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New approaches for the identification of influential and critical nodes in an electric grid

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Abstract: The aftermath of including new technologies in a modern electric system in conjunction with the incessant rise in power demand could pose a risk to the optimal operation of the system. Therefore, it becomes imperative to identify the most influential and critical nodes of such a system to avert future problems in network operation. In this paper, to identify most significant nodes of the system, the authors propose two measures of centrality in accordance with the network structural properties of a power system, namely, degree centrality (DC) and eigenvector centrality (EC). These are expressed considering the admittance matrix that exists among the interconnection of load to load nodes in an electrical power network. A critical node closeness centrality (CNCC) method is also proposed to identify critical nodes of the system. This is done by modifying the conventional closeness centrality (CC) to include the influence of interconnection that exists between network load to load nodes as captured by the admittance matrix between them. A comparative analysis of the proposed techniques with other conventional methods is also carried out. The result of the simulation shows that the proposed methods could serve as alternative tools in the identification of influential and weak nodes in a power system.

Key words: centrality measures, critical nodes, influential nodes, power flow, power system

1. Introduction

The interconnected electric grid permits long-range power delivery for more effective power network operation; nonetheless, it likewise allows the spread of instabilities in the power grid [1]. As a consequence, in the past years, a substantial number of blackouts triggered by intrinsic

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catastrophes and man factors have supervened, which resulted in a series of severe impacts on the performance and safety of the power networks, social stability and economic growth. For example, in 1996, in August to be specific, more than four million people in numerous states, especially in the western parts of the United States of America (USA) remained outside of the power service [2, 3]. Also, a substantial shutdown was initiated in August 2003 in the power grid of Canada and the USA. As a consequence a 61 800 MW disc of power was disconnected to a region that spread up to the greater part of the USA and two provinces of Canada, wholly comprising over fifty million people [4]. The failure of some critical lines or nodes may make a power grid vulnerable to attacks, thus, resulting in a major blackout. Therefore, it has now become very vital to find the most influential vertices and edges to improve the power system reliability and effectiveness by checking and protecting them [5]. By critical nodes, we mean nodes where there are shortages of reactive power, which could result in system voltage drop and power loss. Similarly, some nodes play an important role in the connectivity, and it will cause enormous losses when they breakdown. Earlier studies have shown that, the positions of nodes in the reality network are of great difference. These nodes, when suddenly disconnected from the system, could cause severe damages to its optimum operation. These nodes are denoted as the important or influential nodes of a power system [2].

The early work of most authors on the performance assessment of weak buses in a power grid is centered on traditional power flow methods. Methods like continuation power flow, modal analysis, sensitivity analysis, use of voltage stability indices, and P–V and Q–V curves to mention a few, have generally been used to find a critical node in a power system [6–10]. Even though, these methods give valuable insight, however, the approaches are not without shortcomings, as they did not take into consideration, the interconnection that exists among the elements (loads and generators) and the influence of power network topology [11–14]. Recently, complex networks theory has attracted much attention in considerable number of arenas, which include social networks, management science, natural science, economics, computer science and biological science. Also, physical modeling of the power grid has recently been an interesting area of research to identify vulnerable nodes in a power network using the Gutierrez graph theory/complex network approach 2013 [15].Thus, there are numerous investigations on complex systems and complex networks presently [16–18].

For instance, [16] performed a research based on the susceptibility evaluation procedure of the large power network hinged on the concept of a graph theory. In the paper, the strength and fault propagation mechanism of the electrical network under cascading disturbances or failures were considered using the graph theory suggested by the author. Also, the author of [1] suggested a set of centrality measures (CMs) that were in connection with the matrix Y of the network. The authors in [19] investigated the use of electrical betweenness based on an approach, which captured the loss of load that shows the capacity of electrical networks to supply enough power to electricity users, to evaluate the susceptibility and the size of the largest cluster. The authors in [20] also proposed a metric called community-based mediator (CbM), to identify nodes which are influential in a bulky and complex power network. This index takes into consideration the entropy of a random walk from a bus to each community. The CbM defines how the bus is crucial to connect two or more than two communities of the network. Precisely, a study on the assessment of the node importance is of practical and theoretical consequence, for instance, it can be used in research on public opinion and rumor dynamics and in the control of the disease.

