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Admissibility tests for multidimensional singular fractional continuous-time models

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In this paper we present and discuss a new class of singular fractional systems in a multidimensional state space described by the Roesser continuous-time models. The necessary and sufficient conditions for the asymptotic stability and admissibility by the use of linear matrix inequalities are established. All the obtained results are simulated by some numerical examples to show the applicability and accuracy of our approach.

Key words: singular Roesser model, fractional systems, multidimensional systems, linear matrix inequalities, admissibility

1. Introduction

Analysis and design of multidimensional systems (dD) where ($d \geqslant 2$) has been the subject of much research in the recent decades. Multidimensional systems propagate the state in several independent spatial directions and have applications in systems theory, but also in engineering areas such as circuit theory, digital filtering and image processing [2, 4, 8, 11, 12, 21]. Note that the transition of $1D$ case to the multidimensional case is done naturally by studying the two dimensional models.

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Recently, the stability of multidimensional systems has been considered in many research for several decades. It has attracted the interest of many researchers and practitioners in control theory [3, 5, 8, 13, 16, 18].

To study stability problems, one and two dimensional systems must be implemented; the $1D$ models in dynamic systems are introduced and analyzed in [16, 17] by the use of fundamental notions on the eigenvalues of the dynamic matrix, and in [18], Marir et al. was expressed and developed necessary and sufficient conditions of admissibility for singular fractional order continuous time with $1 \leqslant \alpha < 2$ proposed as a strict $\mathcal{LMI}$ term. Kaczorek in [14] studied the problem of the stability of multidimensional systems with it various applications, and in [2, 3, 12, 15, 20, 21], the authors investigated the stability conditions using new $\mathcal{LMI}$ developed with different approaches for continuous and discrete time system. Moreover, in [19] Tofighi et al. developed results for the stability analysis three-dimensional $(3D)$ systems using an advanced wave model. A robust stabilization conditions of multidimensional hybrid systems described by the Roesser models was developed by Ghamgui et al. in [4], and recently Aissa et al. focuses on the problem of $\mathcal{LMI}$ based on stability conditions for the class of singular continuous and discrete time multidimensional systems described by the Fornasini-Marchesini models.

In this paper, we look at the extension of the work in [18] to characterize the admissibility conditions of multidimensional systems expressed in a set of strict linear matrix inequalities. Numerical examples are given to illustrate the proposed methods.

2. Admissibility of dD singular fractional Roesser models

Based on [17, 18], we recall some needed definitions and properties as the fractional Caputo derivative, Singular Value decomposition and the Kronecker product.

Definition 1 *[18] Let the matrices $A = \left[a_{ij}\right] \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{q\times p}$, so the Kronecker product $A \otimes B$ of matrices A and B is the block matrix*

$$A \otimes B = \left[a_{ij}B\right] \in \mathbb{R}^{mq\times np} \tag{1}$$

for all $i = 1, ..., m$ and $j = 1, .., n$.
The matrix $A \otimes B$ is $(mq \times np)$ matrix with (mn) blocs $[a_{ij}B]$ of order (pq).

Definition 2 *[17] The function given by the formula*

$$\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}\,\mathrm{d}t, \quad \Re(x) > 0$$

is called the Euler gamma function.

Theorem 1 *[18] Let $A \in \mathbb{R}^{m\times n}$ be a real matrix with rank r, then there exist an orthonormal matrices $U \in \mathbb{R}^{m\times n}$, $V \in \mathbb{R}^{m\times n}$ and a rectangular diagonal matrix $\Sigma \in \mathbb{R}^{m\times n}$ with coefficients*

$$\Sigma_{ij} = \begin{cases} \sigma_i & \text{if } i = j, \\ 0 & \text{if } i \neq j \end{cases}$$

such that

$$A = U\Sigma V^T.$$

This factorization of A is called the singular value decomposition of A.

For simplicity of notation, we will denote the expression $A + A^T$ by $\operatorname{sym}(A)$.

In the following, we introduce a general formulation of multidimensional *dD* fractional continuous-time systems described by the Roesser model

$$E_d \begin{bmatrix} D_{t_1}^{\alpha_1} x_1(t_1, t_2, \cdots, t_d) \\ D_{t_2}^{\alpha_2} x_2(t_1, t_2, \cdots, t_d) \\ \vdots \\ D_{t_d}^{\alpha_d} x_d(t_1, t_2, \cdots, t_d) \end{bmatrix} = A_d \begin{bmatrix} x_1(t_1, t_2, \cdots, t_d) \\ x_2(t_1, t_2, \cdots, t_d) \\ \vdots \\ x_d(t_1, t_2, \cdots, t_d) \end{bmatrix} + \begin{bmatrix} B_1 \\ \vdots \\ B_d \end{bmatrix} u(t_1, t_2, \cdots, t_d), \tag{2}$$

where

$$E_d = \begin{bmatrix} E_{11} & E_{12} & \cdots & E_{1d} \\ E_{21} & E_{22} & \cdots & E_{2d} \\ \vdots & \vdots & \ddots & \vdots \\ E_{d1} & E_{d2} & \cdots & E_{dd} \end{bmatrix} \in \mathbb{R}^{n\times n}. \tag{3}$$

E_d can be assumed invertible.

