THE ARCHIVE OF MECHANICAL ENGINEERING

Vol. LI 2004 Number 1

Key words: *pump efficiency, energy-consumption*

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DETERMINATION OF A GENERALISED FORM OF THE LOWER SLOPE REGION OF THE CENTRIFUGAL PUMP EFFICIENCY CHARACTERISTIC CURVE

The article presents criterion of evaluating energy consumption of water pumping. The lower slope of pump efficiency characteristic curve was plotted for the case of: constant gain of rotational speed and variable gain of rotational speed. The unified form of the lower slope of pump efficiency characteristic curve was defined for variable head. The right-side coefficient of pump efficiency correction was defined.

ESSENTIAL NOMENCLATURE

e	– unit energy consumption of pumping unit,
H	– head of pump $[m]$,
H_n	– rated head of pump,
H_{zn}	– geometric head of inflow,
$h^* = \frac{H}{H_n}$	– relative head of pump,
$h^*_{zn} = \frac{H_{zn}}{H_n}$	– relative head of inflow in suction collector,
m	– number of working pumping sets,
n	– rotational speed $[obr/\text{min}]$,
n_n	– rated rotational speed,
$n^* = \frac{n}{n_n}$	– relative rotational speed,

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n^*_{opt} – optimum pump rotational speed where optimum pump efficiency occurs,

$n_q = \frac{n}{60}\frac{\sqrt{Q}}{H^{3/4}}$ – specific speed,

Q – capacity of pumping set [m^3/h],

Q_n – rated capacity of pumping set,

$q^* = \frac{Q}{Q_n}$ – relative capacity of pumping set,

η – pump capacity,

η_f – efficiency of frequency converter,

η_g – efficiency of pumping unit,

$\eta_{\max}$ – maximum pump efficiency for rated head of pump,

η_o – approximating function of the lower slope of pump efficiency

η_{opt} – optimum pump efficiency, assuming maximum value for any head of pump

η_s – efficiency of induction motor,

η_z – efficiency of pumping set,

$\eta^* = \frac{\eta}{\eta_{max}}$ – relative pump efficiency.

1. Introduction

Application of modern power electronic elements in drives of pumping sets allows for development of measures to rationalise energy consumption [13]. Consumption of electricity depends on parameters of a pumping set including a pump and its electric drive. Drive of a pumping set comprises a motor and a motor powering device, that is, a frequency converter or any of other electrical devices that power the motor directly from the three-phase network.

Pumping sets that are in series or parallel connection between the suction and discharge collectors make up a pumping unit. A pumping unit consisting of one pump set is a special case. It is assumed that pumps in a pumping set are of identical rated efficiency Q_n, and identical rated head H_n. For a pumping set to operate properly, it is necessary to fit it with:

- Hydraulic fixtures including check and shut-off valves
- Control and measurement elements including pressure converters, water-meters, manometers
- Control and safety components

Unit energy consumption e of a pumping unit in the process of pumping 1 m^3 water is defined as follows [13]:

$$e = \frac{1}{367}\frac{H}{Q}\sum_{i=1}^{m}\frac{Q_i}{\eta_{zi}}\left[\frac{kWh}{m^3}\right] \tag{1}$$

where:

$$\eta_z = \eta\eta_s\eta_f \tag{2}$$

$$Q = \sum_{i=1}^{m} Q_i \tag{3}$$

Overall efficiency of a pumping unit η_g is specified in the following dependence:

$$\eta_g = \frac{Q}{\sum_{i=1}^{m}\frac{Q_i}{\eta_{zi}}} \tag{4}$$

One of the factors that, rapidly and at a minimum investment, increases efficiency η_g while reducing unit energy consumption e of a pumping unit is the choice of an adequate method of controlling operation of a pumping unit.

In few pumping applications it is reasonable for the pump to function at one rotational speed. Rotational speed n is adjusted through changing frequency of the voltage feeding the induction motor. When frequency of the voltage feeding the induction motor changes within the interval $f \in (35 \div 55)$ Hz, efficiency of the motor η_s and of frequency converter η_f are subject to changes of up to 5% in relation to their maximum value [14]. In such a range of frequency f changes, it is assumed that the impact of efficiency η_s and η_f on efficiency of the pumping set η_z is negligible. Under this assumption, equation (2) indicates that, within the range of frequency $(35 \div 55)$Hz of the power feeding the pump motor, the value of unit energy consumption e is significantly dependent upon the pump efficiency η.

2. Maximum pump efficiency at constant head

The characteristic curve is the set of information concerning variation of pump head H depending on capacity Q, rotational speed n, and pump efficiency η. In general, it can be stated as follows:

$$H = f(Q, n, \eta) \tag{5}$$

Pump design differences influence planar distribution of characteristic curves (H, Q). One of the elements describing the pump properties is the specific speed n_q. Variability of electric power P_{el} in function of capacity Q [5] depends on the value of specific speed n_q. For the purposes of further analysis, such one- and two-stream, as well as multi-grade pumps were considered, whose specific speed n_q, ensure reduced consumption of electric power at declining capacities Q, i.e. $n_q \cong 20...70$..

a)

b)

