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Application of seepage time curve in the assessment of flood embankment tightness

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Abstract: Flood embankments have played an important role in flood protection systems for centuries. Tightness evaluation of water structures should result in determining whether floods may cause seepage incidents, consequently leading to their damage or even destruction. It is assumed that the time of water passage from the river to the protected area under steady-state conditions can be a good indicator of the embankment resistance to long-term water rise. The curve of water passage times shows the values obtained at different ordinates of the dammed water and can be regarded as a characteristic parameter of flood embankment tightness. Determination of the water passage time (T_p) additionally requires finding the trajectory along which this time is the shortest possible. However, there is no need to analyse the unsteady filtration, which requires the determination of an initial condition and the course of time-dependent boundary conditions. Engineers in practice, often use the time T_{pp} which elapses from the beginning of flooding to the occurrence of seepage in the protected area. The relationship between the passage time (T_p) and the seepage onset time (T_{pp}) was analysed on a model example. Practical use of the curve of passage times is showed on the example of the reconstruction of the left-bank Warta embankment in the area of Konin.

Keywords: flood protection system, levee, seepage, time of seepage

INTRODUCTION

Floods are classified as extreme hydrologic incidents that pose a significant threat to people and the economy. The droughts observed in recent years in Poland have resulted in greater interest of researchers and the general public in this factor. However, some more in-depth scientific analyses indicate that it cannot be concluded that in Poland and more broadly in Europe the frequency as well as the size of floods (measured by the volume of the wave) is decreasing due to contemporary climate change [Kundzewicz *et al.* 2018]. Therefore, the development and modernisation of flood protection systems that lead to the reduction of flood risk should still be prioritised in all socio-economic activities. For centuries, flood embankments have been an essential part of protection systems. The most important factor that exposes embankments to destruction during the passage of flood wave is the too low ordinate of the embankment crest. Consequently, it leads to water overflow and washout of the entire structure. Intense seepage through the embankment body or the substratum is the second destructive factor (i.a., Warcholak and Tonder [1998]). Prolonged intensive filtration can result in dragging fine fractions from the body or its substratum (suffosion). This forms caverns which, as they develop over time, lead to hydraulic piping.

Another mechanism of destruction consists in displacing the poorly permeable layer of soil in the landside area. The seepage flow may also cause the slope to lose its stability. Due to the possible occurrence of these phenomena, it is postulated that the seepage through the embankment body and the substratum should be limited.

Engineering analyses seek to answer the question whether the flood embankment is tight enough to prevent any threats to its safety due to filtration. According to the guidelines for designing flood embankments applied in Poland, two characteristic times are calculated – the seepage onset time (T_{pp}) and the time of reaching the steady-state of the seepage flow (T_s) [Borys, Mosiej 2003]. Both these values are difficult to determine. In order to determine T_{pp}, it is necessary to know the initial condition (the state of the system at the beginning of the water rise) and/however, such information is usually not available. To determine the time of reaching the steady-state of the seepage flow, it is necessary to adopt a properly selected criterion of achieving this state. It is proposed to adopt the time of passage (seepage time) from the floodplain to the landside area under steady-state flow conditions as the characteristic time [SROKA 2011]. The parameter is not related to the initial condition or to the shape of the flood wave. Additionally, what is worth mentioning is the fact that it is relatively easy to calculate.

MATERIALS AND METHODS

A typical cross-section through a flood embankment is showed in Figure 1. There are four zones marked here: A – riverbed and floodplain, B – embankment body together with the substratum, C – drainage zone, D – protected area. At low water levels in the river (bank water), the seepage flow, usually of low intensity, goes through the permeable substratum. When water rises, it flows to the floodplain that extends from the edge of the riverbed to the embankment foot. High water level in the river induces seepage in the embankment body and in the substratum, which is directed towards the landside area. Time-varying head water $H_g(t)$ is the key factor affecting the phenomenon of filtration, yet not the only one to be considered. Saturation of the embankment body at the beginning of the flood is also vital. The landside area D is limited by introducing the so-called mathematical boundary. Hydrostatic pressure distribution is usually assumed along this boundary (alternatively, an impermeable boundary can be assumed). Proper distance of the mathematical boundary from the embankment ensures that this simplification has little effect on the process of filtration in the embankment body (cf., i.a. KLIMIUK *et al.* [2006]).

