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Optimal Fiscal Policy in a Small Open Economy: Insights from the Growth Model with Human Capital and Public Debt

Michał Konopczyński*

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Abstract

This paper investigates the linkages between economic growth and fiscal policy under perfect capital mobility. The model incorporates wide range of fiscal policy instruments: the budget deficit, the structure of public debt, public expenditures on education, public consumption, and four tax rates. We prove that two tax rates – on consumption and interest on government bonds held by domestic lenders – are neutral for economic growth: both for the balanced growth path (BGP), and for transitory dynamics. All other parameters of fiscal policy are not neutral. Theoretical results are illustrated with an empirical analysis for Poland based on post-global financial crisis data for the Polish economy (2009–2018). Numerical simulations show that if fiscal policy remains unchanged, Polish economy will converge to the BGP with GDP growing at 2.3%. The best way to accelerate growth is to increase public investment in education. The other budgetary policy instruments are less effective in shaping economic growth.

Keywords: optimal fiscal policy, economic growth, human capital, budget deficit, public debt

JEL Classification: E62, F43, H6, H52, H63, I28

*Poznań University of Economics, Poland; e-mail: michal.konopczynski@ue.poznan.pl; ORCID: 0000-0003-1955-2027

1 Introduction

Theoretical literature on fiscal policy in the growth theory of an open economy is not large (for detailed overview see Konopczyński 2015). It is traditionally based on endogenous growth models with Ramsey-type utility function. Precursors include Nielsen and Sorensen (1991), Rebelo (1992) and Razin and Yuen (1994, 1996), who study dynamic effects of various forms of capital income taxation under perfect capital mobility, and Asea and Turnovsky (1998), who analyze the effects of capital income taxation on economic growth in a stochastic model of a small open economy (SOE). One of the most significant contributions is the monograph by Turnovsky (2009), based on his earlier extensive research. It contains a selection of models of optimal fiscal policy in the SOE under perfect or imperfect mobility of capital. He examines productive government expenditures and 3 types of taxes: taxes on consumption, production and foreign debt of the private sector. Important qualitative differences between closed economy and SOE are exposed. For example, the capital income tax ceases to have any effect on the long-run growth rate of the economy. The equilibrium growth rate is independent of almost all fiscal instruments, including public expenditures. The only tool of fiscal policy that is not neutral is the tax rate on foreign interest income. Another important contribution is Fisher (2010), who investigates fiscal policy shocks in a SOE growth model, where domestic capital accumulation, subject to installation costs, is the engine of economic growth.

While these open-economy models provided valuable insight into the long-run effects of taxes and public spending, they typically suffer from one severe oversimplification: the assumption of permanently balanced government budget (zero deficit and debt), or – at best – incomplete picture of fiscal policy. This unfortunate tradition is likely inherited from closed economy models, where usually Ricardian equivalence holds, and hence the budget deficit is neutral for the long-run rate of growth. To give some more examples, Schmitt-Grohe and Uribe (2003), and Assibey-Yeboah and Mohsin (2014) abstract completely from fiscal policy, i.e., there is no room for public sector. Chatterjee and Turnovsky (2007) assume that the government behaves passively: it runs "continuously balanced budget", and hence there is no public debt, which stands in sharp contrast to the private sector that can borrow abroad. Similarly, Fisher (1995), and Fisher and Terrell (2000), using their own words, "abstract from government policy, there is no distinction between private sector and sovereign' debt". Certainly, there are some exceptions. For example, Turnovsky (2002) argues that with the proportional tax on capital Ricardian equivalence does not hold any more. Despite that in his model fiscal policy remains largely neutral, because "an increase in the tax on capital reduces the growth rate of capital, but leaves the growth rate of consumption unaffected." Another exception is Konopczyński (2014b) who investigates the implications of the size and structure of budget deficit in an open economy under perfect capital mobility, and Konopczyński (2018), who takes the model to the world of imperfect capital mobility. Both these papers clearly prove that, in the SOE context, disregarding government deficit and public debt is unjustified: both the deficit and the debt structure are not neutral for the long-run growth and welfare.

This paper presents a significant modification and generalization of existing models, particularly those presented by Turnovsky (2009) and Konopczyński (2014b). We grant the government an active role by incorporating a broad spectrum of fiscal policy tools. In particular, we assume that public expenditures may exceed revenues (in the growing economy even permanently), whereas public deficit is financed by domestic and foreign debt, independently of the private sector's foreign debt. Human capital is introduced as a separate factor of production, which allows us to investigate the role of public spending on education. There are four types of taxes: on labor, capital, consumption and interest on government bonds held by domestic investors. Public expenditures comprise three categories: public consumption, education, and financial transfers. Incorporating so many elements together in a single setting is novel in theoretical literature. There is a cost to this: our model is so complex that it can be solved only numerically. Because of this complexity, unlike many of Turnovsky's elegant models, the balanced growth path (BGP) is not guaranteed, and if it exists, it may not be unique. We show that the existence and uniqueness of the BGP depends on the values of economic parameters. In particular, an inappropriate fiscal policy may destabilize the economy. However, we have verified that for a relatively wide range of realistic sets of parameter values the economy converges towards a unique BGP along the saddle path. The sensitivity of the BGP can only be investigated numerically. Nevertheless, we put forward some intriguing propositions regarding fiscal policy, which are not present in the existing literature. In particular, in our model two tax rates are neutral for economic growth – on consumption and interest on government bonds: both for the BGP, and for transitory dynamics. All other parameters of fiscal policy are not neutral: any modification of their values shifts the BGP, and also changes the transitory dynamics. In the empirical part of the paper we illustrate these theses with the case of Poland.

The paper is organized as follows. Section 2 presents the details of the model. In Section 3 we derive the balanced growth path and analyze its mathematical properties: existence, uniqueness and stability. Section 4 clarifies the properties of the trajectory of consumption, some of which may be surprising for closed economy modelers. In Section 5 the model is calibrated for Polish economy over the period 2009–2018. Section 6 outlines the baseline scenario. In Section 7 we search for the optimal parameters of fiscal policy. Section 8 summarizes the main theoretical and empirical results. Mathematical proofs are included in the appendix.

2 The model

2.1 Output and factors of production

The output of a representative firm is described by the production function:

$$Y_i = F(K_i, L_i) = AK_i{}^{\alpha}(hL_i)^{\beta}, \quad \text{with} \quad \alpha + \beta = 1,\ \alpha, \beta > 0,\ A > 0, \tag{1}$$

where i is the firm index, K_i denotes the stock of physical capital, L_i represents raw labor, and h is the average stock of human capital per worker: $h = H/L$. Obviously, the aggregate output of the entire economy is: $Y = AK^{\alpha}(hL)^{\beta} = AK^{\alpha}H^{\beta}$, where K is the aggregate stock of capital and L is the supply of labor in the country, by assumption growing exponentially: $L = L_0e^{nt}$. Dividing both sides by L yields the per capita production function:

$$y = \frac{Y}{L} = Ak^{\alpha}h^{\beta}, \tag{2}$$

where $k = K/L$. Firms are maximizing profits in perfectly competitive markets, which implies that the marginal product of capital is equal to the real rental rate:

$$\forall t \quad \frac{\partial Y_i}{\partial K_i} = \alpha AK_i^{\alpha-1}(hL_i)^{\beta} = \frac{\alpha Y_i}{K_i} = w_K = r + \delta_K \tag{3}$$

and, simultaneously, the marginal product of labor equals the real wage rate:

$$\forall t \quad \frac{\partial Y_i}{\partial L_i} = \beta hAK_i^{\alpha}(hL_i)^{\beta-1} = \frac{\beta Y_i}{L_i} = w_L. \tag{4}$$

It follows that

$$\forall t \quad w_K = \frac{\alpha Y}{K} = \frac{\alpha y}{k}, \tag{5}$$

$$\forall t \quad w_L = \frac{\beta Y}{L} = \beta y, \tag{6}$$

so that $\forall t\ w_K k + w_L = y$. The accumulation equations are:

$$\dot{K} = I_K - \delta_K K, \quad 0 < \delta_K < 1, \tag{7}$$

$$\dot{H} = I_H - \delta_H H, \quad 0 < \delta_H < 1, \tag{8}$$

where δ_K and δ_H denote depreciation rates. In per capita terms,

$$\dot{k} = i_K - (n + \delta_K)k, \tag{9}$$

$$\dot{h} = i_H - (n + \delta_H)h. \tag{10}$$

Investment in physical capital is subject to quadratic adjustment costs, which means that in order to attain net investment equal to I_K, one needs to expend

$$\Phi(I_K, K) = I_K \left(1 + \frac{\chi}{2}\frac{I_K}{K}\right), \quad \text{with} \quad \chi > 0. \tag{11a}$$

In per capita terms:

$$\phi(i_K, k) = i_K \left(1 + \frac{\chi}{2}\frac{i_K}{k}\right), \quad \text{where} \quad \chi > 0. \tag{11}$$

We assume that investment in physical capital is financed by the private sector, whereas investment in human capital is financed by the government only. This assumption is motivated by empirical evidence for Poland and many other countries, where private spending on education is very small compared to public expenditures. In Poland, according to Eurostat, private spending on education (all levels) over the last 2 decades was around 0.6–0.7% of GDP, whereas public expenditures on education are equal to 5.0–5.5% of GDP. Similar differences appear in most European countries. We assume the following "linear production function" of new human capital:

$$I_H = \varepsilon G_E, \quad 0 < \varepsilon < 1, \tag{12}$$

where G_E represents public spending on education.