$$A_d = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1d} \\ A_{21} & A_{22} & \cdots & A_{2d} \\ \vdots & \vdots & \ddots & \vdots \\ A_{d1} & A_{d2} & \cdots & A_{dd} \end{bmatrix}] \in \mathbb{R}^{n\times n} \qquad \text{is the dynamic matrix,} \tag{4}$$

$$B = \begin{bmatrix} B_1 \\ \vdots \\ B_d \end{bmatrix} \in \mathbb{R}^{n\times m} \qquad \text{is the control matrix,} \tag{5}$$

$$\begin{bmatrix} x_1(t_1, t_2, \cdots, t_d) \\ x_2(t_1, t_2, \cdots, t_d) \\ \vdots \\ x_d(t_1, t_2, \cdots, t_d) \end{bmatrix} \in \mathbb{R}^{n} \qquad \text{represent the state of subvectors,} \tag{6}$$

$$u\left(t_1, t_2, \cdots, t_d\right) \in \mathbb{R}^m \quad \text{the input vector} \tag{7}$$

$$0 < \alpha_i \leqslant 1 \quad \text{for all } i = \overline{1.d} \tag{8}$$

The unforced system associated to (2) is as follow

$$E_d \begin{bmatrix} D_{t_1}^{\alpha_1} x_1\left(t_1, t_2, \cdots, t_d\right) \\ D_{t_2}^{\alpha_2} x_2\left(t_1, t_2, \cdots, t_d\right) \\ \vdots \\ D_{t_d}^{\alpha_d} x_d\left(t_1, t_2, \cdots, t_d\right) \end{bmatrix} = A_d \begin{bmatrix} x_1\left(t_1, t_2, \cdots, t_d\right) \\ x_2\left(t_1, t_2, \cdots, t_d\right) \\ \vdots \\ x_d\left(t_1, t_2, \cdots, t_d\right) \end{bmatrix}. \tag{9}$$

First we begin by defining the dD Laplace transform. The multiple Laplace transform relates functions $f\left(t_1, t_2, \ldots, t_d\right)$ of the d independent real variables $t_1, t_2, \ldots, t_d$ to a function $F\left(s_1, s_2, \ldots, s_d\right)$ of d independent complex variables $\left(s_1, s_2, \ldots, s_d\right)$ through the equation

$$\begin{aligned} L\left[f\left(t_1, t_2, \ldots, t_d\right)\right] :=& F\left(s_1, s_2, \ldots, s_d\right) \\ &= \int_0^{+\infty} \int_0^{+\infty} \cdots \int_0^{+\infty} \Big[e^{-s_d t_d - s_{d-1} t_{d-1} - \cdots - s_1 t_1} \times \\ & \quad f\left(t_1, t_2, \ldots, t_d\right) \Big] \mathrm{d}t_1 \mathrm{d}t_2 \cdots \mathrm{d}t_d \,. \end{aligned} \tag{10}$$

The function defined by (10) is called the multiple Laplace transform of $f(t_1, t_2, \ldots, t_d)$ where $F(s_1, s_2, \ldots, s_d)$ stands for the s domain representation of the signal $f(t_1, t_2, \ldots, t_d)$.

By the use of the equation (10) to the singular fractional system (9) we deduce the general definition of spectral abscissa which is

$$\gamma_d(E_d, A_d) = \max_{\forall\left(s_1^{\alpha_1}, s_2^{\alpha_2}, \cdots, s_d^{\alpha_d}\right) \in \Gamma_d} \left[\operatorname{Re}(s_1)^{\alpha_1} \quad \operatorname{Re}(s_2)^{\alpha_2} \quad \cdots \quad \operatorname{Re}(s_d)^{\alpha_d} \right], \tag{11}$$

where:

The equation (11) means that the spectral abscissa contain d values, ie: $\gamma_d(E_d, A_d)$ represent a vector of d dimensions that it's entries are $\operatorname{Re}(s_i)^{\alpha_i}$ with $i = \overline{1, d}$ and

$$\Gamma_d = \left\{ \left(s_1^{\alpha_1}; s_2^{\alpha_2}, \cdots, s_d^{\alpha_d} \right) \Big| \det\left(K_d^{\alpha_1, \alpha_2, \cdots, \alpha_d} E_d - A_d \right) = 0 \right\}, \tag{12}$$

$$K_d^{\alpha_1, \alpha_2, \cdots, \alpha_d} = \begin{bmatrix} s_1^{\alpha_1} I_{n_1} & 0 & \cdots & 0 \\ 0 & s_2^{\alpha_2} I_{n_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ \vdots & \vdots & \vdots & s_d^{\alpha_d} I_{n_d} \end{bmatrix} \tag{13}$$

with,

$$\sum_{i=1}^{d} n_i = n. \tag{14}$$

The characteristic polynomial associated at the the equation (9) is given by

$$D(t_1, t_2, \cdots, t_d) = E_d K_d^{\alpha_1, \alpha_2, \cdots, \alpha_d} - A_d \,. \tag{15}$$

We propose in the following a new definition of partial derivatives in the case of fractional multidimensional systems.

$$\begin{aligned} D_{t_i}^{\alpha_i} x\left(t_1, t_2, \cdots, t_d\right) &= \frac{\partial^{\alpha_i}}{\partial t_i^{\alpha_i}} x\left(t_1, t_2, \cdots, t_d\right) \\ &= \frac{1}{\Gamma\left(N_i - \alpha_i\right)} \int_0^{t_i} \frac{x_{t_i}^{(N_i)}(\tau)}{\left(t_i - \tau\right)^{\alpha_i + 1 - N_i}} \mathrm{d}\tau, \end{aligned} \tag{16}$$

where $N_i - 1 \leqslant \alpha_i < N_i, N_i \in \mathbb{N}$, for all $0 < \alpha_i \leqslant 1$, $i = \overline{1.d}$. are the order of fractional partial derivative, $\Gamma(x)$ is the Euler gamma function defined by Definition 2 and

$$x_{t_i}^{(N_i)}(\tau) = \begin{cases} \dfrac{\partial^{N_1} x\left(\tau, t_2, \cdots, t_d\right)}{\partial \tau^{N_1}} & \text{for } \ i = 1, \\ \dfrac{\partial^{N_2} x\left(t_1, \tau, \cdots, t_d\right)}{\partial \tau^{N_2}} & \text{for } \ i = 2, \\ \vdots & \\ \dfrac{\partial^{N_d} x\left(t_1, t_2, \cdots, t_{d-1}, \tau\right)}{\partial \tau^{N_d}} & \text{for } \ i = d. \end{cases}$$

We introduce and discuss the stability analysis of multidimensional in continuous time systems by the use of similar essential definitions and results in the case of integer non derivatives and the fractional derivatives based on the work in [8, 11, 13, 18].

