$;
$p_{jr}^w$ width or depth of the product $j$ on orientation $r$;

$$p_{jr}^w = \begin{cases} p_{j0}^w, & \text{if } r = 0, \text{ width for front orientation,} \\ p_{j1}^w, & \text{if } r = 1, \text{ depth for side orientation,} \end{cases}$$

$p_j^{o_2}$ side orientation binary parameter of the product $j$;

$$p_j^{o_2} = \begin{cases} 1, & \text{if side orientation available for prod. } j, \\ 0, & \text{otherwise;} \end{cases}$$

$f_j^{\min}$ minimum number of SKUs of the product $j$;
$f_j^{\max}$ maximum number of SKUs of the product $j$;
$s_j^{\min}$ minimum number of shelves on which the product $j$ can be allocated;
$s_j^{\max}$ maximum number of shelves on which the product $j$ can be allocated.

Category parameters:

$c_k^m$ minimum category size as a percentage of the shelf length;
$c_k^t$ category size tolerance between shelves in the category as a percentage of the shelf length.

Tag parameters:

$b_t^n$ band name of the tag $t$, $b_t^n = \{H; H^+; V^+\}$;
$b_{tij}^t$ product to shelf compatibility tag.

$$b_{tij}^t = \begin{cases} 1, & \text{if } s_{ti}^t = p_{tj}^t \wedge b_t^n = \{H\}, \\ 0, & \text{otherwise,} \end{cases}$$

$t = 1, ..., T$ – for the horizontal shelf level for big products;

$$b_{tij}^t = \begin{cases} \min(p_{tj}^t; 1) \wedge b_t^n = \{V^+\}, \\ 1, \ \text{if } p_{tj}^t = 1 \wedge s_{ti}^t = p_{tj}^t \wedge b_t^n = \{H^+\}, \\ 0, \ \text{if } p_{tj}^t = 1 \wedge s_{ti}^t \neq p_{tj}^t \wedge b_t^n = \{H^+\}, \\ 1, \ \text{if } p_{tj}^t = 0 \wedge b_t^n = \{H^+\}, \end{cases}$$

$t = 1, ..., T$ – for the horizontal and vertical shelf level for small products.

Decision variables:

$$x_{ijr} = \begin{cases} 1, & \text{if product } j \text{ is placed on shelf } i \\ & \text{on orientation } r, \\ 0, & \text{otherwise} \end{cases}$$

– product placement binary variable, for all $i = 1, ..., S$, $j = 1, ..., P$, $r \in \{0, 1\}$: $x_{ijr} \in \{0, 1\}$,

$f_{ijr}$ the number of SKUs of the product $j$ on the shelf $i$ on orientation $r$;

$$y_j = \begin{cases} 0, & \text{if product } j \text{ is on front orientation,} \\ 1, & \text{if product } j \text{ is on side orientation} \end{cases}$$

– orientation of the product $j$, for all $j = 1, ..., P$: $y_j \in \{0, 1\}$.

Heuristics parameters:

$x_{ij}$ sequence of shelf allocations;
$f_{ij}$ sequence of product allocations.

We start with a precise definition of the problem.

In supply chain logistics, zone picking is a type of order picking. It involves the partition of SKUs into a series of separate zones, with distribution centre staff teams assigned to pick up the orders from inside each designated zone. A zone is allotted to each picker group. For each order, they're in control of selecting all SKUs inside that zone. "Pick and pass" is a term used to describe the process of shifting distinct SKUs from one zone to another. There is only one scheduling period per shift with this strategy.

In this study, the problem of dimensioning the shelf space on the rack is represented by a rack that is visually divided into vertical groups to make it easier for the picker to select the product to the order. Distributors have different ways of arranging their products on shelves. They may arrange product packaging in prominent horizontal or vertical categories based on the product types, packages or weight. Each shelf of an examined rack is tagged horizontally, and each shelf is assigned a relevant tag. The shelves of the rack are also tagged vertically, which means that a tag is assigned to each category and applies to all shelves. These tags indicate which products belong to a particular category location. When the container comes to the zone in the picker's responsibility, the picker will be able to quickly recognise each product type on the rack and pick it to the container.

The main problem can be stated as follows. A certain amount of products must be put on the shelves of a rack in a zone. Vertical categories are used to categorise the products on the rack. Each vertical category is allocated on a rack by the distributor, who assigns the smallest size of the category on the rack. With such categorisation, distribution centres can keep more orderly. In order to develop appropriate racks, distributors typically need to contact potential buyers or analyse the historical data of the movement of products. The goal is to determine the necessary shelf space for each category on a rack that specifies the number of SKUs for each product in order to maximise overall movement/profit.

Distribution centres frequently have a variety of product brands. Product classes or groupings are used to categorise them. Each product category is laid down vertically. Horizontally, the items and shelves are also labelled. At the same time, each product might have several tags. On each shelf, several tags may be issued at the same time.

As an illustration of shelf tags:

- a shelf is used for heavy/light products;
- a shelf is used for a specific product package (box, plastic bag, can, or bottle);
- a shelf is at touch/eye/hat level.

Consider the following as an example. In this study, we create three alternative tags $b_t^n = \{H; H^+; V^+\}$. Tags may be used to tag shelves and items, although each shelf or single product may be tagged by one or many tags:

- $H$ – The shelf in the horizontal layout is exclusively for goods of the specific weight category (heavy or light).
- $H^+$ – The shelf in the horizontal layout is for items that must be put in the specific levels (touch/eye/hat level). This means that some products are placed at a specific level to make them easier for pickers to be found. Other products without the location specificity may also be placed at these levels.
- $V^+$ – In the vertical layout, the shelf is allocated to a certain product category. For the vertical product category, all shelves might be utilised to organise items by type, colour or package.

In real-world distribution centres, this is a regular practice by rack planners. Figure 1 depicts the specificity of the vertical and horizontal bands on a rack in the investigated case. There is the example of vertical categorisation of products into two categories. The lowest shelf is dedicated to big and heavy products. On other shelves, lighter products can be placed.