2.2 The public sector (the government)

The total tax revenue of the government in real terms is:

$$T = \tau_L w_L L + \tau_K w_K K + \tau_C C + \tau_D r D_D, \tag{13}$$

where τ_L, τ_K, τ_C, τ_D are the average tax rates on wages, capital income, consumption, and interest on government bonds held by domestic lenders, respectively. The deficit of the public sector is the difference between total government spending and tax revenue, i.e. in real terms: $J = G + rD - T$, where G is total government spending and D represents total public debt. We assume that the budget deficit is a fixed percentage of GDP, i.e., $J = \xi Y$, where $\xi = const > 0$ is a decision parameter. Therefore, the budgetary rule can be written as:

$$G = T - rD + \xi Y. \tag{14}$$

The deficit is financed by government bonds, which raises the public debt according to the equation: $\dot{D} = \xi Y$. Certain percentage (ω) of bonds is sold to foreign investors, and the remainder is purchased by domestic lenders, i.e.

$$\dot{D}_F = \omega \xi Y, \tag{15}$$

$$\dot{D}_D = (1 - \omega)\xi Y, \tag{16}$$

where D_D and D_F represent domestic and foreign debt of the government, respectively. The government spending consists of three components:

$$G = G_T + G_E + G_C, \tag{17}$$

where G_T denotes cash transfers to the private sector (mainly social transfers, i.e. pensions, various benefits, social assistance etc.), and G_C is public consumption (mainly health care, national defense, and public safety). By assumption, public consumption is proportional to private consumption, whereas expenditures on education are fixed to GDP:

$$G_C = \sigma_C C, \quad 0 < \sigma_C < 1, \tag{18}$$

$$G_E = \gamma_E Y, \quad 0 < \gamma_E < 1. \tag{19}$$

Note that strict deficit rule implies that cash transfers G_T serve as a balancing item. In particular, if the government decides to spend more on education or public consumption (given everything else unchanged), it will entail a reduction in cash transfers. Theoretically, G_T may be a negative number (though it is highly unrealistic): in such case G_T should be interpreted as additional taxation.

2.3 The private sector

The preferences of the representative household are expressed by the intertemporal utility function:

$$U = \int_0^{\infty} \frac{1}{\gamma} \left(c g_C^{\kappa}\right)^{\gamma} e^{-(\rho-n)t} dt, \quad \rho > 0,\ \rho > n, \tag{20}$$

where c denotes private consumption and g_C is public consumption. The elasticity of substitution between both types of consumption is expressed by $\kappa > 0$. A fraction $1/(1-\gamma)$ is equal to the intertemporal elasticity of substitution. We assume that $\gamma < 0$, which is justified on the basis of empirical research; see e.g., Turnovsky (2009), p. 177. The effective rate of discount equal to $\rho - n$ is adopted from Acemoglu (2008), p. 310. It reflects the assumption that a household derives utility from its own consumption and also from the consumption of its descendants (children, grandchildren, etc.), the number of which is growing at an annual rate n. We assume that $\rho > n$. Otherwise, the integral in (20) would not be convergent. Note that public expenditures on education do not appear in the utility function. It reflects the idea that households treat education as a factor of production (just like capital) rather than a utility-enhancing item.

The private sector receives income in the form of remuneration of labor and capital, the interest on domestic public debt, profits on foreign assets B, and cash transfers from the government. The private sector's real disposable income after taxes is defined as:

$$Y_d = (1-\tau_L) w_L L + (1-\tau_K) w_K K + (1-\tau_D) r D_D + rB + G_T. \tag{21}$$

This income is spent on consumption and investment, as well as purchases of government bonds. Any difference is covered by (net) lending/borrowing to/from abroad. Therefore, the instantaneous budget constraint in real terms is expressed as follows: $Y_d = C(1+\tau_C) + \Phi(I_K, K) + \dot{D}_D + \dot{B}$. Substituting Equation (16), and rearranging yields: $\dot{B} = Y_d - C(1+\tau_C) - \Phi(I_K, K) - (1-\omega)\xi Y$. Using Equations (11a), (21), (5) and (6), this budget constraint can be transformed into the per capita form:

$$\dot{b} = (1-\tau_L)\beta y + (1-\tau_K)\alpha y + (1-\tau_D) r d_D + [r-n]b + \\ + g_T - c(1+\tau_C) - i_K\left(1 + \frac{\chi}{2}\frac{i_K}{k}\right) - (1-\omega)\xi y. \quad (22)$$

It is worth emphasizing that the representative agent treats all fiscal variables as exogenous, reasonably assuming that his individual influence on the market is negligible. In particular, when making decisions, he respects the budget constraint (22) treating g_T and d_D as constants.
The private sector chooses its flows of consumption and investment so as to maximize the level of utility expressed by Equation (20), subject to the budget constraint (22). The initial values of variables (endowments) are given by b_0, $k_0 > 0$, $d_0 \geq 0$, $d_{F0} \geq 0$, $d_{D0} \geq 0$ with $d_{F0} + d_{D0} = d_0$. The following fiscal variables are treated by an individual decision-maker as exogenous: g_T, g_C, d_D, d_F. The current value hamiltonian is:

$$H_c = \frac{1}{\gamma}\left(c g_C^{\kappa}\right)^{\gamma} + \lambda_1 \cdot [(1-\tau_L)\beta y + (1-\tau_K)\alpha y + (1-\tau_D) r d_D + [r-n]b + g_T + \\ - c(1+\tau_C) - i_K\left(1+\frac{\chi}{2}\frac{i_K}{k}\right) - \ (1-\omega)\xi y] + \lambda_2 \cdot [i_K - (n+\delta_K)k]. \quad (23)$$

The solution of this optimization problem (details in Appendix A) boils down to the following system of two (non-linear, autonomous) differential equations:

$$\begin{bmatrix} \dot{q}_K \\ \dot{\underline{k}} \end{bmatrix} = \begin{bmatrix} f^1(q_K, \underline{k}) \\ f^2(q_K, \underline{k}) \end{bmatrix} \quad (24)$$

where $\underline{k} = k/y$ denotes the ratio of capital to GDP. The ratio of shadow prices $q_K = \lambda_2/\lambda_1$ represents the market price of capital in relation to the market price of foreign assets. For clarity, let us write down Equation (24) in an explicit form:

$$\begin{bmatrix} \dot{q}_K \\ \dot{\underline{k}} \end{bmatrix} = \begin{bmatrix} [r+\delta_K]\, q_K - (q_K-1)^2/2\chi - W_2\alpha/\underline{k} \\ \beta\left(\frac{q_K-1}{\chi} + \delta_H - \delta_K\right)\underline{k} - \beta\varepsilon\gamma_E A^{1/\beta}\underline{k}^{1/\beta} \end{bmatrix}, \quad (25)$$

where $W_2 = [(1-\tau_L)\beta + (1-\tau_K)\alpha - (1-\omega)\xi] = const.$

3 The balanced growth path (BGP)

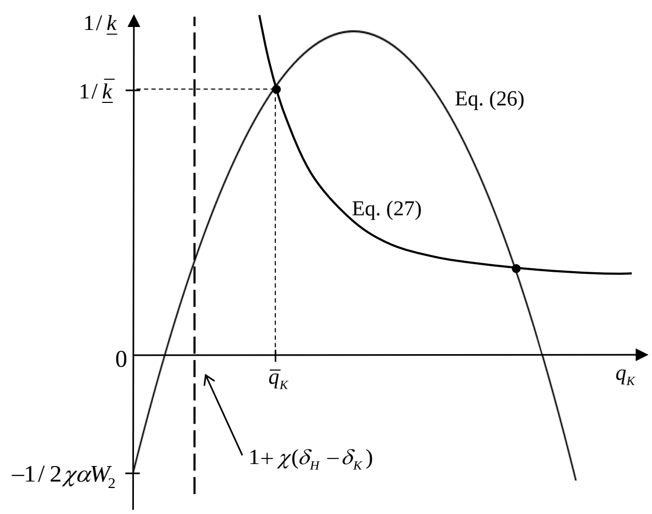
3.1 Existence and uniqueness

In order to find the steady state one must solve the system of equations: $f^i(q_K, \underline{k}) = 0$ $(i = 1, 2)$. Unfortunately, due to its complexity, it cannot be solved analytically. (In other words, an explicit form of the steady-state solution does not exist.) Moreover, potential problems of non-existence or non-uniqueness exist and cannot be eliminated by any simple assumptions. To see why, first notice that $f^1 = 0$ yields (throughout the paper bars over variables denote their steady-state values):

$$\frac{1}{\bar{\underline{k}}} = \frac{r + \delta_K}{\alpha W_2}\bar{q}_K - \frac{(\bar{q}_K - 1)^2}{2\chi\alpha W_2}, \tag{26}$$

which means that there is a parabolic relationship between $1/\underline{k}$ and q_K (see Figure 1).

