Key words: *excavator, Lagrange's equations, generalised co-ordinates*

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ALGORITHM OF CONSTRUCTION OF SINGLE-BUCKET EXCAVATOR MOTION EQUATIONS

The authors present a concept of constructing the equations of motion for a single-bucket pulling excavator in terms of generalised Lagrange's variables. The applied model is based on the assumption that the excavator is a system of rigid solids connected with rotational constrains of ten degrees of freedom. The essence of the proposed algorithm consists in reducing the procedure of constructing the system of excavator's motion equations to multiplication of adequate matrices. One avoids analytical or numerical derivation of the consecutive time derivatives of kinetic and potential energy of the system. The algorithm formulated in such a way may constitute a basis for constructing a numerical program for the analysis of excavator system dynamics. The proposed method of generation of Lagrange's equations can be generalised and applied to a wider class of multibody systems.

1. Introduction

The task of modelling of excavator dynamic system, treated as a system of rigid solids, can be accomplished in many different ways. Recently, many authors have utilised the method presented in [1], [8] in application to constructing the equations of motion of an excavator (for example in the work [2]). The essential feature of this approach is that the motion equations are formulated in the space of Euler's parameters – representing certain relations between Cartesian co-ordinates in the system [1], [8]. The start point in this

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method is formulating the equations of constrains. The differentiated matrix of constrains, called the Jacobi matrix, created on the basis of the equations, makes it possible to introduce the constrains reaction forces into the matrix equations of motion and, in this way, to guarantee that the movements would comply with their mechanical limitations. The algorithm of constructing the motion equations is relatively simple in this method, and there is a possibility of fast, almost automatic generation of the equations. However, computer realization of such an algorithm can be associated with a certain hazard. In numerical calculations, the equations of constrains could not be satisfied quite strictly. It results from the cumulating computational errors, which – sooner or later, depending on the computer's power – cause that the conditions of constrains do not hold any longer, and the system does not realize the mechanical conditions. It is particularly important, for example, in the case of the closed kinematic loop in a single-bucket pulling excavator that consists of the boom's hydraulic cylinder, the boom and the bucket.

The classic approach to the construction of the equations of motion is based on the second kind Lagrange's equations (see [3]). In comparison with the previous one, this method offers the advantage of constrains equations being satisfied virtually automatically, due to the very definition of the generalised co-ordinates. The disadvantage of the method is, however, the complicated process of deriving the equations, which involves multiple differentiations of the functions with respect to the generalised co-ordinates and velocities. The level of difficulties increases dramatically with the number of generalised co-ordinates, especially in the case of three-dimensional model [9].

For several years, the authors have carried out the research on the construction of a dynamic model of an excavator. The experience gained in these research yielded the notion of developing such a method, which would combine the advantages of both above-mentioned methods, i.e. the automatism of the procedure of deriving the equations of motion, characteristic for the Nikravesh method, with the natural simplicity of complying with the constrains, which follows on the very definition of generalised co-ordinates in the Lagrange method. This work presents the proposed concept of automated algorithm for generating excavator motion equations in the generalised co-ordinates, on the basis of Lagrange's equations. The basic principles of the method are presented in this work with an example of a spatial model of a single-bucket excavator.

2. Model structure and basic assumptions

Let us assume that the dynamic system of the excavator consists of the immobile subsoil and of seven rigid solids: the undercarriage (1), the body (2), the boom (3), the arm (4), the bucket (5) and two connectors, (6) and (7), whose mass we neglect (Fig. 1). The solids of the excavator are joined by cylindrical rotational constrains.