The most important buses in the power grid cause more serious or critical impact on the function and structure of the network than other. As the essential tool to determine the influence of buses in complex networks, substantial numbers of CMs have been used frequently. These include eigenvector centrality (EC), degree centrality (DC), closeness centrality (CC), betweenness centrality (BC), LeaderRank (LR), PageRank (PR), among other techniques. More recently, [11] and [12] proposed measures, which captured information that showed an electrical inter-relationship present between various vertices and edges of an electric power grid. This was solely dependent on the matrix Y of the network. The sensitivity of each vertex in the power network is obtained from the application of eigenvalue decomposition on the matrix Y [21, 22] Although, the conventional techniques of using a CM to identify important buses in an electric grid are quite insightful and have assisted power system engineers tremendously, nevertheless, none of these authors have done a thorough comparative analysis of the aforementioned CMs, particularly from the structural characteristics that are intrinsic in the power grid. In this present work, the authors established some theories to demarcate influential nodes from weak nodes in an electric network. One key benefit of the proposed approach is knowing the inherent features or properties of the power system as captured by the matrix Y. Electrical interconnections that occur among different nodes of the power system are easily known. This includes an interrelation between load to load buses, generator to generator buses, and load to generator buses. Also, the proposed technique is free or does not depend on the network loading conditions and thus can assist the system operator in the appropriate planning and operation of the power system should in the event of an incident of any contingencies.

The summary of the contributions of this paper are as follows: firstly, to find the most important nodes of a power system, we present existing CMs, majorly known as degree centrality (DC), closeness centrality (CC) and eigenvector centrality (EC) measures. We also proposed new measures of centrality (degree and eigenvector centralities) centered on the power grid topological characteristics of the system. This is done by modifying the conventional centrality measures (CCMs). A different method to the traditional closeness centrality measure (CCM) is also proposed. This is used to identify critical nodes of the system. The efficacy of the proposed critical node closeness centrality (CNCC) is done by comparing it with a load flow-based modal analysis method. Next, we carried out an in-depth comparative analysis of all the approaches and identify inconsistencies observed in the results obtained.

The rests of this paper are structured as follows: Section 2 presents the mathematical formulations of the existing centrality measures, while the suggested centrality measures centered on the network topological properties of a power grid are presented in Section 3. A load flow-based index for the identification of weak buses in a power system is also given in Section 3. Section 4 presents the simulation results and discussion. Section 5 concludes the work.

2. Problem formulations of the CCM

The problem formulations of the most prominent traditional centrality techniques are presented in this section.

2.1. DC measure

The DC measure signifies the interconnectivity of a vertex to the remainder of the power grid and shows the instant chance for a vertex to utilize its influences on the remainder of the power network [23]. Individuals that possess more links associated with others are more connected to the network. This is due to the fact that they have more sources and access to information than any other individual [24]. An electric power network may be denoted by the graph $G = (V, E, W)$ consisting of a set V, whose elements are named vertices and a set of well-ordered pairs of vertices, E, referred to as edges or links/lines and also, the element $t = (p, q)$ of the edge set E, whose direction is taken to be from p to q. q is taken as the head while p is named the tail of the edge E. Sets W, are considered as weights of edge set elements. It is worth-noting that there is a one-to-one correspondence between set E and set W [11, 12]. The analogy between a Laplacian L and the admittance matrix Y, may be used to formulate the degree centrality of a vertex or node V. Thus, electrical DC can be given as:

$$C_d^Y(V) = \frac{\|Y(V,V)\|}{n-1}\,. \tag{1}$$

2.2. EC measure

This approach makes use of weights of the first eigenvector to allocate a centrality value for every vertex. EC is linked with the matrix A, called the adjacency matrix. This measures the significance of a vertex or node in a network according to its adjacency [1]. It is assumed the network graph $G = (V, E, W)$, and its adjacency matrix A, one eigenvalue λ, and the equivalent eigenvector y satisfy $\lambda t = At$. We can define the centrality of a vertex v as the v-th entry of the eigenvector y associated with the largest eigenvalue $\lambda_{\max}$. The adjacency A may be extracted from the Laplacian:

$$A_t = -Y + \text{Diagonal}(Y), \tag{2}$$

where Y stands for the admittance matrix and Diagonal(Y) depicts the diagonal of the matrix Y. Thus, the EC of the node v as the v-th entry of the eigenvector t is given by:

$$C_E(v) = \|t_v\| = \left\| \frac{1}{\lambda_{\max}} \sum_{j=1}^{N} A(v,j)t_j \right\|. \tag{3}$$

