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Research paper

Buckling of bipolarly prestressed closely-spaced built-up member

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Abstract: The study analysed a bisymmetric closely-spaced built-up member, pin-supported at both ends. It was bipolarly pre-stressed with a displacement (BPCSBM), and loaded with an axial compressive force. Maximum internal gap between the chords was assumed in the section, in which during the stability failure in a classic closely-spaced member, the largest lateral displacements between nodes would potentially occur. As regards the BPCSBM chosen for analysis, the issues of the buckling resistance in the presence of compressive axial load were solved using the energy method, in which the functional minimisation was performed in accordance with the Rayleigh–Ritz algorithm. The problem of BPCSBM stability was also solved using FEM. A spatial shell model was developed. The stability analysis was performed. The analysis resulted in obtaining the buckling load and the member buckling modes. A general conclusion was formulated based on the results obtained: bipolar pre-stressing leads to an increase in buckling resistance of closely-spaced members.

Keywords: buckling of closely-spaced built-up member, back-to-back C-sections, bipolar prestressing, prestressed with a displacement

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1. Introduction

Nowadays there is more and more structures with variable section height encountered in engineering practice. It is usually related to economic reasons, the effect of which is optimization of the shape in terms of load-bearing capacity. In this paper was analysed buckling of innovative bipolar pre-stressed closely-spaced built-up members.

Bipolar pre-stressing of a closely-spaced built-up member [12, 16] consists in a controlled, symmetrical and permanent induced displacement of chords relative to each other. The maximum internal gap between the chords was assumed in the section, in which the largest lateral displacements between nodes would potentially occur during the stability failure in a classic closely-spaced member. While inducing displacement, self-equilibrating pre-stressing was introduced into the member chords. As a result, a bipolarly pre-stressed closely-spaced built-up member (BPCSBM) is obtained. It is a coaxial structure with an induced, non-linear pattern of chords (Fig. 1), which is intended for compression members.

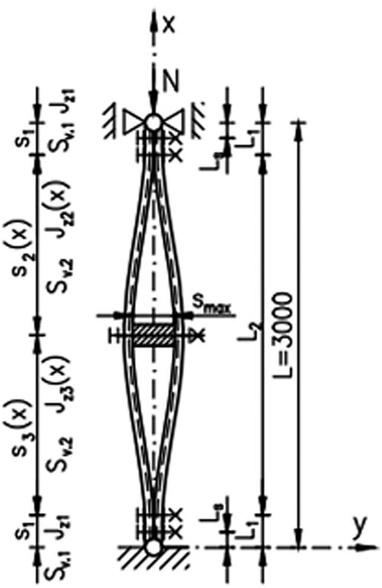

Fig. 1

Bipolar displacement pre-stressing can be used in compression closely-spaced members, especially those with complex bisymmetric sections made from, e.g. pairs of *C*-sections set back-to-back.

The beginning of the search for appropriate relationships describing the load-bearing capacity of built-up sections can be found for example in the works of Engesser [7], Harringx [9], Bleich [4] or Timoshenko and Gere [21]. Later works, on both built-up and closely-spaced built-up members, referred directly to the solutions proposed in [4, 7, 9, 21]. At the threshold of the 21st century among others Bažant [3], Kowal [10] proposed different calculation models to determine the critical load bearing capacity of a compressed member sensitive to shearing.

Classic closely-spaced built-up members in compression have been investigated by many researchers. Slenderness and verification of the code formulas in this respect have been a major issue [2,14,19,20]. Since the end of the 20th century, closely-spaced members, the components of which are made from thin-walled and slender-walled profiles have been analysed [5, 18]. Numerous experimental [17, 18, 22, 23] and numerical [1, 23] studies into classic built-up sections have been conducted.

