CALCULATION MODEL OF PIPELINE RESISTANCE FOR CEMENTED PASTE BACKFILL CONSIDERING THIXOTROPY

The rheological behaviour of cemented paste backfill (CPB) has an important influence on the stability of its transportation in pipelines. In the present study, the time-dependent rheological behaviour of CPB was investigated to elucidate the effects of time and solid content. Experimental results showed that when CPB is subjected to a constant shear rate, the shear stress gradually decreases with time before finally stabilising. When the solid content was 60%~62%, a liquid network structure was the main factor that influenced the thixotropy of CPB, and the solid content had less influence. When the solid content was 64%~66%, a floc network structure was the main factor that influenced the thixotropy of CPB, and the solid content had a more significant influence on the thixotropy than the shear rate. The initial structural stability of CPB increased with the solid content, and this relationship can be described by a power function. Based on the experimental results, a calculation model of pipeline resistance considering thixotropy was proposed. The model was validated by using industrial experimental data. The current study can serve as a design reference for CPB pipeline transportation.

Keywords: thixotropy; cemented paste backfill; pipeline transportation; calculation model; hydraulic gradient

1. Introduction

Mining and mineral processing operations are associated with many challenges, such as surface subsidence and the accumulation of mining waste [1-4]. Cemented paste backfill (CPB) uses tailings to fill the underground extraction area, reducing the discharge of tailings and has become a mainstream mining practice in many countries [5-9]. CPB is usually prepared on the surface and then transported to the underground quarry, so it should have sufficient fluidity and
stability [10]. Determining rheological behaviour is essential for evaluating the fluidity and stability of CPB pipelines [11,12]. In fluid mechanics, yield stress and viscosity are essential parameters for characterising rheological behaviour [13]. Kretser et al. [14] reported that yield stress is a significant indicator for ensuring the successful startup of a pumping system from a static shutdown condition. Viscosity is an indicator of the subsequent pumping requirements and fluidity. However, these two parameters are not used in engineering terminology, and the fluidity of CPB is often assessed concerning the pipeline resistance.

Gao et al. [15] combined a structural fluid test with a particle flow model and proposed a method for optimising the conveying parameters of the CPB pipeline. They used the Herschel-Bulkley model to analyse the conveying resistance of CPB and established a function relating to the resistance and parameters. Bharathan et al. [16] discussed friction coefficient correlations for the hydraulic model and found that the Swamee-Agarwal friction coefficient correlation can accurately predict the pressure loss along the CPB pipeline with an error within 5%. Cheng et al. [17] studied the influence of time and temperature on the rheological behaviour of CPB and proposed the time-temperature equivalent effect, which they used to establish a calculation model for the conveying resistance of the pipeline considering time and temperature [18]. Collected practical engineering cases of CPB from Gobi sand and tailings and proposed a flow loop test for estimating the pressure drop. They found that the Swamee-Agarwal model can effectively predict the pressure drop for the laminar flow of a Bingham plastic fluid in the pipeline.

CPB is formed by mixing cement, tailings, and water into a paste with high solid content and non-Newtonian fluid behaviour [19-23]. Many researchers have investigated the rheological behaviour of CPB. Jiang et al. [24] investigated the effect of three mineral admixtures (i.e., fly ash, slag, and silica fume) on the rheological behaviour of CPB and found that partially replacing the cement with fly ash significantly improves the fluidity, as well as a linear correlation between the thixotropy and plastic viscosity. Guo et al. [25] used the theory of water film thickness to study the fluidity of CPB mixed with hydrotalcite and found that the bulk density of particles increases significantly with the amount of flocculant. Based on observations of inelastic suspended media, Dullaert and Mewis [26] established a general structural dynamics model describing the flow behaviour of thixotropic systems. Barnes [27] described the history of the development of thixotropy and its understanding by the scientific community today. Roussel et al. [28] found that the maximum critical strain is associated with colloidal interactions between cement particles, whereas the minimum critical strain is associated with early hydrates. Mewis and Wagner [29] classified and evaluated existing rheological models for thixotropic suspensions. Xue et al. [30] investigated the rheological behaviour of CPB made over time. It was from ultrafine tailings, and it determined the effects of high temperature, solid content, cement content, and binder type.