Definition 3 *The system defined by the equation (9) is regular if and only if:*

$$\det\left(E_d K_d^{\alpha_1, \alpha_2, \cdots, \alpha_d} - A_d\right) \neq 0 \tag{17}$$

form some $s_i \in \mathbb{C}$, and $0 < \alpha_i \leqslant 1$, $i = \overline{1.d}$.

Definition 4 *The system defined by the equation (9) is said to be impulse-free if and only if it's regular and:*

$$\deg\left(\det\left(E_d K_d^{\alpha_1,\alpha_2,\cdots,\alpha_d} - A_d\right)\right) = rank\, E_d \tag{18}$$

for $0 < \alpha_i \leqslant 1$, $i = \overline{1.d}$.

To study the stability and admissibility conditions of the multidimensional fractional continuous-time system (9) we must introduce some definitions and notions. The first definition is the basic definition of asymptotic stability of multidimensional systems.

Definition 5 *The multidimensional fractional continuous-time system (9), is asymptotically stable if and only if the state $x_i(t_1, t_2, ..., t_d)$ converges to zero for zero input and every bounded initial conditions, i.e.*

$$\lim_{t_1,t_2,\cdots,t_d \to +\infty} \left\| \left(x_i\left(t_1, t_2, \cdots, t_d\right) \right) \right\| = 0 \tag{19}$$

where $i = \overline{1, d}$ *and for*

$$\begin{aligned}
&u\left(t_1, t_2, \ldots, t_d\right) = 0 \quad \mathit{for}\ \ t_i \in \mathbb{R}_+ ,\\
&\sup_{t_1 \in \mathbb{R}_+} \left\|x_1\left(t_1, 0, \ldots, 0\right)\right\| < \infty,\\
&\sup_{t_2 \in \mathbb{R}_+} \left\|x_2\left(0, t_2, \ldots, 0\right)\right\| < \infty,\\
&\vdots\\
&\sup_{t_d \in \mathbb{R}_+} \left\|x_d\left(0, 0, \ldots, t_d\right)\right\| < \infty.
\end{aligned} \tag{20}$$

Based on the obtained results [6, 8] and [13] in multidimensional systems and the fact that the considered models are stable if the stability for all t_i where $i = \overline{1, d}$ is guaranteed since all the variables are supposed independents, For this reason we can give and establish the following results.

Lemma 1 *Let us suppose:* $\arg\left(spec(E_d, A_d)\right) = (\lambda_1\ \ \lambda_2\ \ \cdots\ \ \lambda_d)^T$. *Then the model described by the equation (9) with $u\left(t_1, t_2, \ldots, t_d\right) = 0$ is called stable if*

$$(\lambda_1 \quad \lambda_2 \quad \cdots \quad \lambda_d)^T > \left[\alpha_1 \frac{\pi}{2} \quad \alpha_2 \frac{\pi}{2} \quad \cdots \quad \alpha_d \frac{\pi}{2}\right]^T. \tag{21}$$

Which is imply:

$$\lambda_i > \alpha_i \frac{\pi}{2}$$

for all $0 < \alpha_i \leqslant 1$, $i = \overline{1, d}$, *where*

$$\operatorname{spec}(E_d, A_d) = \left\{\left(s_1^{\alpha_1}; s_2^{\alpha_2}, \cdots, s_d^{\alpha_d}\right) \Big/ \left(s_1^{\alpha_1}; s_2^{\alpha_2}, \cdots, s_d^{\alpha_d}\right) \in \mathbb{C}^d, \right.$$
$$\left. \det\left(E_d K_d^{\alpha_1, \alpha_2, \cdots, \alpha_d} - A_d\right) = 0\right\} \tag{22}$$

denotes the set of finite modes for the pair (E_d, A_d).

Corollary 1 *The multidimensional system* (9) *with* $0 < \alpha_i \leqslant 1$ *for all* $i = \overline{1, d}$ *is asymptotically stable if and only if one of the following equivalents properties are satisfied:*

1. *There exist a matrix* $P_d = P_d^T$ *such that*

$$\operatorname{sym}\{\boldsymbol{\Theta}_d \otimes (A_d P_d)\} \prec 0. \tag{23}$$

2. *There exist a matrix* $Q_d = Q_d^T$ *such that*

$$\operatorname{sym}\left\{\boldsymbol{\Theta}_d \otimes \left(A_d^T Q_d\right)\right\} \prec 0 \tag{24}$$

with

$$\boldsymbol{\Theta}_d = \begin{bmatrix} \Theta_{11} & 0 & \cdots & \cdots & 0 \\ 0 & \Theta_{22} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \Theta_{dd} \end{bmatrix}, \tag{25}$$

where

$$\Theta_{ii} = \begin{bmatrix} \sin\theta_i & \cos\theta_i \\ -\cos\theta_i & \sin\theta_i \end{bmatrix}, \qquad \theta_i = \pi - \alpha_i \frac{\pi}{2} \quad \textit{for all} \;\; i = \overline{1, d}\,. \tag{26}$$

Definition 6 *The multidimensional fractional system* (9) *is called admissible if and only if it is: regular, impulse free and stable.*

According to [18], we extend some results to fractional multidimensional model (9).

Lemma 2 *If the pair* (E_d, A_d) *is regular, then the following statements are satisfied,*

1. *There exist two non singular matrices M and N satisfying*

$$ME_dN = \begin{bmatrix} I_r & 0 \\ 0 & F \end{bmatrix}, \qquad MA_dN = \begin{bmatrix} A_I & 0 \\ 0 & I_{n-r} \end{bmatrix}, \tag{27}$$

where $A_1 \in \mathbb{R}^{r\times r}$ and $F \in \mathbb{R}^{(n-r)\times(n-r)}$ is nilpotent.

2. *The pair (E_d, A_d) is impulse free if and only if* F = 0.

A new results based on $\mathcal{LMI}s$ conditions for the admissibility of dD fractional continuous-time systems are now derived.