The method of the design and description of the geometry in axially compressed BPCSBM with symmetrical support conditions and a bi-symmetric cross-section were discussed in [16]. In the function dependent on the position along the member of x section, in which geometric parameters were determined, a number of quantities were defined. They included the following: the curve $y_0(x)$ of precamber of the member chords in the pre-stressing zone, distance between chords in the clearance $s_i(x)$, moment of inertia $J_{zi}(x)$ relative to the main axes, the equivalent moment of inertia $J_{z,sr}$ and the eccentric $e_i(x)$ of the compressive force action in a single chord. The values assumed to be known included the following: the length of the end segment L_1 and that of the pre-stressing zone L_2, the maximum gap of the chords in the clearance $s_{\max}$, and geometric characteristics of the cross-section of a single chord.

Apart from the papers [12, 16], the content concerning the BPCSBMs analysed in this paper was not found in the literature and there are no direct procedures to design this kind of struts according to EN 1993 (2005) standard. However to determine a buckling capacity it is always necessary to estimate a critical load capacity.

In this paper the analysis was carried out for the axially pre-stressed BPCSBM which was pin-supported on both ends, as shown in Figure 1. To estimate the critical load of the BPCSBM, the variation-energy method was employed. The method included the minimisation of the potential energy functional using the Rayleigh–Ritz algorithm. The BPCSBM stability problem also was solved with *FEM* using commercial software for two pairs of C-sections: $2 \times C120 \times 50 \times 5$ and $2 \times C160 \times 65 \times 6$.

2. Analytical estimation of the critical load of BPCSBM

The total potential energy Π of a pin-supported, axially compressed member with the length of L, extended to include the energy of shear strain, has the form:

$$\Pi = U_1 + U_2 - T_1 \tag{2.1}$$

where: U_1 – the energy of elastic deformation, resulting from the loss of stability for a member with variable flexural stiffness $EJ(x)$ described by the relationship:

$$U_1 = \frac{N^2}{2E} \int\limits_L \frac{[y(x)]^2}{J(x)} \, \mathrm{d}x \tag{2.2}$$

U_2 – energy of shear strain:

$$U_2 = \frac{N^2}{2S_v} \int_L \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 \mathrm{d}x \tag{2.3}$$

T_1 – potential energy of the load, corresponding to the work of the axially pre-stressing force N on the displacement from the original configuration to the current one:

$$T_1 = \frac{N}{2} \int_L \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 \mathrm{d}x \tag{2.4}$$

In the end sections with the length L_1, respectively for $x \in \langle 0; L_1 \rangle$ and for $x \in \langle L - L_1; L \rangle$, the pattern of the member chords is linear, and the spacing of the chords in the clearance $s_1 = 0$. Consequently, the moment of inertia has a constant value of $J_1 = \text{const}$, whereas in the paper [16] the moment of inertia $J_{zi}(x)$ was defined in the pre-stressed zone, respectively for $x \in \langle L_1; 0.5L \rangle$ and for $x \in \langle 0.5L; L-L_1 \rangle$. As a result, the relationship (2.2) will take the following form:

$$U_1 = \frac{N^2}{2EJ_1} \left\{ \int_0^{L_1} [y(x)]^2 \,\mathrm{d}x + \int_{L-L_1}^{L} [y(x)]^2 \,\mathrm{d}x \right\} + \frac{N^2}{2E} \left\{ \int_{L_1}^{0.5L} \frac{[y(x)]^2}{J_2(x)} \mathrm{d}x + \int_{0.5L}^{L-L_1} \frac{[y(x)]^2}{J_3(x)} \mathrm{d}x \right\} \tag{2.5}$$

Due to the fact that the shear stiffness of the BPCSBM at the end rectilinear sections $S_{v.1}$ will be different from the non-linear middle section $S_{v.2}$, the energy of the shear strain according to Eq. (2.3) was written as:

$$U_2 = \frac{N^2}{2S_{v.1}} \left[\int_0^{L_1} \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 \mathrm{d}x + \int_{L-L_1}^{L} \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 \mathrm{d}x \right] + \frac{N^2}{2S_{v.2}} \int_{L_1}^{L-L_1} \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 \mathrm{d}x \tag{2.6}$$

On the basis of a non-linear model [10], a relation describing the shear stiffness S_v of the classic twin-chord batten member was determined [6]. The solution takes into account the amplification of local displacement δ caused by axial forces N in the chord of the twin-chord batten member [6]. The relationship was used to describe the shear stiffness $S_{v.1}$ on the end sections (length L_1) of the BPCSBM.

