10.24425/acs.2022.142845

Archives of Control Sciences
Volume 32(LXVIII), 2022
No. 3, *pages* 535–555

On-line process identification using the Modulating Functions Method and non-asymptotic state estimation

Witold BYRSKI and Michał DRAPAŁA

The paper presents an iterative identification method dedicated for industrial processes. The method consists of two steps. In the first step, a MISO system is identified with the Modulating Functions Method to obtain sub-models with a common denominator. In the second step, the obtained subsystems are re-identified. This procedure enables to obtain the set of models with different denominators of the transfer functions. The algorithm was used for on-line identification of a glass conditioning process. Identification window is divided into intervals, in which the models can be updated based on recent process data, with the use of the integral state observer. Results of the performed simulations for the identified models are compared with the historical process data.

Key words: system identification, modulating functions method, state observers, signal processing, adaptive algorithms

1. Introduction

The problem of on-line system identification is of great practical importance in the industry. Models based on the laws of physics are versatile, but their identification is difficult. Moreover, covering all relevant aspects of an analysed process is often impossible, hence the obtained models may not be enough accurate. On the other hand, linear models identified on-line often can ensure sufficient approximation of system dynamics around specified operating points.

On-line system identification is widely used in various industries. There are many papers illustrating such applications, e.g. [16] presents a universal identification platform for chemical industry, [23] illustrates an algorithm for on-line

W. Byrski (corresponding author, e-mail: wby@agh.edu.pl) and M. Drapała (e-mail: mdrapala@agh.edu.pl) are with Department of Automatic Control and Robotics, AGH University of Science and Technology, Al. Mickiewicza 30, 30-059 Kraków, Poland.

This work was supported by the scientific research funds from the Polish Ministry of Education and Science and AGH UST Agreement no 16.16.120.773 and was also conducted within the research of EC Grant H2020-MSCARISE-2018/824046.

Received 17.06.2020. Revised 10.08.2022.

modeling and control of a blast furnace ironmaking process and [18] gives a description of an expert model predictive controller, based on dynamically updated models, applied for a water deaslination process. An example of software package for signal and system identification is given in [10]. However, industrial applications of on-line identification algorithms are associated with many problems. Available historical data often do not conatin clear signal changes for long time periods. A special selection method, to overcome this difficulty, was proposed in [1]. Furthermore, data used for identification may not be very reliable. Algorithms to deal with corrupted or missing input data and with random input delays were given in [20] and [21] accordingly.

In the case of identifying systems with multiple inputs and outputs, the problem of interactions between them is very significant. If the correlation is strong enough, the system cannot be decomposed into subsystems. In [22] an approch for identification and decomposition of a discrete Multi Input Single Output (MISO) system with unknown time delays is proposed. Individual subsystems are obtained in an iterative optimization procedure. Algorithms for continuous-time MISO systems were described in [6] and [14]. In both papers, the same problem was mentioned. For the standard identification approaches, developed for Single Input Single Output (SISO) systems and adapted to the MISO case, all obtained models have a common denominator.

This problem was also the main motivation of developing the described two-step identification method. In the first step, the initial set of models, with a common denominator, is identified. In the next stage, the single SISO model is re-identified using logged system inputs and the output calculated as a difference between the whole system output and simulated outputs for other subsystems. Similar idea was presented in [17]. The methodology was developed with the aim of application for identifying a glass conditioning process. In the previous paper [5] the initial results for this approach was presented. It is worth noting that the solution utilizes continuous-time models. Unlike many other algorithms [9], it can be applied for high order systems.

The paper is organised as follows: at the beginning the Modulating Functions Method, which is the core identification procedure used in the algorithm, is introduced. Next, the re-identification algorithm is explained in details. Furthermore, an application of the method for glass conditioning is described. Finally, the results of performed experiments are presented and shortly discussed.

2. Modulating functions method

Linear Time Invariant (LTI) MISO system with K inputs can be described as

$$\sum_{i=0}^{n} a_i y^{(i)}(t) = \sum_{k=1}^{K} \sum_{j=0}^{m_k} b_{kj} u_k^{(j)}(t) = \sum_{j=0}^{m_1} b_{1j} u_1^{(j)}(t) + \ldots + \sum_{j=0}^{m_K} b_{Kj} u_K^{(j)}(t). \quad (1)$$

Functions $y^{(i)}, u_1^{(j)}, \ldots, u_K^{(j)}$ are the derivatives of the system inputs and the output given on the interval $[t_0, T_{ID}]$. There are n output derivatives and m_k derivatives for the k-th input, where $m_k \leqslant n, \forall k = 1, \ldots, K$. Parameters $\boldsymbol{a}$ and $\boldsymbol{b}$ are not known and should be identified. Most often, the system inputs u and the output y can be measured, but their derivatives are not known.

Modulating Functions Method (MFM) was introduced in [19]. In the following years, different variants of the method were developed, e.g. the authors in [15] proposed the use of spline functions in their approach. A generalization of the method for fractional-order systems was presented in [8] and for distributed parameters systems in [2].

The approach is based on the rule of integrating by parts. Left and right hand sides of (1) are convoluted with the known modulating function ϕ. Required properties of the function ϕ were described in [4]. In the described method, the Loeb and Cahen functions were used

$$\phi(t) = t^N (h - t)^M, \tag{2}$$

where h is the width of the function support.