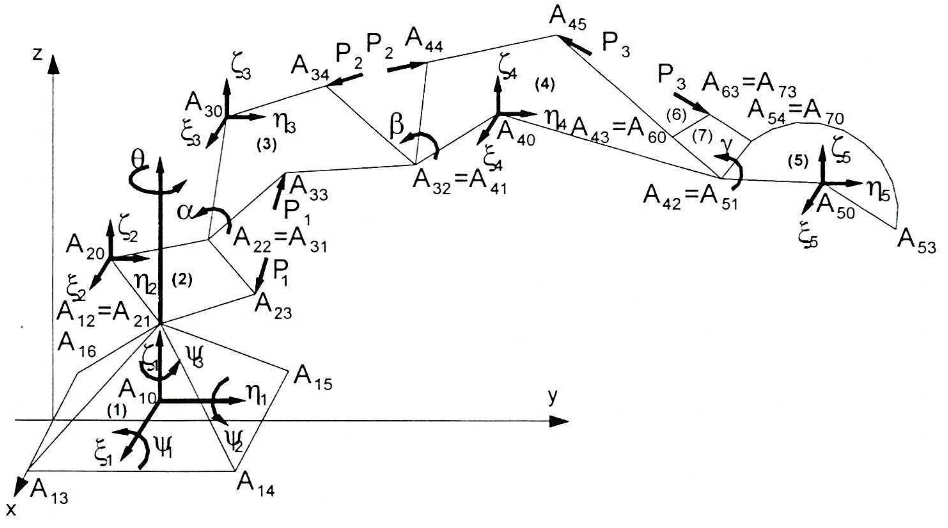

Fig. 1. Functional diagram of mechanical system of single-bucket excavator

A local system of co-ordinates is associated with each of the solids, and is steadily attached to it. Let us introduce the following denotations: $\{\Omega xyz\}$ – global co-ordinate system attached to subsoil, $\{\Omega\xi\eta\zeta\}$ – local co-ordinate system of solid i, where i – index denoting the solid; 0 – subsoil, 1 – undercarriage, 2 – body, 3 – boom, 4 – arm, 5 – bucket, 6, 7 – connectors. The characteristic points of the solids are denoted as A_{ij}, where i – number of the solid, j – index denoting a consecutive point of the solid. The solids are connected by rotational constrains in the points $A_{12} = A_{21}$, $A_{22} = A_{31}$, $A_{32} = A_{41}$, $A_{42} = A_{51}$, $A_{43} = A_{61}$, $A_{63} = A_{73}$, $A_{54} = A_{71}$.

Assuming that the origins of the local co-ordinate systems, in the solids whose mass is taken into consideration, are located at the centres of gravity, A_{i0}, and the directions of the axes are consistent with those of the global Cartesian co-ordinate system $\{\Omega xyz\}$ in the case of zero angle of rotation. The origins of the local systems in the massless connectors are assumed at the points $A_{60} = A_{43}$ and $A_{70} = A_{54}$. The positions of solids' points in the local systems are determined by means of the vectors $r_{ij}^{(k)} = [\xi_k^{Aij}, \eta_k^{Aij}, \zeta_k^{Aij}]^T$, where $k = 0,...7$ denotes the number of the system.

Let $l_{ij} = |r_{ij}^{(i)}|$ denote the distance of point A_{ij} $(i = 1,...7)$ from the origin of the local co-ordinate system, and $\varphi_{ij}^{\xi} = \sphericalangle(\xi_i, r_{ij}^{(i)})$, $\varphi_{ij}^{\eta} = \sphericalangle(\eta_i, r_{ij}^{(i)})$, $\varphi_{ij}^{\zeta} = \sphericalangle(\zeta_i, r_{ij}^{(i)})$ the angles that the vectors defining the position of point A_{ij} in local co-ordinate system make with the axes of the system. We assume that the parameters l_{ij}, φ_{ij}^{ξ}, φ_{ij}^{η}, φ_{ij}^{ζ} are given, and do not change during the motion. Then, the co-ordinates of point Aij in its local co-ordinate system are

$$r_{ij}^{(i)} = l_{ij}\,[\cos\varphi_{ij}^{\xi} \;\; \cos\varphi_{ij}^{\eta} \;\; \cos\varphi_{ij}^{\zeta}]^T \tag{1}$$