Figure 1: The steady state(s)



Second, notice that $f^2 = 0$ yields:

$$\frac{1}{\bar{\underline{k}}} = \left(\frac{\bar{q}_K - 1 + \chi(\delta_H - \delta_K)}{\chi\varepsilon\gamma_E A^{1/\beta}}\right)^{-\beta/\alpha}, \tag{27}$$

which generates a hyperbolic relationship between $1/\underline{k}$ and q_K (see Figure 1). These two curves may intersect once, twice, or have no intersection – depending on the values of parameters of the model. If they intersect once (in other words, if they are tangent to each other), then the steady state is unique. If they intersect twice, then there are 2 different steady states. Finally, if they do not intersect, then a steady state does not exist.

Numerical experiments (presented below) suggest that in case of 2 different steady states, the transversality condition is satisfied for one of them only: the one located to the left. For the lack of space, we will leave this issue without further mathematical investigation.

In practice, in order to find the balanced growth path one must solve the system of Equations (26) and (27) numerically, which is doable once numbers are substituted for all parameters. Obviously, knowing $\bar{q}_K$ and $\bar{\underline{k}}$ allows a straightforward derivation of the steady-state values of all other variables (in other words, the trajectories along the balanced-growth path). For example, the balanced growth rate (the BGR, hereafter labeled $\bar{\varphi}$) can be determined from Equation (A.9):

$$\bar{\varphi} = \bar{\varphi}_y = \bar{\varphi}_k = \bar{\varphi}_h = \frac{(\bar{q}_K - 1)}{\chi} - (n + \delta_K). \tag{28}$$

From Equations (15) and (16), it follows that: $\hat{\underline{d}}_F = \widehat{D}_F - \widehat{Y} = \omega\xi/\underline{d}_F - n - \varphi_y$, $\hat{\underline{d}}_D = (1-\omega)\xi/\underline{d}_D - n - \varphi_y$. As $\dot{\underline{d}}_F = \hat{\underline{d}}_F \cdot \underline{d}_F$ and $\dot{\underline{d}}_D = \hat{\underline{d}}_D \cdot \underline{d}_D$, we have:

$$\dot{\underline{d}}_F = (-n - \varphi_y)\underline{d}_F + \omega\xi, \tag{29}$$

$$\dot{\underline{d}}_D = (-n - \varphi_y)\underline{d}_D + (1-\omega)\xi, \tag{30}$$

whereas the steady-state debt-to-GDP ratios are as follows:

$$\bar{\underline{d}}_F = \frac{\omega\xi}{n + \bar{\varphi}}, \tag{31}$$

$$\bar{\underline{d}}_D = \frac{(1-\omega)\xi}{n + \bar{\varphi}}. \tag{32}$$

3.2 Stability

Proposition 1 (Details in Appendix B)**.** *The decentralized equilibrium (the balanced growth path) has the form of the stable saddle path. The linear approximation of the model yields the following solution (trajectories):*

$$\begin{bmatrix} q_K & \underline{k} \end{bmatrix}^T = \begin{bmatrix} \bar{q}_K & \bar{\underline{k}} \end{bmatrix}^T + s_1 e^{r_1 t} \mathbf{v}^1, \tag{33}$$

where r_1 is the negative eigenvalue, and $\mathbf{v}^1$ is its corresponding eigenvector of the Jacobian matrix of Equation (24) calculated in the equilibrium. The unknown constant s_1 can be obtained by plugging the initial value (endowment) of the ratio of capital to output $\underline{k}(t=0) = \underline{k}_0$ into the second row of Equation (33), which results in the following equation: $\underline{k}_0 = \bar{\underline{k}} + s_1 v_2^1$. Knowing the value of s_1, the first equation of (33) yields the initial value of q_K: $q_{K0} = \bar{q}_K + s_1 v_1^1$.

3.3 Transversality conditions

In Appendix C we prove that the transversality condition (A.f) is satisfied if and only if

$$r > \bar{\varphi} + n, \tag{34}$$

which means that interest rate must be higher than the GDP growth rate along the balanced growth path. The transversality condition (A.e) determines the initial value of consumption:

$$c_0 = \left(b_0 - d_{F0} + \frac{\upsilon}{r - n - \bar{\varphi}} + \frac{\omega \xi y_0}{n + \bar{\varphi}} \right) \frac{r - n - \psi}{1 + \sigma_C}, \tag{35}$$

where $\psi = \dot{c}/c = (r - \rho)/[1 - (1 + \kappa)\gamma] = const$ is the rate of growth of consumption per capita (constant over time), and

$$\upsilon = \left(1 + \omega\xi - \gamma_E - \frac{r\omega\xi}{n + \bar{\varphi}} \right) y_0 - \left(\frac{\bar{q}_K^2 - 1}{2\chi} \right) k_0. \tag{36}$$

In Appendix D we show that condition (A.e) requires the following:

$$r > n + \psi, \tag{37}$$

which imposes a limit on the rate of growth of consumption per capita.

4 Consumption

In Appendix A we prove that the trajectory of private consumption has the following form:

$$c(t) = c_0 \cdot e^{\psi t}, \tag{38}$$

where $\psi = (r - \rho)/[1 - (1 + \kappa)\gamma] = const$ and c_0 is given by Equation (35) together with (36). Importantly, the trajectory of private consumption is a function of virtually all parameters of the economy, as well as initial endowments (b_0 and d_{F0}), with two noteworthy exceptions: τ_C and τ_D. These two tax rates are neutral for consumption trajectory (and thus for welfare). This property of the model is the result of the assumed set of fiscal rules. These tax rates may cease to be neutral, if the government changes its fiscal rules, or stops to comply with them.
There are 6 fiscal parameters which influence the trajectory of consumption: ω, ξ, σ_C, τ_L, τ_K, γ_E. An analytical sensitivity analysis is, however, not feasible, because the BGP can only be calculated numerically – as we concluded above. Thus, if we want to find out, how these parameters influence consumption – and ultimately welfare – first we need to calibrate the model, and then perform numerical sensitivity analysis. However, before we turn to this, we want to expose one important property of the model. Typically, in closed economy models, as well as in most open-economy models,

the BGP is characterized by identical rates of growth of all variables – in particular consumption grows at identical speed as output (GDP). This is not the case in our model, though. Let us explain this issue in detail.

4.1 Why consumption can (indefinitely) grow faster or slower than GDP?

In Appendix D we show that the trajectory of foreign assets along the BGP can be written as:

$$b(t) = \left(b_0 - \Delta d_{F0} + \frac{\upsilon}{r-n-\bar{\varphi}}\right) e^{\psi t} - \frac{\upsilon}{r-n-\bar{\varphi}} e^{\bar{\varphi} t} + \Delta d_{F0} e^{-nt}, \tag{39}$$

where $\Delta d_{F0} = d_{F0} - \omega\xi y_0/(n+\bar{\varphi})$ is the deviation of foreign debt ratio from its equilibrium level in period $t = 0$, and υ is given by Equation (36). It follows that

$$\widehat{b}(t) = \frac{(b_0 - \Delta d_{F0})\,\psi + [\upsilon k_0/(r-n-\bar{\varphi})]\left(\psi - \bar{\varphi} e^{(\bar{\varphi}-\psi)t}\right) - \Delta d_{F0} n e^{-(n+\psi)t}}{(b_0 - \Delta d_{F0}) + [\upsilon k_0/(r-n-\bar{\varphi})]\left(1 - e^{(\bar{\varphi}-\psi)t}\right) - \Delta d_{F0} e^{-(n+\psi)t}}. \tag{40}$$

The growth rate is not constant over time, except for one special case. For the purposes of interpretation, let us assume (realistically) that the rate of growth of GDP is positive, i.e.

$$n + \bar{\varphi} > 0. \tag{41}$$

Then, using L'Hospital's rule it is easy to show that:

A) if $\psi = \bar{\varphi}$, then $\widehat{b}(t) = \psi$,

B) if $\psi > \bar{\varphi}$, then $\widehat{b}(t) \overset{t\to\infty}{\longrightarrow} \psi$,

C) if $\psi < \bar{\varphi}$, then $\widehat{b}(t) \overset{t\to\infty}{\longrightarrow} \bar{\varphi}$.