Utilizing the properties:

$$y_i(t) = \int_0^h y(t - \tau)\phi^{(i)}(\tau)\,\mathrm{d}\tau, \tag{3}$$

$$u_{kj}(t) = \int_0^h u_k(t - \tau)\phi^{(j)}(\tau)\,\mathrm{d}\tau, \tag{4}$$

the differential equation (1) can be transformed into an algebraic one (5)

$$\sum_{i=0}^{n} a_i y_i(t) = \sum_{j=0}^{m_1} b_{1j} u_{1j}(t) + \ldots + \sum_{j=0}^{m_K} b_{Kj} u_{Kj}(t) + \epsilon(t). \tag{5}$$

The term ϵ represents a difference resulting from signal noise and equation error. The minimization problem is typically solved with the use of the Least Squares Method (LSM), e.g. in the case of the Output Error Method (OEM), assuming that $a_0 = 1$, it can be stated as

$$\left(\boldsymbol{M}^T \boldsymbol{M}\right)^{-1} \boldsymbol{M}^T \boldsymbol{y}_0 = [a_1, \ldots, a_n, \boldsymbol{b}_1, \ldots, \boldsymbol{b}_K]^T, \tag{6}$$

where: $\boldsymbol{M} = [-\boldsymbol{y}_1, \ldots, -\boldsymbol{y}_n, \ldots, \boldsymbol{u}_{10}, \ldots, \boldsymbol{u}_{1m_1}, \ldots, \boldsymbol{u}_{K0}, \ldots, \boldsymbol{u}_{Km_K}]$ consists of m column vectors. The dimension of vectors $\boldsymbol{y}_i$ and $\boldsymbol{u}_{kj}$ depends on the identification

time T_{ID}. Vectors $\boldsymbol{b}_k$ have the dimensions of m_k. Different parameter constraints in the OEM or in the Equation Error Method (EEM) give solutions that differ in the value of ϵ. The effect of parameter constraints on identification results was discussed in [4].

In the same work, a different variant of the MFM was described. The squared value of ϵ is treated as an identification performance index

$$\begin{aligned}\epsilon(t) = \boldsymbol{c}^T(t)\boldsymbol{\theta} = \big[&y_0(t), \ldots, y_n(t), -u_{10}(t), \ldots, -u_{1m_1}(t),\\ &\ldots, -u_{K0}(t), \ldots, -u_{Km_K}(t)\big]\big[a_0, \ldots, a_m, \boldsymbol{b}_1, \ldots, \boldsymbol{b}_K\big]^T,\end{aligned} \tag{7}$$

where: $\boldsymbol{\theta} \in R^{n+m_1+\ldots+m_K+K+1}$.

The minimization problem is stated in $L^2[t_0 + h, T_{ID}]$ as

$$\min_{\theta} J^2 = \min \|\epsilon(t)\|^2_{L^2[t_0+h,T]} = \min \|c(t)^T\boldsymbol{\theta}\|^2_{L^2}. \tag{8}$$

Linear constraint vector $\boldsymbol{\eta}$ is introduced to avoid the trivial solution and is some kind of parameters normalization. It is assumed that $\boldsymbol{\eta}^T\boldsymbol{\theta} = 1$. The norm in (8) can be written down as an inner product in the space L^2

$$J^2 = \left\langle \boldsymbol{c}^T(t)\boldsymbol{\theta}, \boldsymbol{c}^T(t)\boldsymbol{\theta} \right\rangle_{L^2} = \boldsymbol{\theta}^T \left\langle \boldsymbol{c}(t), \boldsymbol{c}^T(t) \right\rangle \boldsymbol{\theta} = \boldsymbol{\theta}^T \boldsymbol{G} \boldsymbol{\theta}. \tag{9}$$

The square real Gram matrix $\boldsymbol{G}$ can be expressed as

$$\boldsymbol{G} = \begin{bmatrix} \boldsymbol{YY} & \boldsymbol{YU}_1 & \ldots & \boldsymbol{YU}_K \\ \boldsymbol{U}_1\boldsymbol{Y} & \boldsymbol{U}_1\boldsymbol{U}_1 & \ldots & \boldsymbol{U}_1\boldsymbol{U}_K \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{U}_K\boldsymbol{Y} & \boldsymbol{U}_K\boldsymbol{U}_1 & \ldots & \boldsymbol{U}_K\boldsymbol{U}_K \end{bmatrix}, \tag{10}$$

where:

$$\begin{aligned}
\boldsymbol{YY}(i,j) &= \langle y_i, y_j \rangle \text{ and } i = 0 \ldots n, \quad j = 0 \ldots n,\\
\boldsymbol{YU}_k(i,j) &= -\langle y_i, u_{kj} \rangle \text{ and } k = 1 \ldots K, \quad i = 0 \ldots n, \quad j = 0 \ldots m_k,\\
\boldsymbol{U}_k\boldsymbol{Y}(i,j) &= -\langle u_{ki}, y_j \rangle \text{ and } k = 1 \ldots K, \quad i = 0 \ldots m_k, \quad j = 0 \ldots n,\\
\boldsymbol{U}_k\boldsymbol{U}_l(i,j) &= \langle u_{ki}, u_{lj} \rangle \text{ and } k = 1 \ldots K, \; l = 1 \ldots K, \; i = 0 \ldots m_k, \; j = 0 \ldots m_l.
\end{aligned}$$

Elements of these matrices are created by the inner products in L^2 of $\boldsymbol{c}(t)$ elements, e.g.

$$\langle y_i, u_{kj} \rangle = \int_{t_0+h}^{T_{ID}} y_i(\tau) u_{kj}(\tau) \, \mathrm{d}\tau. \tag{11}$$

The vector of parameters $\boldsymbol{\theta}$ can be found by solving the minimisation task with the use of the Lagrange multiplier technique

$$L = \boldsymbol{\theta}^T \boldsymbol{G}^{-1} \boldsymbol{\theta} + \lambda \left(\boldsymbol{\eta}^T \boldsymbol{\theta} - 1 \right), \tag{12}$$

in the form

$$\boldsymbol{\theta}^0 = \frac{\boldsymbol{G}^{-1} \boldsymbol{\eta}}{\boldsymbol{\eta}^T \boldsymbol{G}^{-1} \boldsymbol{\eta}} . \tag{13}$$

The values of the identified parameters vary depending on the selected constraint vector $\boldsymbol{\eta}$. In the case of constraints in the form $\boldsymbol{\eta} = [1 \ 0 \ldots 0]^T$, the method is analogous to the OEM. In [4] it was also proven that the minimal value of the index (9) is obtained for the constraint vector $\boldsymbol{\eta}$ selected as the eigenvector of the matrix $\boldsymbol{G}$, which corresponds to its minimal eigenvalue.

