Improved Differential Evolution Algorithm to solve multi-objective of optimal power flow problem

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Abstract: This article presents a new efficient optimization technique namely the Multi-Objective Improved Differential Evolution Algorithm (MOIDEA) to solve the multi-objective optimal power flow problem in power systems. The main features of the Differential Evolution (DE) algorithm are simple, easy, and efficient, but sometimes, it is prone to stagnation in the local optima. This paper has proposed many improvements, in the exploration and exploitation processes, to enhance the performance of DE for solving optimal power flow (OPF) problems. The main contributions of the DE algorithm are i) the crossover rate will be changing randomly and continuously for each iteration, ii) all probabilities that have been ignored in the crossover process have been taken, and iii) in selection operation, the mathematical calculations of the mutation process have been taken. Four conflicting objective functions simultaneously have been applied to select the Pareto optimal front for the multi-objective OPF. Fuzzy set theory has been used to extract the best compromise solution. These objective functions that have been considered for setting control variables of the power system are total fuel cost (TFC), total emission (TE), real power losses (RPL), and voltage profile (VP) improvement. The IEEE 30-bus standard system has been used to validate the effectiveness and superiority of the approach proposed based on MATLAB software. Finally, to demonstrate the effectiveness and capability of the MOIDEA, the results obtained by this method will be compared with other recent methods.

Key words: Multi-objective Improved Differential Evolution Algorithm (MOIDEA), optimal power flow (OPF), set of Pareto front solutions, multi-objective function problems, fuel costs considering emissions, fuel costs considering real power losses, fuel costs considering voltage deviation

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1. Introduction

Nowadays, the increasing demand for electricity, the development of technology, and the deregulation of the power system pushed the power system to operate near its operating limits. At the same time, due to financial and political issues, the power systems are slowly installed. The optimal power flow (OPF) is one of the most important issues in the power system operation because of the ability to achieve the most efficient operation and planning in the power system. The main goal of the OPF is to optimize objective functions by setting the control variables while fulfilling equality and inequality constraints. Fuel cost, pollution, losses, the voltage profile, and voltage stability are factors that must be provided in the power system to achieve proper operation and planning.

In the past three decades, meta-heuristics optimization algorithms have been very popular among researchers. These algorithms are inspired by different concepts such as evolutionary, human, and natural. The main reasons to increase the huge number of optimization methods are flexibility, simplicity, local optima avoidance, and derivation-free mechanism. To avoid the local optima solution and discover the global optima, these algorithms are performed by random operators. Lately, many meta-heuristic methods have been applied to the OPF problem with an impressive success such as the Genetic Algorithm (GA) [1], Particle Swarm Optimization (PSO) [2], Modified Artificial Bee Colony (MABC) [3], Differential Evolution (DE) [4], Lightning Attachment Optimization Technique (LAOT) [5], Glowworm Optimization Algorithm (GOA) [6], Salp Swarm Algorithm (SSA) [7], Modified Bacteria Foraging Algorithm (MBFA) [8], Modified Jaya Algorithm (JAYA) [9], Hybrid Firefly and Particle Swarm Optimization (HFPSO) [10], Fruit Fly Optimization Algorithm (FOA) method [11].

Multi-objective optimal power flow (MOOPF) is an important tool in power systems operation planning [12]. Numerous methods have been introduced to solve multi-objective (MO) optimization such as the weighted sum approach [25], $\varepsilon$-constant [15], penalty function method [16], strength Pareto evolutionary algorithm [17], non-dominated sorting genetic algorithm-based approach [18]. The Pareto optimization (PO) is one of the most common methods to solve a multi-objective optimization problem [19]. This method compares a set of conflicting objective functions (OFs) with each other in a multi-objective search space to select the most preferable solutions [20]. One of the most important aspects related to the Pareto front computation in the decision-making process is the selection of the best compromise solution because it is not obtained in an automatic way. Selection of the best compromise solution has been proposed by several methods in the literature, such as a similar philosophy is followed by the entropy method, the fuzzy membership approach, and the pseudo-weight vector approach [33]. Several algorithms are incorporated with the Pareto concept to rank non-dominated solutions and determine the reproduction probability of each individual, such as Harris Hawks Optimization (HHO) [23], Jaya Optimization [24], the Fruit Fly Optimization (FFO) algorithm [25], Harmony Search (HS) algorithm [26], Modified Shuffled Frog Leaping (MSFL) algorithm [27], Artificial Bee Colony (ABC) [28].