$$S_{v.1} = \frac{24\,E\,J_{z,ch}}{L_b^2} \left(1 - \frac{N}{2N_{1cr}}\right) \tag{2.7}$$

where: L_b – distance between high strength friction grip bolts:

$$L_b = L_1 - L_s \tag{2.8}$$

L_1 – length of the end section of BPCSBM, L_s – the distance from the edge of the member to the first bolt connecting the chords, E – Young modulus, $J_{z,ch}$ – minimum moment of inertia of the chord, N_{1cr} – local buckling force of the member chord between battens according to Eq. (2.9).

$$N_{1cr} = \frac{N_{1e} G A_{ch}}{n N_{1e} + A_{ch} G} \tag{2.9}$$

where: G – shear modulus, A_{ch} – cross-sectional area of a single chord, n – shear factor, N_{1e} – Euler's critical force of the member chords between the battens according to Eq. (2.10):

$$N_{1e} = \frac{\pi^2 E J_{z,ch}}{L_b^2} \tag{2.10}$$

In order to describe the shear stiffness of BPCSBM in the pre-stressing zone $S_{v.2}$, the pattern of the member chord was approximated by a straight line. The formula describing the shear stiffness for linearly convergent multi-chord members with battens [8], with the assumption that the separator is a rigid batten ($J_d = \infty$), was modified to account for displacement amplification [10]. That was done following the solution proposed by Kowal [10]:

$$S_{v.2} = \frac{192 E J_{z,ch} \cos\theta}{L_2^2 \left[1 + \left(\dfrac{4e_z + s_{\max}}{4e_z + 3s_{\max}}\right)^2\right]} \left(1 - \frac{N}{2N_{1cr}}\right) \tag{2.11}$$

where: E – Young modulus, $J_{z,ch}$ – minimum moment of inertia of the chord, L_2 – length of the pre-stressing zone, $e_{z,ch}$ – position of the centre of gravity of the cross-section of one chord of the closely-spaced member, $s_{\max}$ – maximum distance in the clearance between chords of the bipolarly pre-stressed closely-spaced member, θ – the angle of inclination of the chords in relation to the vertical, N_{1cr} – local buckling force of the member chord between the battens according to Eq. (2.9), N_{1e} – Euler's buckling force of the member chord between the battens according to (2.10).

Using variational methods in the study of equilibrium states, an approximate form of a particular integral of a differential equation is selected that satisfies the differential equation of the problem in question, or the equation of boundary conditions as exactly as possible. In this study, it was assumed that the function describing the buckling curve will be approximate, whereas the boundary conditions will be exactly satisfied. For the case under consideration, namely nominally pinned support at both ends, it was proposed to adopt the function $y(x)$ that approximates the deflection line in the form of a series:

$$y(x) = \sum_{i=1}^{n} a_i \sin\left(\frac{(2i-1)\pi x}{L}\right) \tag{2.12}$$

The potential energy functional Π can therefore be written as Eq. (2.13).