3. Re-identification idea for MISO systems

The set of MISO transfer function parameters $\boldsymbol{a}$ and $\boldsymbol{b}$ identified with the method described in Section 2 allows obtaining the model, in which all transfer functions of the component SISO subsystems have a common denominator. This feature of the analysed method is often a drawback, because the obtained model can be not accurate enough, especially when the individual sub-models have different time constants. What is more, the MISO model composed of these sub-models is not observable, because the rank of its state matrix is equal to n instead of $K \cdot n$. The developed method was created in response to the mentioned problems. It is based on the below assumptions:

- The SISO models corresponding to the subsequent inputs of the MISO system are successively updated in the following iterations of the method.
- Parameters of the only one SISO subsystem can be updated during a single iteration of the method. The logged k-th input of the system and its simulated output, calculated as a difference between the logged real system output and a sum of simulated outputs for the other SISO subsystems, is used in the identification procedure.
- The method can be performed for the zero initial condition MISO system response.

3.1. Method description

The method is presented in details in the form of Algorithm 1. At the start of the method, a collection of the previously obtained SISO models: $\mathrm{SISO}_1, \ldots, \mathrm{SISO}_K$, (with a common denominator) is needed. These models are then modified in subsequent iterations until the overall MISO model is improved.

Algorithm 1 MISO re-identification

Require: $\mathrm{SISO}_1, \ldots \mathrm{SISO}_K, u_1(t), \ldots u_K(t), y(t)$
Ensure: $\mathrm{SISO}_{opt_1}, \ldots \mathrm{SISO}_{opt_K}$

Step 1

for $k \leftarrow 1$ to K **do**
- simulate the k-th system output $y_{SISO_k}(t)$
- set the optimal models: $\mathrm{SISO}_{opt_k} \leftarrow \mathrm{SISO}_k$

end for
- calculate the MISO output for the initial models $y_{MISO}(t) \leftarrow \sum_{j=1,\ldots K} y_{SISO_j}(t)$
- calculate the performance index $E_0(y_{MISO}(t), y(t))$ for the initial models
- set the current iteration number: $it \leftarrow 1$

Step 2

for $k \leftarrow 1$ to K **do**
- obtain the k-th system output for the identification $y_k(t) \leftarrow y - \sum_{j=1,\ldots K: j \neq k} y_{SISO_j}(t)$
- identify the model SISO_{ID_k} using the LSM or the GSM
- simulate the obtained SISO system output $z_{SISO_k}(t)$

end for

Step 3

for $k \leftarrow 1$ to K **do**
- calculate the MISO output for the obtained models $y_{MISO_k}(t) \leftarrow \sum_{j=1,\ldots K: j \neq k} y_{SISO_j}(t) + z_{SISO_k}(t)$
- calculate the performance index $E_{MISO_k}(y_{MISO_k}(t), y(t))$

end for

Step 4
- select the least performance index $E_{\min} = \min(E_{MISO_k})$ for $k \leftarrow 1, \ldots, K$
- save the current performance index $E_1 \leftarrow E_{\min}$
- set the k counter $k \leftarrow k_{\min}$
- **go to Step 8**

Step 5
- update the k-th model $\mathrm{SISO}_k \leftarrow \mathrm{SISO}_{ID_k}$
- update the k-th optimal model $\mathrm{SISO}_{opt_k} \leftarrow \mathrm{SISO}_{ID_k}$
- update the k-th simulated output $y_{SISO_k}(t) \leftarrow z_{SISO_k}(t)$
- increment the current iteration number $it \leftarrow it + 1$

if $k = K$ **then**
- $k \leftarrow 1$

else
- $k \leftarrow k + 1$

end if

Step 6
- obtain the k-th system output for the identification $y_k(t) \leftarrow y - \sum_{j=1,\ldots K: j \neq k} y_{SISO_j}(t)$
- identify the model SISO_{ID_k} using the LSM or the GSM
- simulate the obtained SISO system output z_{SISO_k}

Step 7

- calculate the MISO output for the obtained models
  $y_{MISO_k} \leftarrow \sum_{j=1,\ldots K: j\neq k} y_{SISO_j}(t) + z_{SISO_k}(t)$
- calculate the performance index $E_{it} \leftarrow E(y_{MISO_k}(t), y(t))$

Step 8

if $E_{it-1} > E_{it}$ **and** $it < iter_{max}$ **then**
- **go to Step 5**

elseif $E_{it-1} > E_{it}$
- update the k-th optimal model $\text{SISO}_{opt_k} \leftarrow \text{SISO}_{ID_k}$
- **return** $\text{SISO}_{opt_1}, \ldots \text{SISO}_{opt_K}$

else
- **return** $\text{SISO}_{opt_1}, \ldots \text{SISO}_{opt_K}$

end if

The new performance index E, defined as a squared difference between the real system output $y(t)$ and the simulated MISO system output $y_{\text{sim}}(t)$, is introduced

$$E(y_{\text{sim}}(t), y(t)) = \int_{t_0}^{t} (y(t) - y_{\text{sim}}(t))^2 \, \mathrm{d}t, \tag{14}$$

to evaluate the obtained solutions.

The new individual SISO_k model is identified using the MFM, for the logged input $u_k(t)$ and the output $y_k(t)$ calculated as a difference between the real system output and the simulated outputs for the other subsystems. The output error problem can be formulated as

$$\left[a_{k1}, \ldots, a_{kn}, b_{k0}, \ldots, b_{km_k}\right] \begin{bmatrix} -\boldsymbol{y}_{k1} \\ \vdots \\ -\boldsymbol{y}_{kn} \\ \boldsymbol{u}_{k0} \\ \vdots \\ \boldsymbol{u}_{km_k} \end{bmatrix} = \boldsymbol{y}_{k0}\,, \tag{15}$$

where: $\boldsymbol{y}_k$ and $\boldsymbol{u}_k$ are the modulated signals for the subsystem, $\boldsymbol{a}_k$ and $\boldsymbol{b}_k$ are the identified parameters. It is assumed that each subsystem has the same rank n. This problem can be solved using the LSM, according to the equation (6) from the previous section.