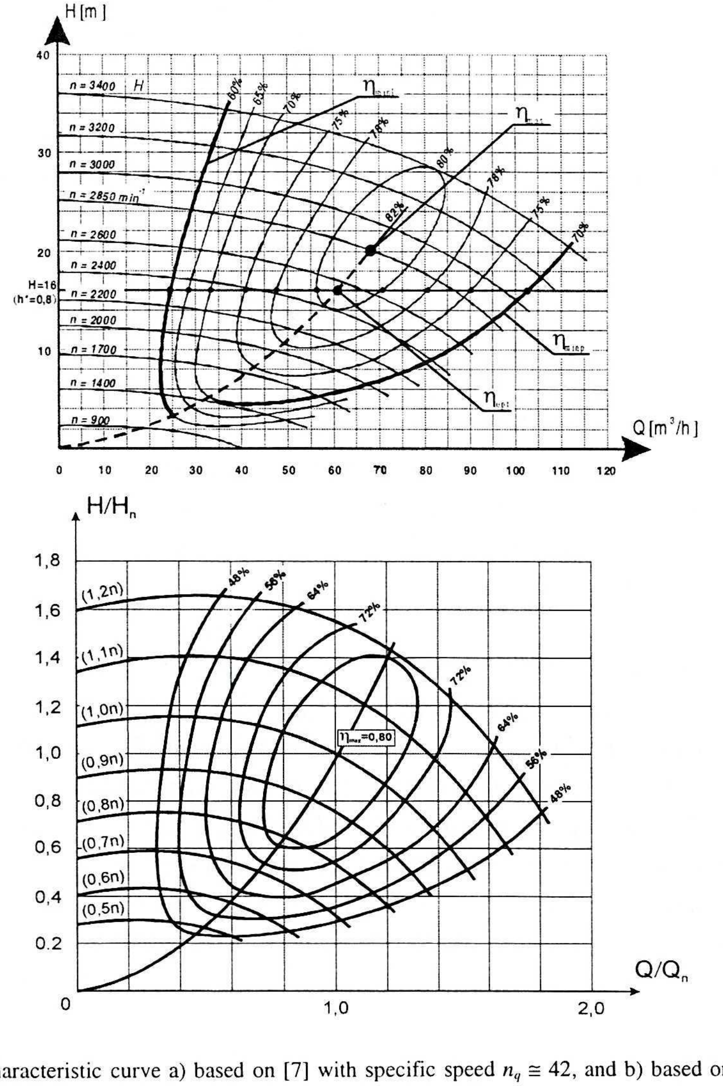


Fig. 1. Pump characteristic curve a) based on [7] with specific speed $n_q \cong 42$, and b) based on [6] with specific speed $n_q \cong 20 \div 30$

The analysis of pump energy consumption is based on characteristic curves. Sample curves are shown in Figure 1 [6], [7]. The criterion of energy consumption assumes minimising unit energy consumption e (1). Low energy consumption occurs when the pump operates in the proximity of maximum values of pump efficiency η. More detailed consideration of pump energy consumption properties requires planar distribution of pump efficiency η (Q,n). Distribution of pump efficiency depends on the method of controlling operation of a pumping set. Most pumping applications feature a method of control that provides for constant head H ($H=const$) of pump throughout the range of pump capacity Q. This control method is taken into account when analysing pump efficiency characteristic curve.

Regulation of relative pump rotational speed n^* in order to stabilise head h^* has a concomitant impact on pump efficiency η. Thus, the knowledge of pump efficiency characteristic curve η in function of rotational speed n^* and capacity q^* at $h^*=const$ becomes a major element of analysis of pumping energy consumption. Pump efficiency characteristic curves $\eta=f(n^*)$ and $\eta=f(q^*)$ at $h^*=const$ are to a degree dependent on the pump type, e.g. one-stream, two-stream, and multi-grade. A correct course of pump efficiency η is obtained on the basis of reliable characteristic curves.

Most applications assume the condition of maintaining constant pressure in the discharge collector. In the case of constant head of inflow h^*_{zn}, constant head of pump h^* obtains [14]. Constant head h^* is obtained by changing capacity q^* of one or several pumps in a set. Capacity q^* is most often changed by varying pump rotational speed n^*.

Sample characteristic curves of pump efficiency at constant heads h^*, flow q^*, and rotational speed n^*, are shown in Fig. 2. Characteristic curves of pump efficiency η were based on the characteristic curve described in [6]. The analysis assumed the range of changes of pump efficiency η within $0{,}48 \div 0{,}8$. Pump efficiency η displays slight variation for various heads h^*, and at constant flow q^* lower than the nominal flow of one (Fig. 2a). For capacity q^* below the nominal value, efficiency differences do not exceed 5%. At flow $q^* > 1$, differences of efficiency values η are significant. Variations among pump efficiencies at identical capacity q^* over nominal flow reach as much as 15% in the range of head $h^* \in (0{,}6;\ 1{,}2)$.

Pump efficiency η is highly dependent on changes of rotational speed n^* (Fig. 2b). Reducing rotational speed n^* by 0.1 while maintaining constant head h^* reduces pump efficiency by 0.3. A substantial drop in pump efficiency is counterproductive from the perspective of operating costs [3].