$$\Pi = \frac{N^2}{2EJ_1}\int_0^{L_1}\left[\sum_{i=1}^{n} a_i \sin\left(\frac{(2i-1)\pi x}{L}\right)\right]^2 \mathrm{d}x \tag{2.13}$$
$$+ \frac{N^2}{2E}\int_{L_1}^{0.5L}\frac{\left[\sum_{i=1}^{n} a_i \sin\left(\frac{(2i-1)\pi x}{L}\right)\right]^2}{J_2(x)}\mathrm{d}x + \frac{N^2}{2E}\int_{0.5L}^{L-L_1}\frac{\left[\sum_{i=1}^{n} a_i \sin\left(\frac{(2i-1)\pi x}{L}\right)\right]^2}{J_3(x)}\mathrm{d}x$$
$$+ \frac{N^2}{2EJ_1}\int_{L-L_1}^{L}\left[\left(\sum_{i=1}^{n} a_i \sin\left(\frac{(2i-1)\pi x}{L}\right)\right)\right]^2 \mathrm{d}x + \frac{N^2}{2S_{v.1}}\int_0^{L_1}\left[\left(\sum_{i=1}^{n} a_i \sin\left(\frac{(2i-1)\pi x}{L}\right)\right)^I\right]^2 \mathrm{d}x$$
$$+ \frac{N^2}{2S_{v.2}}\int_{L_1}^{L-L_1}\left[\left(\sum_{i=1}^{n} a_i \sin\left(\frac{(2i-1)\pi x}{L}\right)\right)^I\right]^2 \mathrm{d}x + \frac{N^2}{2S_{v.1}}\int_{L-L_1}^{L}\left[\left(\sum_{i=1}^{n} a_i \sin\left(\frac{(2i-1)\pi x}{L}\right)\right)^I\right]^2 \mathrm{d}x$$
$$- \frac{N}{2}\int_0^{L}\left[\left(\sum_{i=1}^{n} a_i \sin\left(\frac{(2i-1)\pi x}{L}\right)\right)^I\right]^2 \mathrm{d}x$$

Based on (2.13), a system of homogeneous algebraic equations was obtained. The critical load is the smallest positive root of the equation derived from the comparison to zero of the determinant of the matrix of coefficients for parameters a_i.

3. Finite element analysis of BPCSBM

The BPCSBM buckling problem was solved with *FEM* using the ABAQUS/CAE software. A spatial shell model was developed. An element was created, which consisted of two straight chords with a C cross-section, split at the midspan by a separator. The separator had the form of a rectangular sheet metal with a thickness of t_d, corresponding to the maximum planned spacing of the chords $s_{\max}$. The contacts between the chords and the separator, and also between the chords were defined. The bolt at the midspan of the element connecting the chords and the separator was modelled as a *beam* type connector with a diameter corresponding to that of the bolt.

The four-node S4R finite elements available in the program library were used, with linear shape functions, and reduced numerical integration. At each node, finite elements have three translational, namely $U1$ (direction x), $U2$ (direction y) and $U3$ (direction z), and three rotational degrees of freedom – $UR1$ (rotation around the x axis), $UR2$ (rotation around the y axis), and $UR3$ (rotation around the z axis). In the simulations, the finite element size was assumed to be not larger than 10×10 [mm] and it was chose before the actual simulations on the basis of preliminary tests. Use of smaller ones did not give more accurate results but increased the model and computation time. The number of finite elements (included ties) used in analysis was 13 262 for $2 \times C120 \times 50 \times 5$ and 18 810 for $2 \times C160 \times 65 \times 6$.

The material was defined by materials parameters specified in the relevant standards [15]: Young modulus ($E = 210\,000$ N/mm^2), Poisson's ratio ($\nu = 0.3$) and density ($\rho = 7800$ kg/m^3).

The analysis of compressed BPCSBMs was divided into three computational steps: initial, bipolar pre-stressing and buckling.

Initial is a default step, in which the contact between the separator and the chords of the closely-spaced member was defined. It was choosen a standard surface to surface interaction with contact property options for normal behaviour as a "hard" contact and for tangential behaviour as a penalty friction formulation.

Bipolar pre-stressing, the step was selected in order to obtain non-linear geometry of the closely-spaced member and the state of self-induced stresses. The translational movement of the separator's edges was blocked in all directions ($U1 = U2 = U3 = 0$). The displacement of nodes corresponding to the locations of the pre-stressing bolts in z direction ($U3 = 0.5s_{\max}$) was defined. Boundary conditions used in this step were shown in the Figure 2.