It could be seen from (3) that the centrality of the vertex v is proportional to the summation of centralities of all its adjacent vertices. Thus, the definition selects the eigenvector associated with the maximum eigenvalue $\lambda_{\max}$. This is to ensure that, all the centrality scores, which are all the entries in the eigenvector, are positive.

3. Closeness centrality (CC) measure

CC could be expressed as the general mean of the shortest path between a vertex and all other vertices accessible from it [26]. The CC measure takes into account the notion of speed of communication between vertices in such a manner that the vertex which is “closest” to all others receives the maximum score. That is, the CC measure permits the identification of vertices, which on average require fewer steps to communicate with the other vertices, not only with the first neighbors [27]. In mathematical form, the CC measure of a vertex t, $C_C(t)$ in a network of m vertices is expressed as [24]:

$$C^C(t) = \frac{\sum\limits_{m \varepsilon V \backslash t} S(t,l)}{n-1}, \tag{4}$$

where $s(t,l)$ represents the shortest pathway length between the nodes t and l. This definition of CC indicates a measure of the distance of a specific vertex from other vertices. Subsequently, the inverse of the shortest path was used by some authors to compute the CC measure as follows:

$$C^C(t) = \frac{1}{\sum\limits_{m \varepsilon V \backslash t} S(t,l)}. \tag{5}$$

The electrical CC is defined by

$$C_C^V(v) = \frac{n-1}{\sum\limits_{m \in V/t} S_Z(t,l)}, \tag{6}$$

where $S_Z(t,l)$ represents the shortest electrical distance between the vertices t and l. To take a broad view of the theory in both transmission and distribution systems, the resistance of network lines must be taken into account, which is an important part of the impedance of the line applicable to distribution lines.

4. Suggested modified CM based on network structural characteristic indices (NSCIs)

The suggested approach of the modified CM based on network structural characteristics of an electric network is established based on the notion formulated in [28].

Let's assume that an electric grid is $\eta = (G, Y)$, where the graph $G = (U, E)$ and U are the sets of vertices signifying nodes, E is the set of edges formed by pairs of nodes. Y is the complex value on all the lines or edges $e \in E$. It must be noted that Y is taken as the admittance matrix of $e \in E$. The interrelationship between the admittance Y, voltage V and Ψ is the current injected into the network and found based on Kirchhoff's circuit law and the fundamental circuit law of (7).

$$\Psi = YV. \tag{7}$$

Equation (7) may be expressed as:

$$\Psi = Y_{\text{NETWORK}} V, \tag{8}$$

where Y_{NETWORK} is the network admittance matrix. If η is a connected electrical power network with boundary, we may then find the Schur complement of the response matrix R_Y in Y by partitioning the admittance matrix Y, expressed in terms of generator vertices and load vertices as:

$$Y = \begin{bmatrix} Y_{G-G} Y_{G-L} \\ Y_{L-G} Y_{L-L} \end{bmatrix}, \tag{9}$$

where Y_{G-G} represents the square matrix Y containing the interconnection between boundary (generator) nodes. Y_{G-L} is the $(G \times L)$ admittance matrix relating the boundary nodes with the load nodes. Y_{L-G} represents the transpose of Y_{G-L}. Y_{L-L} is the square admittance matrix containing the interconnectivity between load buses.