Differential Evolution (DE) is one of the important optimization algorithms due to its ability to give the optimal solutions and its superiority of performance over the other recent optimization methods [29]. The DE algorithm was proposed by Storn and Price in 1997 [30]. In this work, the Multi-objective Improved Differential Evolution Algorithm (MOIDEA) has been proposed to
OPF problems by using multiple Pareto front-optimal solutions to find non-dominated solutions. This proposed approach is based on improving the DE variant (DE/best/1) for solving OPF formulations. The set of Pareto front solutions is assessed by fuzzy set theory to find the best compromise solution. Three improvements have been proposed to develop the DE algorithm. These improvements are:

- The values of crossover (CR) and scale factor (F) are not fixed numbers, they are variable numbers ranging from 0–1 for each generation.
- Creating a new process in the crossover stage represents a new vector trail.
- The mutation calculations will be a consideration in the selection process.

These improvements will result in diversity, efficiency, and an increasing rate of accelerating convergence with less iteration. Total fuel cost (TFC) minimization, minimization of total emission (TE), real power losses (RPL) reduction, and voltage profile (VP) improvement have been handled as objective functions.

The rest of this paper can be arranged as follows: the mathematical formulation of OPF is introduced in section 2. Section 3 declares the strategies of multi-objective solutions. Multi-objective based on Improved Differential Evolution will be presented in section 4. Section 5 discusses the simulation results and compares these results with other recent methods of optimization. Finally, the conclusions are presented in section 6.

2. The mathematical formulation of OPF problems

The OPF problem is mathematically demonstrated. The objectives are optimized by adjusting the control-variables values with respect to inequality and equality constraints. The mathematical formulation of the OPF problem can be expressed as follows:

\[
\text{Optimize } f_i(x, u), \quad i = 1, 2, \ldots, N, \\
\text{subjected to } g_j(x, u) = 0, \quad j = 1, 2, \ldots, M, \\
h_k(x, u) \leq 0, \quad k = 1, 2, \ldots, K, 
\]

where: \( f(x, u) \) represents objective functions to be minimized; \( g(x, u) \) and \( h(x, u) \) are the sets of equality and inequality constraints, respectively; \( N, M, \) and \( K \) refer to the number of objectives, equality constraints, and inequality constraints, respectively; \( x \) represents the vectors of state variables and can be symbolized as:

\[
x^T = [P_{G1}, V_{L1}, \ldots, V_{LN}, Q_{G1}, \ldots, Q_{GNG}] \]

where: \( N_L \) and \( NG \) are the numbers of load terminals and generators, respectively; \( P_{Gi} \) is the active power output at the swing bus; \( Q_{Gi} \) is the reactive power output at PV busses; \( V_L \) is the voltage magnitude at P-Q buses; \( u \) is the vectors of control variables and can be defined as below:

\[
u^T = [P_{G2}, \ldots, P_{GNG}, V_{G1}, \ldots, V_{GNG}, T_1, \ldots, T_{NT}, Q_C, \ldots, Q_{CNC}] 
\]

where: \( NC \) and \( NT \) are the numbers of shunt VAR compensation and taps changing transformers, respectively; \( P_G \) and \( V_G \) are the real power generation and the magnitude voltage at PV generator nodes except for the slack generator, respectively; \( T \) denotes the tap setting of regulating transformers; \( Q_c \) is the number of shunt VAR compensation.
2.1. Objective functions

In this article, four objectives are carried out to demonstrate the performance of Improved Differential Evolution (IDE) of multi-objective functions. These objective functions are the total fuel cost (TFC), the total emission (TE), the real power losses (RPL) minimization, and voltage profile (VP) improvement.

a. Total fuel cost (TFC) minimization

The total fuel cost ($f_1$) of generation units can be expressed as:

$$f_1 = \sum_{i=1}^{N_G} \left( a_i P_{G_i}^2 + b_i P_{G_i} + c_i \right) \text{ [$/h]},$$

where: $f_1$ denotes the total fuel cost of the generation units, $P_{G_i}$ is the active power output of the generation units, $N_G$ is the number of the generators, and $a_i$, $b_i$, and $c_i$ represent the fuel cost coefficients of the generating unit $i$.