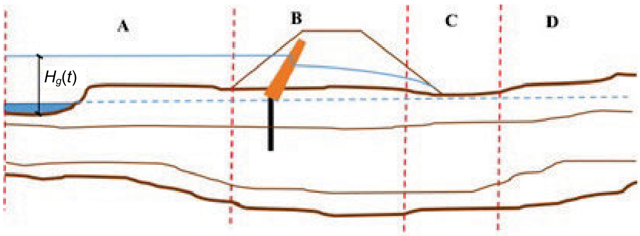

Fig. 1. Scheme of the problem; $H_g(t)$ = time-varying head water, A = riverbed and floodplain, B = embankment body together with the substratum, C = drainage zone, D = protected area; source: SROKA [2011]

For the analysis of the discussed filter flow problem, it is convenient to use a mathematical model that considers both saturated and unsaturated zones. The variable of interest can be the pore water pressure $h(x, t)$ or the piezometric height $H(x, t)$. In the work, the problem was formulated, and a solution was sought in the form of the piezometric height distribution H. This function should satisfy the continuity equation and the motion equation, which lead to the following second-order differential equation:

$$C\frac{\partial H}{\partial t} = \mathrm{div}[\mathbf{E}\ \mathrm{grad}(H)] \tag{1}$$

where: C = retention function including the compressibility effect of water and soil, as well as variable saturation; H = piezometric height; t = time; $\mathbf{E}$ = hydraulic conductivity matrix, it is the saturation function $\mathbf{E} = \mathbf{E}(S)$ in the unsaturated zone (S = degree of saturation), in saturation zone it is constant and denoted by the symbol $\mathbf{K_s}$.

The generalised Darcy's law was applied. In the unsaturated zone, it was assumed that the hydraulic conductivity is expressed as the product of the conductivity matrix at full saturation $\mathbf{K_s}$ and the scalar function of saturation called relative conductivity $k_r(S)$. When there is an unsaturated zone, the values of the retention function depend mainly on changes in saturation and the remaining factors can be neglected [BEAR, VERRUIJT 1987]. Knowing the value of hydraulic conductivity in the saturation zone $\mathbf{K_s}$, it is also necessary to know the function $k_r(S)$ and the retention curve $S(\Psi)$. Numerous empirical formulas have been developed describing the hydraulic conductivity and the retention curve in the unsaturated zone. The most used, due to their good accuracy, are the formulas of Mualem–van Genuchten [VAN GENUCHTEN 1980; ZARADNY 1990]:

$$S(\Psi) = \left[\frac{1}{1+(a\Psi)^n}\right]^m, k_r(S) = \sqrt{S}\left[1-\left(1-S^{1/m}\right)^m\right]^2, \begin{smallmatrix}(m=1-1/n)\\(0<m<1)\end{smallmatrix} \tag{2}$$

where: a = parameter (L^{-1}); Ψ = suction height; m, n = parameters; $k_r(S)$ = relative conductivity; S = degree of saturation.

In Equations (2), the parameters m, n are dimensionless and have no physical interpretation and the parameter a for large values of n will be approximately equal to the inverse of the bubbling pressure. It worth adding that the piezometric height H, the pressure height h and the height of suction Ψ are related to the dependencies: $H = z + h$, $\Psi = -h$ (z-ordinate).

The described approach has two clear advantages over other models (e.g., with a free surface). First, the problem is solved, admittedly described by a non-linear differential Equation (1), but in a known and time-invariant flow area (this was already noticed by BEAR and VERRUIJT [1987]). Secondly, more complex problems e.g., with internal seepage zones, with atmospheric boundary conditions or in which water uptake by the plant root system occurs, can be addressed here without difficulty. Using this approach, even problems with preferential filtration pathways created by burrowing animals or penetrating plant roots are analysed [COBOS-ROA 2015].

However, the complete description of the model still requires the specification of the initial condition i.e., the distribution of the piezometric head over the entire area at the initial moment $H(\mathbf{x},0)$ and the appropriate boundary conditions given as time-dependent functions. For steady-state problems, the above mathematical description is greatly simplified. In Equation (1), the left-hand side is equal to zero, and it is not necessary to know the retention function C. There is also no need to specify the initial condition and time-dependent boundary conditions. The obtained solution $H(\mathbf{x})$ makes it possible to determine the passage time, which is proposed to be adopted as an indicator of embankment tightness.