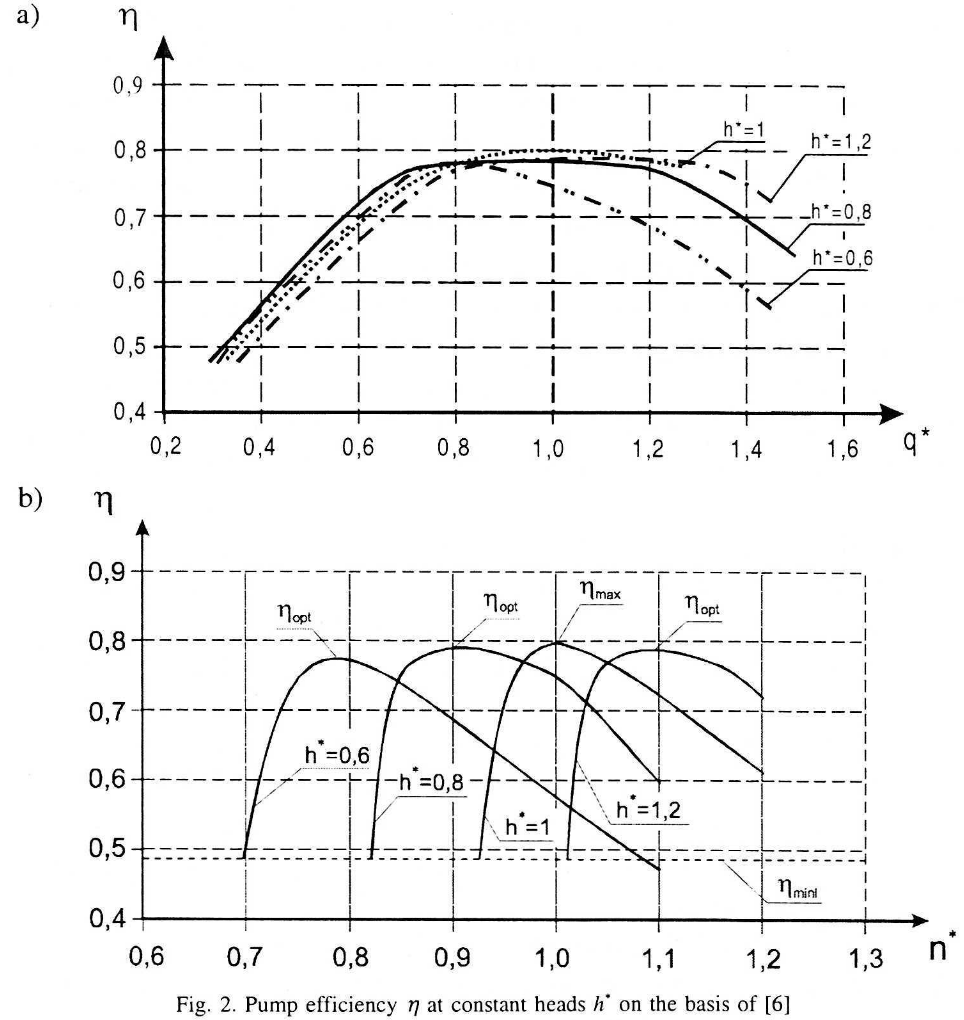


Fig. 2. Pump efficiency η at constant heads h^* on the basis of [6]

3. The lower slope of pump efficiency characteristic curve

Pump efficiency courses in function of rotational speed n^* were analysed on the basis of available characteristic curves for different type pumps. Pump efficiency η was determined at constant head h^*. On the basis of available source materials, the range of pump head changes within the interval $h^* \in (0{,}6 \div 1)$ was assumed.

In order to fully analyse changes of pump efficiency η, the efficiency characteristic curve was divided into two slopes: upper and lower one. Both slopes have their domains delineated by boundary values of rotational speed n^* and the corresponding boundary pump efficiencies η. Boundary values of efficiency for the lower slope are defined by:

- left-side efficiency that corresponds to optimum pump efficiencies η_{opt} situated on a common similarity parabola
- right-side efficiency that corresponds to the curve for minimum efficiency η_{minp} in the region of capacity Q higher than capacities at optimum efficiency η_{opt} (Fig.1).

The optimum efficiency η_{opt}, determined for nominal speed is defined as the maximum efficiency η_{max}. Maximum pump efficiencies η_{max}, assume their values within the range 20% – 90% depending on the pump's nominal power. Large variation among maximum values of pump efficiency make comparative analysis of diverse pumps more difficult. The pump efficiency characteristic curve is further analysed by introducing the notion of relative pump efficiency η^* as the quotient of pump efficiency η and maximum efficiency η_{max}:

$$\eta^* = \frac{\eta}{\eta_{\max}} \tag{6}$$

By analogy to the general definition of relative pump efficiency, the concepts of relative minimum efficiency $\eta^*_{\min p}$ and relative optimum efficiency η^*_{opt} were introduced for the lower slope of pump efficiency characteristic curve and defined as follows:

$$\eta^*_{\min p} = \frac{\eta_{\min p}}{\eta_{\max}} \tag{7}$$

$$\eta^*_{opt} = \frac{\eta_{opt}}{\eta_{\max}} \tag{8}$$

Assuming any value of the head, e.g. $h^* = 0{,}8$ (Fig.1.b), we can plot one characteristic curve of pump efficiency η^* in function of rotational speed n^*. Plotting of efficiency characteristic curve consists in forming point pairs (η^*, n^*) included in the line that corresponds to the head of pump $h^* = 0{,}8$ and intersects with the curves of constant efficiency value. In the characteristic curve, the intersection points of the line $h^* = 0{,}8$ and of the curve corresponding to any constant value of pump efficiency η^*_i make up a matrix of points (η^*_i; n^*_i) that belong to the characteristic curve of pump efficiency η^*. The matrix of points (η^*_i; n^*_i) is defined in Table 1. Boundary values of pump efficiency correspond to the following rotational speeds:

a) optimum rotational speed n^*_{opt} corresponds to pump operation at optimum efficiency η^*_{opt},

b) maximum rotational speed n^*_{max} corresponds to pump operation at minimum efficiency $\eta^*_{\min p}$.

Table 1.

Pump efficiency η^* and its approximating curve η^*_o, defined for diverse frames of reference, assuming $h^* = const$ – lower slope of the pump efficiency characteristic curve

Frame of reference	η^*, η^*_o	η^*_{opt}	η^*_2	....	$\eta^*_{\min p}$
$(N; \eta^*)$, $(N; \eta^*_o)$	n^*	n^*_{opt}	n^*_2	....	$n^*_{\min p}$
$(X; \eta^*)$, $(X; \eta^*_o)$	x	1	2	....	x_{max}
$(Z; \eta^*)$, $(Z; \eta^*_o)$	z	z_1	z_2	....	z_{max}

The number of point pairs (η^*, n^*) varies for each pump type depending on available source material. The set of point pairs (η^*, n^*) plots a non-continuous course of the efficiency characteristic curve η^*. Points of efficiency characteristic curve η^* define the function $\eta^*_o = f_1(x) = f_2(z) = f_3(n^*)$ which approximates the course of efficiency η^*. The function η^*_o, approximating the lower slope of the efficiency characteristic curve η^*, intersects three interpolation centres common to the curve η^* and η^*_o. It is assumed to be the solution of Lagrange's interpolation polynomial (14). The efficiency characteristic curves η^* and η^*_o are determined in three different frames of reference (Fig. 4):

1. The first domain, X, belongs to the set of natural numbers. Element x_i of the domain X is the consecutive number of point (η^*, n^*) determined on the basis of characteristic curve (Fig.1) for variation of rotational speed by a constant value δn^* (table 1). Element $x_1 = 1$ thus corresponds to the first point of the pump efficiency characteristic curve (Fig.4). The subsequent points x_i are defined in the following dependence:

$$x_i = x_1 + \frac{n^*_i - n^*_{opt}}{\delta n^*_p} \quad \text{where } i = 1,2,3,...,max \tag{9}$$

2. The second domain, Z, belongs to the set of natural numbers. Element z_i of the domain Z determines the consecutive number of points (η^*, n^*) calculated on the basis of rotational speed $n^* = 0$. Where the rotational speed $n^* = 0$, element $z_i = 0$. The subsequent point numbers (η^*, n^*) equal multiples of speed changes δn^*. The set of elements $z \in (z_1; z_{max})$ defines the domain of the pump efficiency characteristic curve (Fig. 4) where the pump should operate owing to low energy consumption. Elements of the domain Z fit the following dependence:

$$z_i = \frac{n_i^*}{\delta n^*} \quad \text{where } i \in C \tag{10}$$

where:

C – set of integers,

z_i – consecutive values of computational steps in the domain Z. The value of index $i=0$ defines location of Y-axis of the frame (X, η_o^*). Negative values of parameter i determine the negative portion of the domain X.