Obviously, the vector $r_{ij}^{(i)}$ is invariable during the motion of the system.
Let us also assume that the plane $\{0xy\}$ is the plane of symmetry of the system in its initial position, and that, in an arbitrary position, the characteristic points (centres of mass, points of rotational constrains) of the body, the boom, the arm, the bucket and the connectors lie in the same plane.
We assume that the forces acting on the system are the forces of hydraulic cylinders P_1, P_2, P_3, applied at the points $A_{23}, A_{33}, A_{34}, A_{44}, A_{45}, A_{63}$, respectively, reaction forces of subsoil R_j $(j=1,...4)$ at the points of support of the undercarriage, $A_{13}, A_{14}, A_{15}, A_{16}$, and the reaction force of the ground at the cutting edge of the bucket, A_{53}.
It will be assumed that the position of the excavator in the global co-ordinate system is determined by the vector q of ten generalised co-ordinates, $q = [\psi_1\ \psi_2\ \psi_3\ \theta\ \alpha\ \beta\ \gamma\ x_0\ y_0\ z_0]^T$ where ψ_1, ψ_2, ψ_3 – angles of rotation of the axes of undercarriage's local co-ordinate system with respect to the global system (Brytan's angles) $\theta = \sphericalangle(\xi_1, \xi_2)$ – angle of rotation of body with respect to undercarriage about the axis parallel to the axis ξ_1 passing through the point A_{12}, $\alpha = \sphericalangle(\eta_2, \eta_3)$ – angle of rotation of boom with respect to body about the axis passing through the point A_{22}, $\beta = \sphericalangle(\eta_2, \eta_4)$ – angle of rotation of arm with respect to boom about the axis passing through the point A_{32}, $\gamma = \sphericalangle(\eta_2, \eta_5)$ – angle of rotation of bucket with respect to arm about the axis passing through the point A_{42}, x_0, y_0, z_0 – co-ordinates of undercarriage's mass centre, A_{10}.

3. Transformations of co-ordinate systems

The matrices of rotation of co-ordinate systems, which allow for transformation of position and velocity co-ordinates from one local system to another, play an essential role in the construction of excavator's equations of motion. These will be denoted as follows:

$$\varphi_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos q_i & -\sin q_i \\ 0 & \sin q_i & \cos q_i \end{bmatrix} \text{for } i = 1,\ 5;\ \varphi_i = \begin{bmatrix} \cos q_i & -\sin q_i & 0 \\ \sin q_i & \cos q_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{for } i = 3,\ 4;$$

$$\Phi_2 = \begin{bmatrix} \cos q_2 & 0 & \sin q_2 \\ 0 & 1 & 0 \\ -\sin q_2 & 0 & \cos q_2 \end{bmatrix}; \qquad (2)$$

$$\Phi_6 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\delta_1 & -\sin\delta_1 \\ 0 & \sin\delta_1 & \cos\delta_1 \end{bmatrix},\ \Phi_7 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\delta_2 & -\sin\delta_2 \\ 0 & \sin\delta_2 & \cos\delta_2 \end{bmatrix}$$

where q_i – i^{th} element of generalised co-ordinates vector, δ_1, δ_2 – rotation angles of local systems of the connectors with respect to the arm (functions of the generalised co-ordinate q_7).
The product of matrices of rotation is denoted as $\Phi_{ij} = \Phi_i \cdot \Phi_{i+1} \cdot ... \cdot \Phi_j$ for $i<j$. The matrix Φ_{1i}, for $i>3$, is the matrix of rotation of solid i-2 in a local system with respect to the global system.
Let A1j be the point of the undercarriage with the local system co-ordinates $r_{1j}^{(1)} = [\xi_1^{Aij}, \eta_1^{Aij}, \zeta_1^{Aij}]^T$. Then, the co-ordinates of this point in the global system are

$$r_{1j}^{(0)} = r_{10}^{(0)} + \Phi_{13} r_{1j}^{(1)} \qquad (3)$$

where $r_{10}^{(0)} = [x_0, y_0, z_0]^T$ – vector of position of the undercarriage centre of mass in the global system. If the co-ordinates of point A_{ij} ($i=2,...7$) in its local system are equal to $r_{ij}^{(i)} = [\xi_i^{Aij}, \eta_i^{Aij}, \zeta_i^{Aij}]^T$, then its co-ordinates in the global system are

$$r_{ij}^{(0)} = r_{(i-1)2}^{(0)} + \Phi_{1(i+2)} (r_{ij}^{(i)} - r_{i1}^{(i)}) \qquad (4)$$