Another approach involves the use of the Gauss-Seidel Method (GSM). This procedure often allows obtaining better identification results in terms of the identification index E. It occurs that improving the model step by step can be

beneficial, especially when the matrix $\boldsymbol{M}$ in the equation (6) is ill-conditioned. An example of such a problem will be demonstrated in Subsection 3.3. The Algorithm 2 presents the identification problem being solved with the use of the GSM. The method was implemented according to [12]. The sufficient, but not necessary, condition for the convergence of the GSM is the positive definite matrix $\boldsymbol{M}$.

Algorithm 2 Gauss-Seidel Method applied to the MFM

Require: modulated input and output signals: $\boldsymbol{u}_{k0}, \ldots, \boldsymbol{u}_{km_k}, \boldsymbol{y}_{k0}, \ldots, \boldsymbol{y}_{kn}$, k-th model parameters: $a_{k0}, \ldots, a_{kn}, b_{k0}, \ldots, b_{km_k}$, number of the GSM iterations: $iter_{GS}$

Ensure: updated k-th model parameters $a_{k1}, \ldots, a_{kn}, b_{k0}, \ldots, b_{km_k}$

– Create the matrices:

$\boldsymbol{M} = [-\boldsymbol{y}_{k1} \ldots -\boldsymbol{y}_{kn} \quad \boldsymbol{u}_{k0} \ldots \boldsymbol{u}_{km_k}]$,

$\boldsymbol{Y} = \boldsymbol{M}^T \boldsymbol{y}_{k0}$,

$\boldsymbol{p}_0 = [a_{k1} \ldots a_{kn} \quad b_{k0}, \ldots b_{km_k}]^T$,

$\boldsymbol{L}$ – the lower triangular matrix of $\boldsymbol{M}^T \boldsymbol{M}$,

$\boldsymbol{U} = \boldsymbol{M}^T \boldsymbol{M} - \boldsymbol{L}$.

for $i \leftarrow 0$ to $iter_{GS} - 1$ **do**

– $\boldsymbol{p}_{i+1} = \boldsymbol{L}^{-1}(\boldsymbol{Y} - \boldsymbol{U}\boldsymbol{p}_i)$

end for

– **return** the optimal vector of parameters $\boldsymbol{p}_{iter_{GS}}$

3.2. Simulation example

The developed method is illustrated using the simple example. The given MISO system is composed of two SISO subsystems: SISO_1 and SISO_2. Their transfer functions are given below

$$\begin{aligned} G_1(s) &= \frac{1}{s^3 + 0.2s^2 + 10s + 1}, \\ G_2(s) &= \frac{1}{s^3 + 3s^2 + 2s + 1}. \end{aligned} \tag{16}$$

It is assumed that only the MISO system's output can be measured (a sum of the subsystems' outputs). The control signals $u_1(t)$ and $u_2(t)$ were supplied accordingly to the $SISO_1$ and the $SISO_2$ systems. It is verified if the single sine signal and the single unit step signal allow the identification of all parameters. The signals are given as

$$\begin{aligned} u_1(t) &= \sin\left(0.1t + \frac{\pi}{2}\right), \\ u_2(t) &= \begin{cases} 0, & \text{if } t < 10 \text{ s}, \\ 1, & \text{if } t \geqslant 10 \text{ s}. \end{cases} \end{aligned} \tag{17}$$

The MISO model was initially identified according to the OEM. In the next step, the re-identification procedure was performed using the GSM. The applied parameters for both methods are presented in Table 1. Table 2 presents the identified parameters in comparison with the exact system parameters. Identification results are compared in Fig. 1 and in Table 3 in terms of the Mean Squared Error (MSE).

Table 1: Re-identification method parameters

Parameter	Description	Value
N, M	Loeb-Cahen function parameters	7, 8
h	Filtering function support width	5 s
$iter_{\max}$	Max. re-ident. method iterations	10
$iter_{GS}$	Gauss-Seidel method iterations	5

Table 2: Identified parameters in comparison with their exact values – 3.2

Description	Model	a_{k0}	a_{k1}	a_{k2}	a_{k3}	b_{k0}
Real system	$SISO_1$	1	10	0.2	1	1
	$SISO_2$	1	2	3	1	1
MISO ident.	$SISO_1$	1	4.0173	3.8282	1.5849	0.6941
	$SISO_2$	1	4.0173	3.8282	1.5849	1.0571
Re-ident. procedure	$SISO_1$	1	9.7764	0.5227	1.1699	0.9827
	$SISO_2$	1	1.8616	2.7679	0.4538	1.0005

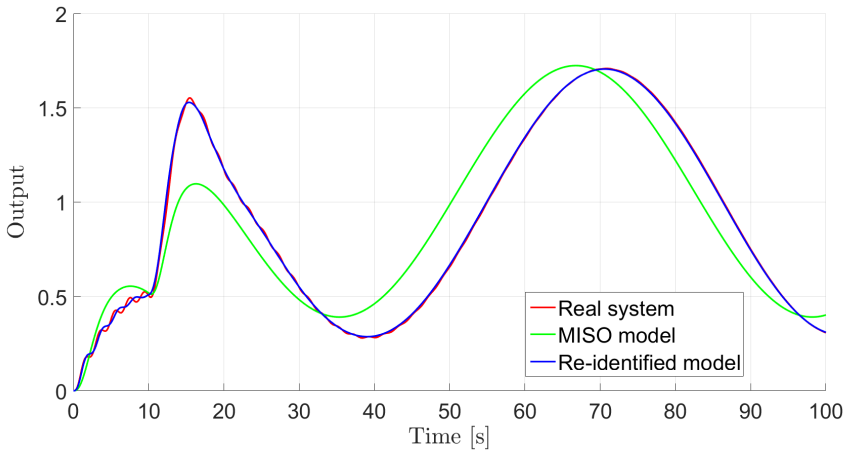

Figure 1: Comparison of the identified models' outputs – 3.2

Table 3: Mean squared error values for the obtained models

Method	MSE value
MISO ident.	0.3805
Re-ident. procedure	0.1290×10^2

3.3. Numerical aspects

As it was pointed out in the previous paragraph, utilizing the GSM often allows to improve the identified model. In the presented example, there are two SISO subsystems (18) forming the MISO model.