b. The total emission (TE) minimization

The second objective is the reduction of polluted gases based on environmental emission minimizations such as $NO_x$ and $SO_2$ issued by fossil-fueled thermal units. This objective function can be expressed as:

$$f_2 = \sum_{i=1}^{N_G} 10^{-2} \left( a_i + \beta_i P_{G_i} + \gamma_i P_{G_i}^2 \right) + \zeta_i \exp \left( \lambda_i P_{G_i} \right) \text{ [ton/h]},$$

where: $f_2$ denotes total emission of generation units, $a_i$, $\beta_i$, $\gamma_i$, $\zeta_i$, and $\lambda_i$ represent the emission coefficients of the generating unit $i$.

c. The real power losses (RPL) minimization

This objective function aims to reduce real power losses in the transmission network of the power system which given by:

$$f_3 = \sum_{k=1}^{N_{TL}} g_{(i,j)} \left( V_i^2 + V_j^2 - 2V_i V_j \cos \delta_{i,j} \right) \text{ [MW]},$$

where: $f_3$ is the total active power losses on transmission lines, $N_{TL}$ is the number of transmission lines, $g_{(i,j)}$ is the transmission conductance.

d. The voltage profiles (VPs) improvement

The voltage profile improvement is the fourth objective function by reducing the voltage deviation from 1.0 p.u. at load buses, which can be expressed as:

$$f_4 = V_d = \sum_{i=1}^{N_L} \left| V_i - 1 \right| \text{ [p.u.]} ,$$

where: $f_4$ is the voltage deviation, $N_L$ represents the numbers of load terminals, $V_i$ denotes the voltage in each load bus of the network.
2.2. Constraints

The constraints of the OPF are classified into two types: equality constraints and inequality constraints. Power balance represents equality constraints and the operating system components represent inequality constraints.

a. Equality constraints

Equality constraints can be expressed as:

\[ P_{G_i} - P_{D_i} = V_i \sum_{j=1}^{N_B} V_j \left( G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij} \right), \]

\[ Q_{G_i} - Q_{D_i} = V_i \sum_{j=1}^{N_B} V_j \left( G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij} \right), \]

where: \( N_B \) denotes the number of the grid terminals; \( \theta_{ij} \) is the difference of the voltage phase angles between the buses \( i \) and \( j \); \( V_i \) and \( V_j \) are the voltages magnitude of the bus \( i \) and \( j \), respectively; \( G_{ij} \) and \( B_{ij} \) indicate the conductance and susceptance connecting the terminal \( i \) and terminal \( j \).

b. Inequality constraints

The inequality constraints can be divided into two groups: the control variables’ limits and state variables’ limits.

i) Control variables’ limits

\[ P_{G_i}^{\text{min}} \leq P_{G_i} \leq P_{G_i}^{\text{max}} \quad i = 2, 3, \ldots, N_G, \]

\[ V_{G_i}^{\text{min}} \leq V_{G_i} \leq V_{G_i}^{\text{max}} \quad i = 2, 3, \ldots, N_G, \]

\[ Q_{C_k}^{\text{min}} \leq Q_{C_k} \leq Q_{C_k}^{\text{max}} \quad k = 1, 2, \ldots, N_C, \]

\[ T_{j}^{\text{min}} \leq T_{j} \leq T_{j}^{\text{max}} \quad j = 1, 2, \ldots, N_T. \]

\( P_{G_i}^{\text{min}} \) and \( P_{G_i}^{\text{max}} \) are the minimum and maximum real power generation at the PV generator nodes except for the slack generator, respectively; \( V_{G_i}^{\text{min}} \) and \( V_{G_i}^{\text{max}} \) are the minimum and maximum voltage magnitude at the PV generator nodes, respectively; \( Q_{C_k}^{\text{min}} \) and \( Q_{C_k}^{\text{max}} \) are the maximum and maximum of the reactive power output of the VAR source, respectively; \( T_{j}^{\text{min}} \) and \( T_{j}^{\text{max}} \) are the minimum and maximum tap settings limit of the \( i \)-th transformer, respectively; \( N_G \), \( N_C \), and \( N_T \) are the number of generators, VAR, and transformers, respectively.