Lemma 3 *The pair (E_d, A_d) is admissible if and only if the pair $\left(E_d^T, A_d^T\right)$ is admissible.*

Proof. To confirm that the pair (E_d, A_d) is admissible if and only if $\left(E_d^T, A_d^T\right)$ is admissible we present the proof as follow

$$\begin{aligned}\det\left(E_dK_d^{\alpha_1,\alpha_2,\cdots,\alpha_d} - A_d\right) &= \det\left(E_dK_d^{\alpha_1,\alpha_2,\cdots,\alpha_d} - A_d\right)^T \\ &= \det\left(E_d^TK_d^{\alpha_1,\alpha_2,\cdots,\alpha_d} - A_d^T\right)\end{aligned} \tag{28}$$

and

$$\begin{aligned}\deg\left(\det\left(E_dK_d^{\alpha_1,\alpha_2,\cdots,\alpha_d} - A_d\right)\right) &= \deg\left(\det\left(E_dK_d^{\alpha_1,\alpha_2,\cdots,\alpha_d} - A_d\right)^T\right) \\ &= \deg\left(\det\left(E_d^TK_d^{\alpha_1,\alpha_2,\cdots,\alpha_d} - A_d^T\right)\right)\end{aligned} \tag{29}$$

which mean that the pair (E_d, A_d) is regular and impulse free if and only if $\left(E_d^t, A_d^T\right)$ is regular and impulse free. By the use of Lemma 2 and Corollary 1 we obtain that the stability of the pair (E_d, A_d) depend on $\widetilde{A_1}$ or $\widetilde{A_1}^T$.

Consequently, we have proved the equivalence of the stability between this two pairs, which complete proof of lemma. □

Theorem 2 *The multidimensional fractional system* (9) *is said to be admissible if and only if there exist a matrix*

$$X_d = \begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1d} \\ X_{21} & X_{22} & \cdots & X_{2d} \\ \vdots & \vdots & \ddots & \vdots \\ X_{d1} & X_{d2} & \cdots & X_{dd} \end{bmatrix} \succ 0 \quad \textit{and} \quad Y = \begin{bmatrix} Y_{11} \\ Y_{21} \\ \vdots \\ Y_{d1} \end{bmatrix} \quad \textit{verifying} \tag{30}$$

1.

$$\operatorname{sym}\left\{\boldsymbol{\Theta_d} \otimes A_d^T\left(X_d E + E_{0d} Y_d^T\right)\right\} \prec 0\,. \tag{31}$$

2.

$$\operatorname{sym}\left\{\boldsymbol{\Theta_d} \otimes A_d\left(X_d E_d^T + E_{0d} Y_d^T\right)\right\} \prec 0\,. \tag{32}$$

3. There exist a matrix $P_d = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1d} \\ P_{21} & P_{22} & \cdots & P_{2d} \\ \vdots & \vdots & \ddots & \vdots \\ P_{d1} & P_{d2} & \cdots & P_{dd} \end{bmatrix}$ *satisfying*

$$E_d^T P_d = P_d E_d^T \geq 0, \tag{33}$$

and

$$\operatorname{sym}\left\{\boldsymbol{\Theta_d} \otimes A_d^T P_d\right\} \prec 0\,, \tag{34}$$

where E_{0d} *is an arbitrary matrix of full column rank, and which satisfies the condition* $E_d^T E_{0d} = 0$*, with*

$$\boldsymbol{\Theta}_d = \begin{bmatrix} \Theta_{11} & 0 & \cdots & \cdots & 0 \\ 0 & \Theta_{22} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \Theta_{dd} \end{bmatrix}, \tag{35}$$

where

$$\Theta_{ii} = \begin{bmatrix} \sin\theta_i & \cos\theta_i \\ -\cos\theta_i & \sin\theta_i \end{bmatrix}, \qquad \theta_i = \pi - \alpha_i \frac{\pi}{2} \quad \textit{for all } \ i = \overline{1, d}\,. \tag{36}$$

Proof. We have to prove that the admissibility and the condition (31) in the previous theorem are equivalent.

1. Sufficient condition: First let us partitioned the matrices X_d, Y_d and A_d in block matrix as follows. Let us suppose that the inequality (31) is satisfied for some matrices

$$X_d = \begin{bmatrix} X & X_{12} & \cdots & X_{1d} \\ X_{21} & X_{22} & \cdots & X_{2d} \\ \vdots & \vdots & \ddots & \vdots \\ X_{d1} & X_{d2} & \cdots & X_{dd} \end{bmatrix} = \left[\begin{array}{c|c} X_I & X_{II} \\ \hline X_{III} & X_{IV} \end{array}\right] \succ 0$$

and

$$Y_d = \begin{bmatrix} Y_{11} \\ Y_{21} \\ \vdots \\ Y_{d1} \end{bmatrix} = \left[\begin{array}{c} Y_I \\ \hline Y_{II} \end{array}\right],$$

$$A = A_d = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1d} \\ A_{21} & A_{22} & \cdots & A_{2d} \\ \vdots & \vdots & \ddots & \vdots \\ A_{d1} & A_{d2} & \cdots & A_{dd} \end{bmatrix} = \left[\begin{array}{c|c} A_I & A_{II} \\ \hline A_{III} & A_{IV} \end{array}\right].$$

By the use of singular value decomposition of the matrix E_d, We assure the existence of two non-singular matrices M and N such that

$$ME_dN = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \quad MA_dN = \begin{bmatrix} \widetilde{A}_I & \widetilde{A}_{II} \\ \widetilde{A}_{III} & \widetilde{A}_{IV} \end{bmatrix} \quad \text{and} \quad E_0 = M^T \begin{bmatrix} 0 \\ I_{n-r} \end{bmatrix} \tag{37}$$

with $r = \operatorname{rank} E_d$.