$$\begin{aligned} G_1(s) &= \frac{1}{s^2 + 3s + 2}, \\ G_2(s) &= \frac{1}{s^2 + 10s + 15}. \end{aligned} \tag{18}$$

Control signals for these models are given by:

$$\begin{aligned} u_1(t) &= \sin(0.2t), \\ u_2(t) &= \sin(0.4t). \end{aligned} \tag{19}$$

Similarly to the previous case, only sum of both components can be measured.

The identification procedure was carried out, assuming the direct LSM for the parameters' re-identification. The coefficients of the method, presented in Table 1, are the same as in the previous case. The obtained results are depicted in Table 4. It can be seen, that none of the models identified using this method is stable. Simulation result for the re-identified models was worse than the initial value of the performance index, so the procedure was aborted. Such a solution results

Table 4: Identified parameters in comparison with their exact values – 3.3

Description	Model	a_{k0}	a_{k1}	a_{k2}	b_{k0}
Real system	$SISO_1$	1	3	2	1
	$SISO_2$	1	10	15	1
MISO ident.	$SISO_1$	1	1.8093	4.9997	0.6102
	$SISO_2$	1	1.8093	4.9997	0.1235
Re-ident. LSM	$SISO_1$	1	1.8093	4.9997	0.6102
	$SISO_2$	1	1.1797	1.1525	-2.0804×10^9
Re-ident. GSM	$SISO_1$	1	1.3098	2.8730	0.4513
	$SISO_2$	1	0.3065	4.7710	0.0173

from the numerical reasons. The matrix $\boldsymbol{M}$ composed of the modulated signals is ill-conditioned. Moreover, it is not positive definite, so the GSM convergence cannot be provided. Despite that, the iterative method gives better result. The vector of optimal parameters was obtained in the fourth iteration of the method. Although, their values are significantly different that the exact ones, the simulated system response, presented in Fig. 2, is similar to the original one. The value of mean squared error is equal to 8.9240×10^{-3}.

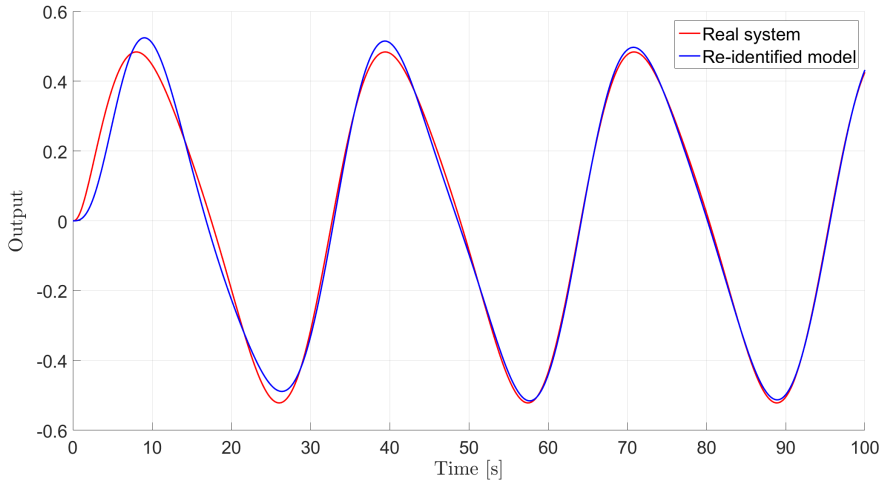

Figure 2: Comparison of the real system's and the model's output – 3.3

Another important problem, regarding the performance indices, is worth discussing here. The term J, mentioned in the previous section for the Equation Error Method (EEM), refers to the difference between two sides of the equation for the modulated signals. It is possible, especially when the identified system's dimension is not known, that despite minimizing the index J, the simulation results for the identified model are unsatisfactory. Therefore, another performance index E was adopted in the developed method. The simulation method demands greater computational effort, but can be beneficial especially in the case where a model of real plant is identified and, its structure is unknown.

4. Algorithm for industrial processes

The algorithm described in the paper was created with a view to on-line application for industrial processes, in response to problems that have arisen during identification of a glass conditioning process. As it was pointed out in Section 1, the results obtained for the process, with the use of the initial version

of the method, can be found in [5]. In comparison with the previous work, the algorithm was significantly changed, e.g. the state observer is applied for the whole MISO system's matrix, not for the single SISO subsystems. Moreover, the iterative GSM is applied for solving the output error problem in the current method.

4.1. Algorithm description

The described algorithm is based on the following assumptions, that result from features of the analysed problem.

- The identification procedure is intended to be performed on-line, although the experiments presented in the next section were performed off-line based on the historical process data. The identification time is divided into several intervals of width T.
- The initial model is obtained after n_{start} intervals in two steps. Initially, the MISO model identification is performed according to the description in Section 2. Then, the model is re-identified with the use of the method described in Section 3. The initial model can be obtained only around the operating point t_0, assuming zero initial condition.
- The model's parameters can be changed on-line, if the squared difference between the real system and the current model for the last n_{reident} intervals is greater than the threshold value tr. If the updated model gives better result in terms of the MSE, its parameters are updated.
- The initial state conditions for the model, in the subsequent intervals, are obtained with the use of the exact state observer and the current model of the system. The observation interval T_{OB} can have different width than T.