ii) State variables’ limits

\[ P_{G_i}^{\text{min}} \leq P_{G_i} \leq P_{G_i}^{\text{max}}, \]

\[ V_{L_q}^{\text{min}} \leq V_{L_q} \leq V_{L_q}^{\text{max}} \quad q = 1, 2, \ldots, N_L, \]

\[ Q_{G_i}^{\text{min}} \leq Q_{G_i} \leq Q_{G_i}^{\text{max}} \quad i = 1, 2, \ldots, N_G, \]

\[ S_{L_m}^{\text{min}} \leq S_{L_m} \leq S_{L_m}^{\text{max}} \quad m = 1, 2, \ldots, N_L. \]

where \( V_{L_q}^{\text{min}} \) and \( V_{L_q}^{\text{max}} \) are the minimum and maximum load voltage of the \( i \)-th bus. \( S_{L_m} \) is the apparent power flow limit of the \( i \)-th branch.
3. The strategy of multi-objective solutions

Multi-objective optimization is the simultaneous optimization of many objective functions which includes mostly non-commensurable and conflicting objectives. Due to these conflicting objective functions, a set of optimal solutions will appear instead of one optimal solution which is called Pareto-optimal solutions. The approach of Pareto optimal represents one of the effective methods to solve multi-objective problems. According to these objective functions, Pareto dominance can be divided into dominated and non-dominated solutions. The decision-maker is responsible to choose the best compromise solution of the non-dominated solutions. The fuzzy set theory, centroid concept, and entropy criterions are proposed strategies to choose the best compromise solution [31]. The fuzzy decision-maker is the most known approach to choosing the best compromise and using it widely [22, 26, 27, 32]. In this paper, the fuzzy decision-making approach has been chosen to determine the best compromise solution from the final Pareto front as follows:

3.1. Pareto optimization approach

The set of acceptable solutions will be found in the Pareto optimization. The solution \( X_1 \) is assumed to dominate the solution \( X_2 \) if the two conditions have been satisfied [33]:

\[
\forall i \in \{1, 2, \ldots, n\}: \quad F_i (X_1) \leq F_i (X_2),
\]

\[
\forall j \in \{1, 2, \ldots, n\}: \quad F_j (X_1) \leq F_j (X_2).
\]

(18)

The solutions that could not overcome each other are Pareto optimal solutions and called a set of dominant solutions. These Pareto sets are stored and updated to solve multi-objective problems.

3.2. Selection of the best compromise solution

The numerical values of multi-objective problems are not of the same kind and in different ranges often. Therefore, the conversion of the numerical values in a similar range became necessary. Equation (19) represents the value of corresponding membership function for each objective function (Fig. 1):

\[
u^k_i = \begin{cases} 
1 & F_i \leq F_i^{\min} \\
\frac{F_i^{\max} - F_i}{F_i^{\max} - F_i^{\min}} & F_i^{\min} < F_i < F_i^{\max} \\
0 & F_i \geq F_i^{\max}
\end{cases}
\]

(19)

where \( F_i^{\min} \) and \( F_i^{\max} \) represent the minimum and the maximum value of the objective function \( F_i \) among all non-dominated solutions. The membership function \( (u^k) \) of each non-dominated solution \( (k) \) can be calculated as:

\[
u_i^k = \frac{\sum_{i=1}^{N_{obj}} u_i^k}{\sum_{k=1}^{M} \sum_{i=1}^{N_{obj}} u_i^k},
\]

(20)
Improved Differential Evolution Algorithm to solve multi-objective

where: $M$ is the total number of non-dominated solutions, $u_i^k$ is the weight factor of the $i$-th objective function, the maximum value of $u_i^k$ represents the best compromise solution [24].