Let us also see what happens over time with the ratio of foreign assets to GDP. Note that

$$\begin{aligned} \underline{b}(t) = \left(\underline{b}_0 - \underline{d}_{F0} + \frac{\upsilon}{(r-n-\bar{\varphi})y_0} + \frac{\omega\xi}{n+\bar{\varphi}}\right) e^{(\psi-\bar{\varphi})t} + \\ - \frac{\upsilon}{(r-n-\bar{\varphi})y_0} + \left(\underline{d}_{F0} - \frac{\omega\xi}{n+\bar{\varphi}}\right) e^{-(n+\bar{\varphi})t}. \end{aligned} \tag{42}$$

It follows that

A)
$$\psi = \bar{\varphi} \Rightarrow \lim_{t\to\infty} \underline{b}(t) = \underline{b}_0 - \underline{d}_{F0} + \frac{\omega\xi}{n+\bar{\varphi}} \tag{43a}$$

B)

$$\psi > \bar{\varphi} \Rightarrow \lim_{t\to\infty} \underline{b}(t) = +\infty \tag{43b}$$

C)

$$\psi < \bar{\varphi} \Rightarrow \lim_{t\to\infty} \underline{b}(t) = \frac{-\upsilon}{(r-n-\bar{\varphi})y_0} \tag{43c}$$

In case A the growth rate of consumption (both private and public) is exactly the same as that of GDP. In this case we can observe an interesting relationship between public finances and private sector assets. Put simply, any amount of money borrowed abroad by the government eventually becomes foreign assets of the private sector. Equation (31) implies that the right-hand side of implication (43a) may be written as $\lim_{t\to\infty}(\underline{b}(t) - \underline{b}_0) = \underline{d}_F - \underline{d}_{F0}$. This means that if the government's foreign debt rises over time by a certain number of percentage points, then private sector's foreign assets will over time rise by the exact same number of percentage points. Funds borrowed abroad by the government will be de facto accumulated by the private sector in the form of foreign assets. As a result, the net international investment position (NIIP) of the country does not change over time. Of course, this is only the special case which requires $\psi = \bar{\varphi}$. The other two cases are completely different.

In case B $\psi > \bar{\varphi}$, so that consumption grows faster than capital and production. Maintaining a high rate of consumption growth is possible through revenues from foreign assets accumulated by the private sector. Due to the relatively low rate of capital accumulation, balancing the flow of incomes with the flow of expenditures requires relatively low consumption in the initial phase – in particular c_0 must be relatively low. Consumers are sufficiently patient here (the discount ratio ρ is low enough). In the initial phase they accept low consumption, and patiently, year after year, invest their savings into foreign assets. After enough time their fate changes diametrically: they have accumulated enough wealth to be able to finance ever-growing consumption – and not only ever-growing, but growing faster than GDP.

As a real-world illustration, consider the case of Norway or Saudi Arabia: over the past few decades, consumption in these countries has grown more slowly than GDP. Both countries are still accumulating huge foreign assets: they invest in the US, EU, Japan, China, etc. One day oil and gas will disappear, but both countries will likely enjoy high consumption (perhaps growing even faster than GDP) financed by income from foreign assets: stocks, bonds, real estate, etc. In fact, in the extreme case, if they will accumulate enough foreign assets, they will not have to produce anything, they will just consume. After all, if a person can be a rentier, why not the whole country? Notice that the NIIP of Norway has already exceeded 300% of GDP! Rough estimate (based on "stylized facts"): if the GDP in developed countries is around 1/3 of the productive capital, and the capital share in GDP is about 40% (60% is the share of labor), then Norway already earns an equivalent of 40% of GDP from foreign assets. If Norway's NIIP continues to grow and one day reaches 1000% of GDP, the income from these assets will probably exceed GDP!

In case C consumption growth is slower than GDP: $\psi < \bar{\varphi}$. Consumers are impatient (the discount ratio ρ is high). In the initial phase they demand high consumption, which must be financed by borrowing abroad. Over time the stock of foreign assets $b(t)$ grows to infinitely large negative number. However, this does not mean that indebtedness in relation to GDP is rising indefinitely (that would violate the transversality conditions). The debt-to-GDP ratio stabilizes at a finite level expressed by formula (43c). Think of the United States as an illustration. Consumers in the last 3 decades have chosen to consume so much (plus the government chose to finance very costly wars) that the country (as a whole) had to borrow a lot from abroad. The US NIIP is deteriorating and in 2021 it is already minus 65% of GDP. There are, of course, countries with much greater debt, e.g. Ireland –170%, Greece –180% of GDP.

5 Calibration for Poland

Table 1 summarizes the first part of calibration. It contains the set of parameters and the initial values (endowments) together with short explanation. Generally, these values are based on statistics for the last decade, i.e. the period 2009–2018, which we consider the starting "point" (endowment). The data comes from the Eurostat database, the National Bank of Poland, the Central Statistical Office of Poland, the Kiel Institute for the World Economy, and some empirical literature regarding OECD countries.

Table 1: The first part of calibration

Parameters & endowments	Sources of data and explanations
Technology	
$\alpha = \frac{2}{3}$, $\beta = \frac{1}{3}$	The review of empirical literature: Mankiw, Romer, Weil (1992), Bernanke, Gurkaynak (2002), Willman (2002), Balisteri et al. (2003), and studies focusing on Poland: Cichy (2008) and Growiec (2012).
$\chi = 13$	The review of empirical and theoretical literature, e.g. Hayashi (1982), Caballero, Engel (1999), Cooper and Haltiwanger (2006).
$\delta_K = 4\%$ $\delta_H = 1.5\%$	δ_K is difficult to estimate for Poland, due to rapid economic transformation which resulted in huge amount of obsolete machinery, infrastructure, etc. inherited from the centrally "planned" economy. In various research papers regarding OECD countries, physical capital depreciation varies from approximately 3.5% to 7%. As the focus of our analysis is on the long run, we set the depreciation rate at a rather low level of $\delta_K = 4\%$. The rate of human capital depreciation is borrowed from Manuelli and Seshadri (2005) and Arrazola and de Hevia (2004).

Table 1: The first part of calibration cont.

Parameters & endowments	Sources of data and explanations
The utility function and demographics	
$\kappa = 0.27$	The average of values estimated by Turnovsky (1999, 2004), Park and Philippopoulos (2004), Dhont and Heylen (2009).
$\rho = 0.04$	The metanalysis by Nijkamp and Percoco (2006) of 42 previous analyses, and European Commission (2002).
$\gamma = -1$	The comprehensive meta-analysis by Havranek et al. (2013) of 169 previous analyses.
$n = 0\%$	Demographic forecasts for Poland published by the Central Statistical Office of Poland.
Fiscal policy	
$\sigma_C = 30.2\%$	According to Eurostat, during the period 2009–2018 public consumption as a share of GDP amounted to an average of 18.1%, while private consumption was on average 60.0% of GDP. Thus, $\sigma_C = 18.1\%/60.0\% = 30.2\%$.
$\gamma_E = 5.25\%$	Public expenditures on education (as percent of GDP) during the period 2009–2018.
$\xi = 3.8\%$	The average deficit of the public sector in the period 2009–2018 (according to Eurostat methodology).
$\omega = 0.483$	The average share of foreign debt in public debt during the period 2009–2018.
$\tau_K = 22.13\%$, $\tau_L = 19.38\%$, $\tau_D = 19\%$, $\tau_C = 19.79\%$	Calibrated so as to be consistent with the tax revenue statistics (shares of GDP; averages for 2009–2018) published by Eurostat.
The initial values (endowments)	
$k_0 = 300$	The initial stock of capital per capita is set arbitrarily (as a numeraire); 300 is convenient, as it yields $y_0 = 100$ and hence the initial values of all the other variables are identical to their percentage shares of GDP.
$\underline{b}_0 = -56.2\%$ $\underline{d}_{F0} = 25.2\%$	Statistical data for Poland published by the National Bank of Poland (NBP): net international investment position (NIIP) of the private sector and the public sector; the average values over 2009–2018.
$\underline{d}_{D0} = 26.9\%$	The difference between the public debt (the average value over 2009–2018, i.e. 52.1%) and the $\underline{d}_{F0}$.