The rheological behaviour of CPB is consistent with the Bingham and Herschel-Buckley (H-B) models, but these models do not account for time dependence [31,32]. Time-dependent rheological behaviour has an important influence on the pipeline transportation of CPB [17,33,34]. Researchers have found that the yield stress and viscosity change over time [17,30,33], without accounting for the time dependence leads to inaccurate calculations of losses due to pipeline transmission resistance.

The objective of the present study was to clarify the coupling effect of time and solid content on the rheological behaviour of CPB. Experiments were performed with different solid contents and shear rates, and the evolution in the shear stress over time with different solid contents was analysed. A model was developed for calculating the pipeline resistance considering the effects of time and solid content on rheological behaviour and evaluated against industrial test data.
2. Materials and methods

2.1. Materials

CPB was mixed from unclassified tailings (i.e., all particle sizes) from Gaoguanying Iron Mine in Tangshan City, Hebei Province, China; P.O 42.5 Portland cement according to the China Common Portland Cement Standard; and tap water. A laser-diffraction particle size analyzer (Malvern) was used to detect the particle size distribution of the tailings and cement, which is shown in Fig. 1. The important particle size parameters for the unclassified tailings were \(d(50) = 13.37 \, \mu m\) and \(d(90) = 35.58 \, \mu m\). The important particle size parameters for the cement were \(d(50) = 14.82 \, \mu m\) and \(d(90) = 42.94 \, \mu m\). Table 1 presents the chemical composition of the tailings.

![Fig. 1. Particle size distribution](image)

<table>
<thead>
<tr>
<th>Chemical composition</th>
<th>TFe</th>
<th>SiO₂</th>
<th>CaO</th>
<th>MgO</th>
<th>Al₂O₃</th>
<th>Others</th>
</tr>
</thead>
<tbody>
<tr>
<td>Content (%)</td>
<td>6.00</td>
<td>67.58</td>
<td>4.04</td>
<td>5.60</td>
<td>7.30</td>
<td>9.48</td>
</tr>
</tbody>
</table>

2.2. Sample Preparation

The CPB experimental samples were prepared with a solid content of 60%, 62%, 64%, and 66% and a cement-tailing (c/t) ratio of 1:4. Previous exploratory experiments showed that these proportions resulted in good fluidity and stability. The cement, tailings, and tap water were poured into a container and then stirred for 3 min to produce the CPB samples.
2.3. Apparatus and experimental procedures

The thixotropic behaviour of CPB reflects the response of the internal structure to time and shear action [35,36]. Some scholars believe that the thixotropic area can indicate the structural buildup of cement paste [37]. However, the thixotropic area only provides a qualitative analysis of the thixotropic behaviour of CPB; it cannot provide an accurate calculation of the time-dependent rheological behaviour. Some researchers have found that shear stress curves with time can be obtained at a fixed shear rate and that two shear stresses can be obtained before and after thixotropy [38,39]. However, if regression analysis is applied to shear stress at multiple shear rates, the yield stress can be obtained, as shown in Fig. 2. The different transverse coordinates and shear stresses show the effects of time and shear rate on the rheological behaviour of CPB.

The rheological behaviour of CPB was tested using a rheometer (HAAKE Viscotester IQ) and a four-bladed rotor (FL22 4B/SS-01170440). After the material stabilised, the rotor was slowly placed in the sample. The test started with the controlled shear rate method. The change in shear stress with time was obtained by collecting data (i.e., shear rate, shear stress, and time) with a computerised monitoring system. The shear rate was set to 30 s$^{-1}$, 60 s$^{-1}$, 90 s$^{-1}$, and 120 s$^{-1}$, and the shear time was set to 1000 s. The measurement was repeated with reconstituted samples until a reproducible stable result was obtained.

3. Results and discussion

3.1. Effect of time and solid content on shear stress

Fig. 3 shows the changes in shear stress of CPB with different solid content with time under the condition of a shear rate of 60 s$^{-1}$. As shown in Fig. 3(a), when the shear rate was fixed, the floc network structure of CPB was gradually destroyed with increasing shear time, and the shear stress gradually decreased. The shear stress stabilised at a certain value when the shear damage
and CPB recovery reached equilibrium. The rheological behaviour did not show significant time dependence at low solid contents. As the solid content increased, the internal particle content increased, which made the floc network structure more stable, and the shear stress increased.