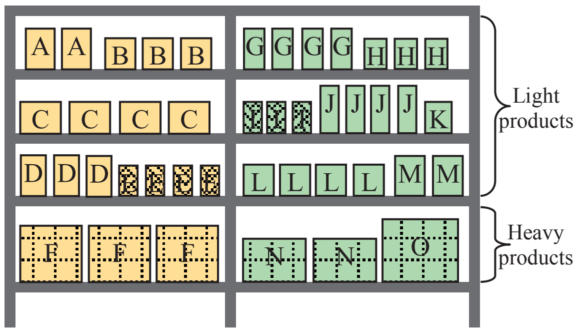


Fig. 1. Rack with vertical and horizontal bands: 2 vertical categories (yellow, green), 2 horizontal (light, heavy) categories

In the given rack, the following levels are specified: lowest level for heavy products, touch level for light products, eye level for light products, and the highest shelf without a tag (all light products can be placed there). The following tags are given for some of the products on a rack:

- there are two vertical categories ($V^+$);
- the lowest level is for heavy products of both categories ($H$);
- all shelves except the lowest one are for lightweight products of both categories ($H$);
- the product I in the second category ($V^+$) is a lightweight ($H$) at eye-level ($H^+$);
- the product E in the first category ($V^+$) is a lightweight ($H$) at touch level ($H^+$);

Different tags could be applied to the products simultaneously. The tags $b_t^n = \{H; H^+; V^+\}$ characterise only a type of grouping. There could be multiple groups of the defined tag. Some products could be grouped in a cluster, and therefore, they should be placed on one shelf together (e.g. charger and a cable). In this way, distributors can better prevent out-of-stock situations by stocking shelves with extra relevant products aside. Some products could be not assigned to any shelf level; only a vertical group should be defined.

A single product category is represented by several shelves. Each item might potentially be put on many shelves. Products can have multiple SKUs. Manufacturers and distributors care about the number of product SKUs. In order to make the goods visible to the picker, the distributor decides on the minimum and the maximum number of shelves for the product, as well as the minimum and the maximum number of SKUs for each product on the shelf. The products that are picked frequently get the most SKUs. If the product is put on many shelves, its supply limit determines its maximum availability.

If the product is packaged in a box, it can be positioned on the shelf in two ways: front and side. Obviously, orientation is not used for bottles because the width and depth are identical, and rotating the bottle does not reduce the overall width occupied by items on the shelf. By default, all goods are oriented in the front. As a result, the orientation binary parameter decides whether the product may be laid on its side depending on the packaging and label visibility displayed on the package.

In this study, just the front visible SKU row is explored. The number of vertical SKUs and depth SKUs are not considered. Because the lower shelves of a rack are often deeper, the shelf depth changes, but the product depth and shelf depth are only considered for the front SKU row. If the product's depth on the shelf is surpassed both front and side orientations yet is available, the object might be rotated on this shelf or placed on the deeper shelf in this circumstance.

To solve the problem, the distributor must first decide whether the product should be placed on the shelf, next define the number of SKUs of each product that should be on each shelf, then decide whether it should be placed on the front or side orientation, and take into account a set of constraints divided into four categories: shelf constraints, product constraints, orientation constraints, and bands constraints. The goal of space on the rack distribution and product arrangement is to maximise total rack movement/profit. Having a larger number of SKUs for their brands also benefits manufacturers. With the extra shelf space on the rack, they can sell more products within the distribution centre.

We formulate the problem using the decision variables listed below for the space distribution problem definition provided.

$x_{ijr}$ – if the product $j$ is placed on the shelf $i$ on orientation $r$;

$f_{ijr}$ – the number of SKUs of the product $j$ on the shelf $i$ on orientation $r$;

$y_j$ – orientation of the product $j$.

The problem can be formulated as follows:

$$\max \sum_{j=1}^{P} \sum_{i=1}^{S} \sum_{r=0}^{1} p_j^u f_{ijr} \tag{1}$$

subject to:

Rack shelf constraints:

- shelf length limit

$$\forall(i) \left[ \sum_{j=1}^{P} \sum_{r=0}^{1} p_{jr}^w f_{ijr} \le s_i^w \right]; \tag{2}$$

- shelf depth if a product is placed on front orientation

$$\forall(i, j, p_{j1}^w > s_i^d)[f_{ij0} = 0]; \tag{3}$$

- shelf depth if a product is placed on side orientation

$$\forall(i, j, p_{j0}^w > s_i^d)[f_{ij1} = 0]. \tag{4}$$

Product constraints:

- minimum and maximum number of products

$$\forall(i, j) \left[ f_j^{\min} x_{ijr} \le \sum_{r=0}^{1} f_{ijr} \le f_j^{\max} x_{ijr} \right]; \tag{5}$$

- supply limit if the same product is placed on multiple shelves

$$\forall(j) \left[ \sum_{i=1}^{S} \sum_{r=0}^{1} f_{ijr} \le p_j^s \right]; \tag{6}$$

- product is placed on the shelf

$$\forall(i, j, r) \left[ f_{ijr} \le x_{ijr} f_j^{\max} \right]; \tag{7}$$

- if products are grouped into clusters, they are placed on the same shelf

$$\forall(i)\, \forall(a, b : p_a^l = p_b^l, a, b = 1, ..., P) \left[ \sum_{r=0}^{1} x_{iar} = \sum_{r=0}^{1} x_{ibr} \right]. \tag{8}$$

Multi-shelves constraints:

- minimum and maximum number of shelves

$$\forall(j) \left[ s_j^{\min} \le \sum_{i=1}^{S} \sum_{r=0}^{1} x_{ijr} \le s_j^{\max} \right]; \tag{9}$$

- if the product is placed on multiple shelves, the next shelf only is available

$$\forall(j)\, \forall(a, b\colon |a-b| \ne 1 \wedge a < b,\ a, b = 1, ..., S) \left[ \sum_{r=0}^{1} x_{ajr} + \sum_{r=0}^{1} x_{bjr} \le 1 \right]. \tag{10}$$

Orientation constraints:

- side orientation is possible for the product

$$\forall(i, j) \left[ y_j \le p_j^{o_2} \right]; \tag{11}$$

- only one orientation (front or side) is available for the product

$$\forall(i,j)\left[\sum_{r=0}^{1} x_{ijr} \leq 1\right]. \tag{12}$$

Rack bands constraints:
- tags compatibility for the shelves and products

$$\forall(i,j)\left[\prod_{t=1}^{T} b_{tij}^{t} \geq \sum_{r=0}^{1} x_{ijr}\right]; \tag{13}$$

- minimum category size if the category exists on the shelf

$$\forall(i,k)\left[\left(\sum_{\substack{j=1,\\ p_j^k=k}}^{P}\sum_{r=0}^{1} p_{jr}^{w} f_{ijr} \geq \left[s_i^l \cdot c_k^m\right]\right) \vee \left(\sum_{\substack{j=1,\\ p_j^k=k}}^{P}\sum_{r=0}^{1} f_{ijr} = 0\right)\right]; \tag{14}$$