Using these relationships, we can express the co-ordinates of the centres of mass A_{i0} ($i=2,...5$) in the following form

$$r_{i0}^{(0)} = r_{(i-1)0}^{(0)} + \Phi_{1(i+1)} r_{(i-1)2}^{(i-1)} - \Phi_{1(i+2)} r_{i1}^{(i)} \qquad (5)$$

For example, taking the undercarriage point A_{23} where the force of cylinder P_1 is applied, and the centre of the body's mass A_{20} we have

$$r_{23}^{(0)} = r_{12}^{(0)} + \Phi_{14}(r_{23}^{(2)} - r_{21}^{(2)})$$

$$r_{20}^{(0)} = r_{10}^{(0)} + \Phi_{13} r_{12}^{(1)} - \Phi_{14} r_{21}^{(2)}$$

The above relationships are represented graphically in Figure 2.

a) b)

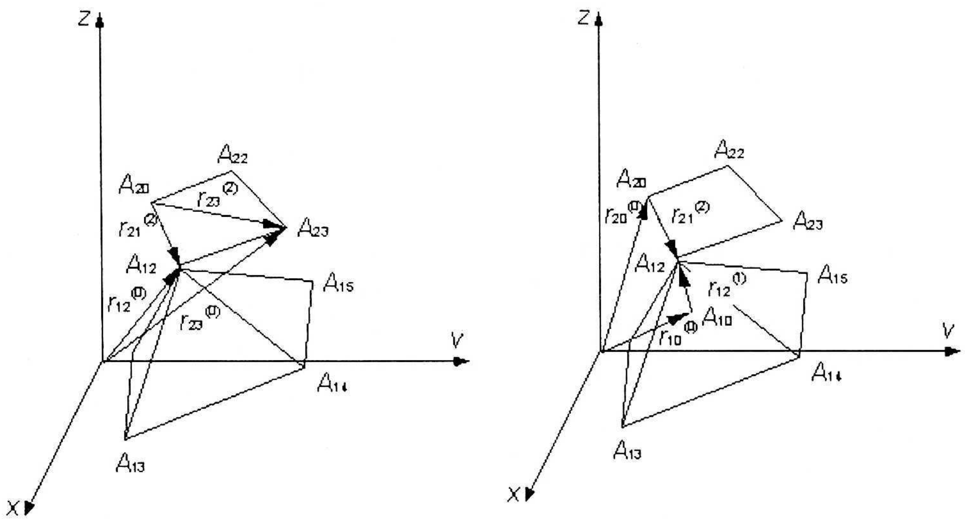

Fig. 2. Example graphical interpretation of relationships (5): a) undercarriage point A_{23} where the force of cylinder P_1 is applied, b) centre of the body's mass A_{20}

4. Equations of motion of excavator system

The description of co-ordinates of the excavator's mechanical system points was utilised in the construction of motion equations based on Lagrange's equations of type II [6], [7] in the form

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right) - \frac{\partial T}{\partial q} + \frac{\partial V}{\partial q} = Q \tag{6}$$

where: T – kinetic energy of the system, V – potential energy, Q – vector of generalised forces. In our case, equation (6) is a system of ten scalar differential equations of second order. Assuming that kinetic energy does not depend explicitly on time, we can represent these equations in a form more convenient for numerical calculations

$$\ddot{q} = \left[\frac{\partial^2 T}{\partial \dot{q}^2}\right]^{-1} \left\{Q - \frac{\partial V}{\partial q} + \frac{\partial T}{\partial q} - \frac{\partial}{\partial q^T}(\frac{\partial T}{\partial \dot{q}})\dot{q}\right\} \tag{7}$$