To complete the calibration, first we must estimate the real rate of return on capital (r). From (3), it follows that $r = \alpha \cdot Y/K - \delta_K$. The ratio of Y/K is difficult to estimate for Poland – major problems have been exposed by Konopczyński (2014a). In short, the data available for Poland reflect a fraction of all productive capital: the "gross value of fixed assets". Therefore, following Konopczyński (2014a), we will apply the average ratio from the whole set of OECD countries in the Database on Capital Stocks in OECD Countries constructed by Kiel Institute for the World Economy, i.e., we set $Y/K = 1/3$. Thus $r = 1/3 \cdot 1/3 - 0.04 = 7.11\%$, which is very close to most long-run estimates for developed countries. For example, Campbell et al. (2001) report that the average real rate of return on stocks in the U.S. over the period 1900-1995 was 7%.

Let us do some growth accounting. Note that

$$\widehat{K} = \frac{\dot{K}}{K} = \frac{I_K}{K} - \delta_K = \frac{I_K}{Y}\frac{Y}{K} - \delta_K. \tag{44}$$

According to Eurostat, gross fixed capital formation in Poland in the period 2009–2018 was on average 19.45% of GDP. Thus, Equation (44) yields $\widehat{K} = 0.1945/3 - 0.04 = 2.48\%$. The average GDP growth rate in Poland during the period 2009–2018 was 3.46%. Knowing this, we can estimate the growth rate of human capital, on the basis of the aggregate production function $Y = AK^{\alpha}(hL)^{\beta} = AK^{\alpha}H^{\beta}$, which implies that $\widehat{H} = \frac{\widehat{Y} - \alpha\widehat{K}}{\beta} = \frac{3.46\% - \frac{1}{3}\cdot 2.48\%}{2/3} = 3.95\%$.

These figures imply that in the period 2009–2018, economic growth in Poland was driven primarily by fast accumulation of human capital, and only secondarily by the accumulation of physical capital. An impressive increase in human capital in Poland is a well-known "stylized fact" confirmed by sharp increase in the number of graduates, PhDs, etc. Moreover, note that Polish economy is not on the balanced growth path (BGP) yet, though it may be gradually converging to the BGP.

For simulations it is necessary to set the value of the total factor productivity A. Substituting (12) and (19) into Equation (8), and dividing both sides by H yields $\widehat{H} = \varepsilon\gamma_E Y/H - \delta_H$. Therefore,

$$\frac{Y}{H} = \frac{\widehat{H} + \delta_H}{\varepsilon\gamma_E}. \tag{45}$$

Meanwhile, dividing both sides of the production function $Y = AK^{\alpha}H^{\beta}$ by H and K, respectively, yields:

$$\frac{Y}{H} = A\left(\frac{K}{H}\right)^{\alpha}, \tag{46}$$

$$\frac{Y}{K} = A\left(\frac{K}{H}\right)^{-\beta}. \tag{47}$$

It follows that $\frac{K}{H} = \left(A\frac{K}{Y}\right)^{1/\beta}$. Using this together with (46) in Equation (45) yields:

$$\frac{\widehat{H} + \delta_H}{\gamma_E} = \varepsilon A^{1/\beta}\left(\frac{K}{Y}\right)^{\alpha/\beta}. \tag{48}$$

We have already calibrated almost all parameters and ratios in this formula – there are only 2 remaining "unknowns": ε and A. Note that they are, unfortunately, bound together in Equation (48). Thus, there is no way to find/calibrate these values within our model, i.e. without reaching to other models or econometric evidence. Therefore, as a reasonable value, we set $A = 0.49$, which is the arithmetic average of estimates obtained in our recent studies with similar production function. It follows from Equation (48) that $\varepsilon = 1.7468$. Using this value in Equation (45) yields $Y/H = 0.5941$.

To perform the simulations, we should also assume certain initial (endowment) values of K and H. Without loss of generality, we can normalize the initial value of GDP to 100, i.e. set $Y(0) = 100$. Thus, given the above (initial) ratios of $Y/K = 1/3$ and $Y/H = 0.5941$ we get: $K(0) = 300$ and $H(0) = 168.3237$.

6 The baseline scenario

The baseline scenario is defined as the BGP generated by the set of parameters obtained above – on the basis of actual statistics over the period 2009–2018. Using the procedure described at the end of Section 3 we have numerically obtained the BGR in the baseline scenario, which is equal to 2.31% – more than 1 percentage point less than the average growth rate during the period 2009–2018. The rate of growth of consumption equals $\psi = 1.37\%$, and thus we have case C (see Section 4.1) with relatively impatient consumers. The transversality conditions imply the following initial level of consumption (in relation to GDP): $\underline{c}_0 = 58.93\%$, which happens to be very close to the factual statistical share of private consumption in GDP over 2009–2018 (60.0%). The debt-to-GDP ratios along the BGP converge to the following values: $\underline{\bar{d}}_F = 79.4\%$, $\underline{\bar{d}}_D = 85.0\%$, $\underline{\bar{b}} = -13.4$.

We are now ready to simulate the effects of changes in fiscal policy.

7 Optimal fiscal policy

The model incorporates quite a few parameters of fiscal policy which are subject to the decision of the government: public expenditures on education as percentage of GDP (γ_E), public consumption in relation to private consumption (σ_C), the budget deficit in relation to GDP (ξ), the foreigners' share of public debt (ω) and four tax rates (τ_L, τ_K, τ_C, τ_D). Table 2 summarizes the results of simulations. It contains the values of selected variables along the balanced growth path. In each instance we modified the value of just one parameter, assuming that all remaining parameters have the baseline values. Recall that two tax rates (τ_C and τ_D) are neutral for the balanced growth path, so they are omitted in the table.

The most significant are the effects of increasing education spending. If the government permanently raises public spending on education by 1 pp of GDP (from recent 5.25% to 6.25%), the GDP rate of growth (the BGR) increases from 2.3% to almost 3%! Yet another 1 pp of GDP raises the rate of growth to almost 3.7%.

Table 2: Alternative fiscal policies

	The BGR	The initial level of consumption (share of GDP)	Debt-to-GDP ratios			
	$\bar{\varphi}$	$\underline{c}_0$	$\underline{\bar{d}}$	$\underline{\bar{d}}_F$	$\underline{\bar{d}}_D$	$\underline{\bar{b}}$
The baseline scenario: $\gamma_E = 5.25\%$, $\sigma_C = 30.2\%$, $\xi = 3.8\%$, $\omega = 48.3\%$, $\tau_L = 19.38\%$, $\tau_K = 22.13\%$	2.31%	58.9%	164%	79.4%	85.0%	-13.4
Public spending on education						
$\gamma_E = 6.25\%$	2.99%	64.1%	127%	61.4%	65.7%	-14.7
$\gamma_E = 7.25\%$	3.66%	70.9%	104%	50.1%	53.7%	-16.4
Public consumption						
$\sigma_C = 20.2\%$	2.31%	63.8%	164%	79.4%	85.0%	-13.4
Government deficit						
$\xi = 2.8\%$	2.32%	59.0%	120%	58.2%	62.3%	-13.6
$\xi = 1.8\%$	2.34%	59.1%	77%	37.2%	39.8%	-13.9
$\xi = 0\%$	2.36%	59.3%	0	0	0	-14.3
Foreigners' share of public debt						
$\omega = 0\%$	2.27%	58.6%	167%	0	167%	-14.1
$\omega = 100\%$	2.36%	59.3%	161%	161%	0	-12.7
Taxes on labor						
$\tau_L = 14.38\%$	2.39%	59.5%	159%	76.8%	82.3%	-13.6
Taxes on capital						
$\tau_K = 17.13\%$	2.35%	59.2%	162%	78.1%	83.6%	-13.5

Such dynamic economy requires education spending equal to as much as 7.25% of GDP – the value which is not beyond the reach – even in Europe there are countries which invest well above 6% of GDP in education, e.g. Iceland (7.3% in 2018), Sweden (6.9%), Denmark (6.4%), Belgium and Estonia (6.2%). Note that this scenario is also very beneficial for the government budget: public debt converges to 104% of GDP rather than 164% (in the baseline scenario).

Welfare implications of increased spending on education are huge. First, recall that the trajectory of consumption is given by Equation (38): $c(t) = c_0 \cdot e^{\psi t}$, where the rate of growth of consumption along the BGP is $\psi = \frac{r-\rho}{1-(1+\kappa)\gamma} = const$ and c_0 is given by Equation (35) together with (36). Note that increased spending on education has no influence on ψ, but it raises c_0. In particular, if the government raises public spending on education from recent 5.25% to 6.25%, c_0 increases by as much as 8.7%. It implies that, since ψ is unchanged, the entire trajectory of consumption shifts 8.7% upwards. Reducing the size of public consumption in our model changes neither the BGR nor debt indicators. In fact, there is only one structural change: private consumption increases as households are forced to compensate the loss. Obviously, it may have welfare implications: the value of utility given by Equation (20) may change, because the trajectory of private consumption shifts up, whereas the trajectory of public consumption shifts down. In particular, in the scenario presented in Table 2 (with $\sigma_C = 20.2\%$ instead of $\sigma_C = 30.2\%$), the trajectory of private consumption shifts up by 8.3%, whereas the trajectory of public consumption shifts down by 27.5%. Substituting these numbers together with $\kappa = 0.27$ into Equation (20) implies that these changes actually slightly reduce welfare (utility U).