- category size tolerance

$$\forall(k)\left[\max_{i=1,\dots,S}\left(\sum_{\substack{j=1,\\ p_j^k=k}}^{P}\sum_{r=0}^{1} p_{jr}^{w} f_{ijr}\right) - \min_{i=1,\dots,S}\left(\sum_{\substack{j=1,\\ p_j^k=k}}^{P}\sum_{r=0}^{1} p_{jr}^{w} f_{ijr}\right) \leq \left[\max_{i=1,\dots,S}\left(s_i^l\right)\cdot c_k^t\right]\right]. \tag{15}$$

Relationships constraints:
- SKU relationships

$$\forall(i,j,r)\left[f_{ijr} \geq x_{ijr}\right]; \tag{16}$$

- SKU and orientation relationships

$$\forall(i,j)\left[f_{ij0} \leq (1-y_j) f_j^{\max}\right]; \tag{17}$$

- SKU and orientation relationships

$$\forall(i,j)\left[f_{ij1} \leq y_j f_j^{\max}\right]. \tag{18}$$

Decision variables:
- the product placed is on the shelf

$$\forall(i,j,r)\left[x_{ijr} \in \{0,1\}\right]; \tag{19}$$

- the number of SKUs

$$\forall(i,j,r)\left[f_{ijr} = \{f_j^{\min} \dots f_j^{\max}\}\right]; \tag{20}$$

- orientation

$$\forall(j)\left[y_j \in \{0,1\}\right]. \tag{21}$$

## Heuristics development

In this research, two heuristics to reach a solution to the shelf space distribution problem on the rack with vertical and horizontal product categories are proposed and implemented. The algorithmic thinking skills allow identifying the logical steps in order to find the solution. While the solution approaches can become complicated, conceptually, they're quite simple. We describe below two algorithms designed to solve the problem at hand. Then we will present a computational comparison of these algorithms. The proposed heuristics solution process is represented along these lines.

Let $x_{ij}$ is a sequence of shelf allocations, consequently, $x_{ij} = \sum_{r=0}^{1} x_{ijr}$ binary value means if the product is placed on the shelf. Let $f_{ij}$ is a sequence of product allocations, consequently $f_{ij} = \sum_{r=0}^{1} f_{ijr}$, an integer positive value means how many SKUs of the product are placed on the shelf. Variable $y_j$ is used to determine the front or side orientation of the product.

- Estimate how many could be the sets of shelf allocation sequences for each of the product orientations (front or side) based on the orientation (11)–(12) and cluster (8) constraints.
- If there are too many sets of shelf allocation sequences, analyse the input data and, based on intuition and product distribution knowledge, determine less proficient sets of sequences. For example, if there are 4 shelves and 5 products of one category should be placed there, maybe there is no reason to include in the sets of shelf allocation sequences sets by 4 products on the shelf because 1 shelf will be empty. Of course, there could be a solution with the empty shelf but analyse if there couldn't be more profitable allocations and exclude the fewer ones. Exclude also shelf allocation sequences which are expected to give not so good solutions based on the

minimum category size (14) and category size tolerance (15) constraints. The goal of this step is to intuitively select a set of shelf allocation sequences that are expected to give a good solution. They will be processed in later steps. The output of this step should be, e.g. "generate sets of shelf allocation sequences by 2 or 3 products", "don't generate sets of shelf allocation sequences by 1 or 4 or 5 products". Moreover, there could be different reasons why distributors decide to allocate or not allocate products on the shelves or to allocate on one shelf more products than on the other.

- Using the sets of shelf allocation sequences from the previous step, generate a set of shelf allocation sequences that allows for placing the products on the shelves with regard to the compatibility tags $b_t^n = \{H; H^+; V^+\}$, i.e. allocate the initially found shelf allocation sequences on each shelf and exclude incorrect product to shelf allocations. Next, exclude allocations that do not satisfy shelf tags compatibility (13), minimum and maximum number of shelves (9) and next shelf (10) constraints.
- Generate a set of product allocation sequences for each of the shelf sequences considering allocation product on the shelf (7), minimum and maximum numbers of products (5), shelf length (2), shelf depth (3)–(4), supply limit (6) constraints.
- From the achieved set of product allocation sequences, exclude the sequences that do not satisfy product allocation on multiple shelves, i.e. minimum category size (14) and category size tolerance (15) constraints.
- The rest relationship constraints in this approach will also be satisfied; therefore, checking them is not included in these steps.

The number of shelf and product sequences can be approximated after these preparation stages are completed. This is necessary in order to estimate how many of them can be generated and checked in the main solution finding step in a not relatively long time. Because of this, only a portion of all possible sequences that will provide a satisfactory result is generated based on the intuitive retail experience. Furthermore, the future expected movement/profit of a product allocation sequence might be evaluated in advance using the minimum and maximum numbers of product parameters. As a result, some steering parameters could be adjusted in order to decrease the time of running this algorithm.

Steering parameters are the following:

- Parameter 1: The list of numbers of products to be used while generating sequences of shelf allocations.
- Parameter 2: Number of generated product allocations for each shelf to be checked.
- Parameter 3: The step for going through product allocations on each shelf.
- Parameter 4: Number of generated product allocations for each category to be checked.
- Parameter 5: The step for going through product allocations in each category.
- Parameter 6: Minimum and maximum width range for each category to be checked, the sum of maximum widths must exceed the shelf width in order to have a better solution; other category widths outside the defined range are not considered.
- Parameter 7: Input movement/profit for each category to be checked, the product allocations with the profit below it are not considered.
- Parameter 8: Total movement/profit for a solution on all shelves, the allocations with the profit below it are not considered.

All shelf allocations that satisfy the defined constraints will be examined, but the number of product allocations (with different numbers of SKUs) will be reduced by steering parameters. The steering parameters must be clearly analysed because, in some cases, little size and worse total movement/profit of one category may result in a significantly better movement/profit for the other category, i.e. the total movement/profit will be greater than if each of the categories occupies approximately equal space.