The total kinetic energy of system T is the sum of the kinetic energy T_1 of translation motion of mass centres of solids in the system, and the kinetic energy T_2 of rotary motion about the instantaneous rotation axes passing through the centres of masses in the system, that is $T = T_1 + T_2$. The energy of translation motion has the form

$$T_1 = \tfrac{1}{2}\sum_{i=1}^{5} m_i v_{i0}^T v_{i0}$$

where m_i, v_{i0} $(i = 1,...5)$ – mass and velocity of translation motion of mass centre of i^{th} solid.
In equation (7), the derivatives of this part of kinetic energy, expressed by the derivatives of translation motion velocity, are equal to:

$$\frac{\partial}{\partial q_l}(\frac{\partial T_1}{\partial \dot{q}_k})\dot{q}_l = \sum_{i=1}^{5} m_i \left[\frac{\partial v_{i0}^T}{\partial q_l}\frac{\partial v_{io}}{\partial \dot{q}_k} + v_{i0}^T \frac{\partial^2 v_{i0}}{\partial q_l \partial \dot{q}_k}\right] \tag{8}$$

$$\frac{\partial T_1}{\partial q_k} = \sum_{i=1}^{5} m_i v_{i0}^T \frac{\partial v_{i0}}{\partial q_k}, \quad \frac{\partial^2 T_1}{\partial \dot{q}_l \partial \dot{q}_k} = \sum_{i=1}^{5} m_i \frac{\partial v_{i0}^T}{\partial \dot{q}_l}\frac{\partial v_{i0}}{\partial \dot{q}_k} \tag{9}$$

The velocities of translation motion of mass centres can be expressed as functions of velocity of the undercarriage mass centre and time derivatives of rotation matrices of the appropriate local systems:

$$v_{i0} = \dot{r}_{10}^{(0)} = [\dot{x}_0, \dot{y}_0, \dot{z}_0]^T \tag{10}$$

$$v_{i0} = \dot{r}_{i0}^{(0)} = v_{(i-1)0} + \dot{\Phi}_{1(i+1)} r_{(i-1)2}^{(i-1)} - \dot{\Phi}_{1(i+2)} r_{i1}^{i}, \quad (i = 2,...,5) \tag{11}$$

In expression (11), only the rotation matrices depend on the co-ordinates and the generalised velocities. Then, determination of the velocity derivatives (10) and (11) that appear in equations (8) and (9) can be reduced to differentiation of the rotation matrix.
The energy of rotary motion of the considered system has the form

$$T_2 = \frac{1}{2} \sum_{i=1}^{5} \omega_i^T I_i \omega_i$$

where $\omega_i = [\omega_{i\xi} \quad \omega_{i\eta} \quad \omega_{i\zeta}]^T$ – angular velocity of i^{th} solid in local co-ordinate system, and

$$I_i = \begin{bmatrix} I_{i\xi} & -I_{i\xi\eta} & -I_{i\xi\zeta} \\ -I_{i\xi\eta} & I_{i\eta} & -I_{i\eta\zeta} \\ -I_{i\xi\zeta} & -I_{i\eta\zeta} & I_{i\zeta} \end{bmatrix}$$

the matrix of mass moments of inertia of i^{th} solid (invariable during the motion).

The appropriate derivatives of kinetic energy of rotary motion are equal to:

$$\frac{\partial}{\partial q_i} \frac{\partial T_2}{\partial \dot{q}_k} = \sum_{i=1}^{5} \left(\frac{\partial \omega_i^T}{\partial q_i} I_i \frac{\partial \omega_i}{\partial \dot{q}_k} + \omega_i^T I_i \frac{\partial^2 \omega_i}{\partial q_i \partial \dot{q}_k} \right) \tag{12}$$

$$\frac{\partial T_2}{\partial q_k} = \sum_{i=1}^{5} \omega_i^T I_i \frac{\partial \omega_i}{\partial q_k}, \quad \frac{\partial^2 T_2}{\partial \dot{q}_i \partial \dot{p}_k} = \sum_{i=1}^{5} \frac{\partial \omega_i^T}{\partial \dot{q}_i} I_i \frac{\partial \omega_i}{\partial \dot{q}_k} \tag{13}$$