The baseline scenario indicates that Poland cannot maintain recent level of budget deficit (on average 3.8% of GDP), because over time public debt would reach almost 164% of GDP, which strongly exceeds a constitutional 60% ceiling on public debt. Note that there is no inflation in the model. Introducing "inflation tax" could reduce the long-run debt-to-GDP ratio significantly: even moderate inflation of 3% per year could cut this figure by half. Konopczyński (2015) discusses such scenarios in chapter 5. Moreover, our simulations suggest that reducing public deficit is not harmful for economic growth – provided that expenditures on education are at the same level. In fact, cutting the budget deficit by 1 pp of GDP rises the BGR by approximately 0.02 pp. According to the simulations, the best policy is to run balanced budget (at least on average over the long run). Welfare effects are not big, though: eliminating public debt completely raises the trajectory of consumption by 0.6%.

The financing structure of public debt is not irrelevant for economic growth: the bigger the foreigners' share of public debt, the higher the BGR. Therefore, from the point of view of maximizing economic growth, the optimal strategy is to finance public debt entirely from foreign sources. However, it should be remembered that such a strategy makes the economy more vulnerable to external shocks, speculation, etc. Our model does not take such risks into account.

Last but not least, the income taxes on labor and capital are too high. Reducing the efficient tax rate on wages by 5 pp not only slightly speeds up economic growth – it also improves public debt indicators: public debt converges to 159% of GDP rather than 164%. The trajectory of consumption shifts up by 1.0%. Cutting the efficient tax rate on capital is also beneficial, although the effect is about twice smaller compared to taxes on labor: the trajectory of consumption shifts up by 0.5%. It's important to remember that if the government needs to compensate the reduction in income taxes, it may do so by rising taxes on consumption, which are neutral for economic growth.

8 Conclusions and discussion

Introducing a broad spectrum of fiscal policy instruments into the otherwise rather standard open economy growth model leads to interesting theoretical conclusions. In the long run the economy converges towards the balanced growth path (BGP) which, however, may not be unique. The very existence and uniqueness of the BGP hinges on the values of parameters characterizing the economy. Our calculations suggests that for a wide range of reasonable (realistic) sets of values of parameters the existence and uniqueness is guaranteed. In such cases, the economy converges towards the BGP along the saddle path. The rate of growth along the BGP (the balanced growth rate, BGR), as well as the saddle path (transitory dynamics) can only be established numerically. Despite that nuisance, certain general theoretical propositions regarding fiscal policy have been formulated. For example, we have proved that two tax rates – on consumption and interest on government bonds held by domestic lenders – are neutral for economic growth: both for the BGP, and for transitory dynamics. All other parameters of fiscal policy are not neutral: any modification in their values shifts the BGP, and it also changes the transitory dynamics. Generally, somewhat annoyingly, the relationship between the BGP and parameters of fiscal policy is ambiguous: it may be negative, positive or neutral, depending on set of values of other parameters. Due to relative complexity of the model, these relationships cannot be determined by standard analytical methods – numerical simulations are necessary.

As an empirical illustration, we have calibrated the model for Poland and performed the sensitivity analysis. The calibration was based on the period 2009–2018. We found that over that period economic growth (on average 3.46% annually) was driven primarily by intense accumulation of human capital (growing at 3.95% per year), and secondarily by the accumulation of capital (2.48% annually). Compared to the results reported by Konopczyński (2014a), which were based on an earlier period (2000–2011), we found that human capital remains the main factor of growth, though its importance has slightly decreased.

The baseline (continuation) scenario suggests that Poland will converge to the balanced growth path with the BGR equal to 2.31%, which is significantly less than in the reference period. However, economic growth may be accelerated, if fiscal policy is appropriately adjusted. The best method to accomplish permanent increase in economic growth is to increase investment in education: raising public spending on education by 2 percentage points of GDP (from recent 5.25% to 7.25% of GDP) would boost the BGR from 2.3% to almost 3.7% and create large welfare benefits in the long run. Reducing public deficit and income taxes (especially on wages) would also accelerate growth, although the effects are far less significant.

On the one hand, the model presented in this paper captures many features typical for a small open economy heavily integrated with the outside world. On the other hand, it incorporates wide range of fiscal policy instruments, with relatively simple and transparent fiscal rules. Therefore, it can easily be applied – perhaps with some modifications – for most small countries around the world – in particular Eastern Europe. Note, however, that the underlying assumption of perfect capital mobility makes the eurozone countries the best suited sample. A reliable analysis of countries which do not satisfy that assumption requires a significant modification of the model: replacing perfect capital mobility with some alternative assumptions about the interest rates.

Regarding the empirical part of the paper, it is worth remembering that the model neglects two important phenomena which undoubtedly had (in fact, still have) big impact on Polish economy. On the one hand, Poland has attracted large capital inflows in the form of FDI, portfolio investment, and – last but not least – EU convergence funds. On the other hand, there is large emigration from Poland to other EU countries, only partially offset by temporary workers (mainly from Ukraine). These two facts are not included in our model, but they undoubtedly offset one another out – at least partly.

Finally, it's worth remembering that we were analyzing fiscal policy by comparing different paths of balanced growth: in mathematical terms, steady states. Changes in fiscal policy also cause temporary effects: the so-called transitory dynamics that is beyond the scope of this article.

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Appendix A The solution of the optimization problem

The optimal solution must satisfy the following (necessary and sufficient) conditions, including two transversality conditions:

$$\forall t \quad \frac{\partial H_c}{\partial c} = 0, \tag{A.a}$$

$$\forall t \quad \frac{\partial H_c}{\partial i_K} = 0, \tag{A.b}$$

$$\dot{\lambda}_1 = \frac{-\partial H_c}{\partial b} + \lambda_1(\rho - n), \tag{A.c}$$

$$\dot{\lambda}_2 = -\frac{\partial H_c}{\partial k} + \lambda_2(\rho - n), \tag{A.d}$$

$$\lim_{t\to\infty} e^{-(\rho-n)t}\lambda_1(t)b(t) = 0, \tag{A.e}$$

$$\lim_{t\to\infty} e^{-(\rho-n)t}\lambda_2(t)k(t) = 0. \tag{A.f}$$

Condition (A.a) can be written as

$$\lambda_1(1 + \tau_C) = c^{\gamma-1} g_C^{\kappa\gamma}, \tag{A.1}$$

which means that the shadow price of wealth (in the form of bonds), adjusted for the size of consumption tax must be (for all t) equal to the marginal utility of private consumption. Log-differentiating this equation with respect to t yields (throughout the paper hats over variables denote rates of growth):

$$\widehat{\lambda}_1 = (\gamma - 1)\widehat{c} + \kappa\gamma\widehat{g}_C. \tag{A.2}$$

Equation (18) implies that private and public consumption per capita grow at identical rates, say ψ. Thus $\widehat{g}_C = \widehat{c} = \psi$. Condition (A.c) can be written as:

$$\widehat{\lambda}_1 = \frac{\dot{\lambda}_1}{\lambda_1} = \rho - r. \tag{A.3}$$

Substituting Equation (A.3) into Equation (A.2), and using $\widehat{g}_C = \widehat{c} = \psi$ yields:

$$\psi = \frac{\dot{c}}{c} = \frac{r-\rho}{1-(1+\kappa)\gamma} = const. \tag{A.4}$$

Thus, the optimum trajectory of private consumption per capita is given by Equation (38). Log-differentiating the production function (2) yields:

$$\varphi_y = \widehat{y} = \alpha\widehat{k} + \beta\widehat{h} = \alpha\varphi_k + \beta\varphi_h. \tag{A.5}$$

In what follows, it is convenient to use certain variables expressed as shares in GDP. We denote these shares with an underline, e.g., $\underline{k} = k/y$, $\underline{c} = c/y$, $\underline{d}_D = d_D/y$, etc. Dividing both sides of Equation (10) by h, and substituting Equations (12) and (19) yields:

$$\varphi_h = \widehat{h} = \frac{\varepsilon\gamma_E}{\underline{h}} - (n+\delta_H). \tag{A.6}$$