Fig. 1. Membership function [24]

4. Multi-objective Improved Differential Evolution Algorithm (MOIDEA)

The Improved Differential Evolution (IDE) algorithm is an improved version of the Differential Evolution (DE) algorithm. This proposed algorithm proved its effectiveness to solve single-objective optimal power flow applied on the IEEE 30-bus and IEEE 57-bus [34, 35]. In this paper, the authors prove the effectiveness of the proposed algorithm to solve the multi-objective OPF. Three main improvements to the original DE algorithm have been proposed as follows:

a. Crossover rate CR

In the original DE algorithm, the crossover rate is a user-specified constant within the range $[0, 1]$ only once for all iterations. In this section, the crossover rate will be changing randomly and continuously for each iteration. This improvement leads to giving more diversity and efficiency, expedites the convergence and maintains the exploratory feature of the trail process. It can be described as follows:

$$[CR] = \begin{bmatrix} \text{rand} \{0, 1\}_{1,1} & \ldots & \text{rand} \{0, 1\}_{1,D} \\ \text{rand} \{0, 1\}_{2,1} & \ldots & \text{rand} \{0, 1\}_{2,D} \\ \vdots & \ddots & \vdots \\ \text{rand} \{0, 1\}_{NP,1} & \ldots & \text{rand} \{0, 1\}_{NP,D} \end{bmatrix}$$

$CR$ is the crossover rate within $[0, 1]$, $\text{rand} \{0, 1\}$ is the matrix of random numbers in the range $[0–1]$.

b. Crossover operation

To increase the population size $NP$ and cover all potential solutions, the proposed process has been carried out on the original DE algorithm. In this process, all probabilities that have
been ignored in the crossover process will be taken into consideration according to the following formula:

\[
Y_{ij}(G) = \begin{cases} 
V_{ij}(G) & \text{if } j \neq j_{\text{rand}} \text{ or } \left( \text{rand}_j(0, 1) \right) \geq C_r, \\
X_{ij}(G) & \text{otherwise}
\end{cases}
\]  

(22)

\(Y_{ij}(G)\) refers to the new trail vector that will be added to the calculations in the selection process \(j_{\text{rand}}\) and is randomly chosen in the range \([1, D]\), \(C_r\) is the crossover probability \([0, 1]\).

This modification leads to producing new genes in the mutation process, and therefore, increases the probability of the exploration for the search space.

c. Mutation operation

In the DE algorithm, during the selection operation, the mathematical calculations of the mutation process have not been taken into consideration. These calculations have been taken into consideration at the selection stage, and therefore, increased the probability to select the best control variable by comparing the vectors of the target, mutant, trail, and new trail. This improvement can be expressed by Eq. (13).

\[
X_{ij}(G + 1) = \begin{cases} 
Y_{ij}(G) \rightarrow f(Y_{ij}(t)) \leq (f(U_{ij}(t)) \text{ and } f(V_{ij}(t)) \text{ and } f(X_{ij}(t))) \\
U_{ij}(G) \rightarrow f(U_{ij}(t)) \leq (f(V_{ij}(t)) \text{ and } f(X_{ij}(t))) \\
V_{ij}(G) \rightarrow f(V_{ij}(t)) \leq f(Y_{ij}(t)) \\
X_{ij}(G) & \text{otherwise}
\end{cases}
\]  

(23)

\(Y_{ij}(G), U_{ij}(G), V_{ij}(G)\) and \(X_{ij}(G)\) are the components of the new trail, trail, mutant, and target vectors, \(X_{ij}(G + 1)\) is the target vector at the current iteration. These improvements expedite the convergence and give more ability to produce good genes in the subsequent generations. The procedure of multi-objective function optimization stops whenever the number of non-dominated solutions equals the predetermined number. The structure of control variables that used to solve OPF problems is shown in Fig. 2. Figure 3 illustrates the flowchart of the MOIDEA.

<table>
<thead>
<tr>
<th>Generator slack bus [MW]</th>
<th>Generator voltages [p.u.]</th>
<th>Reactive power output [MVAr]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_{G_1})</td>
<td>(V_{G_1})</td>
<td>(Q_{G_1})</td>
</tr>
<tr>
<td>(\ldots)</td>
<td>(\ldots)</td>
<td>(\ldots)</td>
</tr>
</tbody>
</table>

(a)

<table>
<thead>
<tr>
<th>Generator output [MW]</th>
<th>Generator voltages [p.u.]</th>
<th>Transformer settings</th>
<th>Shunt elements [MVAr]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_{G_{NC}})</td>
<td>(V_{G_{NC}})</td>
<td>(T_1)</td>
<td>(Q_{1})</td>
</tr>
<tr>
<td>(\ldots)</td>
<td>(\ldots)</td>
<td>(\ldots)</td>
<td>(\ldots)</td>
</tr>
</tbody>
</table>

(b)

Fig. 2. Structure of: control variable (a); state variable (b)
5. Simulation results

To investigate the efficiency and performance of the proposed MOIDEA, the IEEE 30-bus test system, which has 6 generators, 41 transmission lines, 4 off-nominal tap ratio transformers and 9 shunt VAR compensators, has been employed. The system demand is 283.4 [MW]. To demonstrate the effectiveness of the proposed MOIDEA, three cases have been applied to solve the multi-objective optimization problem for every two objectives as shown below.