The association between angular velocities of the solids in their local systems and the rotation matrices is defined by the relations:

$$\omega_1 = \omega_{30} + \Phi_3^T (\omega_{20} + \Phi_2^T \omega_{10}) \tag{14}$$

$$\omega_i = \omega_{(i+2)0} + \Phi_{i+2}{}^T \omega_{i-1} \quad (i = 2,...,5) \tag{15}$$

where

$$\omega_{10} = [\dot{\psi}_1 \ 0 \ 0]^T, \ \omega_{20} = [0 \ \dot{\psi}_2 \ 0]^T, \ \omega_{30} = [0 \ 0 \ \dot{\psi}_2]^T, \ \omega_{40} = [0 \ 0 \ \dot{\theta}]^T,$$
$$\omega_{50} = [\dot{\alpha} \ 0 \ 0]^T, \ \omega_{60} = [\dot{\beta} \ 0 \ 0]^T, \ \omega_{70} = [\dot{\gamma} \ 0 \ 0]^T.$$

Similarly as in the case of kinetic energy of translational motion of mass centres, determination of the angular velocity derivatives with respect to the generalised co-ordinates with generalised velocities consists in differentiation of the appropriate rotation matrices.

Potential energy of the system has the form

$$V = g\sum_{i=1}^{5} m_i z_{i0}$$

where: g – acceleration of gravity, $z_{i0} = r_{i0z}^{(0)}$ – co-ordinates z of mass centres in the global system.
The derivative of energy with respect to the vector of generalised co-ordinates equals

$$\frac{\partial V}{\partial q_k} = g\sum_{i=1}^{5} m_i \frac{\partial z_{i0}}{\partial q_k} \tag{16}$$

The derivatives of these co-ordinates, appearing in equation (2), are the third components of the derivatives of vectors of mass centre positions with respect to generalised co-ordinates.

$$\frac{\partial r_{10}^{(0)}}{\partial q^T} = \begin{bmatrix} 0 & . & . & 0 & 1 & 0 & 0 \\ 0 & . & . & 0 & 0 & 1 & 0 \\ 0 & . & . & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\frac{\partial r_{i0}^{(0)}}{\partial q_k} = \frac{\partial r_{(i-1)0}^{(0)}}{\partial q_k} + \frac{\partial \Phi_{1(i+1)}}{\partial q_k} r_{(i-1)2}^{(i-1)} - \frac{\partial \Phi_{1(i+2)}}{\partial q_k} \mathrm{r}_{j1}^{(i)} \quad (i = 2,\dots,5)$$

The generalised forces Q that appear in Lagrange's equation (7) result from the action of forces in hydraulic cylinders, the reaction forces of undercarriage outriggers, and the reaction force of the ground at the cutting edge of the bucket. The components of vector Q are the coefficients in the expressions for virtual work of the above forces on virtual translations of the generalised co-ordinates. The virtual work of these forces equals

$$L = P_1^2\delta r_{23}^{(0)} + P_1^3\delta r_{33}^{(0)} + P_2^3\delta r_{34}^{(0)} + P_2^4\delta r_{44}^{(0)} + P_3^4\delta r_{45}^{(0)} + P_3^6\delta r_{63}^{(0)} + \\ + \sum_{j=1}^{4} R_j\delta r_{1j}^{(0)} + R_5\delta r_{52}^{(0)} \tag{17}$$

where: P_n^i (n = 1,2,3, i = 2,3,4,6) – force of n^{th} hydraulic cylinder acting on i^{th} solid, R_j – reaction force acting on j^{th} outrigger (j = 1,...4), R_5 – reaction force of the ground at the bucket edge.
In order to determine the generalised forces, it is necessary to express the virtual translations by variations of the generalised co-ordinates:

$$\delta r_{23}^{(0)} = \sum_{k=1}^{10}\left[\frac{\partial r_{12}^{(0)}}{\partial q_k} + \frac{\partial \Phi_{14}}{\partial q_k}(r_{23}^{(2)} - r_{21}^{(2)})\right]\delta \dot{q}_k,\quad \delta r_{33}^{(0)} = \sum_{k=1}^{10}\left[\frac{\partial r_{22}^{(0)}}{\partial q_k} + \frac{\partial \Phi_{15}}{\partial q_k}(r_{33}^{(3)} - r_{31}^{(3)})\right]\delta q_k$$