From the non-singularity of the matrix M, E_{0d} is a full rank matrix which satisfied the equation $E^T E_0 = 0$. Let us put the following equations

$$X = M^T \begin{bmatrix} \widehat{X}_I & \widehat{X}_{II} \\ \widehat{X}_{III} & \widehat{X}_{IV} \end{bmatrix} M, \qquad Y = N^{-T} \begin{bmatrix} \widehat{Y}_I \\ \widehat{Y}_{II} \end{bmatrix}. \tag{38}$$

The equations (37) and (38) yields

$$A_d^T \left(X_d E_d + E_{0d} Y^T \right) = N^{-T} \widehat{A}_d N^{-1} \tag{39}$$

with

$$\widehat{A}_d = \begin{bmatrix} \widetilde{A}_I^T \widehat{X}_I + \widetilde{A}_{II}^T \widehat{Y}_I^T & \widetilde{A}_{III}^T \widehat{Y}_{II} \\ \widetilde{A}_{II}^T \widehat{X}_I + \widetilde{A}_{IV}^T \widehat{X}_{II}^T + \widetilde{A}_{IV}^T \widehat{Y}_I^T & \widetilde{A}_{IV}^T \widehat{Y}_{II}^T \end{bmatrix} \tag{40}$$

and by the use of some properties of the Kronecker product, we then obtain

$$\begin{aligned} \boldsymbol{\Theta}_d \otimes \left(A_d^T \left(X_d E_d + E_{0d} Y_d^T \right) \right) &= (I_{2d} \cdot \boldsymbol{\Theta}_d) \otimes \left(N^{-T} \cdot \left(\widehat{A}_d N^{-1} \right) \right) \\ &= \begin{bmatrix} N^{-T} & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & N^{-T} \end{bmatrix} (\boldsymbol{\Theta}_d \otimes \widehat{A}) \begin{bmatrix} N^{-1} & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & N^{-1} \end{bmatrix} \end{aligned} \tag{41}$$

with

$$\boldsymbol{\Theta}_d \otimes \widehat{A}_d = \begin{bmatrix} \widehat{A}_{11} & \widehat{A}_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\widehat{A}_{12} & \widehat{A}_{11} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \widehat{A}_{21} & \widehat{A}_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\widehat{A}_{22} & \widehat{A}_{21} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \ddots & \ddots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \widehat{A}_{d1} & \widehat{A}_{d2} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\widehat{A}_{d2} & \widehat{A}_{d1} \end{bmatrix}$$

and for all $i = \overline{1, d}$

$$\widehat{A}_{i1} = \begin{bmatrix} (\widetilde{A}_I^T \widehat{X}_I + \widetilde{A}_{III}^T \widehat{Y}_I^T) \sin(\theta_i) & \widetilde{A}_{III}^T \widehat{Y}_{II} \sin(\theta_i) \\ (\widetilde{A}_{II}^T \widehat{X}_I + \widetilde{A}_{IV}^T \widehat{X}_I^T + \widetilde{A}_{IV}^T \widehat{Y}_I^T) \sin(\theta_i) & \widetilde{A}_{IV}^T \widehat{Y}_{II}^T \sin(\theta_i) \end{bmatrix}, \tag{42}$$

$$\widehat{A}_{i2} = \begin{bmatrix} (\widetilde{A}_I^T \widehat{X}_I + \widetilde{A}_{III}^T \widehat{Y}_I^T) \cos(\theta_i) & \widetilde{A}_{III}^T \widehat{Y}_{II} \cos(\theta_i) \\ (\widetilde{A}_{II}^T \widehat{X}_{1d} + \widetilde{A}_{IV}^T \widehat{X}_I^T + \widetilde{A}_{IV}^T \widehat{Y}_I^T) \cos(\theta_i) & \widetilde{A}_{IV}^T \widehat{Y}_{II}^T \cos(\theta_i) \end{bmatrix}. \tag{43}$$

Then

$$\begin{aligned} &\operatorname{sym}\left\{\boldsymbol{\Theta}_d \otimes A_d^T \left(X_d E_d + E_{0d} Y_d^T\right)\right\} \\ &= \begin{bmatrix} N^{-T} & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & N^{-T} \end{bmatrix} \begin{bmatrix} \mathcal{H}_1 & 0 & 0 & 0 \\ 0 & \mathcal{H}_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \mathcal{H}_d \end{bmatrix} \begin{bmatrix} N^{-1} & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & N^{-1} \end{bmatrix}, \end{aligned} \tag{44}$$

where

$$\mathcal{H}_i = \begin{bmatrix} \widehat{A}_{i1} + \widehat{A}_{i1}^T & \widehat{A}_{i2} - \widehat{A}_{12}^T \\ \widehat{A}_{i2}^T - \widehat{A}_{i2} & \widehat{A}_{i1} + \widehat{A}_{i1}^T \end{bmatrix}.$$

We finally deduce that

$$\operatorname{sym}\left\{\boldsymbol{\Theta}_d \otimes A_d^T \left(X_d E_d + E_{0d} Y_d^T\right)\right\} \prec 0 \tag{45}$$

which implies that $\widehat{A}_{i1} + \widehat{A}_{i1}^T \prec 0$ for all $i = \overline{1, d}$.
Where

$$\widehat{A}_{i1} + \widehat{A}_{i1}^T = \begin{bmatrix} \cdots & \cdots \\ \cdots & (\widetilde{A}_{IV}^T \widehat{Y}_{II}^T + \widehat{Y}_{II} \widetilde{A}_{IV}) \sin(\theta_i) \end{bmatrix}$$

we conclude that for every $i = \overline{1.d}$, we have

$$(\widetilde{A}_{IV}^T \widehat{Y}_{II}^T + \widehat{Y}_{II} \widetilde{A}_{IV}) \sin(\theta_i) \prec 0$$

since $\sin(\theta_i) \prec 0$.

Therefore $\widetilde{A}_{IV}$ is a non-singular matrix which means that the system (9) is regular and impulse free.