- In later steps, according to intuitive rules specified for each heuristic, reduce the number of product allocations so that only a small part of them will be analysed. Next, find the best solution among them from the generated product allocations.
  - H1 – For each shelf in each category for each parent shelf allocation, generate product allocations so that they will be in not descending order of category width, next in not ascending order of category movement/profit. Do not take the product sequences outside the category width defined by parameter 6. As well as, do not take the product sequences with the category movement/profit below one defined by parameter 7. Take the defined by steering parameter 2 number of product allocations for each shelf but go through all possible product allocations with shelf step defined as parameter 3. Product sequences on each shelf for each category are completed. Combine them with the consequent product sequences of other categories. The resulted product sequences sort in *not ascending order of movement/profit*; next, sort the parts inside sorted ones in *not ascending order of profit ratio.* Take the defined by steering parameter 4 number of product allocations for each category but go through all possible product allocations with

category step defined as parameter 5. Do not take the product sequences with a total movement/profit below one defined by parameter 8.

– H2 – For each shelf in each category for each parent shelf allocation, generate product allocations so that they will be in not descending order of category width, next in not ascending order of category movement/profit. Do not take the product sequences outside the category width defined by parameter 6. As well as, do not take the product sequences with the category movement/profit below one defined by parameter 7. Take the defined by steering parameter 2 number of product allocations for each shelf but go through all possible product allocations with shelf step defined as parameter 3. Product sequences on each shelf for each category are completed. Combine them with the consequent product sequences of other categories. The resulted product sequences sort in *not ascending order of profit ratio;* next, sort the parts inside sorted ones in *not ascending order of movement/profit.* Take the defined by steering parameter 4 number of product allocations for each category but go through all possible product allocations with category step defined as parameter 5. Do not take the product sequences with a total movement/profit below one defined by parameter 8.

N.B. Parameter 1 is used for the generation of shelf sequences before generating the product sequences. Heuristics H1 and H2 explain how to deal with product sequences; therefore, parameter 1 is not mentioned here. Heuristics H1 and H2 differ from each other by order of sorting in the second part (values in italic). Because of this, different steering parameter values might be applied to them, and different results could be obtained.

Step parameters 3 and 5 are implemented because if the number of solutions is very high, the consequent solutions distinguish slightly. Therefore in order to diversify the solutions to be checked, we take not consequent values but values with the defined step.

In our notation, profit ratio means the ratio of the total movement/profit of the products allocated on a rack divided by occupied space, i.e., free space on each shelf, is not taken into calculations.

Choosing the correct approach is frequently the key to locating the ideal solutions. Distributors appreciate creative and unusual ideas. The above described step-by-step approach allows for completing the product shelf space distribution task. It could be observed that the initial rack space distribution problem is broken into parts because its steps are simple and easy to perform.

## Computational experiments

Due to the fact that not all problems can be treated as linear, the best solution could not be discovered. We investigated if the recommended principles within the heuristics produce good results in order to apply these heuristics to more complex rack space distribution problems in the future. CPLEX solver is used to verify that the proper result is obtained by heuristics while solving a problem.

Experiments were performed on 10, 15, and 20-product sets which must be allocated on a 4-shelf rack with the shelf lengths 250 cm, 375 cm, 500 cm, 625 cm, and 750 cm. Product sets with different product parameters (width, depth, movement/profit, etc.) were prepared. For these products, 2 vertical categories on the rack were specified.

The experiments were conducted using the computer with the following parameters:

- Processor: AMD Ryzen 5 1600 Six-Core Processor 3.20 GHz.
- RAM: 16 GB.
- System type: 64-bit Operation System, x64-based processor.
- Windows 10.

Heuristics were developed in MS SQL Server 2008 R2. Microsoft SQL Server Management Studio: 10.50.4000.0. The optimal solution was found using IBM ILOG CPLEX Optimization Studio Version: 12.7.1.0.

Table 1 shows the performance of the developed heuristics H1 and H2 compared to the commercial

Table 1
Performance of the developed heuristics

| Products | Shelf width | Profit ratio of H1 | Profit ratio of H2 | Time of H1 [min] | Time of H2 [min] |
|---|---|---|---|---|---|
| 10 | 250 | 100.00% | 100.00% | 0.03 | 0.03 |
| | 375 | 100.00% | 100.00% | 0.20 | 0.21 |
| | 500 | 100.00% | 100.00% | 0.07 | 0.07 |
| | 625 | 100.00% | 100.00% | 0.16 | 0.16 |
| | 750 | 100.00% | 100.00% | 0.73 | 0.72 |
| 15 | 250 | 100.00% | 100.00% | 0.25 | 0.47 |
| | 375 | 100.00% | 100.00% | 0.11 | 0.11 |
| | 500 | 98.82% | 99.12% | 2.27 | 2.54 |
| | 625 | 100.00% | 100.00% | 0.05 | 0.05 |
| | 750 | 100.00% | 100.00% | 0.33 | 0.33 |
| 20 | 250 | 100.00% | 100.00% | 2.99 | 3.72 |
| | 375 | 99.22% | 99.39% | 7.35 | 7.89 |
| | 500 | 92.58% | 92.58% | 0.19 | 0.19 |
| | 625 | 98.91% | 98.91% | 1.46 | 1.46 |
| | 750 | 99.06% | 99.06% | 3.84 | 26.44 |

CPLEX solver solution. The columns with profit ratio mean movement/profit found by heuristics divided by the optimal movement/profit found by CPLEX solver. Such metric has been calculated for each heuristic for each test instance.

It could be observed that both heuristics found the optimal solution for the 1st product set of 10 products on all shelf lengths. For the 2nd product set of 15 products, both heuristics found the optimal solution on the four shelf lengths. For the rest 500 cm shelf length, heuristics H2 was slightly better than heuristics H1, H2 achieved a profit ratio of 99.12% compared to 98.82% achieved by H1. For the 3rd product set of 20 products, both heuristics found the optimal solution only for the shortest shelves rack. For the 374 cm shelf, heuristics H2 was also slightly better than heuristics H1 giving the 99.39% profit ratio compared to 99.22% given by heuristics H1. The results of the other shelf lengths were the same for both heuristics.

All results are equal to or higher than 92.58%; in 10 from 15 tests for each heuristic, both of them found optimal solutions. This proves the rationality of the heuristics proposed and the reasonableness of the steering parameters, which strongly reduce the number of solutions to be generated without the making result worse. Only profitable solutions are generated to be checked in later steps of the heuristics.

The solution time of CPLEX was from 1 to 4 seconds. It could be noticed in Table 1 that most of the test instances with optimal solutions, except the 20 products set on 250 cm shelf lengths, found the solution in less than a minute. The solution time for the largest instance (20 products on 750 cm shelf length) of H2 was increased up to 26.44 minutes.