Also, by re-writing and partitioning (8), we have:

$$\begin{bmatrix} \Psi_G \\ \Psi L \end{bmatrix} = \begin{bmatrix} Y_{G-G} Y_{G-L} \\ Y_{L-G} Y_{L-L} \end{bmatrix} \begin{bmatrix} V_G \\ V_L \end{bmatrix}, \tag{10}$$

$$\Psi_G = Y_{G-G} V_G + Y_{G-L} V_L\,, \tag{11}$$

$$\Psi_L = Y_{L-G} V_G + Y_{L-L} V_L\,, \tag{12}$$

$$V_G = [Y_{G-G}]^{-1} \left([\Psi_G] - [Y_{G-L}]\,[V_L] \right). \tag{13}$$

By substituting (13) into (12)

$$[\Psi_L] = [Y_{L-G}]\,[Y_{G-G}]^{-1}\,[\Psi_G] + \Big([Y_{L-L}] - [Y_{L-G}]\,[Y_{G-G}]^{-1}\,[Y_{G-L}] \Big)\,[V_L]\,. \tag{14}$$

In which case, we define

$$R_Y = [Y_{L-L}] - [Y_{L-G}]\,[Y_{G-G}]^{-1}\,[Y_{G-L}] = \Gamma_{L-L} \tag{15}$$

as the Schur complement of the submatrix Y_{G-G} in the admittance matrix Y.

Next, by combining (14) and (13),

$$\begin{bmatrix} V_G \\ \Psi_L \end{bmatrix} = \begin{bmatrix} [Y_{G-G}]^{-1} - [Y_{G-G}]^{-1}\,[Y_{G-L}] \\ [Y_{L-G}]\,[Y_{G-G}]^{-1}\,[Y_{L-L}] - [Y_{L-G}]\,[Y_{G-G}]^{-1}\,[Y_{G-L}] \end{bmatrix} \begin{bmatrix} \Psi_G \\ V_L \end{bmatrix}, \tag{16}$$

we may re-write (16) as:

$$\begin{bmatrix} V_G \\ \Psi_L \end{bmatrix} = \begin{bmatrix} Y_{G-G}^{-1} \Pi_{G-L} \\ \Pi_{G-L}^{T} \Gamma_{L-L} \end{bmatrix} \begin{bmatrix} \Psi_G \\ V_L \end{bmatrix}, \tag{17}$$

where Π_{G-L} is the electrical interconnectivity that occurs between load and generator nodes. Π_{G-L}^{-1} is the negative transpose of $[Y_{G-G}]^{-1}$ that represents the impedance between the generator nodes in an electric grid. Ψ_G and Ψ_L are the generator and load injected currents, respectively. V_G and V_L represent the complex voltages of the generator and load nodes, respectively. The response matrix $\mathfrak{R}_{LL} = \Gamma_{LL}$ is the total load node equivalent admittance. It depicts electrical interconnectivity that occurs between load nodes of the power grid $\eta = (G, Y)$

Thus, from (16) and (17),

$$\Pi_{G-L} = -\left[Y_{G-G}^{-1}\right]\,[Y_{G-L}]\,, \tag{18}$$

$$\Pi_{G-L}^{T} = [Y_{L-G}]\,[Y_{G-G}]^{-1}, \tag{19}$$

$$\Gamma_{L-L} = [Y_{L-L}] - [Y_{L-G}]\,[Y_{G-G}]^{-1}\,[Y_{G-L}]\,. \tag{20}$$

Eigenvalue decomposition may then be applied on the matrix $\Re_{LL}$:

$$\Re_{LL} = \Gamma_{L-L} = M\Phi M^{*} = \sum_{j=1}^{N} m_j \lambda_j m_j^{*}, \tag{21}$$

where M represents the orthonormal matrix with the associated right eigenvectors m_j of the matrix $\Re_{LL}$. m_j^{*} is the left eigenvector of $\Re_{LL}$. Φ is the diagonal matrix with the eigenvalue λ_j as its diagonal elements. Since, the response matrix $\Re_{LL}$ is a symmetrical matrix, (21) may be written as:

$$\Re_{LL} = \Gamma_{L-L} = [m_1 \ \ldots \ m_N] \begin{bmatrix} \lambda_1 & & & & \\ & \cdot & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \lambda_N \end{bmatrix} \begin{bmatrix} m_1 \\ \\ \\ \\ m_N \end{bmatrix}. \tag{22}$$

4.1. Proposed DC measure based on NSCI

The suggested approach of the DC measure based on the NSCI is formulated in line with the established index of (22) as:

$$D_Y(l) = \frac{\|\Re(l,l)\|}{N_L - 1}, \tag{23}$$

where l is the load (interior) nodes and N_{L-1} is used as a normalization factor. N_L represents the total number of load nodes in the system. $D_Y(l)$ represents the suggested electrical degree centrality measure for the load (interior) node of the electric power network $\eta = (G, Y)$.