Since the system (9) is regular and impulse free, there exist invertible matrices L, R

$$LE_dR = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \qquad LA_dR = \begin{bmatrix} \widetilde{A}_I & 0 \\ 0 & I_{n-r} \end{bmatrix}. \tag{46}$$

Note that

$$X_d = L^T \left[\begin{array}{c|c} \widehat{X}_I & \widehat{X}_{II} \\ \hline \widehat{X}_{III} & \widehat{X}_{IV} \end{array}\right] L, \qquad Y = R^{-T} \left[\begin{array}{c} \widehat{Y}_I \\ \hline \widehat{Y}_{II} \end{array}\right], \qquad E_{0d} = L^T \begin{bmatrix} 0 \\ I_{n-r} \end{bmatrix} \tag{47}$$

from the equations (46), (47) and the inequality (32) the following result is deduced

$$\begin{bmatrix} N^{-T} & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & N^{-T} \end{bmatrix} \begin{bmatrix} \Psi_1 & 0 & 0 & 0 \\ 0 & \Psi_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \Psi_d \end{bmatrix} \begin{bmatrix} N^{-1} & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & N^{-1} \end{bmatrix} \prec 0 \tag{48}$$

with

$$\Psi_i = \begin{bmatrix} \Phi_{i1} & \Phi_{i2} \\ \Phi_{i2}^T & \Phi_{i1} \end{bmatrix},$$

$$\begin{aligned} \Phi_{i1} &= \begin{bmatrix} \left(\widetilde{A}_I^T \widehat{X}_I + \widehat{X}_I^T \widetilde{A}_I\right) \sin(\theta_i) & \left(\widehat{X}_{II} + \widehat{Y}_I\right) \sin(\theta_i) \\ \left(\widehat{X}_{II}^T + \widehat{Y}_I^T\right) \sin(\theta_i) & \left(\widehat{Y}_{II}^T + \widehat{Y}_{II}\right) \sin(\theta_i) \end{bmatrix}, \\ \Phi_{i2} &= \begin{bmatrix} \left(\widetilde{A}_I^T \widehat{X}_I - \widehat{X}_I^T \widetilde{A}_I\right) \cos(\theta_i) & -\left(\widehat{X}_{II} + \widehat{Y}_I\right) \cos(\theta_i) \\ \left(\widehat{X}_{II}^T + \widehat{Y}_I^T\right) \cos(\theta_i) & \left(\widehat{Y}_{II}^T - \widehat{Y}_{II}\right) \cos(\theta_i) \end{bmatrix}. \end{aligned} \tag{49}$$

For all $i = \overline{1, d}$.

The inequality (48) implies that we have $\begin{bmatrix} \Psi_1 & 0 & 0 & 0 \\ 0 & \Psi_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \Psi_d \end{bmatrix} \prec 0.$

In other ways $\Psi_i \prec 0$, $\forall i = \overline{1, d}$.

This, is equivalents to

$$\begin{bmatrix} \mathcal{W}_1 & 0 & 0 & \cdots & 0 \\ 0 & \mathcal{W}_2 & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & \cdots & 0 \\ \vdots & \vdots & 0 & \ddots & 0 \\ 0 & 0 & 0 & \cdots & \mathcal{W}_d \end{bmatrix} \prec 0 \tag{50}$$

with

$$\mathcal{W}_i = \begin{bmatrix} \left(\widetilde{A}_I^T \widehat{X}_I + \widehat{X}_I^T \widetilde{A}_I\right) \sin\theta_i & \left(\widetilde{A}_I^T \widehat{X}_I - \widehat{X}_I^T \widetilde{A}_I\right) \sin\theta_i & \left(\widehat{X}_{II} + \widehat{Y}_I\right) \sin\theta_i & -\left(\widehat{X}_{II} + \widehat{Y}_I\right) \cos\theta_i \\ \left(\widehat{X}_I^T \widetilde{A}_I - \widetilde{A}_I^T X_I\right) \cos\theta_i & \left(\widetilde{A}_I^T \widehat{X}_I + \widehat{X}_I^T \widetilde{A}_I\right) \sin\theta_i & \left(\widehat{X}_{II} + \widehat{Y}_I\right) \cos\theta_i & \left(\widehat{X}_{II} + \widehat{Y}_I\right) \sin\theta_i \\ \left(\widehat{X}_{II}^T + \widehat{Y}_I^T\right) \sin\theta_i & \left(\widehat{X}_{II}^T + \widehat{Y}_I^T\right) \cos\theta_i & \left(\widehat{Y}_{II}^T + \widehat{Y}_{II}\right) \sin\theta_i & \left(\widehat{Y}_{II}^T - \widehat{Y}_{II}\right) \cos\theta_i \\ -\left(\widehat{X}_{II}^T + \widehat{Y}_I^T\right) \cos\theta_i & \left(\widehat{X}_{II}^T + \widehat{Y}_I^T\right) \sin\theta_i & \left(\widehat{Y}_{II} - \widehat{Y}_{II}^T\right) \cos\theta_i & \left(\widehat{Y}_{II}^T + Y_{II}\right) \sin\theta_i \end{bmatrix} \prec 0. \tag{51}$$

Using the inequality (51) we deduce that for all $i = \overline{1, d}$,

$$\begin{bmatrix} \left(\widetilde{A}_I^T \widehat{X}_I + \widehat{X}_I^T \widetilde{A}_I\right) \sin\theta_i & \left(\widetilde{A}_I^T \widehat{X}_I - \widehat{X}_I^T \widetilde{A}_I\right) \cos\theta_i \\ \left(\widehat{X}_I^T \widetilde{A}_I - A_I^T \widehat{X}_I\right) \cos\theta_i & \left(\widetilde{A}_I^T \widehat{X}_I + \widehat{X}_I^T \widetilde{A}_I\right) \sin\theta_i \end{bmatrix} \prec 0. \tag{52}$$

Finally, relations (50), (51) and (52) confirms the asymptotic stability of the system (9) since $\widehat{X}_I \succ 0$. As a results, the system (9) is admissible (regular, impulse free and stable).