The aim of the algorithm is to provide the precise linear model of the process, suitable for control purposes. It can be useful for predicting the system output after n_{start} intervals. Example results of such prediction are presented in Section 5.

The identified MISO system can be presented in the state-space form (20). The state-space matrices are divided into blocks corresponding to the suitable SISO sub-systems. It is assumed that there are K SISO sub-models and each of these models has the same rank n.

$$\boldsymbol{A} = \underset{(K\cdot n\times K\cdot n)}{\begin{bmatrix} \boldsymbol{A}_1 & \dots & \boldsymbol{0} \\ \vdots & \ddots & \vdots \\ \boldsymbol{0} & \dots & \boldsymbol{A}_K \end{bmatrix}}, \qquad \boldsymbol{B} = \underset{(K\cdot n\times K)}{\begin{bmatrix} \boldsymbol{B}_1 & \dots & \boldsymbol{0} \\ \vdots & \ddots & \vdots \\ \boldsymbol{0} & \dots & \boldsymbol{B}_K \end{bmatrix}},$$
$$\boldsymbol{C} = \underset{(1\times K\cdot n)}{\left[\left[\, 0 \;\dots\; 1 \,\right] \;\dots\; \left[\, 0 \;\dots\; 1 \,\right]\right]}, \qquad \boldsymbol{D} = \underset{(1\times K)}{\left[0 \;\dots\; 0\right]}, \tag{20}$$

where:

$$\boldsymbol{A}_k = \begin{bmatrix} 0 & \dots & 0 & -\dfrac{a_{k0}}{a_{kn}} \\ 1 & \ddots & \vdots & \vdots \\ \vdots & \ddots & 0 & -\dfrac{a_{kn-2}}{a_{kn}} \\ 0 & \dots & 1 & -\dfrac{a_{kn-1}}{a_{kn}} \end{bmatrix}, \qquad \boldsymbol{B}_k = \begin{bmatrix} \dfrac{b_{k0}}{a_{kn}} \\ \vdots \\ \dfrac{b_{kn-1}}{a_{kn}} \end{bmatrix}.$$

4.2. The exact state observer

The use of the exact state observer in the developed method arises from the necessity of performing system simulation with a non-zero initial condition. This simulation is performed to obtain the performance index value or to predict the system output in the next interval. In contrast to the standard asymptotic observers, the exact state observer, applied for the finite observation window, guarantees obtaining the real value of the observed state at the time T_{OB}. The theory of the exact state observers was described in [3]. The general formula is given as

$$x(T_{OB}) = \int_0^{T_{OB}} \boldsymbol{G}_1(t) y(t)\,\mathrm{d}t + \int_0^{T_{OB}} \boldsymbol{G}_2(t) u(t)\,\mathrm{d}t, \tag{21}$$

where:

$$\boldsymbol{M}_0 = \int_0^{T_{OB}} e^{\boldsymbol{A}^T\tau} \boldsymbol{C}^T \boldsymbol{C} e^{\boldsymbol{A}\tau}\,\mathrm{d}\tau, \qquad \boldsymbol{G}_1(t) = e^{\boldsymbol{A}T_{OB}} \boldsymbol{M}_0^{-1} e^{\boldsymbol{A}^T t} \boldsymbol{C}^T,$$

$$\boldsymbol{G}_2(t) = e^{\boldsymbol{A}T_{OB}} \boldsymbol{M}_0^{-1} \left[\int_0^{t} e^{\boldsymbol{A}^T\tau} \boldsymbol{C}^T \boldsymbol{C} e^{\boldsymbol{A}\tau}\,\mathrm{d}\tau \right] e^{-\boldsymbol{A}t} \boldsymbol{B}.$$

As it was mentioned previously, in the implemented on-line procedure, the expanding identification window is divided into intervals, in which the model's parameters can be updated. The equation for the system observed state value at the end of each interval for the moving observation window version is given as

$$x(t_p) = \int_{t_p - T_{OB}}^{t_p} \boldsymbol{G}_1\left(T_{OB} - t_p + t\right) y(t)\,\mathrm{d}t + \int_{t_p - T_{OB}}^{t_p} \boldsymbol{G}_2\left(T_{OB} - t_p + t\right) u(t)\,\mathrm{d}t, \tag{22}$$

where successive time moments are given by

$$t_p = t_0 + p \cdot T - (t_0 \quad \textbf{\textit{modulo}} \quad T), \qquad p = n_{\text{start}}, n_{\text{start}} + 1, \dots.$$

5. Experimental results

The results of the described algorithm applied to the data collected from a glass conditioning installation are presented in this section. First of all, the analysed industrial process is briefly described and obtained results are discussed in the next part. The presented method is intended to be implemented in real industrial control systems, but for now only the off-line experiments based on the previously collected historical data was performed. However, the algorithm operation remains the same in both cases.

5.1. Brief description of the process

Glass conditioning process is carried out after melting a glass batch. The goal of this operation is to obtain a desired glass temperature profile along the forehearth. Molten glass flows out of a glass furnace, over working end zones and finally gets to forehearths. Typical forehearth is a long channel divided into several zones. Each zone is equipped with gas burners and in some of them cooling valves or cooling dampers are also installed. The desired glass temperature in each zone depends on a type of currently produced containers and should be stabilised with accuracy up to 1°C.

The diagram of a typical forehearth is presented in Fig. 3. Molten glass temperatures in zones 1–3 are controlled by PID controllers. For the first two zones, gas-air mixture pressure and cooling valve position are manipulated variables. In the third zone, only the gas-air mixture pressure can be adjusted. The glass temperature in the last spout zone can be controlled manually. Each forehearth zone is controlled regardless of neighbouring ones.