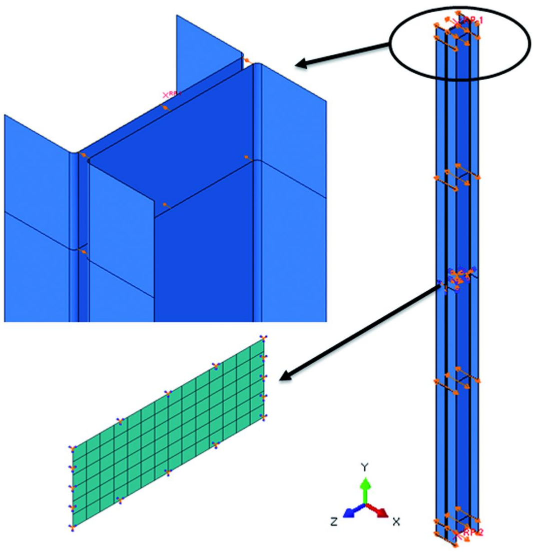

Fig. 2

Buckling analysis, the step was created to analyse the stability of the compressed BPCSBM. The support was modelled as a pinned support at the member bottom, and a roller support at the member top – boundary conditions were shown in the Figure 3. The supports were made in top and bottom reference points connected with C-profiles edges by coupling constraints. The compressive load was defined as a concentrated force with a nominal value of 1 N, applied axially to the top reference point. As a result of defining the contact between the BPCSBM components, in the computations it was necessary to account for geometric non-linearity. The linear buckling procedure with contact nonlinear analysis (CNA) was carried out.

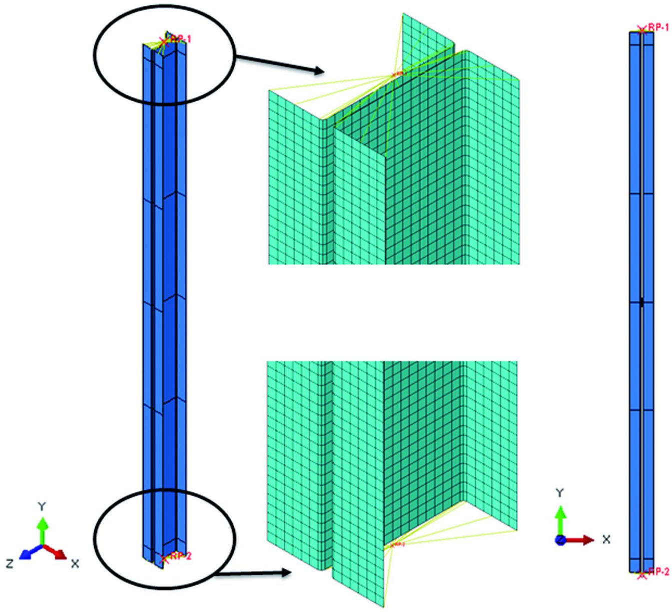

Fig. 3

Due to the contact issue the buckling problem was solved with subspace eigenvalue extraction method. The basic idea of the subspace iteration method is a simultaneous inverse power iteration. A subspace is defined as a small set of base vectors. The set of m vectors defined the m-dimensional subspace out of the n dimensions defined by the variables in the finite element model. Created in this way subspace is transformed by iteration into the space with some of the lowest eigenvectors of the all system. The operation called a generalized "inverse power sweep" with m vectors, involves the solution of the complete set of linear stiffness equations for several right-hand-side vectors. The eigenvectors defined in the reduced space can be transformed back to the full space of the structural problem.

Geometric nonlinearity (the nonlinear effects of large deformations and displacements) activated in second step as the effect of contact definition has been considered in buckle step too.

4. Results

Theoretical and numerical analyses were carried out for BPCSBMs with lengths of $L = 3.0$ m, made from pairs of cold-formed C-profiles with cross-sections $C120 \times 50 \times 5$ and $C160 \times 65 \times 6$. The pre-stressing zone L_2 ranged $0.5L \div 0.8L$, i.e. 1500 to 2400 mm, at increments of $0.1L$ (300 mm). Thickness of the separator $t_d = s_{\max}$ changed in the range of 8÷16 mm at 4 mm increments. The width of the separator was assumed to be $b = 50$ mm. For the cases analysed, the function $y(x)$ was adopted to approximate the deflection line using the first term in the series (2.12):