The problem can only be solved by using numerical methods such as the finite element method (FEM) or the finite difference method (FDM). Probably the first work on the application of FEM to solve the described problem was published by NEUMANN [1973]. Since then, many scientific papers have been published and computer software has been developed (e.g., MIROSŁAW-ŚWIĄTEK [2002], SROKA *et al.* 2004]. Currently, numerous commercial programs are available (e.g., HYDRUS, SEEP/W), and it is also possible to obtain free software packages. The study uses its own FEM software that allows for analysing 2D filtration problems in saturated and unsaturated zones. The software package uses classic triangular elements with linear base functions. The built-in data generator allows for creating a mesh with appropriately refined zones. The results are presented graphically in the form of the distribution of piezometric heights, pressures, hydraulic gradients. At the post-processing stage, it is also possible to calculate the trajectories of moving water particles. Since the velocity within the applied element is constant, good mapping of the trajectory requires the use of a dense FEM mesh. A detailed description of the software can be found in the work of SROKA *et al.* [2004], complemented with the examples of calculations as well as the software itself on CD.

It should be added that the phenomenon of hysteresis occurs during the filtration in the unsaturated zone. The irrigation and drainage processes follow different paths. Other phenomena accompanying filtration are related to the transport of small soil skeleton particles in pores, their deposition (clogging) or dragging away from the porous medium – the embankment body or the substratum (suffosion). Consolidation and contraction are other phenomena related to the movement of water in soil pores. However, they were not included in this analysis. It should be remembered that the use of sophisticated models, taking into account the above-mentioned phenomena, requires a lot of detailed data on the porous medium, initial conditions and the course of many physical processes. Even for physical models created under laboratory conditions, it is difficult to carry out all necessary measurements (e.g., when modelling mechanical suffosion [POPIELSKI 2000]). Flood embankments are linear objects of considerable length (tens or hundreds of kilometres), and the recognition of geotechnical parameters is usually poor. There are also no monitoring systems, which makes the practical use of such complex models very difficult or even impossible to apply in practice.

Flood embankments only dam water only periodically. They do not have to be perfectly tight. However, the intense seepage flow may compromise their safety (suffosion, hydraulic piping). In engineering analyses, the seepage onset time (T_{pp}) is used as one criterion to determine whether a hazard exists [BORYS, MOSIEJ 2003]. When the duration of flood is shorter than the time, it is assumed that there is no risk. Otherwise, it is necessary to perform detailed calculations of seepage flow. However, calculating the T_{pp} is difficult for most practical problems, as it requires a simulation (most often to be carried out on a numerical model) of the course of unsteady seepage flow in the embankment body and the substratum. Therefore, it is necessary not only to recognise well the filtration and retention properties of the soil, but also the piezometric height distribution at the initial moment, in the entire flow area (initial condition). Due to the time-consuming nature of a detailed analysis, it is advisable only for the embankments of special importance. On the other hand, practice shows that even such objects lack necessary data to perform appropriate calculations (e.g., WOSIEWICZ and SROKA [2001]).

According to the author, flood embankment tightness may be characterise by the time T_p needed for seepage water to pass from the floodplain to the protected area under steady-state flow conditions. The parameter is not related to the initial condition (moisture distribution in the embankment body) or to the shape of the flood wave. Additionally, which is worth noting, it is relatively easy to calculate. When determining the passage time, the following boundary conditions are proposed:

– no flow on the riverside slope above the headwater, on the crest and along the landside slope (above the possible seepage face);
– the designed ordinate of the water level or the value corresponding to the permissible flooding adopted for the internal drainage or the drainage ditch;
– the ground level in the landside area treated as an impermeable boundary, however allowing occurrence of the seepage face (pressure not greater than zero).

Once the distribution of piezometric heights has been determined, it is necessary to find the trajectory along which the water passage time from the floodplain to the protected area (landside area, drainage ditch, seepage face into the landside slope) is the shortest [SROKA 2004; 2011]. By calculating the passage times for different water ordinates in the river, starting from the ordinate of the embankment foot up to the maximum ordinate of damming, a useful characteristic of a given structure can be obtained – the seepage time curve. In order to determine the relationship between the seepage onset time T_{pp} used so far in engineering practice and the proposed passage time T_p, analyses were carried out on a model example – a dam without drainage and sealing elements, erected on an impermeable substrate. Application of the seepage time curve in practical engineering problems is shown on the example of evaluation of sealing alternatives for the modernised embankment of the Warta River in the area of Konin.