Case 1: Fuel cost considering emission
Case 2: Fuel cost considering real power loss
Case 3: Fuel cost considering voltage deviation

5.1. Case 1: fuel cost considering emission

In this case, the fuel cost of generating units and emission have been minimized simultaneously to reach an optimal operating point, in the economic sense, for the power system. The fuzzy set theory is the strategy used to select the best compromise solutions using Pareto front solutions with a population size of 200 pollinators. The best compromise solutions for fuel and emission...
cost obtained by the developed framework are $832.4283 [\$/h]$ and $0.2336 [\text{ton/h}]$, respectively. Figure 4(a) shows the best Pareto front solutions obtained from the proposed MOIDEA for total fuel cost, considering the emission of the IEEE 30-bus system. The proposed approach provided the results better than the other method for all three objective functions because of the modifications of the original DE. Table 1 confirmed the advantage of this method. The best Pareto front solutions obtained by the developed framework with other recent optimization techniques have been presented in Table 1.

5.2. Case 2: fuel cost considering real power losses

The set of Pareto front solutions obtained by the proposed approach illustrated the relationship between the total fuel cost of generation units and real transmission power losses which have been minimized together simultaneously, as shown in Fig. 4(b). The results of the comparison of this
Table 1. Comparison results with other recent optimization algorithms for Case 1 to Case 3

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed approach</td>
<td>832.4283</td>
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<td>831.8407</td>
</tr>
<tr>
<td>MOIDEA</td>
<td>832.4655</td>
<td>0.2351</td>
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<td>MPIO-PFM [36]</td>
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<td>831.6652</td>
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<tr>
<td>MOICA [43]</td>
<td>NA</td>
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<td>DE [44]</td>
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</tr>
<tr>
<td>BHBO [45]</td>
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<td>NA</td>
</tr>
<tr>
<td>BBO [46]</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>NKEA [43]</td>
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<tr>
<td>BB-MOPSO [43]</td>
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<td>MNSGA-II [43]</td>
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</table>
method with other recent optimization methods are in Table 1. The best compromise solution obtained by this algorithm for total fuel cost and real power transmission loss are 831.8407 \$/h and 5.1699 [MW], respectively.

5.3. Case 3: fuel cost considering voltage deviation

The set dominant points of Pareto-optimal solutions are illustrated in Fig. 4(c). Table 1 compares the best compromise value for the total fuel cost of generation units and voltage deviation of busses calculated by the MOIDEA with other recent algorithms. The best results for fuel cost and voltage deviation according to the proposed algorithm are 802.4803 \$/h and 0.1452 [p.u.].

In Fig. 4, Pareto optimal solutions are distributed very well for Cases 1–3. It is difficult to obtain the optimal solution with the MOOPF. Moreover, the best compromise solutions (BCs) obtained by the proposed approach of Case 1 can get better solutions than compared to other algorithms such as Modified pigeon-inspired and constraint-objective sorting rule (MPIO-COSR) [36], Non-dominated Sorting GA-II (NSGA-II) [36], a modified pigeon-inspired optimization algorithm with the commonly used penalty function method (MPIO-PFM) [36], Enhanced Self-adaptive Differential Evolution (ESDE) [37], Backtracking Search Optimization Algorithm (BSA) [38], Nondominated Sorting GA-III (NSGA-III) [39], Multi-Objective Particle Swarm Optimization (MOPSO) [39], Novel Hybrid Bat Algorithm (NHBA) [40], and Multi-Objective Dragonfly Algorithm (MODA) [41] is given in Table 1. In addition, the BCs of the MOIDEA achieve better solutions than other optimization algorithms such as MPIO-PFM [36], NSGA-II [36], NSGA-III [39], MOPSO [39], as shown in Table 1. Case 3 cannot demonstrate that the proposed approach achieves the best solution amongst the rest of the algorithms because the best compromises (BCs) of the MOIDEA are not dominated by the other algorithms, as shown in Table 1.