$$y(x) = a_i \sin\left(\frac{\pi x}{L}\right) \tag{4.1}$$

The element utilised for the sake of comparison was a classic, closely-spaced member with a length of $L = 3.0$ m. Classic closely-spaced members were butt-jointed at four sites with bolts spaced at $L_b = 950$ mm. The spacing of connectors, analogous to the arrangement of bolts in the spatial structures of the "Zachod" system, was applied [10]. For $C160 \times 65 \times 6$ C-section pairs, two M12 bolts were used at each of the four sites of connection. For $C120{\times}50{\times}5$ C- section pairs, one M16 bolt was employed at the connection sites. The buckling resistance of the classic closely-spaced members was estimated using the Engesser's formula $N_{cr,d}^{Eng}$:

$$N_{cr,d}^{Eng} = \frac{N_e}{1 + \dfrac{N_e L_b^2}{24EJ_{z,ch}}} \tag{4.2}$$

The buckling resistance of the classic closely-spaced members for $2 \times C120 \times 50 \times 5$ is $N_{cr,d}^{Eng} = 169.1$ kN and for $2 \times C160 \times 65 \times 6$ it is $N_{cr,d}^{Eng} = 444.9$ kN.

Table 1 shows the buckling resistance and percentage differences between the energy method (*EM*) and *FEM* solution. Buckling resistances obtained through the analytical method (*EM*) for the pre-stressing range $L_2 = 1500{\div}2400$ mm give a very good approx-

Table 1. Buckling resistance N_{cr} estimated using the energy method (*EM*) and obtained with *FEM*

Section		$2 \times C120 \times 50 \times 5$				$2 \times C160 \times 65 \times 6$			
t_d	L_z	1500	1800	2100	2400	1500	1800	2100	2400
8	N_{cr}^{EM} [kN]	199.6	206.4	206.9	200.6	512.1	526.7	525.8	508.5
	N_{cr}^{FEM} [kN]	213.9	216.2	215.3	210.2	547.7	547.7	545.1	534.9
	N_{cr}^{EM} vs. N_{cr}^{FEM} [%]	–6.5	–4.3	–3.7	–4.4	–6.5	–4.2	–3.5	–4.9
12	N_{cr}^{EM} [kN]	209.7	219.4	222.1	216.8	–	553.0	556.3	541.2
	N_{cr}^{FEM} [kN]	226.5	231.4	232.3	227.7	–	579.3	574.8	568.8
	N_{cr}^{EM} vs. N_{cr}^{FEM} [%]	–7.4	–5.0	–4.2	–4.7	–	–4.5	–3.2	–4.8
16	N_{cr}^{EM} [kN]	–	232.4	237.4	233.3	–	–	587.2	574.4
	N_{cr}^{FEM} [kN]	–	247.3	246.5	246.5	–	–	612.2	605.6
	N_{cr}^{EM} vs. N_{cr}^{FEM} [%]	–	–6.0	–5.1	–5.4	–	–	–4.1	–5.1

imation of the *FEM* solution with a difference of the order of 3.2÷7.4%. Analyses were carried out exclusively for those BPCSBMs, for which the application of pre-stressing did not lead to the section failure in the elastic range (under the assumption that the C-sections were made of S355 steel). As regards the omitted cases, the use of a separator coupled with the proposed length of the pre-stressing zone resulted in plastic yield of the section at the stage of BPCSBM formation.

Figure 4 shows a graphic comparison of the increase in buckling resistance for BPCSBM. The comparison was made for each of the analysed maximum distances between the chords $s_{\max}$, equated with the thickness of the element inducing bipolar pre-stressing due to displacement t_d. The comparisons were drawn only for the same length of the pre-stressing zone L_2. The increase in buckling resistance N_{cr}^{EM} of BPCSBM was estimated in relation to buckling resistance $N_{cr,d}^{Eng}$ of classic closely-spaced members. The highest gain in buckling resistance (over 40%) was found for a BPCSBM built from a pair of $C120 \times 50 \times 5$ C-sections, with the length of the pre-stressing zone $L_2 = 2100$ mm, and the thickness of the separator $t_d = 16$ mm.