The change in shear stress before and after thixotropy can be used to characterise CPB [30] and is given by

\[ \Delta \tau = \tau_1 - \tau_2 \]  

Where \( \Delta \tau \) (Pa) is the change in shear stress, \( \tau_1 \) (Pa) is the shear stress before thixotropy, and \( \tau_2 \) (Pa) is the shear stress after thixotropy. Fig. 3(b) shows that increasing the solid content stabilised the internal floc network structure of the CPB and increased thixotropy. At solid contents of 60% and 62%, the high free water content meant that the liquid network was the main structure and thixotropic behaviour was not clear. After thixotropy, the change in shear stress increased from 39 to 50 Pa, using Eq. (2) to calculate the shear stress growth rate, which is 28%. When the solid content was 64% and 66%, the floc network was the main structure, and thixotropic behaviour produced qualitative change. After thixotropy, the shear stress increased from 159 to 375 Pa for a growth rate of 136%. Thus, the interparticle floc network structure was the main influence on the thixotropy of CPB.

\[ p = \frac{\Delta \tau}{\tau_1} \]  

where \( p \) (%) is the growth rate, \( \Delta \tau \) (Pa) is the change in shear stress, and \( \tau_1 \) (Pa) is the shear stress before thixotropy.

![Fig. 3. Stress relaxation curves at different solid contents: (a) stress relaxation curve and (b) key parameters analysis](image)

**3.2. Effect of solid content on initial shear stress in CPB**

The initial stress indicated the initial structural stability. Fig. 4 shows when the solid content is low, the interior of CPB is mainly a liquid network structure, and the liquid network structure is unstable, so the shear stress is low. With the increase of solid content, the content of solid
particles in CPB increases, and the floc network structure formed by solid particles increases, so the overall structure of the CPB is more stable and generates larger shear stress. Eq. (3) was used in regression analysis to obtain the relationship between the initial stress and solid content:

$$\tau_i = a \times c^m$$  \hspace{1cm} (3)

where $\tau_i$ (Pa) is the initial stress, $a$ and $m$ are model parameters, and $c$ (%) is the solid content. Table 2 presents the regression parameters.

![Fig. 4. Initial stress versus solid content](image)

<table>
<thead>
<tr>
<th>Shear rate (s$^{-1}$)</th>
<th>$a$</th>
<th>$m$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30  \text{ s}^{-1}</td>
<td>$3.8 \times 10^{-30}$</td>
<td>17.66</td>
<td>0.99</td>
</tr>
<tr>
<td>60  \text{ s}^{-1}</td>
<td>$1.78 \times 10^{-30}$</td>
<td>17.88</td>
<td>0.99</td>
</tr>
<tr>
<td>90  \text{ s}^{-1}</td>
<td>$2.14 \times 10^{-32}$</td>
<td>18.98</td>
<td>0.99</td>
</tr>
<tr>
<td>120 \text{ s}^{-1}</td>
<td>$1.71 \times 10^{-32}$</td>
<td>19.08</td>
<td>0.99</td>
</tr>
</tbody>
</table>

### 3.3. Effect of shear rate on shear stress

As shown in Fig. 5(a), with the increase in shearing time, the internal structure of CPB is gradually destroyed, resulting in a gradual decrease of shear stress, and when the equilibrium between failure and recovery is reached, the shear stress remains stable. When the shear rate is large, the CPB is subjected to a large shearing action, resulting in a large shear stress. From Fig. 5(b), with the increase in shear rate, CPB is subjected to greater shearing action, and the internal floc network structure is damaged more seriously, so the change of shear stress is greater. When the shear rate was increased from 90 to 120 s$^{-1}$, the change in stress increased from 375 to 497 Pa for a growth rate of only 33%. This indicates that the shear rate has less influence on the thixotropy of CPB than the solid content.
3.4. Coupling effect of time and solid content on rheological behaviour

CPB is commonly regarded as a Bingham fluid. Thus, the resulting rheological curve (Fig. 6) was regressed by using the Bingham model [30], which is given by

\[ \tau = \tau_0 + \eta \dot{\gamma} \]  

(4)

where \( \tau \) (Pa) is the shear stress, \( \tau_0 \) (Pa) is the yield stress, \( \eta \) (Pa·s) is the plastic viscosity, and \( \dot{\gamma} \) (s\(^{-1}\)) is the shear rate.