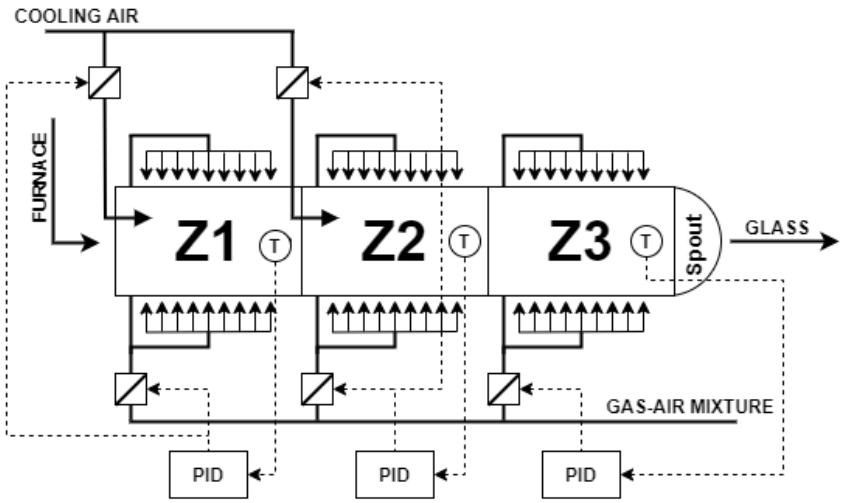

Figure 3: Forehearth control system

Ensuring the proper temperature control, during the conditioning process, is especially important when a production is changed. Not only the temperature set points in the foreherath's zones can be changed, but also the amount of glass flowing through the forehearth (glass pull rate). It significantly affects the dynamics of the process. Moreover, the current glass pull rate cannot be measured in the analysed installation. Approximate values of this parameter are provided by plant operators, usually delayed and can be only used as a guide. Hence, the difficulties in the process identification occur. In the paper, the adaptive method for lumped-parameter model was presented. Other common approach assumes the use of partial differential equations [11,13]. An application of Laguerre functions for on-line identification of the process model was presented in [7].

5.2. Description and results of the performed experiments

Experiments were performed based on the historical process data originating from two zones of the same forehearth. In both zones, there are two system inputs and one output (the measured temperature). The SISO subsystems have the following inputs:

- SISO_1 – the temperature measured in the previous forehearth zone,
- SISO_2 – the gas-air mixture pressure.

In both experimental cases, changes of the temperature set points and the glass pull rate were noticeable. Coefficients of the MFM were adjusted separately for each case. Their values are presented in Table 6 for the first data set and in Table 8 for the second set. In both cases, the constraint vector $\boldsymbol{\eta}$ was selected as the eigenvector of the Gram matrix corresponding to its minimal eigenvalue. It guarantees minimisation of the performance index J, if the vector $\boldsymbol{\eta}$ is taken from the unit ball. Common coefficients of the developed algorithm can be found in Table 5. The GSM was utilized for solving the re-identification problem as previously.

Table 5: Parameters of the identification method

Parameter	Description	Value
T	Model identification interval	$1000s$
T_{OB}	Observation interval	$500s$
n_{start}	Number of initial identification intervals	5
n_{reident}	Number of re-identification intervals	2
tr	Model change threshold	500
$iter_{\max}$	Max. re-ident. method iterations	10
$iter_{GS}$	Gauss-Seidel method iterations	5

Table 6: MFM coeficients – 1. data set

Parameter	Description	Value
N, M	Loeb-Cahen function parameters	3, 4
n	SISO_k transfer function denominator order	2
m	SISO_k transfer function numerator order	0
h	Filtering function support width	$75s$

The logged system inputs are presented in Fig. 4 and the obtained results in Fig. 5 for the first data set. Analogously, for the second data set, the system inputs

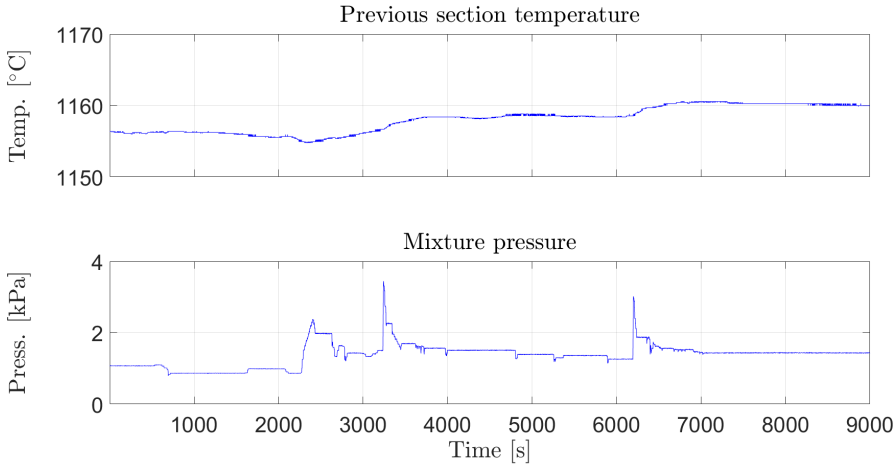

Figure 4: System inputs – 1. data set

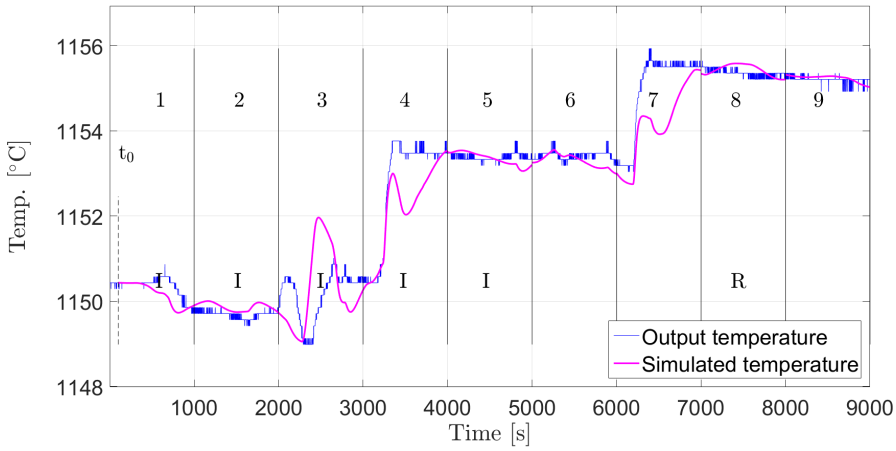

Figure 5: Experimental results – 1. data set

Table 7: Identified model parameters – 1. data set

Int.	Model	a_{k0}	a_{k1}	a_{k2}	b_{k0}
1-7	$SISO_1$	1	63.96	1.14×10^3	0.99
	$SISO_2$	1	67.21	1.50×10^3	2.30
8-9	$SISO_1$	1	63.96	1.14×10^3	0.99
	$SISO_2$	1	77.82	1.47×10^3	2.68