A target node of the network is identified from (23) by considering the vertex with the maximum value of $D_Y(l)$.

4.2. Proposed eigenvector centrality measure

The matrix $\Re_{LL}$ of (18) is of great significance and forms the basis on which the modified EC measure is established. Let's suppose we have the graph $G = (V, E)$ of the network $\eta = (G, Y)$, one eigenvalue, λ, with its adjacency matrix A, the associated eigenvector m fulfils

$$\Re_{LL} m = \lambda m. \tag{24}$$

Worth noting is the fact that the suggested technique based on the network structural features of a power system is a function of the electrical interconnection pattern that occurs between the load nodes.

Given m_i is the eigenvector of the j-th vertex of the connected and weighted network, associated with the maximum eigenvalue $\lambda_{\max}$, the EC of the bus j, taking into consideration the interior vertices, is given as:

$$C_{mE}^{\Re_{LL}}(j) = \|m_j\| = \left\| \frac{1}{\lambda_{\max}} \sum_{i=1}^{N} \Re_{LL}(j,i) m_i \right\|. \tag{25}$$

The definition of the EC of (25) also selects the eigenvector associated with the highest eigenvalue to keep all the centrality scores to be positive. The load node j that gives the highest value of $C_{mE}^{\Re_{LL}}$ is considered as the important node of the network $\eta = (G, Y)$.

4.3. Proposed critical node closeness centrality measure

The suggested approach of the CNCC measure considering the power grid topological features of the system is also expressed in accordance with the matrix of (18) and as an improved form of the traditional electrical CC. Suppose N stands for the overall number of load vertices in an electrical network and i and j represent the nodes, then, the suggested CC is formulated in terms of the interrelationship that occurs between load to load nodes determined by the matrix Y of the network, as follows:

$$C_C^i(i) = \frac{N_L - 1}{\sum\limits_{c \in I/i} \Re_{LL}(i, j)}. \tag{26}$$

The CC measure for the power network also identifies the relative closeness of a load bus with the maximum closeness to other buses.

4.4. Modal analysis

The mathematical formulations as presented in [29] are also adopted in this study to find critical nodes of a power system.

5. Simulation results and discussion

To demonstrate the efficacy of all the approaches presented, the results of two numerical examples are presented in this section. The proposed approaches are illustrated using the IEEE 5-bus and IEEE 57-bus power systems. The IEEE 5-bus power system consist of two (2) generator buses, three load buses and seven (7) transmission lines. Similarly, the standard IEEE 57-bus power system comprises seven (7) generator buses and fifty (50) load nodes. Simulation of results are done using MATLAB R2013a.

5.1. Test Case I: The standard IEEE 5-bus power system

This section presents the results of both the conventional and the suggested modified DC and EC measures. The aim is to identify the most central node, otherwise denoted as the most important node of the network. These nodes are often called because their removal can deteriorate the strength of the system considerably and may cause a huge loss to network performance. The result of the simulation obtained for the relative significance of nodes, taking into consideration both the traditional electrical DC and the proposed electrical DC measures for the IEEE 5-bus test system is presented in Table 1. Similarly, Table 2 shows the simulation results for both the traditional and suggested EC measures for the IEEE 5-bus test system. The load node that corresponds to the highest DC is taken as the most influential of the network. The values of the conventional DC measure and that of the suggested method are obtained from the network topological perspective of the electric grid. It is worth of noting that only PQ nodes are taken into consideration in this work. For the conventional degree centrality, Eq. (1) is used to find the electrical degree centrality of each load node. Load node 4 of column 2 of Table 1 is characterized by the largest number of incident links. When compared with other load nodes, node 4 has a maximum value of 10.987 and is thus considered to be the most influential of the IEEE 5-bus test system. Similarly, for the suggested degree centrality, to compute values of the centrality for each load node, first, the response matrix of Eq. (22) is computed. This matrix captures the interconnectivity n that occurs between several load nodes of the grid. The power network is arranged sequentially so that the generator nodes form the boundary vertices while the load nodes form the interior vertices. Table 1 indicates that node 4 of column 4 shows the highest degree centrality value of 19.7508. This implies that, in the proposed method, load node 4 has the highest number of incident links compared with load nodes 3 and 5 of the IEEE 5-bus power system.