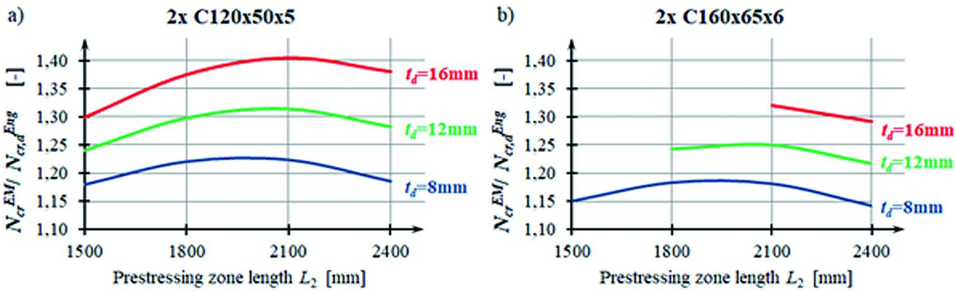

Fig. 4

The largest increase in buckling resistance was observed for BPCSBMs pre-stressed using a separator with a thickness of:

– $t_d = 8$ mm for the length of the pre-stressing zone $L_2 = 1800$ mm;
– $t_d = 12$ mm for the length of the pre-stressing zone $L_2 = 1800$ mm, for $C120\times50\times5$, and $L_2 = 2100$ mm for $C160 \times 65 \times 6$;
– $t_d = 16$ mm for the length of the pre-stressing zone $L_2 = 2100$ mm, for $C120\times50\times5$, and$L_2 = 2100$ mm for $C160 \times 65 \times 6$.

Figure 5 shows the buckling mode of a bipolarly pre-stressed closely-spaced member made of a pair of C-sections $C120 \times 50 \times 5$, with the pre-stressing zone of $L_2 = 2100$ mm, and the separator having a thickness of $t_d = s_{\max} = 12$ mm.

The buckling modes of some BPCSBMs analysed in this paper are shown in Figure 5. The buckling modes of other BPCSBMs analysed in this paper are analogous to the example. BPCSBMs under go flexural buckling with respect to the immaterial axis of the cross-section, assuming the sine half-wave form.

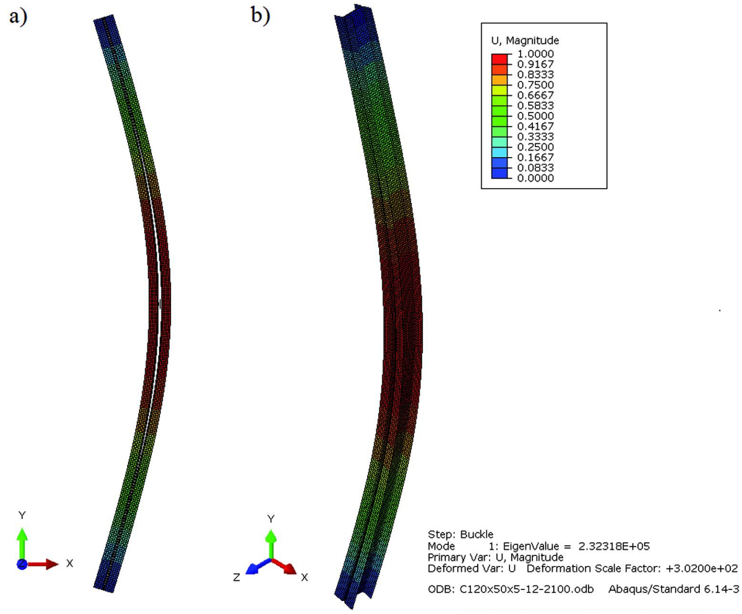

Fig. 5

5. Summary and conclusions

The use of a separator, which induces bipolar pre-stressing due to displacement, leads to an increase in buckling resistance in closely-spaced built-up members.

The increase in buckling resistance can be regulated by changing the separator thickness and/or the length of the pre-stressing zone.

It is possible to select the separator thickness t_d and length of the pre-stressing zone L_2 in such a way so that the BPCSBM resistance is not decided by the member stability, but by the resistance of the section.