$$\delta r_{34}^{(0)} = \sum_{k=1}^{10}\left[\frac{\partial r_{22}^{(0)}}{\partial q_k} + \frac{\partial \Phi_{15}}{\partial q_k}(r_{34}^{(3)} - r_{31}^{(3)})\right]\delta q_k,\quad \delta r_{44}^{(0)} = \sum_{k=1}^{10}\left[\frac{\partial r_{32}^{(0)}}{\partial q_k} + \frac{\partial \Phi_{16}}{\partial q_k}(r_{44}^{(4)} - r_{41}^{(4)})\right]\delta q_k$$

$$\delta r_{45}^{(0)} = \sum_{k=1}^{10}\left[\frac{\partial r_{32}^{(0)}}{\partial q_k} + \frac{\partial \Phi_{16}}{\partial q_k}(r_{45}^{(4)} - r_{41}^{(4)})\right]\delta q_k,\quad \delta r_{63}^{(0)} = \sum_{k=1}^{10}\left[\frac{\partial r_{43}^{(0)}}{\partial q_k} + \frac{\partial (\Phi_{16}\Phi_8)}{\partial q_k} r_{63}^{(6)}\right]\delta q_k. \tag{18}$$

$$\delta r_{1j}^{(0)} = \sum_{k=1}^{10}\left[\frac{\partial r_{10}^{(10)}}{\partial q_k} + \frac{\partial \Phi_{13}}{\partial q_k}(r_{1j}^{(1)}\right]\delta q_k,\ (j = 1,\dots,4),$$

$$\delta r_{52}^{(0)} = \sum_{k=1}^{10}\left[\frac{\partial r_{42}^{(0)}}{\partial q_k} + \frac{\partial \Phi_{17}}{\partial q_k}(r_{52}^{(4)} - r_{51}^{(4)})\right]\delta q_k$$

Determination of the equations of motion is completed after obtaining appropriate derivatives of the rotation matrices Φ_i and Φ_{1j} $(i = 1,\dots6)$. The consecutive steps of this operation constitute the following algorithm:

1. Determination of the first and the second derivative of rotation matrices F_i with respect to these generalised co-ordinates that the matrix depends on, that is

$$\frac{\partial \Phi_i}{\partial q_i},\ \frac{\partial^2 \Phi_i}{\partial q_i^2} \quad \text{for } i = 1,\dots,9$$

2. Determination of double derivatives of rotation matrices Φ_{1i} with respect to generalised co-ordinates and time

$$\frac{\partial \dot{\Phi}_{1i}}{\partial \dot{q}_k} = \Phi_{1(k-1)}\frac{\partial \Phi_k}{\partial q_k}\Phi_{(k+1)i} \quad \text{for } i = 2,\dots,5\,,$$

where we assume that $\Phi_{ij} = E$ for $i > j$ and $\Phi_{ii} = \Phi_i$.

The above relationship results from differentiation of matrix Φ_{1j} with respect to time

$$\dot{\Phi}_{1j} = \frac{\partial \Phi_1}{\partial q_1}\Phi_{2j}\dot{q}_1 + \dots + \Phi_{1(k-1)}\frac{\partial \Phi_k}{\partial q_k}\Phi_{(k+1)j}\dot{q}_k + \dots + \Phi_{1(j-1)}\frac{\partial_j}{\partial q_j}\dot{q}_j$$

The derivative of the above function with respect to the generalised velocity is equal to the matrix coefficient appropriate for this generalised velocity

3. Determination of time derivatives of rotation matrices Φ_{1i}

$$\dot{\Phi}_{1i} = \sum_{k=1}^{i} \frac{\partial \dot{\Phi}_{1j}}{\partial \dot{q}_k} \dot{q}_k$$

4. Determination of mixed derivatives of rotation matrices Φ_{1i} with respect to time, generalised co-ordinates and velocities.