Table 1. DC for the IEEE 5-bus

Load bus	**Conventional degree centrality**	**Ranking order**	**Proposed degree centrality**	**Ranking order**
3	10.1985	2nd	18.3432	2nd
4	**10.1987**	**1st**	**19.7508**	**1st**
5	2.9552	3rd	4.4565	3rd

Table 2. EC for the IEEE 5-bus

Load bus	**Conventional degree centrality**	**Ranking order**	**Proposed degree centrality**	**Ranking order**
3	0.0580	3rd	0.6867	2nd
4	**15.3764**	**1st**	**0.7199**	**1st**
5	13.1345	2nd	0.1003	3rd

In the same vein, to identify most central nodes of the IEEE 5-bus test-system using the conventional eigenvector centrality, the adjacency matrix is first computed using Eq. (2). Thereafter, the technique of eigenvalue decomposition is applied to the adjacency matrix formed. The eigenvector of each load node associated with the biggest eigenvalue is determined using Eq. (3). Table 2 shows the results of the simulation obtained for both the traditional and suggested eigenvector centrality methods. Table 2 shows that, bus 4 of the IEEE 5-bus test system has the highest value for both the conventional and proposed eigenvector centralities of 15.3764 and 0.7199, respectively. This implies that, if, for example, load node 4 is abruptly disconnected from the power grid, the aftermath could be very disastrous as it could lead to the system becoming vulnerable to voltage instability as a result of a high relative impact of this node compared with other nodes. The results of the CCM are also presented in Table 3. Bus 4 is ranked as the most influential of the IEEE 5-bus system with a CCM value of 9.9002, as presented in column 2, row 2 of Table 3.

Table 3. Conventional closeness centrality for the IEEE 5-bus

Load bus	**Conventional closeness centrality**	**Ranking order**
3	9.6877	2nd
4	**9.9002**	**1st**
5	8.7501	3rd

Identification of critical nodes are done using the conventional load flow centered on modal analysis and the suggested CNCC. To identify the critical node of the systems under consideration based on the modal analysis approach, Eigenvalue Decomposition (ED) method is employed and is used on the Jacobian matrix. The critical mode that is associated with the least eigenvalue of the system is determined, and the critical node liable to voltage instability is afterward identified using the participation factors. The node with the maximum value of the participation factor is considered the critical node. Also, to find the critical bus of the power network using the suggested CNCC, the CCM is modified to include the influence of the interconnectivity that occurs between load to load buses of a power system as captured by the matrix Y between them.

The proposed CNCC is of immense benefit as there is no need to run a load flow solution before the weak node of the system is identified. This is because power flow equations are nonlinear and have to be solved iteratively. The use of iterative techniques only provides an arithmetical solution of a power flow without supplying info on the structural interrelationship among nodes that guides the solution that ensued. As such, it could be cumbersome and laborious to use the load flow-based technique. To detect critical nodes of the IEEE 5-bus power system, the matrix Y of the power network is first calculated. Subsequently, the index $\Re_{LL}$ of Eq. (18) is computed. The CC value for each load bus is then found using Eq. (24). The result obtained using both the modal analysis and the proposed CNCC in the identification of the critical node is presented in Table 4. With both techniques, bus 5 was ranked highest and thus, it is considered as the critical node of the IEEE 5-bus test system. Bus 5 is also a potential bus to install the reactive

Table 4. Modal analysis and the proposed CNCC IEEE 5-bus

Load bus	**Modal analysis**	**Voltage mag. (pu)**	**Ranking order**	**Proposed critical node closeness centrality**	**Ranking order**
3	0.3267	1.0611	3rd	0.0544	2nd
4	0.3651	1.0543	2nd	0.0506	3rd
5	**0.3882**	**1.0435**	**1st**	**0.2234**	**1st**

power compensator. This bus also has the least voltage magnitude of 1.0435 pu. Table 5 shows comparative results obtained for all the approaches studied in this paper considering the IEEE 5-bus power system.