RESULTS AND DISCUSSION

MODEL PROBLEM

For the purpose of the study, the author considered the seepage through the embankment erected on an impermeable substrate (Fig. 2), caused by a sudden rise of the water table in the river. The analysis was carried out in order to compare the passage time under steady-state conditions at the maximum ordinate of the flood wave T_p, with the seepage onset time determined for the unsteady flow T_{pp}. It was assumed that the flooding process occurs suddenly, but not abruptly – approx. two orders of magnitude shorter than the passage time T_{pp}. The simulation was preceded by a preliminary stage, in which saturation in the embankment foot was increased from the initial value (negative pressure) to full saturation (pressure equal to zero). Then, the upper water level H_g increased from zero to its maximum value. This enabled a continuous transition from the initial piezometric height distribution $H(\mathbf{x},0)$ to the states achieved over time. In this way, an oscillation-free solution was obtained, and the inter-

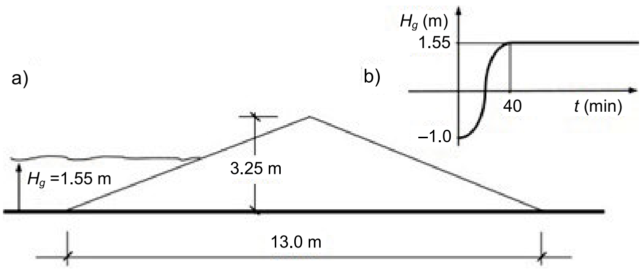

Fig. 2. Diagram of the problem: a) flood embankment, b) hydrograph of head water; H_g = upper water level, t = time; source: SROKA [2011]

pretation doubts as to whether the structure subjected to shock loading would be damaged were removed.

The assumed embankment was made of homogeneous and isotropic sand with a seepage coefficient k_s = 9.44·10^{-5} m·s^{-1}. Retention characteristics and conductivity in the unsaturated zone were adopted according to Equations (2) with the following values of necessary parameters: θ_r = 0.075, θ_s = 0.287, a = 2.96 m^{-1}, n = 4.45.

The solution to the problem was obtained using the finite element method with the use of seepage modelling including both saturated and unsaturated zones. Own made software was used for calculations [Sroka *et al.* 2004].

The hydrostatic pressure distribution in the body was assumed at the initial moment. Suction pressure head Y = 1.0 m was assumed along the embankment base. These pressures at the initial moment ranged from pF = 2.0 at the embankment base to pF = 2.6 at its crest. It is worth pointing out that the dam body was relatively highly saturated. The calculations were performed with a constant time step Δt equal to 120 s until the zero-pressure curve reached the landside slope. The seepage onset time T_{pp} was 18.7 h. This value is consistent with the results of the research on the Hele–Shaw model carried out in the 1970s by Katkowska.

Calculations for the steady-state flow were carried out, assuming the maximum ordinate of damming in the floodplain. The discretisation used was identical with the one applied in the unsteady flow analysis (number of elements le = 1835, number of nodes lw = 982). It searched for the trajectory along which the shortest passage time T_p from the floodplain to the protected area was found. It amounted to 2.89 d. Therefore, this value is almost four times higher than the time T_{pp}. Figure 3 shows the pressure head distribution obtained and the trajectory along which the water flows fastest from the upstream site to the protected area.

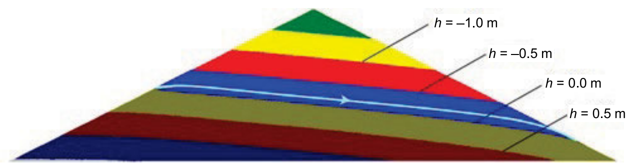

Fig. 3. Solution obtained for steady-state flow – pressure head distribution and trajectory associated with the shortest passage time; h = pressure height; source: own elaboration

RECONSTRUCTION OF THE LEFT-BANK FLOOD EMBANKMENT OF THE WARTA RIVER IN THE AREA OF KONIN

The assessment of the technical condition of the left-bank of the Warta River flood embankment in the area of Konin showed the need for seepage protection along a section of approx. 1.4 km located within the city [Wosiewicz, Sroka 2001]. Two options of modernisation were considered:

- I – sheet pile, placed at the foot of the riverside slope assembled with a bentonite mat spread on the slope (Fig. 4a);
- II – core in the embankment body axis made by injection (Fig. 4b).