Also, note that dividing both sides of Equation (2) by y yields $A\underline{k}^{\alpha}\underline{h}^{\beta} = 1$, which implies that $\underline{k}$ and $\underline{h}$ are always (not only in the stationary state, but always) linked by the following non-linear relationship:

$$\underline{h} = \frac{1}{A^{1/\beta}\underline{k}^{\alpha/\beta}}. \tag{A.7}$$

Condition (A.b) can be written as:

$$q_K = \frac{\lambda_2}{\lambda_1} = 1 + \frac{\chi i_K}{k}. \tag{A.8}$$

The ratio of shadow prices $q_K = \lambda_2/\lambda_1$ can be roughly interpreted as the market price of capital in relation to the market price of private foreign assets (or debt). According to Equation (A.8), it must be equal to the marginal cost of an additional unit of investment (augmented by the adjustment cost). Dividing both sides of Equation (9) by k, and using Equation (A.8), we obtain the growth rate of k:

$$\varphi_k = \widehat{k} = \frac{(q_K - 1)}{\chi} - (n+\delta_K). \tag{A.9}$$

This growth rate is not constant, as it is related to the trajectory $q_K(t)$. Therefore, at this stage, the trajectory $k(t)$ must be written in a general form:

$$k(t) = k_0 \exp\left(\int_0^t \varphi_k(s)ds\right). \tag{A.10}$$

To determine the path of $q_K(t)$, we shall use Equation (A.d). Having regard to Equations (A.3) and (A.8), and using Equation (5), it can be written as:

$-\dot{\lambda}_2 = \lambda_1 W_2 \frac{\partial y}{\partial k} + \lambda_1 \frac{\chi}{2} \left(\frac{i_K}{k}\right)^2 - \lambda_2(\rho + \delta_K)$, where

$$W_2 = (1-\tau_L)\beta + (1-\tau_K)\alpha - (1-\omega)\xi = const. \tag{A.11}$$

Dividing both sides by λ_2, and using Equations (A.3) and (A.8) together with Equation (1), after some manipulation we obtain:

$$\dot{q}_K = [r + \delta_K]\, q_K - \frac{(q_K - 1)^2}{2\chi} - W_2 \alpha / \underline{k}. \tag{A.12}$$

Note that (A.12) is a differential equation of the following form $\dot{q}_K = f_K(q_K, \underline{k})$. To close the emerging system of differential equations we need some more equations. First, let us use the very definition of $\underline{k} = k/y$. Taking time derivative yields: $\dot{\underline{k}} = \dot{k}/y - \varphi_y \underline{k}$, which may be written as:

$$\dot{\underline{k}} = (\varphi_k - \varphi_y)\underline{k}. \tag{A.13}$$

Analogously,

$$\dot{\underline{h}} = (\varphi_h - \varphi_y)\underline{h}. \tag{A.14}$$

Note that all rates of growth φ_i $(i = y, k, h)$ are functions of q_K and $\underline{h}$. Therefore, Equations (A.12), (A.13) and (A.14) constitute a system of three differential (non-linear, autonomous) equations with the following structure:

$$\begin{bmatrix} \dot{q}_K \\ \dot{\underline{k}} \\ \dot{\underline{h}} \end{bmatrix} = \begin{bmatrix} f^1(q_K, \underline{k}) \\ f^2(q_K, \underline{h}, \underline{k}) \\ f^3(q_K, \underline{h}) \end{bmatrix}. \tag{A.15}$$

It is worth noting that (A.7) allows to reduce this system to Equation (24).

Appendix B Details of Proposition 1

The system of equations (25) is nonlinear. Therefore, we will investigate local stability of equilibrium applying a standard method of first-order linearization about the equilibrium. Accordingly, non-linear functions f^i in Equation (24) are approximated as follows:

$$f^i(q_K, \underline{k}) \approx \left.\frac{\partial f^i}{\partial q_K}\right|_E \cdot \widetilde{q}_K + \left.\frac{\partial f^i}{\partial \underline{k}}\right|_E \cdot \tilde{\underline{k}}, \quad i = 1, 2,$$

where tilde denotes deviations from the steady state, i.e., $\widetilde{q}_K = q_K - \bar{q}_K$, $\tilde{\underline{k}} = \underline{k} - \bar{\underline{k}}$. The linear approximation of Equation (25) about the equilibrium has the following form:

$$\begin{bmatrix} \dot{\widetilde{q}}_K & \dot{\tilde{\underline{k}}} \end{bmatrix}^T = \mathbf{M} \begin{bmatrix} \widetilde{q}_K & \tilde{\underline{k}} \end{bmatrix}^T, \tag{B.1}$$

with the matrix of values of partial derivatives (Jacobian) calculated in the equilibrium:

$$\mathbf{M} = \begin{bmatrix} r - n - \bar{\varphi} & W_2 \alpha / \bar{\underline{k}}^2 \\ \beta \bar{\underline{k}} / \chi & -\alpha(\bar{\varphi} + n + \delta_H) \end{bmatrix}.$$

The general solution of the linear system of equations (B.1) can be written as:

$$\begin{bmatrix} q_K & \underline{k} \end{bmatrix}^T = \begin{bmatrix} \bar{q}_K & \bar{\underline{k}} \end{bmatrix}^T + \sum_{i=1}^{2} s_i e^{r_i t} \mathbf{v}^i, \tag{B.2}$$

where r_i are the eigenvalues of the matrix $\mathbf{M}$, $\mathbf{v}^i$ are its eigenvectors, and s_i are unknown constants dependent on the starting point (endowments). The local stability of the equilibrium depends on the signs of the eigenvalues of $\mathbf{M}$. The product of these eigenvalues is equal to $\det \mathbf{M}$, whereas their sum is equal to $\operatorname{tr} \mathbf{M}$. All four elements of matrix $\mathbf{M}$ have predetermined signs: $\mathbf{M} = \left[\begin{smallmatrix} + & + \\ + & - \end{smallmatrix}\right]$, with the first one following from the transversality condition (34); see Appendix C. It follows that $\det \mathbf{M} < 0$, which entails that $\mathbf{M}$ has one negative and one positive real eigenvalue. (Both eigenvalues are necessarily real numbers, because complex eigenvalues always come in conjugate pairs, and so their product cannot be a negative real number.) Therefore, the equilibrium has the form of the stable saddle path, with one variable immediately "jumping" to accommodate any shock instantly, whereas another variable evolves continuously over time. Obviously, the "jump" variable is the ratio of shadow prices q_K, whereas the "smooth" variable is the ratio of capital to GDP, $\underline{k}$. If we denote positive eigenvalue as r_2, then $s_2 = 0$, and the solution (B.2) boils down to Equation (33), where r_1 is the negative eigenvalue, and $\mathbf{v}^1$ is its corresponding eigenvector.

Appendix C Proof that the transversality condition (A.f) is satisfied if and only if $r > \bar{\varphi} + n$

Substituting $\lambda_2(t) = q_K(t)\lambda_1(t)$ into (A.f) yields:

$$\lim_{t \to \infty} e^{-(\rho - n)t} \lambda_1(t) q_K(t) k(t) = 0. \tag{C.1}$$

From Equations (A.9) and (A.10), it follows that the trajectory of capital has the following form:

$$k(t) = k_0 e^{\int_0^t \left(\frac{q_K(s)-1}{\chi}\right) ds} e^{-(n+\delta_K)t}. \tag{C.2}$$

Meanwhile, Equation (A.3) implies that $\dot{\lambda}_1 / \lambda_1 = \rho - r$. Thus, the trajectory $\lambda_1(t)$ is of the form:

$$\lambda_1(t) = \lambda_1(0) e^{\rho t} e^{-rt}. \tag{C.3}$$

Using Equation (C.2) and (C.3), condition (C.1) can be written as:

$$\lambda_1(0)k_0 \lim_{t\to\infty}\left\{q_K(t)e^{-\delta_K t}e^{-rt}e^{\int_0^t\left(\frac{q_K(s)-1}{\chi}\right)ds}\right\}=0,$$

which is equivalent to:

$$\lambda_1(0)k_0 \lim_{t\to\infty}\left\{q_K(t)e^{-\left(r+\delta_K+\frac{1}{\chi}\right)t}e^{\frac{1}{\chi}\int_0^t q_K(s)ds}\right\}=0. \tag{C.4}$$

In order to examine this condition, we need to know the trajectory of variable $q_K(s)$. Because the model is non-linear, we will use the approximate trajectories obtained by solving the linearized model. From the system of equations (B.2) we know that:

$$q_K(t)=\bar{q}_K+s_1e^{r_1t}v_1^1, \tag{C.5}$$

where $r_1<0$. It follows that $\int_0^t q_K(s)ds=\bar{q}_Kt-\dfrac{s_1v_1^1}{r_1}+\dfrac{s_1v_1^1}{r_1}e^{r_1t}$, which implies that

$$e^{\frac{1}{\chi}\int_0^t q_K(s)ds}=e^{\frac{\bar{q}_K}{\chi}t}e^{-\frac{s_1v_1^1}{\chi r_1}}e^{\frac{s_1v_1^1}{\chi r_1}e^{r_1t}}. \tag{C.6}$$