3. Domain N of rotational speed assumes values in the set of real numbers R. The origin of $0N$ and $0Z$ axes is common. The set of elements of rotational speed $n^* \in (n^*_{\text{nim}\,l}; n^*_{opt})$ determines the domain of the pump efficiency characteristic curve, where the pump ought to operate due to low energy consumption.

Domains X and Z of the function η_o^* are closely related to the domain of rotational speed N. The initial values x_1 and z_1 are related to the boundary value of rotational speed n^*_{opt}. The consecutive points x_i and z_i relate to the values of rotational speed n_i that meet the following condition:

$$n_i^* = n^*_{opt} + (i-1)\,\delta n_p^* \quad \text{where } i = 1,2,3,...,max \tag{11}$$

where:

δn_p^* – increment of relative value of rotational speed.

A sample analysis of the lower slope of the characteristic curve of pump efficiency η^* was based on the characteristic curve of a one-grade, one-stream pump type 10A25A (made by the Warsaw Pump and Fixtures Company, $P_n = 55$kW, $n_n = 2950$ rpm). The head $h^* = 0{,}8$ was assumed. The characteristic curve of pump efficiency is plotted through distribution of the set of the pump efficiency point pairs η^* and of the corresponding rotational speed n^*. The matrix of elements (η^*, n^*) forms the basis for defining changes of pump efficiency $1\eta^*$ in function of rotational speed n^* and in the domain X (Fig.3). The pump efficiency η^* is approximated with the curve η_o^*. Absolute error relating to deviation of the approximating curve η_o^* from the actual pump efficiency characteristic curve η^* does not exceed 2%. Similar results were obtained for all available changes of the pump efficiency on the basis of characteristic curves. The approximation shown in Figure 3 results in the approximating function η_o^* defined as follows:

$$\eta_o^* = -\,0{,}0006x^2 + 0{,}0023x + 0{,}9956 \tag{12}$$

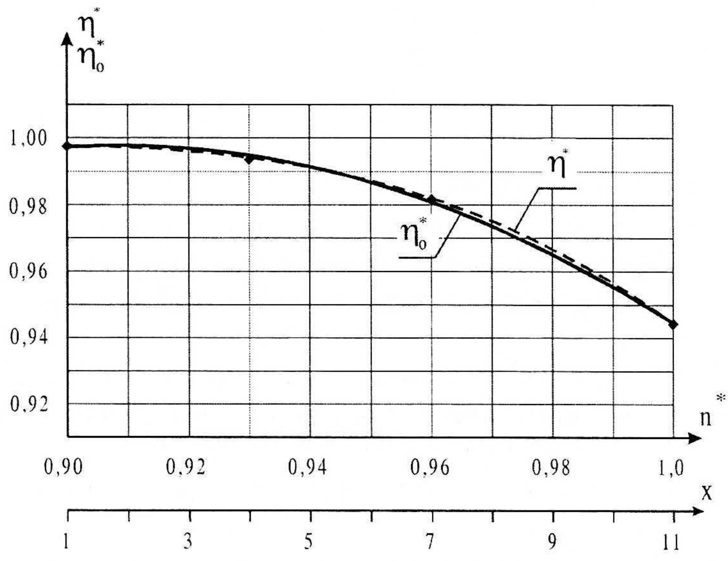


Fig. 3. The lower slope of the characteristic curve of efficiency η^* of the pump 10A25A ($n_q \cong 38$), and the approximating curve η^*_o in function of rotational speed n^* and of argument of the domain X, at $h^* = 0{,}8$

Approximation of pump efficiency η^*_o becomes a quadratic function regardless of what characteristic curves for diverse type pumps are applied. The generalised dependence of the curve η^*_o approximating the lower slope of the pump efficiency is thus defined as follows:

$$\eta^*_o = ax^2 + bx + c \tag{13}$$

To precisely determine the value of three factors a, b, c, one needs to determine three points in the characteristic curve that are located in the line defining identical head h^*, called interpolation centres. From the analytical perspective, any interpolation centres can be selected. The first centre is assumed to correspond to the optimum efficiency η^*_{opt}, and the third centre – to the minimum efficiency $\eta^*_{\min p}$. A random second interpolation centre can be chosen that belongs in the range of efficiency (η^*_{opt}; $\eta^*_{\min p}$). The interpolation centres feature appropriate values of elements x_i in the domain X:

x_1 – argument of the approximating function η^*_o for the first interpolation centre,

x_2 – argument of the approximating function η^*_o for the second interpolation centre,

x_{max} – argument of the approximating function η^*_o for the third interpolation centre.

The accepted interpolation centres are defined by the following point pairs $(\eta^*_{opt}; x_1)$, $(\eta^*_{o2}; x_2)$, $(\eta^*_{\min p}; x_{\max})$. The three interpolation centres are components of Lagrange's interpolation polynomial, defined as follows:

$$\eta^*_o = \sum_{i=1}^{n} f(x_i) \prod_{\substack{j=1 \\ j \neq 1}}^{n} \frac{x - x_j}{x_i - x_j} \tag{14}$$

where:

x_i, x_j – value of the argument of function $f(x)$,

$f(x_i)$ – value of function f(x) in the point x_i.