$$\frac{\partial \dot{\Phi}_{1j}}{\partial \dot{q}_k} = \Phi_{1(k-1)} \frac{\partial^2 \Phi_k}{\partial q_k^2} \Phi_{(k+1)i}, \quad \frac{\partial \dot{\Phi}_{1j}}{\partial \dot{q}_i \partial q_k} = \Phi_{1(l-1)} \frac{\partial \Phi_l}{\partial q_l} \Phi_{(l+1)(k-1)} \frac{\partial \Phi_k}{\partial q_k} \Phi_{(k+1)i} \quad \text{for } l<k$$

$$\frac{\partial \dot{\Phi}_{1i}}{\partial \dot{q}_i \partial q_k} = \frac{\partial \dot{\Phi}_{1i}}{\partial \dot{q}_k \partial q_l} \quad \text{for } l>k$$

Justification of the above relationships is similar to that in the step 2 of the algorithm.

5. Determination of mixed derivatives of rotation matrices Φ_{1i} with respect to time and generalised co-ordinates.

$$\frac{\partial \dot{\Phi}_{1i}}{\partial q_k} = \sum_{k=1}^{i} \frac{\partial \Phi_{1j}}{\partial \dot{q}_l \partial q_k} \dot{q}_l$$

It must be noticed that the first step of the algorithm consists only in simple, analytical differentiation of the matrices. The remaining steps can be reduced to multiplication of matrices, which can be easily performed numerically. The full algorithm of determining motion equations of an excavator consists of the following steps:

1. For the given geometrical parameters of the excavator, determine the positions of the characteristic points in the generalised co-ordinates, according to formulae (1).
2. For the given generalised co-ordinates, determine the rotation matrices Φ_i and Φ_{ij}, $(i,j = 1,...7)$ according to formulae (2).
3. Following the steps of the previously described algorithm, determine the derivatives of the rotation matrices.
4. Determine the velocities of translational motion of mass centres using formulae (10) and (11), and the velocities of rotary motion of the solids by means of formulae (14) and (15).
5. Determine the derivatives of kinetic energy of translational and rotary motion from the derived formulae (10), (11), (14) and (15).

6. Determine the derivatives of kinetic energy of translational and rotary motion from the formulae (8), (9), (11) and (13).
7. Determine the derivatives of potential energy with respect to the generalised co-ordinates using formula (16).
8. Determine the generalised forces as the coefficients of appropriate virtual translations by substituting expression (18) into equation (17).

Finally, substitute the obtained expressions into equation (7)

5. Conclusion

The procedure of analytical (or symbolic) determination of motion equations in the Lagrange's form is associated with multiple derivations of the derivatives of kinetic and potential energy functions. Nowadays, when computational power of contemporary computers increases dramatically, this method might be effective even in the cases of systems having a great number of degrees of freedom. However, the proposed algorithm seems conceptually more simple, and is directly oriented at computer implementation. The presented concept of constructing the motion equations makes it possible to significantly simplify the process of determining the equations of motion, and still preserves maximum accuracy of calculations. This is due to the fact that the application of the generalised co-ordinates guarantees permanent fulfilment of constrains conditions. In the proposed algorithm, the main computational effort is associated with determination of the derivatives of rotation matrices in the local systems. This task, however, can be accomplished by a simple multiplication of appropriate matrices.

The presented method of constructing an excavator's equations of motion has a general character, and can be well applied to a variety of similar types of mechanical systems.

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Algorytm budowy równań ruchu układu dynamicznego koparki jednonaczyniowej

Streszczenie

W pracy sformułowano algorytm budowy równań ruchu modelu koparki jednonaczyniowej podsiębiernej w zmiennych uogólnionych Lagrange'a. W modelu przyjmuje się, że koparka jest układem brył sztywnych połączonych więzami obrotowymi o dziesięciu stopniach swobody. Istota algorytmu polega na sprowadzeniu procedury budowy równań ruchu układu do procesu mnożenia odpowiednich macierzy bez konieczności analitycznego lub numerycznego wyznaczania kolejnych pochodnych energii kinetycznej i potencjalnej układu względem współrzędnych i prędkości uogólnionych. Tak sformułowany algorytm może stanowić podstawę budowy programu numerycznego do analizy dynamiki układu koparki. Przedstawiona metoda generacji równań Lagrange'a może być uogólniona i stosowana do szerszej klasy układów wieloczłonowych.