Table 2 presents the steering parameters of the developed heuristics H1 and H2 correspondingly. The percentage given in the table does not mean the comparison of checked solutions to all possible solutions but to the number of solutions received after processing with the reduction parameters. So at first, the reduced number of solutions were generated on the defined range of numbers of products: for shelf sequences (steering parameter 1), for the range of the category widths (steering parameter 6), which are greater than movement/profit for each category (steering parameter 7), which are greater than total movement/profit (steering parameter 8). Here the number of solutions to be compared with is received. Next, the reduced number of solutions is taken to be checked: for each shelf (steering parameter 2) and for each category (steering parameter 4). Next, the ratios of taken solutions to total reduced solutions are calculated in columns "checked allocations on shelf", "checked allocations for category". 100% means that the reduction of the solution numbers with the help of steering parameters 1, 7, 8 is enough, so steering parameters 2 and 4 are not applied. It could be concluded that the mentioned steering parameters 1, 7, 8 are enough for small instances. But when the number of solutions is very large, the additional steering parameters 2 and 4 should be applied.

Table 2
Numbers steering parameters of the developed heuristics H1 and H2

| Products | Shelf width | Checked alloc. on shelf 1 | Checked alloc. on shelf 2 | Checked alloc. on shelf 3 | Checked alloc. on shelf 4 | Checked alloc. category 1 | Checked alloc. for category 2 |
|---|---|---|---|---|---|---|---|
| 10 | 250 | 100% | 100% | 100% | 100% | 100% | 100% |
| | 375 | 100% | 100% | 100% | 100% | 100% | 100% |
| | 500 | 100% | 100% | 100% | 100% | 100% | 100% |
| | 625 | 100% | 100% | 100% | 100% | 100% | 100% |
| | 750 | 100% | 100% | 100% | 100% | 100% | 100% |
| 15 | 250 | 100% | 100% | 100% | 100% | 50% | 100% |
| | 375 | 100% | 100% | 100% | 100% | 29% | 100% |
| | 500 | 100% | 100% | 100% | 100% | 3% | 90% |
| | 625 | 100% | 100% | 100% | 100% | 100% | 100% |
| | 750 | 100% | 100% | 100% | 100% | 100% | 100% |
| 20 | 250 | 100% | 100% | 100% | 100% | 37% | 100% |
| | 375 | 100% | 100% | 100% | 100% | 28% | 3% |
| | 500 | 100% | 100% | 100% | 100% | 100% | 100% |
| | 625 | 100% | 88% | 100% | 100% | 100% | 100% |
| | 750 | 100% | 37% | 100% | 57% | 4% | 4% |

For the last largest instance, only 4% of reduced numbers of solutions for each category were enough to get the solution. The reduction of solutions for shelf 2 and shelf 4 do not worse the result. Most of the tests show 100%. This means that where there is possible to get the result solution fast enough, we do not reduce the number of solutions with the steering parameters 2 and 4 in order to get the result of higher quality.

The numerical steering parameters in Table 2 are the same for both heuristics because the same input parameters were used for both heuristics. In such a way, the goal was to compare the heuristics themselves because a larger number of solution numbers may result in a better solution but the time increase. Therefore we tried to use the same initial parameters for both heuristics.

Table 3 shows the movement/profit steering parameters of the developed heuristics. The input movement/profit below which the separate solutions for a category were not taken is calculated compared to the sum of average profits of all categories in all reduced partial solutions received after applying steering parameters 1, 7, 8. So at this step, we can approximate what the average profit of the category is; we could take slightly lower than it for one category and slightly greater for another more profitable category. There is no reason to generate solutions with a movement/profit significantly lower than the average for all categories. This principle is also applied in both heuristics.

Table 3
Profit steering parameters of the developed heuristics H1 and H2

| Products | Shelf width | Profit input ratio for category 1 | Profit input ratio for category 2 | Profit input ratio |
|---|---|---|---|---|
| 10 | 250 | 138% | 69% | 77% |
| | 375 | 96% | 125% | 51% |
| | 500 | 84% | 141% | 43% |
| | 625 | 82% | 118% | 42% |
| | 750 | 72% | 103% | 37% |
| 15 | 250 | 130% | 91% | 72% |
| | 375 | 139% | 108% | 76% |
| | 500 | 137% | 95% | 74% |
| | 625 | 144% | 98% | 79% |
| | 750 | 146% | 93% | 82% |
| 20 | 250 | 151% | 55% | 108% |
| | 375 | 152% | 52% | 106% |
| | 500 | 143% | 55% | 97% |
| | 625 | 147% | 67% | 97% |
| | 750 | 123% | 101% | 82% |

The values of profit input ratios greater than 100% mean that there were so many partial solutions that we took only that which could get greater total movement/profit than the sum of average profits of all categories.

It could be noted that there are no test instances where we took partial solutions, which could give less movement/profit than the sum of average profits of all categories (all profit input ratios are greater than 100%). So the proposed approach of excluding less profitable solutions is very valuable. For the set of 10 products, the input profit was equal to the sum of average profits of all categories in all tested widths. For the rest test instances, the input profit was slightly higher (up to 24% for the 20 products on a 250 cm shelf. When the instance is larger, we increase the input profit in order to decrease the number of solutions.

Profit input ratios for separate categories show the ratio of the profit of the category to the average category profit. The values greater than 100% show that we generated solutions for this category higher than the average category profit. The values below 100% show that solutions with category profits lower than average category profit were taken. Increasing the movement/profit of the more profitable category and decreasing the movement/profit of the less profitable one results in higher total movement/profit. Only for two instances (15 products on 375 cm; 20 products on 750 cm) the movement/profit of both categories was higher than the average profit by category. In other instances, increasing movement/profit by both categories gave either worse results or did not find the result at all. Therefore we made the decision to increase one category's movement/profit and decrease the other one.

The total number of shelf allocations in a general case is $(r+1)^{PS} = 3^{PS}$.

Number 3 represents the possible allocations of the product: (1) if it is not placed on the shelf, (2) if it is placed on the shelf in the front orientation, (3) if it is placed on the shelf in the side orientation. All products may be placed on all shelves simultaneously.

The total number of product allocations for each number of products in a general case is

$$\prod_{j=1}^{P} \left(f_j^{\max} - f_j^{\min} + 1\right)^S .$$

Table 4 shows the numbers of generated allocations and solutions in heuristics H1 and H2. Table 5 shows the numbers of all possible shelf and product allocations in general cases calculated using the formulas above. The numbers of H1, H2 could be compared to the total possible numbers of solutions.