Developing Lagrange's polynomial for $n = 3$, and according to the equation (13), we can define the parameters a, b, c that determine the approximating function are defined as follows:

$$\left.\begin{aligned} a &= \frac{(\eta^*_{opt} - \eta^*_{min\,p})(x_1 - x_2) - (\eta^*_{opt} - \eta^*_{o2})(x_1 - x_{max})}{(x_2 - x_1)(x_{max} - x_2)(x_{max} - x_1)} \\ b &= \frac{x_1^2(\eta^*_{opt} - \eta^*_{o2}) + x_2^2(\eta^*_{opt} - \eta^*_{min\,p}) + x_3^2(\eta^*_{o2} - \eta^*_{opt})}{(x_2 - x_1)(x_3 - x_2)(x_3 - x_1)} \\ c &= \frac{x_2^2(x_1\eta^*_{minp} - x_3\eta^*_{minp}) + x_1^2(x_3\eta^*_{o2} - x_2\eta^*_{min\,p}) + x_3^2(x_2\eta^*_{opt} - x_1\eta^*_{o2})}{(x_2 - x_1)(x_3 - x_2)(x_3 - x_1)} \end{aligned}\right\} \tag{15}$$

Transformations of the dependence (15) result in a final equation that defines the following parameters a, b, c:

$$\left.\begin{aligned} a &= \frac{(\eta^*_{opt} - \eta^*_{min\,p})(x_1 - x_2) - (\eta^*_{opt} - \eta^*_{o2})(x_1 - x_3)}{(x_2 - x_1)(x_{max} - x_2)(x_{max} - x_1)} \\ b &= \frac{\eta^*_{opt} - \eta^*_{min\,p}}{x_1 - x_{max}} - a(x_1 + x_{max}) \\ c &= \eta^*_{opt} - ax_1^2 - bx_1 \end{aligned}\right\} \tag{16}$$

The characteristic curve of the approximated efficiency η^*_o is plotted in the domain X. The domain X determines the subsequent numbering of point pairs $(\eta^*_i; n^*_i)$, that is, $x_i = i = 1, 2, 3, \ldots, x_{max}$, which belong to the range $x \in (x_1, x_{max})$. The maximum value of parameter x_{max} depends on the maximum value of rotational speed $n^*_{\max}$, and on the increase of rotational speed δn^*_p in the following way:

$$x_{max} = \frac{n^*_{max} - n^*_{opt}}{\delta n^*_p} + 1 \quad \text{where } x_{max} \in N \tag{17}$$

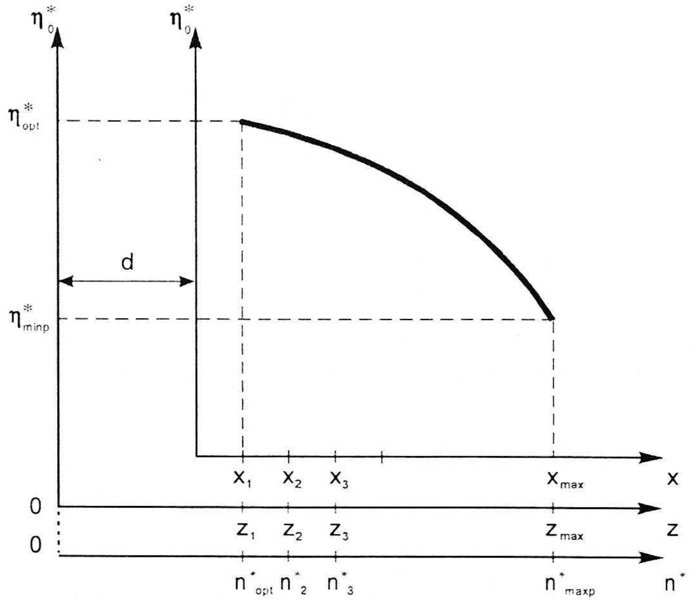


Fig. 4. The curve approximating the lower slope of the pump efficiency characteristic curve, shown in three frames of reference: *X*, *Z*, and *N*

Minimum value of parameter $x_1 = 1$ obtains for a point in the pump efficiency characteristic curve which is determined by the optimum value of rotational speed n^*_{opt}. Rotational speed n^*_{opt} obtained for the optimum pump efficiency η^*_{opt} is variable and depends on head h^* of water. As a result, the origin of the reference system $(X;\ \eta^*_o)$ is moved by the value d (Fig. 4) in relation to the frame of reference $(N;\ \eta^*_o)$ and $(Z;\ \eta^*_o)$. The length of section d is defined in the following dependence:

$$d = \frac{n^*_{opt}}{\delta n^*_p} - 1 = z_1 - 1 \tag{18}$$

With regard to unambiguous courses, it becomes essential to define parameters a, b and c in equation (13). Generalised determination of parameters a, b and c is possible when the curve approximating the efficiency η^*_o is reduced from the frame of reference $(X;\ \eta^*_o)$ to the frame of reference $(Z;\ \eta^*_o)$, and then to $(N;\ \eta^*_o)$. A change of domain of the curve approximating the efficiency η^*_o reduces the analysis of the efficiency characteristic curve to actual conditions as expressed by the rotational speed of pump n^*. The transfer of argument X in relation to the frame of reference $(Z;\ \eta_o^*)$ by the value d determines the following dependence between the arguments of function η_o^* included in domains X and Z:

$$x = z - z_1 + 1 \tag{19}$$

The origins of frames of reference $(Z;\ \eta_o^*)$ and $(N;\ \eta_o^*)$ are common. Transformation of domain $X \rightarrow Z$ (Fab.4) changes the argument of quadratic function (13) to what follows:

$$\eta_o^* = a(z - z_1 + 1)^2 + b(z - z_1 + 1) + c \quad \text{where } z \geq z_1 \tag{20}$$

Elements of the domain Z of the approximating function η_o^* belong to the set of natural numbers, range $z \in (z_1;\ z_{max})$. Values of elements z vary depending on rotational speed n^* and on speed gain δn_p^* (10). The range of rotational speed changes n^* depends on head h^* and is within the range $n^* \in (n_{opt}^*;\ n_{max}^*)$. The values of argument of the approximating function η_o^* for points being interpolation centres are as follows:

$z_1 = \dfrac{n_{opt}^*}{\delta n_p^*}$ – number of computational steps in the range of rotational speed $n^* \in (0; n_{opt}^*)$,

$z_2 = \dfrac{n_2^*}{\delta n_p^*}$ – number of computational steps in the range of rotational speed $n^* \in (0; n_2^*)$,

$z_{max} = \dfrac{n_{max}^*}{\delta n_p^*}$ – number of computational steps in the range of rotational speed $n^* \in (0; n_{max}^*)$.