2. Necessary condition: Let us assume that the system (9) is admissible, then applying the equation (46) and Lemma 1, which gives us $spec(E_d, A_d) = spec(\widetilde{A}_{1d})$ and

$$\arg\left(\text{spec}(\widetilde{A_{1d}})\right) > \begin{bmatrix} \alpha_1 \frac{\pi}{2} & \alpha_2 \frac{\pi}{2} & \cdots & \alpha_d \frac{\pi}{2} \end{bmatrix}. \tag{53}$$

According to the Corollary 1, there exist a matrix $\widetilde{X}_I > 0$ such that

$$\mathrm{sym}\left\{\mathbf{\Theta}_d \otimes \left(\widetilde{A_1}^T \widetilde{X_1}\right)\right\} \prec 0 \tag{54}$$

and

$$E_{0d} = L^T \begin{bmatrix} 0 \\ I_{n-r} \end{bmatrix}, \qquad Y_d = R^{-T} \begin{bmatrix} \widetilde{Y}_I \\ \widetilde{Y}_{II} \end{bmatrix}, \qquad X_d = L^T \begin{bmatrix} \widetilde{X}_{1d} & 0 \\ 0 & I_{n-r} \end{bmatrix} L. \tag{55}$$

From the equations (46) and (55) we obtain

$$\begin{aligned} &\mathrm{sym}\left\{\mathbf{\Theta_d} \otimes \left(A_d^T \left(X_d E_d + E_{0d} Y_d^T\right)\right)\right\} \\ &= \begin{bmatrix} R^{-T} & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & R^{-T} \end{bmatrix} \times \left[\mathrm{diag}\left(\mathcal{A}_i + \mathcal{A}_i^T\right)\right] \\ &\times \begin{bmatrix} R^{-1} & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & R^{-1} \end{bmatrix}, \end{aligned} \tag{56}$$

where the matrices $\mathcal{A}_1$ and $\mathcal{A}_2$ are defined as follows

$$\mathcal{A}_i = \begin{bmatrix} \widetilde{A_{1d}}^T \widetilde{X}_{1d} \sin\theta_i & 0 & \widetilde{A_{1d}}^T \widetilde{X}_{1d} \cos\theta_i & 0 \\ 0 & -I_{n-r}\sin\theta_i & 0 & -I_{n-r}\cos\theta_1 \\ -\widetilde{A_{1d}}^T \widetilde{X}_{1d} \cos\theta_i & 0 & \widetilde{A_{1d}}^T \widetilde{X}_{1d} \sin\theta_i & 0 \\ 0 & I_{n-r}\cos\theta_i & 0 & -I_{n-r}\sin\theta_i \end{bmatrix}. \tag{57}$$

Let us note that

$$\mathcal{A}_i + \mathcal{A}_i^T = \begin{bmatrix} \Lambda_i & \Lambda_{i1} \\ \Lambda_{i1}^T & \Lambda_i \end{bmatrix} \tag{58}$$

with

$$\Lambda_i = \begin{bmatrix} \left(\widetilde{A}_1^T \widetilde{X}_{1d} + \widetilde{X}_{1d}\widetilde{A}_{1d}\right)\sin\theta_i & 0 \\ 0 & -2I_{n-r}\sin\theta_i \end{bmatrix}, \tag{59}$$

$$\Lambda_{i1} = \begin{bmatrix} \left(\widetilde{A}_{1d}^T \widetilde{X}_{1d} - \widetilde{X}_{1d}\widetilde{A}_{1d}\right)\cos\theta_i & 0 \\ 0 & 0 \end{bmatrix}. \tag{60}$$

To prove the inequality (32) of Theorem 2, necessitate to prove the two following conditions

$$\Lambda_i \prec 0$$

and

$$\Lambda_i - \Lambda_{i1}\Lambda_i^{-1}\Lambda_{i1}^T \prec 0.$$

We will prove one of the previous conditions for all $i = \overline{1,d}$ because of the similarity. From the inequality (54) we have directly $\Lambda_i \prec 0\ \forall i = \overline{1,d}$ and also

$$\begin{aligned}
&\Lambda_i - \Lambda_{i1}\Lambda_i^{-1}\Lambda_{i1}^T = \\
&\begin{bmatrix} \left(\widetilde{A}_{1d}^T\widetilde{X}_{1d} + \widetilde{X}_{1d}\widetilde{A}_{1d}\right)\sin\theta_i & 0 \\ 0 & -2I_{n-r}\sin\theta_i \end{bmatrix} - \begin{bmatrix} \left(\widetilde{A}_{1d}^T\widetilde{X}_{1d} - \widetilde{X}_{1d}\widetilde{A}_{1d}\right)\cos\theta_i & 0 \\ 0 & 0 \end{bmatrix} \\
&\times \begin{bmatrix} \left(\widetilde{A}_{1d}^T\widetilde{X}_{1d} + \widetilde{X}_{1d}\widetilde{A}_{1d}\right)^{-1}\dfrac{1}{\sin\theta_i} & 0 \\ 0 & \dfrac{-1}{2\sin\theta_i}I_{n-r} \end{bmatrix} \begin{bmatrix} \left(\widetilde{A}_{1d}^T\widetilde{X}_{1d} - \widetilde{X}_{1d}\widetilde{A}_{1d}\right)\cos\theta_i & 0 \\ 0 & 0 \end{bmatrix} \\
&= \begin{bmatrix} \Omega_i & 0 \\ 0 & -2I_{n-r}\sin\theta_i \end{bmatrix},
\end{aligned} \tag{61}$$

where

$$\begin{aligned}
\Omega_i = {} & \left(\widetilde{A}_{1d}^T\widetilde{X}_{1d} + \widetilde{X}_{1d}\widetilde{A}_{1d}\right)\sin\theta_i - \left(\widetilde{A}_{1d}^T\widetilde{X}_{1d} - \widetilde{X}_{1d}\widetilde{A}_{1d}\right)\left(\widetilde{A}_{1d}^T\widetilde{X}_{1d} + \widetilde{X}_{1d}\widetilde{A}_{1d}\right)^{-1} \\
& \times \left(\widetilde{X}_{1d}\widetilde{A}_{1d} - \widetilde{A}_{1d}^T\widetilde{X}_{1d}\right)\frac{\cos^2\theta_i}{\sin\theta_i}.
\end{aligned} \tag{62}$$

From (54) we have $\Omega_i \prec 0\ \forall i = \overline{1,d}$, which implying that $\Lambda_i - \Lambda_{i1}\Lambda_i^{-1}\Lambda_{i1}^T \prec 0$ and guaranties the relation (31).