Using (C.6), we can rewrite condition (C.4) as:

$$\lambda_1(0)k_0e^{-\frac{s_1v_1^1}{\chi r_1}}\lim_{t\to\infty}\left\{q_K(t)e^{-\left(r+\delta_K-\frac{\bar{q}_K-1}{\chi}\right)t}e^{\frac{s_1v_1^1}{\chi r_1}e^{r_1t}}\right\}=0. \tag{C.7}$$

Note that $r_1<0$ means that $\lim_{t\to\infty}e^{\frac{s_1v_1^1}{\chi r_1}e^{r_1t}}=1$. Therefore, condition (C.7) is satisfied if and only if:

$$\lim_{t\to\infty}\left\{q_K(t)e^{-\left(r+\delta_K-\frac{\bar{q}_K-1}{\chi}\right)t}\right\}=0.$$

Using Equation (C.6) we can rewrite this condition as:

$$\bar{q}_K\lim_{t\to\infty}e^{\left(\frac{\bar{q}_K-1}{\chi}-\bar{r}-\delta_K\right)t}+s_1v_1^1\lim_{t\to\infty}\left\{e^{\left(r_1+\frac{\bar{q}_K-1}{\chi}-\bar{r}-\delta_K\right)t}\right\}=0. \tag{C.8}$$

It is straightforward to show that condition (C.8) holds if and only if, $\lim_{t\to\infty}e^{\left(\frac{\bar{q}_K-1}{\chi}-\bar{r}-\delta_K\right)t}=0$. This equality holds if and only if $\frac{\bar{q}_K-1}{\chi}-\bar{r}-\delta_K<0$, which can be written in a more intuitive form:

$$\bar{r}>\bar{\varphi}\ +n. \tag{C.9}$$

Appendix D Further details

The transversality condition (A.e) determines the initial value of consumption. In order to demonstrate this we need to solve the budget constraint (22) along the BGP. Fiscal rules (14) and (17) imply that $g_T = t + \xi y - rd - g_C - g_E$. Substituting it into Equation (22) yields:

$$\dot{b} = (1-\tau_L)w_L + (1-\tau_K)w_K k + (1-\tau_D)rd_D + [r-n]b + \\ + t - rd - g_C - g_E - c(1+\tau_C) - i_K\left(1+\frac{\chi}{2}\frac{i_K}{k}\right) + \omega\xi y. \quad \text{(D.1)}$$

From Equation (13) it follows that $t = \tau_L w_L + \tau_K w_K k + \tau_D rd_D + \tau_C c$, so that Equation (D.1) can be reduced to:

$$\dot{b} = w + w_K k + rd_D + (r-n)b - rd - g_C - g_E - c - i_K\left(1+\frac{\chi}{2}\frac{i_K}{k}\right) + \omega\xi y.$$

Recall that $w + w_K k = y$ and $d - d_D = d_F$. Therefore, the budget constraint takes the following form:

$$\dot{b} = (1+\omega\xi)y - c - g_C - g_E - i_K\left(1+\frac{\chi}{2}\frac{i_K}{k}\right) + (r-n)b - rd_F. \quad \text{(D.2)}$$

Substituting fiscal rules (18) and (19) into Equation (D.2) yields:

$$\dot{b} = (1+\omega\xi-\gamma_E)y - (1+\sigma_C)c - i_K\left(1+\frac{\chi}{2}\frac{i_K}{k}\right) + (r-n)b - rd_F. \quad \text{(D.3)}$$

Moreover, (A.8) implies that $i_K/k = (q_K-1)/\chi$. Therefore Equation (D.3) can be written as:

$$\dot{b} = (1+\omega\xi-\gamma_E)y - (1+\sigma_C)c - \left(\frac{q_K^2-1}{2\chi}\right)k + (r-n)b - rd_F. \quad \text{(D.4)}$$

Recall that $c(t) = c_0 \cdot e^{\psi t}$, where $\psi = \frac{r-\rho}{1-(1+\kappa)\gamma} = const$. This is, however, the only simple element of Equation (D.4). All other trajectories on the right-hand side of this equation are far more complex; see e.g. (C.2) and (C.6). Substituting these trajectories into (D.4) leads to an equation which is not solvable analytically. Therefore let us consider the economy which is on the BGP from the very beginning (loosely speaking, we may think about the economy which has already fully converged towards the BGP, and we start our calculations at and appropriate moment of time). Thus, we will substitute what follows: $k(t) = k_0 \cdot e^{\bar{\varphi}t}$, $y(t) = y_0 \cdot e^{\bar{\varphi}t}$.
(Note that k_0 represents initial endowment, whereas y_0 is calculated as follows. By definition, $\underline{k} = k/y$, and so $y = k/\underline{k}$, For $t = t_0$, we have $y_0 = k_0/\underline{k}_0$. However, we have

assumed that the economy is on the BGP from $t = t_0$, therefore $y_0 = k_0/\bar{\underline{k}}$, where $\bar{\underline{k}}$ is the capital-to-GDP ratio on the BGP, which can be obtained by numerically solving the system of Equations (26) and (27).)

Now, let us determine the trajectory $d_F(t)$ along the BGP. It follows from Equation (15) that

$$\dot{d}_F = \omega\xi y - n d_F = \omega\xi y_0 e^{\bar{\varphi}t} - n d_F. \tag{D.5}$$

The general solution of this equation is:

$$d_F(t) = s_3 e^{-nt} + \frac{\omega\xi y_0}{n+\bar{\varphi}} e^{\bar{\varphi}t}, \tag{D.6}$$

where the unknown constant s_3 is a function of the initial foreign debt ratio, $d_F(t=0) = d_{F0}$. Substituting d_{F0} into Equation (D.6) yields:

$$s_3 = d_{F0} - \frac{\omega\xi y_0}{n+\bar{\varphi}}. \tag{D.7}$$

In order to find the analytical form of the trajectory of $b(t)$ along the BGP, we need to substitute (D.6) together with $k(t) = k_0 \cdot e^{\bar{\varphi}t}$, $y(t) = y_0 \cdot e^{\bar{\varphi}t}$, and $c(t) = c_0 \cdot e^{\psi t}$ into Equation (D.4). After rearrangement we get:

$$\dot{b} = (r-n)b + \upsilon e^{\bar{\varphi}t} - (1+\sigma_C)c_0 e^{\psi t} - r s_3 e^{-nt}, \tag{D.8}$$

where

$$\upsilon = \left(1 + \omega\xi - \gamma_E - \frac{r\omega\xi}{n+\bar{\varphi}}\right) y_0 - \left(\frac{\bar{q}_K^2 - 1}{2\chi}\right) k_0. \tag{D.9}$$

The general solution of Equation (D.8) takes the form:

$$b(t) = S e^{(r-n)t} - \frac{\upsilon}{r-n-\bar{\varphi}} e^{\bar{\varphi}t} + \frac{c_0(1+\sigma_C)}{r-n-\psi} e^{\psi t} + s_3 e^{-nt}, \tag{D.10}$$

where the unknown constant S is a function of the initial value of $b(t=0) = b_0$. Substituting b_0 into Equation (D.10) yields:

$$S = b_0 - d_{F0} + \frac{\upsilon}{r-n-\bar{\varphi}} - \frac{c_0(1+\sigma_C)}{r-n-\psi} + \frac{\omega\xi y_0}{n+\bar{\varphi}}. \tag{D.11}$$

Substituting (C.3) and (D.10) with (D.11) into the transversality condition (A.e) yields

$$\lambda_1(0) \cdot \lim_{t\to\infty} \left\{ S - \frac{\upsilon}{r-n-\bar{\varphi}} e^{(n-r+\bar{\varphi})t} + \frac{c_0(1+\sigma_C)}{r-n-\psi} e^{(n-r+\psi)t} + s_3 e^{-rt} \right\} = 0, \tag{D.12}$$

which is satisfied if and only if three conditions are fulfilled:

$$S = 0, \tag{D.13}$$

$$r > n + \bar{\varphi}, \tag{D.14}$$

$$r > n + \psi, \tag{D.15}$$

which implies that the interest rate must simply be sufficiently high. Importantly, Equation (D.13) determines the initial amount of consumption:

$$c_0 = \left(b_0 - d_{F0} + \frac{\upsilon}{r - n - \bar{\varphi}} + \frac{\omega \xi y_0}{n + \bar{\varphi}}\right) \frac{r - n - \psi}{1 + \sigma_C}. \tag{D.16}$$

Therefore, the trajectory of foreign assets can be written as follows:

$$b(t) = \frac{c_0(1 + \sigma_C)}{r - n - \psi} e^{\psi t} - \frac{\upsilon}{r - n - \bar{\varphi}} e^{\bar{\varphi} t} + s_3 e^{-nt}, \tag{D.17}$$

or, equivalently, Equation (39).