Table 5. Comparison of all the techniques for the IEEE 5-bus

	Conventional Centrality Measures			**Proposed Centrality Measures**			**Modal analysis**
Load bus	**Degree**	**Eigenvector**	**Closeness**	**Degree**	**Eigenvector**	**CNCC**	
3	10.1985	0.0580	9.6877	18.3432	0.6867	0.0544	0.3267
4	10.1987	**15.3764**	**9.9002**	**19.7508**	**0.7199**	0.0506	0.3651
5	2.9552	13.1345	8.7501	4.4565	0.1003	**0.2234**	**0.3882**

5.2. Test Case III: IEEE 57-bus power system

The result of relative significance of nodes, with both the traditional and proposed electrical DC CM for the IEEE 57-bus power system is as shown in Table 6. The vertex (node) with the highest DC is taken as most influential of the system. In this paper, the first twenty (20) load nodes of the IEEE 57-bus that have maximum values of the DC compared with other load buses are selected. It is worth noting is that, the values of both the existing and suggested DC measures are obtained from the viewpoint of structural properties of the power grid. Beginning with the load bus having the maximum number of connections to other buses of the IEEE 57-bus network, with the conventional DC presented in column 2 of Table 6, load node 22 has the maximum number of incident links with a DC value of 1.6209. Hence, this node is considered the most influential of the PQ nodes of the IEEE 57-bus test system. Similarly, with the suggested methods of electrical DCs presented in column 4 of Table 6, load node 22 is also picked as the most influential node (having a maximum DC value of 1.6209) of the IEEE 57-bus power system. Similar results are also obtained for other techniques involving the traditional and proposed EC, as presented in Table 7.

The traditional and proposed EC also pick bus 22 as the most important of the IEEE 57-bus test system, being the bus with the largest DC value of 0.7690 and 0.7681, respectively. The results of CC are also presented in Table 8. Bus 22 has the largest value of 0.7690 and thus ranked highest.

To find the critical bus of the IEEE 57-bus power system, the values of both left and right eigenvectors of the system are computed. This is later used to find the participation factor of each load bus depending on the critical mode (bus with the least eigenvalue) of the system. For the conventional modal analysis approach, the first three critical load buses are identified as 31, 33 and 32 and presented in Table 9. Their participation factor values, that is, for buses 31, 33 and 32 are found to be 0.1876, 0.1563 and 0.1529, respectively. The proposed CNCC method, whose result is presented in Table 9, also identified load buses 31, 33 and 32 as the first three critical nodes of the system. These three buses, 31, 33 and 32, have the largest values of 10.0647, 8.4902 and 7.8475, respectively.

Table 6. Degree centrality for the IEEE 57-bus system

Load bus	**Conventional degree centrality**	**Ranking order**	**Load bus**	**Proposed degree centrality**	**Ranking order**
22	1.6209	1st	22	1.6209	1st
38	1.3238	2nd	38	1.3238	2nd
13	1.3141	3rd	13	1.3141	3rd
15	1.1858	4th	23	1.0429	4th
23	1.0429	5th	48	1.0302	5th
48	1.0302	6th	36	0.9631	6th
36	0.9631	7th	14	0.9399	7th
8	0.9613	8th	37	0.9284	8th
14	0.9399	9th	15	0.8997	9th
37	0.9284	10th	10	0.8459	10th
10	0.8459	11th	47	0.8428	11th
47	0.8428	12th	8	0.7828	12th
9	0.6900	13th	11	0.5704	13th
11	0.5704	14th	29	0.5699	14th
29	0.5699	15th	9	0.5222	15th
46	0.4852	16th	46	0.4852	16th
24	0.4573	17th	24	0.4573	17th
35	0.4496	18th	35	0.4496	18th
12	0.4446	19th	12	0.4446	19th
26	0.4282	20th	26	0.4282	20th

Table 7. Eigenvector centrality of the IEEE 57-bus system

Load bus	**Conventional eigenvector centrality**	**Ranking order**	**Load bus**	**Proposed eigenvector centrality**	**Ranking order**
22	0.7690	1st	22	0.7681	1st
23	0.5120	2nd	23	0.5110	2nd
38	0.3634	3rd	38	0.3651	3rd
48	0.0874	4th	48	0.0910	4th
44	0.0470	5th	44	0.0473	5th

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Table 7 [cont.]