The change of domain $X \rightarrow Z$ of the approximating function η_o^* defines values of argument x for interpolation centres in the following way:

$$\left.\begin{aligned} x_1 &= 1 \\ x_2 &= z_2 - z_1 + 1 \\ x_3 &= z_{max} - z_1 + 1 \end{aligned}\right\} \tag{21}$$

Finally, parameters a, b, c (15) of the approximating function η_o^* assume the following form in the domain Z:

$$\left.\begin{aligned} a &= \frac{(\eta^*_{opt} - \eta^*_{min\,p})(z_1 - z_2) - (\eta^*_{opt} - \eta^*_{o2})(z_1 - z_{max})}{(z_2 - z_1)(z_{max} - z_2)(z_{max} - z_1)} \\ b &= \frac{\eta^*_{opt} - \eta^*_{min\,p}}{z_1 - z_{max}} - a(z_{max} - z_1 + 2) \\ c &= \eta^*_{opt} - a - b \end{aligned}\right\} \qquad (22)$$

Parameters a, b and c, resulting from equation (22), display constant values at constant head of pump $h^* = const$.

3.1. Unified form of the lower slope of the efficiency characteristic curve

Within the range of pump rotational speed $(n^*_{opt}; n^*_{max})$, the number of points x depends on increments of rotational speed δn^*_p, which assumes the value x_{max}. At diverse heads h^*, variable ranges of rotational speed $(n^*_{opt}; n^*_{max})$ are obtained. As a result, each head h^* corresponds to a variable number of elements in the set of domain $X \in (x_1; x_{max})$.

It is assumed that the maximum number of consecutive points x_{max} is identical for different ranges of rotational speed $(n^*_{opt}; n^*_{max})$ of the pump under examination. The number of points x_{max} can be random, finite, and higher than 10. The assumption is correct within the range of the following pump efficiency changes $(\Delta\eta^*)_{max\,p}$:

$$(\Delta\eta^*)_{max\,p} = \eta^*_{opt} - \eta^*_{min\,p} \quad \text{where } h^* = const \qquad (23)$$

The boundary values of efficiency η^* correspond to boundary values of rotational speed within the range $n^* \in (n^*_{opt}; n^*_{max})$. For the family k of the pump efficiency η^* characteristic curves at varied heads h^*, the following equation, defining the maximum rotational speed n^*_{max} is obtained:

$$n^*_{max}(h^*) = n^*_{opt}(h^*) + (x_{max} - 1)\delta n^*_{hp} \quad \text{where } h^* \in (0{,}6; 1) \qquad (24)$$

with:

δn^*_{hp} – gain in pump rotational speed assuming constant value at one head h^* (Fig.7).

For each head h^*, the pump operates within the assumed maximum range of efficiency changes $(\Delta\eta^*)_{max\,p}$. Maximum efficiency changes $(\Delta\eta^*)_{max\,p}$ correspond to the following range of maximum changes of rotational speed $\Delta n^*_{max\,p}$:

$$\Delta n^*_{max}(h^*) = n^*_{max}(h^*) - n^*_{opt}(h^*) \quad \text{where } h^* \in (0{,}6;\ 1) \tag{25}$$

For identical values of the parameter x_{max}, defined at various heads h^*, variable values of speed gain δn^*_{hp} are determined according to the equation:

$$\delta n^*_{hp} = \frac{\Delta n^*_{max\,p}(h^*)}{x_{max} - 1} \tag{26}$$

Maintaining the condition of the constant parameter x_{max}, we analysed the characteristic curve of pump efficiency η^* for three heads $h^* = 1$, $h^* = 0{,}8$, $h = 0{,}6$. The characteristic curves of pump efficiency were plotted on the basis of the curve presented in [6]. The characteristic curves of pump efficiency η^* are approximated with the function η^*_o in the domain X. Individual dependences are described in the following equations:

$$\left.\begin{aligned} \eta^*_o &= -\,0{,}0002x^2 - 0{,}0064x + 1{,}0036 \quad \text{where } h = 1 \\ \eta^*_o &= -\,0{,}0001x^2 - 0{,}0087x + 1{,}0293 \quad \text{where } = 0{,}8 \\ \eta^*_o &= -\,0{,}0006x^2 - 0{,}0111x + 1{,}0163 \quad \text{where } h = 0{,}6 \end{aligned}\right\} \tag{27}$$

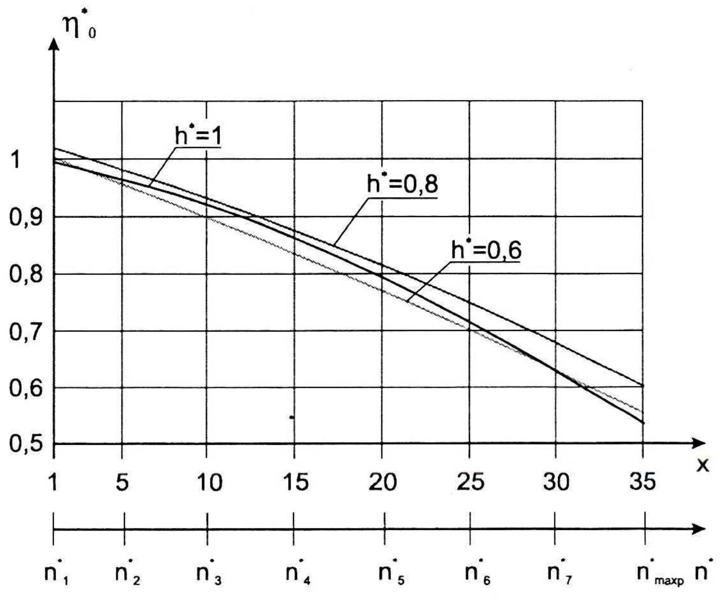


Fig. 5. Lower slope of the characteristic curve of pump efficiency η^{*o} as per [6], where $h^* = 1$, $h^* = 0{,}8$, $h^* = 0{,}6$ assuming identical values of argument x_{max}. Rotational speeds n^*_1 and n^*_{max} at diverse heads are defined in the dependence (28)

Dependences (27) are determined for different domains of rotational speed $n^* \in (n_1^*, n_{max}^*)$. The boundary values of rotational speeds n_1^* and n_{max}^* for selected heads h^* are defined in the following dependences:

$$\begin{array}{lll} n_1^* = 1 & n_{max}^* = 1{,}34 & \text{where } h^* = 1 \\ n_1^* = 0{,}9 & n_{max}^* = 1{,}24 & \text{where } h^* = 0{,}8 \\ n_1^* = 0{,}78 & n_{max}^* = 1{,}12 & \text{where } h^* = 0{,}6 \end{array} \tag{28}$$

The courses of the pump efficiency characteristic curves as functions η_o^* at various heads are illustrated in Figure 5. The differences among values of particular functions η_o^* (Fig.5) for identical argument are below 5% of the relative pump efficiency. Based on these results, we can be conclude that it is possible to determine the lower slope of the pump efficiency η^* characteristic curve for any value of head h^* on the basis of unified function defined by the dependence (13).