Table 4
Numbers of generated allocations and solutions in heuristics H1, H2

| Products | Shelf width | Number of generated allocations to be checked | Number of solutions H1 | Number of solutions H2 |
|---|---|---|---|---|
| 10 | 250 | $8.73 \cdot 10^5$ | $4.60 \cdot 10^1$ | $4.60 \cdot 10^1$ |
| | 375 | $3.06 \cdot 10^7$ | $2.40 \cdot 10^1$ | $2.40 \cdot 10^1$ |
| | 500 | $9.02 \cdot 10^5$ | $3.32 \cdot 10^2$ | $3.32 \cdot 10^2$ |
| | 625 | $7.20 \cdot 10^6$ | $4.63 \cdot 10^4$ | $4.63 \cdot 10^4$ |
| | 750 | $5.40 \cdot 10^5$ | $4.30 \cdot 10^5$ | $4.30 \cdot 10^5$ |
| 15 | 250 | $5.47 \cdot 10^7$ | $2.45 \cdot 10^4$ | $1.78 \cdot 10^5$ |
| | 375 | $1.27 \cdot 10^7$ | $1.00 \cdot 10^1$ | $5.56 \cdot 10^2$ |
| | 500 | $3.68 \cdot 10^9$ | $2.20 \cdot 10^1$ | $1.24 \cdot 10^5$ |
| | 625 | $1.30 \cdot 10^6$ | $2.92 \cdot 10^2$ | $2.92 \cdot 10^2$ |
| | 750 | $6.69 \cdot 10^6$ | $6.22 \cdot 10^4$ | $6.22 \cdot 10^4$ |
| 20 | 250 | $3.71 \cdot 10^7$ | $1.43 \cdot 10^5$ | $5.01 \cdot 10^5$ |
| | 375 | $1.17 \cdot 10^9$ | $4.57 \cdot 10^3$ | $2.36 \cdot 10^5$ |
| | 500 | $1.01 \cdot 10^7$ | $2.21 \cdot 10^3$ | $2.21 \cdot 10^3$ |
| | 625 | $2.97 \cdot 10^5$ | $8.31 \cdot 10^4$ | $8.31 \cdot 10^4$ |
| | 750 | $1.13 \cdot 10^{10}$ | $8.09 \cdot 10^5$ | $8.07 \cdot 10^6$ |

Table 5
Numbers of all possible shelf and product allocations in the general case

| Products | Number of shelf allocations | Number of product allocations |
|---|---|---|
| 10 | $1.22 \cdot 10^{19}$ | $1.10 \cdot 10^{52}$ |
| 15 | $4.24 \cdot 10^{28}$ | $1.15 \cdot 10^{78}$ |
| 20 | $1.48 \cdot 10^{38}$ | $1.21 \cdot 10^{104}$ |

For example, for the largest instance, the number of all possible product allocations on all shelf widths is $1.21 \cdot 10^{104}$. But for heuristics, about a million solutions to be checked were enough to get the result solution. Moreover, for 250 cm shelf width, even 100 thousand solutions to be checked allowed to get an optimal solution by both heuristics. This proves that intuitive rules implemented in heuristics are of great reasonability.

By means of this case study, we show how shelf space distribution planning can be improved in real distribution centres. It could be useful for the examination of particular cases (number of products, categories, shelves, the rack widths) in a real-world context.

## Conclusions and future research

A distribution centre is typically the ideal solution for rapidly moving products in and out of a facility on a per-order basis. The sort of facility that is required is ultimately determined by the nature of the company's operations. The light assembly of products, processing, quality control, repackaging, and other operations necessary to fulfil orders may be included in certain distribution centres.

The process of product management in a distribution centre is not a simple or quick one to accomplish. It requires a lot of effort and time. Allocating certain quantities of products in horizontal and vertical categories has some intrinsic benefits. The product could be found quickly on the shelf of the rack and added to the order. Then it could be quickly packed and shipped. So more orders could be processed.

The supply chain motto is to provide the right items, at the right time, at the right place. The customer-focused distribution centre serves as a link between the supplier and the client.

In this research, the problem of dimensioning the shelf space on the rack with vertical and horizontal product categorisation in a distribution centre is focused on maximising the product movement/profit. Distribution centre's leaders are interested in expert solutions for allocation of products to shelves which allow packers quickly find the product on the shelf, pick it to the order container and send the container to the next picking area, which results in the processing of more orders and as a result the increase of gained movement/profit.

In this research, we developed two heuristics to solve the proposed problem of dimensioning the shelf space on the rack with vertical categories optimally or near optimally on different problem instances. There were 8 steering parameters that allowed to reduce the search space implemented in heuristics. Among them were parameters that decrease the number of products on the shelves (steering parameter 1), the category with a range for assigning most space for the most profitable products within the category (steering parameters 6), the number of generated product allocations (steering parameters 2 and 4), the step parameters for the intensity of solution diversification (steering parameters 3 and 5), and the movement/profit below which the solutions were not generated (steering parameters 7 and 8). The principles in the proposed heuristics could be applied for solving other shelf space on the rack dimensioning problems as well as non-linear models for which an optimal solution could not be found.

There were 15 testing data for each heuristic. Their quality was compared to the solution found by CPLEX solver. In 10 from 15 tests, both heuristics found optimal solutions without exploring the whole solution space. For the rest test sets, the solutions received by heuristics were equal to or higher than 92.58%. Both heuristics are executed without checking the whole solutions space. Despite this fact, they could find an optimal or near-optimal solution.

The maximum number of shelf allocations checked by heuristics H1 and H2 was $1.13 \cdot 10^{10}$. The maximum number of product allocations checked by heuristics H1 was $8.09 \cdot 10^5$, and the same parameter checked by heuristics H2 was $8.07 \cdot 10^6$. At the same time, the total number of the shelf and product allocations (in the whole solution space) were $1.48 \cdot 10^{38}$ and $1.21 \cdot 10^{104}$, consequently.

This proves that the rules applied in the proposed heuristics allow significantly reducing solution space without worsening the gained result. The solution time of heuristics was fast enough for most instances (varying from 2 seconds to approximately 7 minutes). For the last largest instance, it increased up to 26.44 minutes.

When distribution centre leaders put related products within reach of one another, they use rack-making procedures. Modern supply chains with improved data and intelligence can identify product demand months ahead of time, plan appropriately, and deliver products right when they're required. Of course, the method leaders complete calculations are influenced by various circumstances and conditions. Because the best-selling products obtain the most SKUs, the quantity of SKUs is crucial. Therefore we propose several heuristics.