Finally we have proved the equivalent between admissibility and the condition (31).

3. To ensure the relation between admissibility condition (31) and (32), Lemma 3 is used to get this conditions.

Remark 1 *The equivalence between first and third proposition in this theorem yields directly from the equality of two sets:*

$$\Upsilon_{d1} = \left\{X_d \in \mathbb{R}^{n\times n} : E_d^T X_d = X_d^T E_d,\ \ E^T X_d \geqslant 0,\ \ \operatorname{rank} E_d^T X_d = r\right\}, \tag{63}$$

$$\Upsilon_{d2} = \left\{X_d = P_d E_d + E_{0d} Q_d,\ \ P_d \succ 0,\ \ P_d \in \mathbb{R}^{n\times n},\ \ Q \in \mathbb{R}^{(n-r)\times n}\right\} \tag{64}$$

This equivalence completes the proof of our main theorem.

We revisit the admissibility conditions in theorem 2 for the case where $d = 2$ and we present a numerical experiments example to illustrate the $2D$ case by simulating an electrical circuit problem.

Example 1 Let us consider the multidimensional system with $d = 2$, as in [17] the example of an electrical circuit which represents the long transmission line with the distributed element described by the following figure

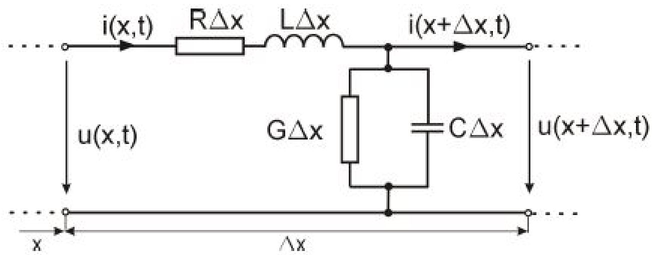

Figure 1: The RLC circuit

For the describing circuit in Figure 1, the development of equations that describe the current and voltage in this line as a function of time t and space variable x,

$$\begin{aligned} -D_x^{\alpha} u(x,t) &= Ri(x,t) + LD_t^{\beta} i(x,t), \\ -D_x^{\alpha} i(x,t) &= Gi(x,t) + CD_t^{\beta} u(x,t), \end{aligned} \tag{65}$$

where $u(x,t)$ is the voltage, and $i(x,t)$ is the current at the point x from the beginning of the line for time t; R is distributed resistance, L is distributed inductance, G is distributed conductance, and C is distributed capacitance of the transmission line; $0 < \alpha \leqslant 1$ and $0 < \beta \leqslant 1$ are fractional (real) orders with respect to the spatial variable x and time t.

An equivalent matrix system which describes the previous figure is given by,

$$\begin{bmatrix} 1 & 0 & 0 & L \\ 0 & 1 & C & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} D_{t_1}^{\alpha} x_h(t_1,t_2) \\ D_{t_2}^{\beta} x_v(t_1,t_2) \end{bmatrix} = \begin{bmatrix} 0 & -R & 0 & 0 \\ -G & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_h(t_1,t_2) \\ x_v(t_1,t_2) \end{bmatrix} + \begin{bmatrix} B_h \\ B_v \end{bmatrix} u(t_1,t_2) \tag{66}$$

with:

$$x_h(t_1,t_2) = x_v(t_1,t_2) = \begin{bmatrix} u(t_1,t_2) \\ i(t_1,t_2) \end{bmatrix},$$

where $u(t_1, t_2)$ is the voltage and $i(t_1, t_2)$ is the current at the point t_1 from the beginning of the line, for time t_2.

For $C = 0.00007F/m$, $R = 0.009\Omega/m, G = 0.08\Omega^{-1}/m$, $L = 0.02H/m$ and $\alpha = 0.5$, $\beta = 0.6$, The considered system (2) with

$$u(t_1, t_2) = \begin{bmatrix} 44.2614 & -24.6661 & 16.8733 & 0.0189 \end{bmatrix} \begin{bmatrix} x_h(t_1, t_2) \\ x_v(t_1, t_2) \end{bmatrix}$$

in this case the system (66) will be as follow

$$E \begin{bmatrix} D^{\alpha}_{t_1} x_h\left(t_1, t_2\right) \\ D^{\beta}_{t_2} x_v\left(t_1, t_2\right) \end{bmatrix} = A \begin{bmatrix} x_h\left(t_1, t_2\right) \\ x_v\left(t_1, t_2\right) \end{bmatrix} \tag{67}$$

with

$$E = \begin{bmatrix} 1 & 0 & 0 & 0.02 \\ 0 & 1 & 0.00007 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{68}$$

$$A = \begin{bmatrix} 0.1328 & -0.0740 & 0.0506 & -0.0899 \\ -0.0800 & 0 & 0 & 0 \\ 4.5409 & -1.9733 & 0.3499 & 0.0015 \\ 0 & 1.0000 & 0 & -1.0000 \end{bmatrix}. \tag{69}$$

The $\mathcal{LMI}s$ defined in (31) and (32) (Theorem 2) are used and the considered system (67) is admissible, the proposed feasible solution is

$$X = \begin{bmatrix} 1.7034 & -0.6852 & 0.0000 & -0.0000 \\ -0.6852 & 2.9106 & -0.0000 & -0.0000 \\ 0.0000 & -0.0000 & 1.0000 & 0.0000 \\ -0.0000 & -0.0000 & 0.0000 & 1.0000 \end{bmatrix} \tag{70}$$

and

$$Y = \begin{bmatrix} -0.2801 & -0.2435 \\ 0.1250 & -0.2141 \\ -0.0180 & -0.0146 \\ 0.0373 & 0.3689 \end{bmatrix}. \tag{71}$$

3. Concluding remarks

In this work, the general fractional multidimensional system described by the Roesser model is presented and analyzed, and new extended results on the stability and admissibility conditions based on the Caputo derivative are introduced. New approach by the use of linear matrix inequality $\mathcal{LMI}$ and Kronecker product are then derived. The obtained results are illustrated by some numerical examples in the case of a two dimensional state space to show the applicability of our method.

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