Load bus	**Conventional eigenvector centrality**	**Ranking order**	**Load bus**	**Proposed eigenvector centrality**	**Ranking order**
21	0.0418	6th	21	0.0418	6th
37	0.0379	7th	37	0.0382	7th
47	0.0319	8th	47	0.0333	8th
24	0.0141	9th	49	0.0197	9th
36	0.0098	10th	24	0.0149	10th
49	0.0097	11th	36	0.0099	11th
39	0.0072	12th	39	0.0072	12th
46	0.0038	13th	46	0.0043	13th
45	0.0027	14th	45	0.0028	14th
26	0.0024	15th	26	0.0026	15th
40	0.0014	16th	40	0.0015	16th
35	0.0012	17th	35	0.0025	17th
13	0.0007	18th	13	0.0024	18th
14	0.0005	19th	14	0.0015	19th
15	0.0005	20th	15	0.0009	20th

Table 8. Closeness centrality of the IEEE 57-bus system

Load bus	**Conventional closeness centrality**	**Ranking order**	**Load bus**	**Conventional closeness centrality**	**Ranking order**
22	0.7690	1st	49	0.0097	11th
23	0.5120	2nd	39	0.0072	12th
38	0.3634	3rd	46	0.0038	13th
48	0.0874	4th	45	0.0027	14th
44	0.0470	5th	26	0.0024	15th
21	0.0418	6th	40	0.0014	16th
37	0.0379	7th	35	0.0012	17th
47	0.0319	8th	13	0.0007	18th
24	0.0141	9th	14	0.0005	19th
36	0.0098	10th	20	0.0004	20th

Table 9. Modal analysis and the proposed CNCC of the IEEE 57-bus system

Load bus	Modal analysis	Ranking order	Load bus	Proposed CNCC	Ranking order
31	0.1876	1st	31	10.0647	1st
33	0.1563	2nd	33	8.4902	2nd
32	0.1529	3rd	32	7.8475	3rd
30	0.1477	4th	30	4.9290	4th
25	0.1132	5th	25	4.9152	5th
34	0.0279	6th	34	4.1937	6th
35	0.0212	7th	54	4.0834	7th
40	0.0167	8th	40	4.0066	8th
36	0.0166	9th	24	3.1020	9th
24	0.0144	10th	36	3.0144	10th
39	0.0137	11th	39	2.0768	11th
37	0.0135	12th	57	2.0670	12th
26	0.0120	13th	20	2.0010	13th
57	0.0116	14th	21	1.9398	14th
56	0.0097	15th	56	1.6521	15th
42	0.0079	16th	42	1.4098	16th
23	0.0071	17th	23	1.2578	17th
21	0.0067	18th	41	1.0234	18th
22	0.0062	19th	27	1.0096	19th
38	0.0060	20th	38	1.0022	20th

6. Conclusion

In this paper, the efficacy of the suggested methods of DC and CC in the identification of the most important nodes of the electrical power grid has been investigated. This suggested technique is compared with other traditional centrality methods of DC, EC and CC. A method centered on the network structural characteristics of a power system named CNCC is also proposed to identify critical buses that are susceptible to voltage collapse in a power system. This is afterward compared with a power flow-based modal analysis method. An in-depth comparative analysis of all the approaches considered is also discussed. The performance of all the techniques investigated and presented is tested over the IEEE 5 bus and IEEE 57-bus power systems. The results of the simulation obtained show that the proposed DC and EC could be a good tool to identify the most influential nodes of the system as it captures information related to the network topological properties of the system. Also, the proposed CNCC gives information, which is not contained in the conventional method such as the network interconnection that exists between various nodes (load to load, load to generator, among others) of the power system and the inherent structural properties of such a network. Overall, the proposed methods could serve as an alternative tool to the conventional ones in identifying the most important nodes and critical nodes of the system. The implementation of this work will help power system engineers in the proper planning and operation of the power system.

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