Table 8: MFM coeficients – 2. data set

Parameter	Description	Value
N, M	Loeb-Cahen function parameters	5, 6
n	$SISO_k$ transfer function denominator order	3
m	$SISO_k$ transfer function numerator order	0
h	Filtering function support width	$150s$

are depicted in Fig. 6 and the results in Fig. 7. In both cases, the input delay for the $SISO_1$ model was included. This delay depends on the glass pull rate and the length of the zone. The adopted values were $260s$ for the first data set and $667s$ for the second. Identified parameters of the models in the subsequent intervals can be found in Table 7 and in Table 9. The intervals, for which the initial model was obtained are marked as ***I***, while the intervals for which the previously re-identified model was applied for the first time are marked as ***R***.

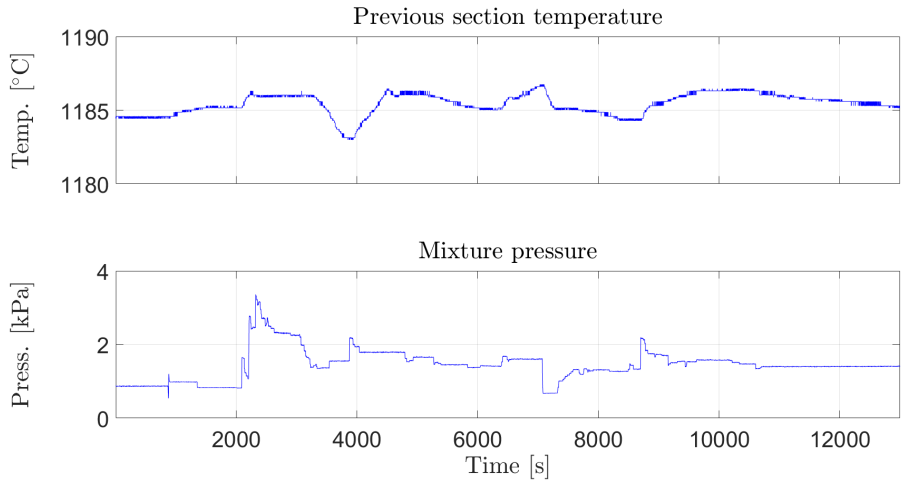

Figure 6: System inputs – 2. data set

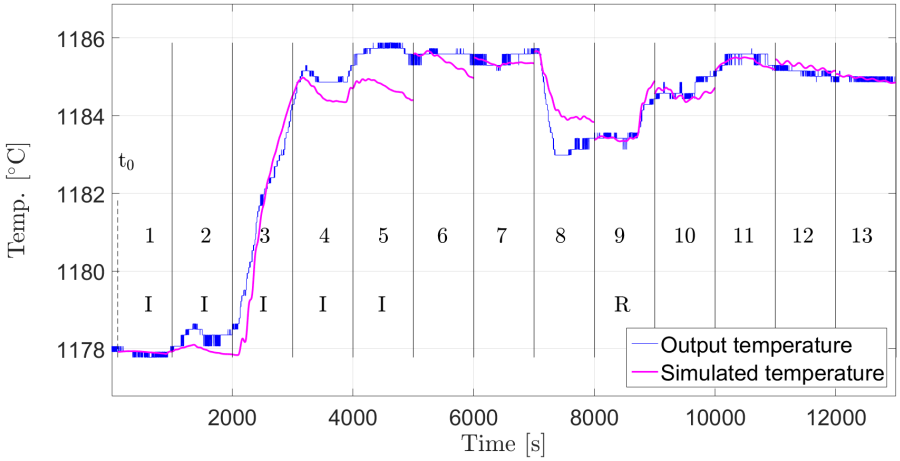

Figure 7: Experimental results – 2. data set

Table 9: Identified model parameters – 2. data set

Int.	Model	a_{k0}	a_{k1}	a_{k2}	a_{k3}	b_{k0}
1–8	$SISO_1$	1	2.10	3.05×10^3	2.07×10^6	1.43
	$SISO_2$	1	1.08×10^3	3.13×10^3	2.68×10^5	7.48
9–13	$SISO_1$	1	2.10	3.05×10^3	2.07×10^6	1.43
	$SISO_2$	1	7.73×10^2	3.62×10^3	2.04×10^5	9.53

The performed experiments proved the relevance of the proposed method. The obtained results are not perfect, but the nature of temperature changes was reflected. The mean squared error value was equal to 0.32 for the first data set and 0.20 for the second. In the last identification intervals the offset between the real system output and the simulation is virtually zero, which indicates that the model correction algorithm works properly.

6. Conclusions

The described identification method uses very advanced mathematical methodology but gives the results that seems to be very useful in practical applications. The performed experiments proved that the dynamics of the industrial process can be accurately reflected with the use of adaptive linear models. Certain disadvantage of the method is that the initial model can be obtained only after

n_{start} intervals. However, this is not a big drawback for the analysed application. The accurate model is especially needed in the final part of the production change process, after a certain period of time, to avoid temperature overshoots. In the future, the algorithm can be modified. One of the ideas involves using a linear programming method in the re-identification part of the algorithm. Considering additional constraints for the identified parameters could give another way to influence the dynamics of the obtained models.

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