The embankment rests on a layer of compacted and medium-compacted sandy soils that are alluvial deposits. The body of the structure is made of loose and medium-compacted sands. The most unfavourable (in terms of seepage) cross-section located approx. 600 m below the Toruński bridge in Konin was

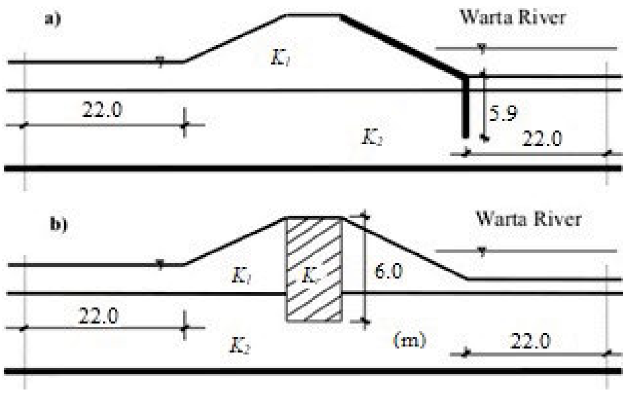

Fig. 4. Scheme of modernisation: a) option I, b) option II; source: Sroka [2011]

selected for calculations (Fig. 5). In the cross-section, the ordinate of the design water table (1%) is approx. 1 m above the landside area. Two soil layers were distinguished. The first one was the embankment body and the topsoil to a depth of 1.2 m, while the second was the subsoil underneath, up to the impermeable layer, assumed to be 12 m below the ground level. The tests of the top layer material showed that it was highly conductive with the seepage coefficient of 16.8 m·d^{-1}. The unsaturated conductivity was determined using formulas identical to those in the model problem. The required parameters were taken from the literature (for loose sand – Hoverkamp *et al.* [1977]):

- layer I – θ_r = 0.075, θ_s = 0.33, a = 2.96 m^{-1}, n = 4.45, k_s = 1.9·10^{-4} m·s^{-1};
- layer II – θ_r = 0.075, θ_s = 0.28, a = 2.96 m^{-1}, n = 4.45, k_s = 9.72·10^{-5} m·s^{-1}.

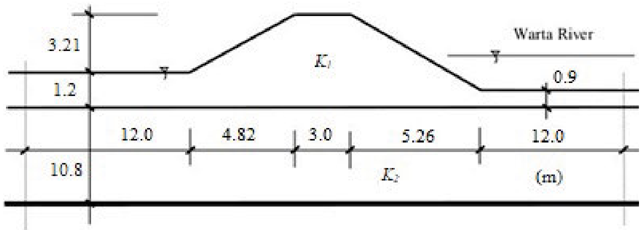

Fig. 5. Cross-section of the embankment – situation before modernisation; K_1 = seepage coefficient in the embankment body and the topsoil, K_2 = seepage coefficient in the subsoil; source: Sroka [2011]

For the existing state, the discretisation of the area was performed using a mesh with over 2,300 nodes and over 4,500 elements. Vertical boundaries were assumed impermeable and located 12 m from the embankment foot. The landside area was assumed to have a piezometric level equal to the ground level. Calculations showed that water seeps from the riverbed to the landside area very fast. Even with a low water level in the floodplain, to a depth of 0.66 m, the time T_p is only 9 days. In the case, the water table ordinate in the river is only 0.36 m above the protected area. With the higher head water, corresponding to the design water (1%), the passage time is only three days.

In the modernisation option I, the seepage flow changed significantly after the introduction of sealing elements. Calculations were made that assumed the water level ranging from 0.66 m up to 2.74 m in the floodplain. The passage times obtained from calculations compared with the time of water level exceedance for the flood in 1997 (before modernisation) and for the flood in 2010 are showed in Figure 6. For this option,

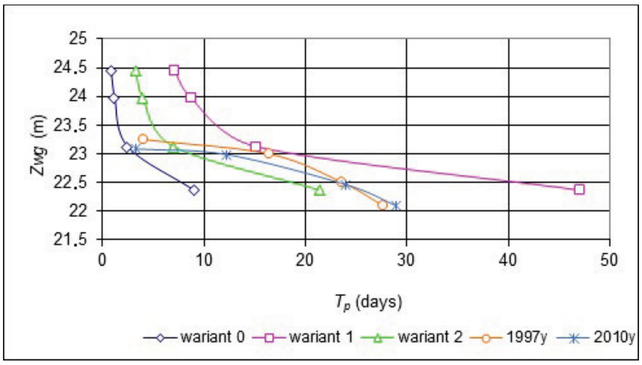

Fig. 6. Passage times T_p and time curve of water level exceedance for the flood in 1997 and in 2010; *Zwg* = water level; source: own study

passage times, depending on the level of water, increased five to nine times.