The advantages of proposed heuristics:

- The proposed approach implements a definite procedure based on the real product distribution experience.
- It can be easily understood by the distributor without programming knowledge.
- New important steps easily could be added with regard to the changing environment and requirements.
- On each step is easy to debug if errors occur or the step was implemented due to a mistake or inconsistency of the initial requirement.
- It is not as time-consuming as other precise algorithms.
- It can be applied to solve large problems of dimensioning the shelf space on the rack in distribution centres.

This shelf space distribution experience and viewpoint form the foundation for a procedure that will be used to discover solutions to other dimensioning the shelf space issues. The basic approach is essential because it can be used to a wide range of issues, including those whose solutions are written in a different computer language. It is expected that the knowledge gained from this study would be useful to practitioners working on the allocation of products on the racks in distribution centers.

This research has some limitations. The proposed heuristics cannot necessarily be applied to all dimensioning the rack shelf space problems because real-life problems vary significantly by the requirements and input data. But the insights gained from this research can then help the distributors and other researchers to develop additional ideas and implement into practice new intuitive solutions.

The results of this research provide insights for further studies. The following research questions could be set:

- What other product space distribution rules exist for working with the data?
- What other relationships exist among the supply chain data values?
- What could other new facts be revealed?
- What other rules of selecting only profitable product allocations could exist?

Last but not least, we recommend comparing the developed heuristics with other optimisation methods which do not use heuristics.

**Acknowledgements**

The studies were realised with a support from statutory activity financed by Polish Ministry of Science and Higher Education (0613/SBAD/4710).

## References

Al-Araidah O., Dalalah D., Azeez M.A.A. and Khasawneh M.T. (2017), A heuristic for clustering and picking small items considering safe reach of the order picker, *European Journal of Industrial Engineering*, Vol. 11, No. 2, pp. 256–269. DOI: 10.1504/EJIE.2017.083256

Bahrami B., Aghezzaf E.H. and Limère V. (2019), Enhancing the order picking process through a new storage assignment strategy in forward-reserve area, *International Journal of Production Research*, Vol. 57, No. 21, pp. 6593–6614. DOI: 10.1080/00207543.2019.1567953

Bartholdi III J.J. and Hackman S.T. (1998), *Warehouse & Distribution Science: Release 0.92*, Atlanta, GA, The Supply Chain and Logistics Institute, School of Industrial and Systems Engineering, Georgia Institute of Technology, Available at: papers3://publication/uuid/7F731EF1-7AB3-46A4-BC4A-514E4B78D73B

Borin N., Farris P.W. and Freeland J.R. (1994), A Model for Determining Retail Product Category Assortment and Shelf Space Allocation, *Decision Sciences*, Vol. 25, No. 3, pp. 359–384. DOI: 10.1111/j.1540-5915.1994.tb01848.x

Calzavara M., Glock C.H., Grosse E.H. and Sgarbossa F. (2019), An integrated storage assignment method for manual order picking warehouses considering cost, workload and posture, *International Journal of Production Research*, Vol. 57, No. 8, pp. 2392–2408. DOI: 10.1080/00207543.2018.1518609

Castelli M. and Vanneschi L. (2014), Genetic algorithm with variable neighborhood search for the optimal allocation of goods in shop shelves, *Operations Research Letters*, Vol. 42, No. 5, pp. 355–360. DOI: 10.1016/j.orl.2014.06.002

Chang F.L., Liu Z.X., Xin Z. and Liu D.D. (2007), Research on the order picking optimisation problem of the automated warehouse, *Xitong Gongcheng Lilun Yu Shijian/System Engineering Theory and Practice*, Vol. 27, No. 2, pp. 139–143. DOI: 10.1016/s1874-8651(08)60015-0

Chen W., Zhang Y. and Zhou Y. (2022), Integrated scheduling of zone picking and vehicle routing problem with time windows in the front warehouse mode, *Computers and Industrial Engineering*, Vol. 163, pp. 107823. DOI: 10.1016/j.cie.2021.107823

Czerniachowska K., Sachpazidu-Wójcicka K., Sulikowski P., Hernes M. and Rot A. (2021), Genetic algorithm for the retailers' shelf space allocation profit maximisation problem, *Applied Sciences (Switzerland)*, Vol. 11, No. 14, pp. 6401. DOI: 10.3390/app11146401

Czerniachowska, K. (2022), A genetic algorithm for the retail shelf space allocation problem with virtual segments, *OPSEARCH*, Vol. 59, No. 1, pp. 364–412. DOI: 10.1007/s12597-021-00551-3

Czerniachowska K. and Hernes M. (2021), Simulated annealing hyper-heuristic for a shelf space allocation on symmetrical planograms problem, *Symmetry*, Vol. 13, No. 7. DOI: 10.3390/sym13071182

Davarzani H. and Norrman A. (2015), Toward a relevant agenda for warehousing research: literature review and practitioners' input, *Logistics Research*, Vol. 8, No. 1. DOI: 10.1007/s12159-014-0120-1

de Koster R., Le-Duc T. and Roodbergen K.J. (2007), Design and control of warehouse order picking: A literature review, *European Journal of Operational Research*, Vol. 182, No. 2, pp. 481–501. DOI: 10.1016/j.ejor.2006.07.009

de Vries J., de Koster R. and Stam D. (2016), Aligning Order Picking Methods, Incentive Systems, and Regulatory Focus to Increase Performance, *Production and Operations Management*, Vol. 25 No. 8, pp. 1363–1376. DOI: 10.1111/poms.12547

Emmett S. (2005), *Excellence in warehouse management: How to minimise costs and maximise value*, John Wiley & Sons Ltd.