Parameters a, b and c in the dependence (13) are defined with equations (22) under conditions corresponding to head $h = 1$. The unified nature of the function (13) consists in the fact that, applying the same parameters a, b and c, distribution of the lower slope of the efficiency characteristic curve can be defined for the remaining heads h^* in the range (0,6 ; 1).

Moving the argument of the function defined in the dependence (13) between the domains $X \rightarrow Z$, and with regard to the equation (19), we can express the unified form of pump efficiency η_o^* as follows:

$$\eta_o^* = a(z - z_1 + 1)^2 + b(z - z_1 + 1) + c \tag{29}$$

Arguments a,b,c of the dependence (29) are defined for the gain in rotational speed $\delta n_{h=1}^*$ determined for head $h^* = 1$.

The characteristic curves of pump efficiency η_o^* in the domain Z are shown in Figure 6 for the heads $h^* = 0{,}6$, $h^* = 0{,}8$, $h = 1$. Each of the resulting efficiency characteristic curves was plotted under the identical condition of constant value of argument x_{max}. Value of argument x_{max} corresponds to the value of argument $\Delta z_{max} = z_{max} - z_1$ in the domain Z. Values of arguments x_{max} and Δz_{max} do not change throughout the range of head h^*.

One can assume any error of pump efficiency $\Delta\eta_p^*$ that would meet the condition $\Delta\eta_p^* < (\Delta\eta^*)_{max\,p}$. The assumed changes of pump efficiency $\Delta\eta_p^*$, correspond to the range of variation of argument Δz in the domain Z, and to the following change of relative rotational speed Δn_{hp}^* in the domain N:

$$\Delta n_{hp}^* = n_{oblp}^*(h) - n_{opt}^*(h) \tag{30}$$

where:

$n^*_{oblp}(h^*)$ – computational rotational speed where the pump operates at minimum computational efficiency η^*_{oblp} that meets the condition $\eta^*_{oblp} > \eta^*_{min\,p}$.

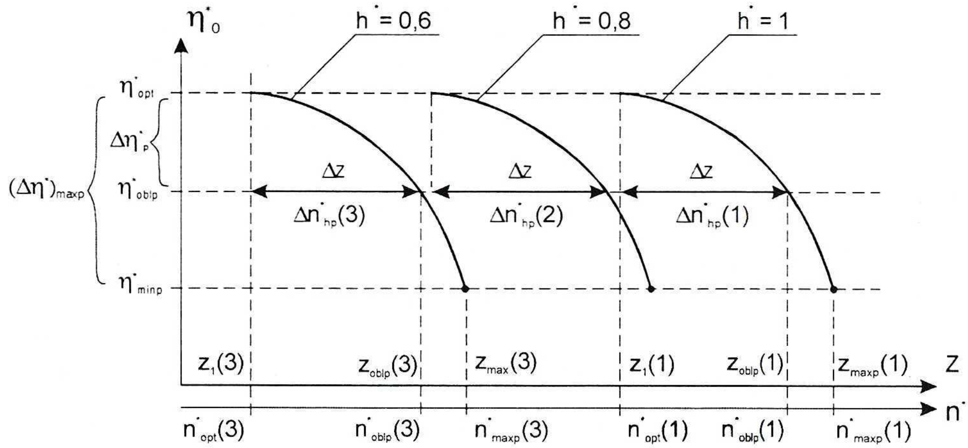


Fig. 6. Pump efficiency η^* in the domains Z and N for three random heads h^*

Fig.7 shows the field of pump operation in the range of head (0,6; 1). The curve corresponding to the minimum pump efficiency $\eta^*_{min\,p}$ is related to the last argument $z_{max}(h^*)$ in the domain Z. The curve defining the similarity parabola of optimum efficiency η^*_{opt} is related to the first argument $z_1(h^*)$.

The unified form of the function η^*_o (13) of pump efficiency determines identical range of changes of argument Δz, at various heads h^* in the field of changes of pump efficiency $\Delta\eta^*_p$ (Fig.6 and Fig.7). Fig.7 presents distribution of argument Δz of identical value at diverse heads h^*. For each head h^*, there is an argument Δz corresponding to the following change of rotational speed Δn^*_{hp}:

$$\Delta n^*_{hp} = \Delta z \cdot \delta n^*_{hp} \qquad (31)$$

Fig. 7 indicates that constant value of efficiency error $\Delta\eta^*_p$ corresponds to a constant value of argument Δz. On the other hand, changes of rotational speed Δn^*_{hp} and of gain in rotational speed δn^*_{hp} increase with growing values of head h^*. Estimating the value of variable Δz on the basis of dependence (29), one can define permissible change of the pump rotational speed Δn^*_{hp} (31) as compared to the optimum rotational speed n^*_{opt}.