Calculations for the modernisation option II assumed that the core would extend to a depth of 6.0 m from the embankment crest (Fig. 4b), and its seepage coefficient was two orders of magnitude lower than that of the sands forming the embankment body. The remaining core parameters (soil III), needed for the description of hydraulic conductivity in the unsaturated zone, were taken from the literature as for clay:

ground III – θ_r = 0.068, θ_s = 0.38, a = 0.8 m^{-1}, n = 1.09, k_s = 1.9·10^{-6} m·s^{-1}

For the option II, the passage time was longer than the existing state by two to four times (Fig. 6). In order to illustrate the results, Figure 7 shows some of the determined trajectories and associated passage times for selected water levels in the floodplain.

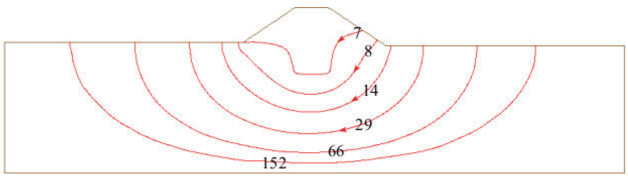

Fig. 7. Trajectory layout: modernisation option II, depth of water in the floodplain 1.4 m, passage time in days; source: Sroka [2011]

The 1997 summer flood in Konin reached the maximum level equal to the design water, and the levels above the bank water lasted for about a month. The hydrograph of water levels in the analysed cross-section was obtained from the flood wave transformation model for the section from Jeziorsko reservoir to Pyzdry [Laks, Kałuża 2006]. On this basis, the time curve of water level exceedance was compiled and compared to the calculated curves of passage times for the existing state and for both modernisation options (see Fig. 6). The layout of the presented curves indicates that, under given conditions, seepage through the embankment will occur during this flood because the time of water level exceedance is longer than the passage time divided by four (value indicated in the model problem). The site manager informed that during the flood in 1997, the water level in the pond located in the immediate vicinity of the embankment (in the City Park) increased up to the ordinate of the landside area, which indicates intense seepage into the protected area. Calculations for the model problem justify the prediction that seepage in the landside area should not occur when the time of water level exceedance is four times shorter than the passage time. It is unlikely to seal the analysed embankment in such a way that no seepage will occur during a flood lasting almost a month. The structure should therefore be equipped with an appropriate drainage system.

The results of the analyses presented above were used by the investor Board of Melioration and Water Facilities in Greater Poland (Pol. Wielkopolski Zarząd Melioracji i Urządzeń Wodnych) for selecting a modernisation variant. Sealing made of a pile and a bentonite mat on the slope was selected for implementation (option I), and it significantly increased the passage time compared to option II.

After the modernisation of the embankment, another flood occurred in 2010 [Laks 2017]. The peak level was slightly lower than during the flood in 1997, and the water damming time in the floodplain was similar – approx. 1 month (Fig. 6). Although the embankment had been sealed, it was observed that its foot was wet on the side of the protected area, which indirectly confirms the conclusions of the analysis carried out above (it is advisable to add drainage to the system from the side of the protected area).

CONCLUSIONS

The passage time curve can be a useful characteristic of flood embankment tightness. It represents the shortest flow times from the floodplain to the landside area under steady-state flow conditions with acceptable (assumed, designed) water levels in the landside and with various water levels in the river. Comparison with the hydrograph for the flood wave provides information on the possibility of seepage or flooding in the landside. It is also worth pointing out that passage times can be determined without much difficulty.

The passage time (T_p), which is a characteristic of the structure itself (in terms of tightness), is not the same as the seepage onset time (T_{pp}), which also depends on the initial state and the shape of the flood wave. The results of the analysis for the model example indicate that, when the water level rise is rapid, T_{pp} can be about four times shorter than T_p. The short passage time in comparison to the duration of flood indicates the risk of intrinsic erosion in the embankment body or the substratum, which may eventually lead to the destruction of the structure.

It should be emphasised that as a result of flood incidents, the properties of soil in the embankment body and in the substrate may change significantly due to suffosion or clogging processes. Appropriate geotechnical tests should then be performed, and the obtained data used to recalculate water passage times. As a rule, analyses related to the estimation of flood risks should be systematically updated since hydrological conditions are constantly changing. Some of them result from local socio-economic activities within the catchment area and some from global climate change.

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