Erol H., Bulkan S., and Agaoglu M. (2015), A Heuristic Approach for Shelf Space Allocation Problem, *Journal of Military and Information Science*, Vol. 4, No. 1, p. 38. DOI: 10.17858/jmisci.89213

Franzke T., Grosse E.H., Glock C.H. and Elbert R. (2017), An investigation of the effects of storage assignment and picker routing on the occurrence of picker blocking in manual picker-to-parts warehouses, *International Journal of Logistics Management*, Vol. 28, No. 3, pp. 841–863. DOI: 10.1108/IJLM-04-2016-0095

Gajjar H.K. and Adil G.K. (2010), A piecewise linearisation for retail shelf space allocation problem and a local search heuristic, *Annals of Operations Research*, Vol. 179, No. 1, pp. 149–167. DOI: 10.1007/ s10479-008-0455-6

Garbacz M., and Łopuszyński M. (2015), Optimization of order picking in warehouse, part 1 and 2 ([Optymalizacja procesu kompletacji w magazynie, część 1 i 2], *Logistyka*, Vol. 6, pp. 628–647

Hansen J.M., Raut S. and Swami S. (2010), Retail Shelf Allocation: A Comparative Analysis of Heuristic and Meta-Heuristic Approaches, *Journal of Retailing*, Vol. 86, No. 1, pp. 94–105. DOI: 10.1016/j.jretai.2010.01.004

Ho Y.C. and Liu C.F. (2005), A design methodology for converting a regular warehouse into a zone-picking warehouse, *Journal of the Chinese Institute of Industrial Engineers*, Vol. 22, No. 4, pp. 332–345. DOI: 10.1080/10170660509509303

Ho Y.C., Wee H.M. and Chen H.C. (2007), A geometric design of zone-picking in a distribution warehouse, Lecture Notes in Computer Science (Including Subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics), Vol. 4707 LNCS, Part 3, pp. 625–636. DOI: 10.1007/978-3-540-74484-9_54

Hong S., Johnson A.L. and Peters B.A. (2012), Large-scale order batching in parallel-aisle picking systems, *IIE Transactions (Institute of Industrial Engineers)*, Vol. 44 No. 2, pp. 88–106. DOI: 10.1080/0740817X.2011.588994

Hübner A.H. and Kuhn H. (2011), Retail Shelf Space Management Model with Space-Elastic Demand and Consumer-Driven Substitution Effects, *SSRN Electronic Journal*. DOI: 10.2139/ssrn.1534665

Hwang H., Choi B. and Lee G. (2009), A genetic algorithm approach to an integrated problem of shelf space design and item allocation, *Computers and Industrial Engineering*, Vol. 56, No. 3, pp. 809–820. DOI: 10.1016/j.cie.2008.09.012

Kostrzewski A., and Kostrzewski M., (2014), Selected problems connected to the processes of order-picking in a warehouse ([Wybrane problemy związane z procesami kompletacji w magazynie], *Gospodarka Materiałowa i Logistyka*, Vol. 5, pp. 327–338

Kumar M. and Kulkarni A.J. (2019), Socio-inspired optimisation metaheuristics: A review, *Studies in Computational Intelligence*, Vol. 828, pp. 241–265. DOI: 10.1007/978-981-13-6569-0_12

Landa-Silva D., Marikar F. and Le K. (2009), Heuristic approach for automated shelf space allocation, *Proceedings of the ACM Symposium on Applied Computing*, pp. 922–928. DOI: 10.1145/1529282.1529482

Lim A., Rodrigues B. and Zhang X. (2004), Metaheuristics with Local Search Techniques for Retail Shelf-Space Optimization, *Management Science*, Vol. 50, No. 1, pp. 117–131. DOI: 10.1287/mnsc.1030.0165

Manzini R., (2012), *Warehousing in the Global Supply Chain*, London: Spinger-Verlag. DOI: 10.1007/ 978-1-4471-2274-6

Önüt S., Tuzkaya U.R. and Dogaç B. (2008), A particle swarm optimisation algorithm for the multiple-level warehouse layout design problem, *Computers and Industrial Engineering*, Vol. 54, No. 4, pp. 783–799. DOI: 10.1016/j.cie.2007.10.012

Ostermeier M., Holzapfel A., Kuhn H. and Schubert D. (2020), Integrated Zone Picking and Vehicle Routing Operations with Restricted Intermediate Storage, *SSRN Electronic Journal*, pp. 1–38. DOI: 10.2139/ssrn.3663157

Parikh, P.J. and Meller, R.D. (2008), Selecting between batch and zone order picking strategies in a distribution center, *Transportation Research Part E: Logistics and Transportation Review*, Vol. 44, No. 5, pp. 1664–1678. DOI: 10.1016/j.tre.2007.03.002

Parikh P.J. and Meller R.D. (2009), Estimating picker blocking in wide-aisle order picking systems, *IIE Transactions*, Vol. 41, No. 3, pp. 232–246. DOI: 10.1080/07408170802108518

Parikh P.J. and Meller R.D. (2010a), A note on worker blocking in narrow-aisle order picking systems when pick time is non-deterministic, *IIE Transactions*, Vol. 42, No. 6, pp. 392–404. DOI: 10.1080/07408170903171043

Parikh P.J. and Meller R.D. (2010b), A travel-time model for a person-onboard order picking system, *European Journal of Operational Research*, Vol. 200, No. 2, pp. 385–394. DOI: 10.1016/j.ejor.2008.12.031

Redmer A. (2020), Analysis of the length of order-picking paths determined using the s–shape method, *Logforum*, Vol. 16, No. 1, pp. 33–46. DOI: 10.17270/J.LOG.2020.379

Reeves C.R. (Ed.). (1993), *Modern heuristic techniques for combinatorial problems*, John Wiley & Sons Inc., NY, USA.

Schultz K.L., Juran D.C. and Boudreau J.W. (1999), Effects of low inventory on the development of productivity norms, *Management Science*, Vol. 45, No. 12, pp. 1664–1678. DOI: 10.1287/mnsc.45.12.1664

Tanaka K., Ihara A. and Zhang J. (2019), Introducing Parallel Zone Picking to Warehouse Batch Picking Systems, *Advances in Computer Science Research*, August. DOI: 10.2991/msbda-19.2019.69

Urban T.L. (1998), An inventory-theoretic approach to product assortment and shelf-space allocation, *Journal of Retailing*, Vol. 74, No. 1, pp. 15–35. DOI: 10.1016/S0022-4359(99)80086-4

Van Nieuwenhuyse I. and de Koster R.B.M. (2009), Evaluating order throughput time in 2-block warehouses with time window batching, *International Journal of Production Economics*, Vol. 121, No. 2, pp. 654–664. DOI: 10.1016/j.ijpe.2009.01.013

Wu Y., Zhou C., Wu Y. and Kong X.T.R. (2017), Zone merge sequencing in an automated order picking system, *International Journal of Production Research*, Vol. 55, No. 21, pp. 6500-6515. DOI: 10.1080/00207543.2016.1264641

Yang M.H. and Chen W.C. (1999), Study on shelf space allocation and management, *International Journal of Production Economics*, Vol. 60–61, pp. 309–317. DOI: 10.1016/S0925-5273(98)00134-0

Yang M.H. (2001), Efficient algorithm to allocate shelf space, *European Journal of Operational Research*, Vol. 131, No. 1, pp. 107–118. DOI: 10.1016/S0377-2217(99)00448-8