Further analytical, numerical and experimental tests are planned to investigate resistance and stability of the BPCSBM, in particular with other chord sections, especially slender-walled ones, different separator thicknesses, and the pre-stressing zone lengths.

It is possible to use the Eurocode 3 Standard [15] to design BPCSBMs as closely-spaced built up members with critical force N_{cr} and stiffness S_v according this paper and [6, 16], but it need further tests planned by author.

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Stateczność osiowo ściskanego pręta bliskogałęziowego sprężonego bipolarnie

Słowa kluczowe: wyboczenie pręta bliskogałęziowego, przekrój złożony z pary ceowników, sprężenie bipolarne, sprężenie przemieszczeniem

Streszczenie:

W pracy analizowano obustronnie przegubowo podparty bisymetryczny pręt bliskogałęziowy bipolarnie sprężony przemieszczeniemi obciążony osiową siłą ściskającą. Bipolarne sprężenie polega na wprowadzeniu trwałego, symetrycznego, kontrolowanego przemieszczenia gałęzi ściskanego pręta bliskogałęziowego względem siebie. Maksymalne rozchylenie gałęzi w świetle przewidziano w przekroju, w którym przy utracie stateczności klasycznego pręta bliskogałęziowego potencjalnie wystąpiłyby największe przemieszczenia między węzłami. Bipolarnie sprężony pręt bliskogałęziowy (PBSB) jest zatem prętem bliskogałęziowym o prostej osi i gałęziach zakrzywionych w strefie sprężenia (L_2), powstałym w wyniku wprowadzenia samozrównoważonego układu naprężeń wstępnych w elemencie.

Dla wytypowanego do analiz PBSB zagadnienie nośności krytycznej pod ściskającym obciążeniem osiowym rozwiązano przy zastosowaniu metody energetycznej z minimalizacją funkcjonału według Rayleigha–Ritza.

Przy budowie funkcjonału energii potencjalnej ustroju sztywność postaciowa PBSB w strefie sprężenia została opisana wzorem na sztywność postaciową dwuzbieżnych liniowo prętów dwugałęziowych, zmodyfikowanym o amplifikację przemieszczeń i wskaźnik imperfekcji wstępnie wygiętego pręta ściskanego.

Zaproponowano przyjęcie funkcji $y(x)$ aproksymującej linię ugięcia w postaci szeregu spełniającego warunki brzegowe rozpatrywanego pręta.

Zagadnienie stateczności PBSB rozwiązano również MES. Wykonano model przestrzenny, powłokowy. Przyjęto model materiału izotropowego idealnie sprężysto-plastycznego, zdefiniowany poprzez moduł Younga, współczynnik Poissona, granicę plastyczności i gęstość. Za kryterium określenia zakresu pracy sprężystej materiału przyjęto warunki przestrzennego wykorzystania naprężeń określone na podstawie hipotezy wytrzymałościowej Hubera-Misesa-Hencky'ego. Przeprowadzono analizę CNA uwzględniającą nieliniowość geometryczną jako efekt zastosowania kontaktu. Otrzymano mnożnik obciążenia krytycznego i postacie wyboczenia pręta.

Rozważania analityczne i numeryczne prowadzono dla PBSB o długości $L = 3$ m, wykonanych z par ceowników formowanych na zimno o przekrojach $C120\times50\times5$ i $C160\times65\times6$, długościach L_2 strefy sprężenia z przedziału 1,2÷2,4 m ze skokiem co 0,3 m i grubościach t_d elementu dystansowego z zakresu 8÷16 mm ze skokiem co 4 mm.

Uzyskano dobrą zgodność wyników otrzymanych drogą analityczną z symulacjami prowadzonymi metodą elementów skończonych (różnice utrzymywały się w granicach 3÷9%).

Na podstawie przeprowadzonych badań stwierdzono, że:

1) zastosowanie w prętach bliskogałęziowych elementu dystansowego, wywołującego bipolarne sprężenie przemieszczeniem, wpływa na wzrost nośności krytycznej;

2) wzrost nośności krytycznej można regulować poprzez zmiany grubości elementu dystansowego i/lub długości strefy sprężenia.

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