It is interesting to note that the final solution of the MOIDEA for the three cases has not violated the limits of constraints, keeping the control variables within permissible limits. Table 2 presents a summary of the obtained optimal setting of control variables for the best solution in the MOIDEA method. Remarkably, several methods that have been applied in the OPF do not constantly respect the entire boundaries and violated the feasibility limits. Therefore, Table 3 and Fig. 5 illustrated the solution feasibility of the MOIDEA for three cases.
<table>
<thead>
<tr>
<th>Item</th>
<th>Max</th>
<th>Min</th>
<th>Initial</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
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</thead>
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<td>$P_1$</td>
<td>50</td>
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<td>116.5440</td>
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<td>20</td>
<td>34.6228</td>
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<td>21.4564</td>
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<td>20</td>
<td>26.2468</td>
<td>28.6865</td>
<td>12.0778</td>
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<tr>
<td>$P_{13}$</td>
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<td>20</td>
<td>26.7211</td>
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<td>12.0307</td>
</tr>
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<td>1.1</td>
<td>1.05</td>
<td>1.0984</td>
<td>1.0996</td>
<td>1.0608</td>
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<tr>
<td>$V_2$</td>
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<td>1.0915</td>
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<td>$Q_{c23}$</td>
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<td>2.5057</td>
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<td>$Q_{c24}$</td>
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<tr>
<td>$Q_{c29}$</td>
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<td>0.2526</td>
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<td>Fuel cost [$/h]</td>
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<td>832.4283</td>
<td>831.8407</td>
<td>802.4803</td>
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<tr>
<td>Emission [ton/h]</td>
<td>0.2430</td>
<td>0.2336</td>
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<td>0.3286</td>
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<tr>
<td>Active power losses [MW]</td>
<td>5.6803</td>
<td>5.5393</td>
<td>5.1699</td>
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<td>Voltage deviation [p.u.]</td>
<td>1.1747</td>
<td>1.1228</td>
<td>1.3761</td>
<td>0.1452</td>
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Table 3. Reactive power of the generators obtained by MOIDEA for three Cases

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<tr>
<th>Unit number</th>
<th>$Q_g$ [MVar]</th>
<th>Min</th>
<th>Max</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
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<tr>
<td>1</td>
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<td>200</td>
<td>–14.96</td>
<td>–9.61</td>
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<td>28.59</td>
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<td>5</td>
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<td>80</td>
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<td>31.79</td>
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<td>–15</td>
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<td>30.90</td>
<td>24.19</td>
<td>7.03</td>
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</tbody>
</table>

6. Conclusions

This article proposes a developed optimization algorithm of the Multi-objective Improved Differential Evolution Algorithm (MOIDEA) for solving the multi-objective optimal power flow problem. Four conflicting objective functions have been considered, namely: total fuel cost (TFC) of generation units, total emission (TE), real power loss (RPL) of transmission lines, and voltage profile (VP) improvement at all busses whilst satisfying equality and inequality constraints. The Pareto optimization approach has been used to solve the multi-objective OPF problem by determining a set of non-dominated solutions (Pareto front) and by using fuzzy set theory to select the best compromise solution. The Multi-objective Improved Differential Evolution Algorithm – MOIDEA – has been applied to the IEEE 30-test bus system with four cases to validate the efficiency of the approach proposed. The results of the comparison demonstrate the effectiveness and the superiority of the MOIDEA method over other recent optimization methods to solve the multi-objective optimal power flow problem, as well as its ability to solve two objective function problems by selecting a set of non-dominated feasible solutions. The solution feasibility of the MOIDEA has been eligible. The dispersion of the solutions in the MOIDEA is relatively low due to the strong convergence of Pareto front solutions.

In future work, the proposed approach, MOIDEA, can be used to solve more difficult power systems with more control variables, such as the, IEEE 118-bus, and IEEE 300-bus systems. In addition, it can be used with multiple objective functions, such as the three-objective function, four-objective function, and five-objective function. Also, we can use more methods to determine the best compromise solutions, such as the centroid method, and entropy method.

References