To optimise the amount of data, a point was selected every 100 s for regression analysis. In total, 44 regression analyses were performed. The yield stress and plastic viscosity were obtained at different times.

Fig. 7 shows the yield stress and plastic viscosity curves over time with different solid contents. As the shear time increased, the yield stress and plastic viscosity gradually decreased and then stabilised.

The changes in the yield stress and plastic viscosity over time were observed to show similar patterns. Eq. (5) was constructed to characterise the change in yield stress over time:

\[ \tau_0 = k_1 + k_2 \times k_3^t \]  

(5)

where \( \tau_0 \) (Pa) is the yield stress, \( k_1, k_2 \) and \( k_3 \) are regression parameters related to the yield stress, and \( t \) (s) is the time. Table 3 presents the regression parameters of the yield stress.

Eq. (6) was constructed to characterise the change in plastic viscosity over time:

\[ \eta = s_1 + s_2 \times s_3^t \]  

(6)

where \( \eta \) (Pa·s) is the plastic viscosity, \( s_1, s_2 \) and \( s_3 \) are regression parameters related to the plastic viscosity, and \( t \) (s) is the time. Table 4 presents the regression parameters of the plastic viscosity.
Fig. 6. Stress relaxation curves at different solid content: (a) 60%, (b) 62%, (c) 64% and (d) 66%

Fig. 7. Curves of the (a) yield stress and (b) plastic viscosity with time
Correlation analysis revealed strong correlations between the solid content and the parameters $k_1$, $k_2$, $s_1$, and $s_2$. Meanwhile, $k_3$ and $s_3$ were constants independent of the solid content, as shown in Fig. 8. The relationships between the regression parameters and solid content are given in Eqs. (7)-(10):

\[
\begin{align*}
  k_1 &= 2.29c^2 - 255.90c + 7133.36 \quad (7) \\
  k_2 &= 3.38c^2 - 406.42c + 12243.53 \quad (8) \\
  s_1 &= 0.016c^2 - 1.88c + 55.26 \quad (9) \\
  s_2 &= 0.05c^2 - 5.95c + 177.40 \quad (10)
\end{align*}
\]

where $k_1$ and $k_2$ are regression parameters related to the yield stress, $s_1$ and $s_2$ are regression parameters related to the plastic viscosity, and $c$ (%) is the solid content. The correlation coefficients $R^2$ for the regression parameters $k_1$, $k_2$, $s_1$, and $s_2$ were 0.99, 0.99, 0.96, and 0.99, respectively, which indicates high correlation. Eqs. (7) and (8) were substituted into Eq. (5) to obtain the coupling effect of time and solid content on the yield stress:

\[
\tau_0 = 2.29c^2 - 255.90c + (3.38c^2 - 406.42c + 12243.53) \times 0.99' + 7133.36 \quad (11)
\]

Eqs. (9) and (10) were substituted into Eq. (6) to obtain the coupling effect of time and solid content on the plastic viscosity:

\[
\eta = 0.016c^2 - 1.88c + (0.05c^2 - 5.95c + 177.40) \times 0.99' + 55.26 \quad (12)
\]
3.5. Coupling effect of time and solid content on pipeline resistance

3.5.1. Calculation model

The Bingham equation expresses the relationship between the rheological parameters and pipeline resistance. The pipeline resistance can be calculated as follows [39-41]:

$$ i = \frac{16}{3D} r_0 + \frac{32\nu}{D^2} \eta $$

However, Eq. (13) does not consider the effect of thixotropy on the rheological behaviour of CPB. Eqs. (11) and (12) were inserted into Eq. (13) to obtain an equation for calculating the pipeline resistance considering time and solid content:

Fig. 8. Regression parameters versus solid content. (a) $k_1$ versus solid content, (b) $k_2$ versus solid content, (c) $s_1$ versus solid content, and (d) $s_2$ versus solid content.
\[ i = \frac{16}{3D} \left\{ 2.29c^2 - 255.90c + (3.38c^2 - 406.42c + 12243.53) \times 0.99' + 7133.36 \right\} + \frac{32\nu}{D^2} \left\{ 0.016c^2 - 1.88c + (0.05c^2 - 5.95c + 177.40) \times 0.99' + 55.26 \right\} \]  (14)

### 3.5.2. Model applicability

Experimental data were collected from the literature to verify the applicability of the proposed model given by Eq. (14) [42]. The solid content was 70%, the pipe diameter was 150 mm, and the pipe length was 250 m. Then, the Bingham equation and Eq. (14) were each used to calculate the pipeline resistance. However, the hydraulic gradient calculated by Eq. (14) is a variable value, so Eq. (15) is used to calculate the average hydraulic gradient. The average hydraulic gradient was calculated as follows:

\[ i_a = \frac{\sum_{k=1}^{n} i_k}{n} \]  (15)

where \( i_a \) (Pa·m\(^{-1}\)) is the average hydraulic gradient, \( i_k \) (Pa·m\(^{-1}\)) is the hydraulic gradient at certain intervals, and \( n \) is the number of hydraulic gradient calculations.

Table 5 presents the results. Some errors were observed because of the different materials used. The Bingham equation had an average error of 57.82%, whereas Eq. (14) had an average error of 32.91%. Thus, the proposed model was demonstrated to be more accurate than the Bingham equation. When the flow rate was low, Eq. (14) resulted in a larger error than the Bingham equation. When the flow rate was large, Eq. (14) resulted in a smaller error than the Bingham equation. This can be explained by the thixotropic behaviour of CPB being more pronounced at higher flow rates. Therefore, the proposed model is suitable for working conditions with high flow rates.

### Table 5

<table>
<thead>
<tr>
<th>Velocity (m·s(^{-1}))</th>
<th>Measured values (Pa·m(^{-1}))</th>
<th>Bingham equation (Pa·m(^{-1}))</th>
<th>Error value (%)</th>
<th>Average error (%)</th>
<th>Eq. (14) (Pa·m(^{-1}))</th>
<th>Error value (%)</th>
<th>Average error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.70</td>
<td>3932.00</td>
<td>3662.59</td>
<td>6.84</td>
<td>57.82</td>
<td>2361.83</td>
<td>39.94</td>
<td></td>
</tr>
<tr>
<td>1.40</td>
<td>4709.00</td>
<td>6772.25</td>
<td>43.81</td>
<td></td>
<td>4995.45</td>
<td>6.07</td>
<td></td>
</tr>
<tr>
<td>2.10</td>
<td>5612.00</td>
<td>9881.91</td>
<td>76.07</td>
<td></td>
<td>7257.78</td>
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<tr>
<td>2.80</td>
<td>6351.00</td>
<td>12991.57</td>
<td>104.55</td>
<td></td>
<td>9926.84</td>
<td>56.30</td>
<td></td>
</tr>
</tbody>
</table>

### 4. Conclusions

The present study investigated the influence of time and solid content on the rheological behaviour of CPB. A calculation model was developed that considers the coupling effect of time and solid content on the pipeline resistance. The main conclusions are as follows:
(1) At a constant shear rate, the shear stress gradually decreased with time and eventually stabilised. When the solid content is constant, the greater the shear rate, the greater the shear stress.

(2) In the range of solid content of 60%~66%, the thixotropy of CPB is mainly affected by the solid content. When the solid content is low, the thixotropy of CPB is small, and when the solid content is high, the thixotropy is large. The yield stress and plastic viscosity gradually decreased with increasing shear time and eventually stabilised.

(3) A model based on the Bingham equation was developed for calculating the resistance to pipeline transportation considering thixotropy and validated. According to the calculation results, the model in this paper is more suitable for the calculation of pipeline transportation resistance of CPB with a high concentration and flow rate of 1~2 m/s, and has certain reference significance for mine filling under this working condition.

Despite the obtained results, there are still some limitations due to the limitation of materials. Further research is necessary to investigate the effect of thixotropy on the rheological behaviour of CPB. Future work is required to investigate the mechanisms of thixotropy at the microscale and to develop numerical models for quantitative analysis of the thixotropy of CPB.

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References