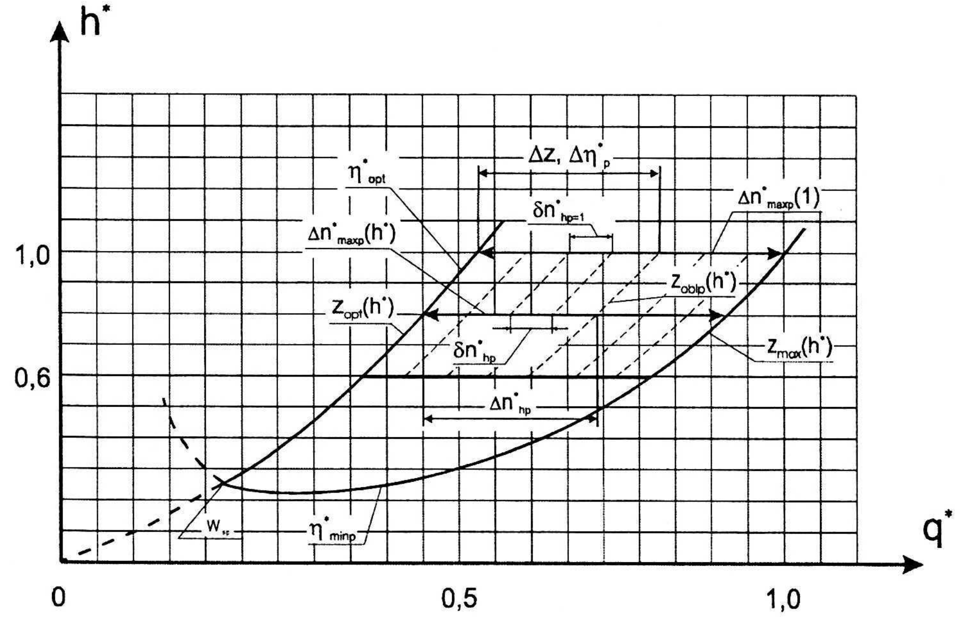


Fig. 7. Distribution of changes of rotational speed $\Delta n^*_{max\,p}$ (h^*), parameter Δz, increase of rotational speed δn^*_{hp} in the field of changes of head $h^* \in (0{,}6;\ 1)$ for the pump based on the characteristic curve, Fig. 1a

Changes of rotational speed Δn^*_{hp} form the basis for defining maximum computational rotational speed of the pump n^*_{oblp} following the equation (30). The computational speed n^*_{oblp} is assumed to correspond to the computational value of argument z_{obl}. Assuming:

$$t = z_{oblp} - z_1 + 1 = \Delta z + 1 \tag{32}$$

and according to (32), (29), one obtains the following dependence in the point of computational speed efficiency η^*_{obl}:

$$at^2 + bt + (c - \eta^*_{oblp}) = 0 \tag{33}$$

The quadratic equation (33) solves for the following zero loci $t_{1/2}$:

$$t_{1/2} = \frac{-b \pm \sqrt{b^2 - 4a(c - \eta^*_{oblp})}}{2a} \tag{34}$$

Zero loci t_1 and t_2 that constitute solutions to the equation (32) are located to the left and right of the argument t_0 which corresponds to the optimum pump efficiency η^*_{opt} (Fig.8). Therefore, the dependence (32) for the lower slope of pump efficiency characteristic curve solves for the value of argument $t_x = t_{1/2}$, which finds itself to the right of argument t_0, and is defined in the following way:

$$\left.\begin{aligned} t_x = t_1 \quad \text{where } t_1 > t_2 \\ t_x = t_2 \quad \text{where } t_2 > t_1 \end{aligned}\right\} \tag{35}$$

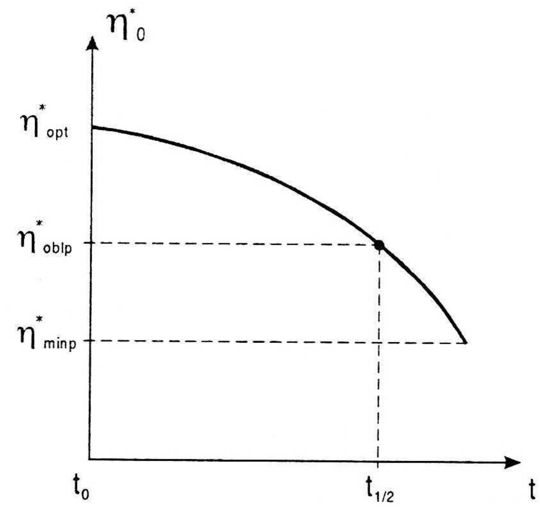


Fig. 8 One of the zero loci t_x of the lower characteristic curve corresponding to the computational efficiency η^*_{oblp}

Following the dependence (32), the maximum gain of computational steps Δz for the assumed minimum computational efficiency η^*_{oblp} is defined as follows:

$$\Delta z = t_x - 1 \tag{36}$$

Taking into account equations (30), (31), one can determine the computational value of rotational speed n^*_{oblp} as follows:

$$n^*_{oblp} = n^*_{opt} + \Delta n^*_{hp} = n^*_{opt} + (t_x - 1)\,\delta n^*_{hp} \tag{37}$$

The dependence (37) defines the computational pump rotational speed η^*_{oblp} when the pump operates at assumed minimum computational efficiency η^*_{oblp}. The resulting computational pump rotational speed η^*_{opt} and the optimum rotational speed η^*_{opt} define the permissible range of the pump rotational speed (η^*_{opt}; n^*_{oblp}) for the lower slope of the pump efficiency η^* characteristic curve for the assumed changes of relative pump efficiency $\Delta\eta^*_p$ where $h^* = const.$

Unification of the pump efficiency η^* function is essential for the purposes of establishing the criterion of pump operation control. It becomes possible to determine an energy-saving method of controlling the pump rotational speed n^* in the assumed range of pump efficiency $\eta_{tp} \in (\eta^*_{opt}; \eta^*_{oblp})$ at diverse heads $h^* \in (0{,}6; 1)$.

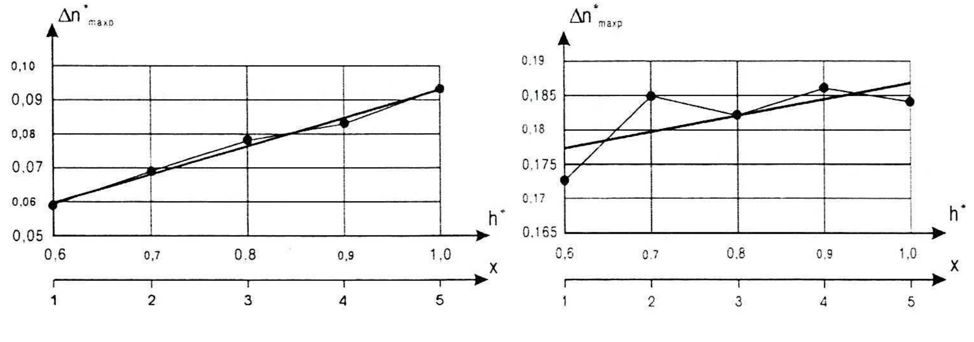
3.2. Right-side coefficient of pump efficiency correction

Each determination of computational rotational speed η^*_{oblp} according to the dependence (37) implies a change of speed gain δn^*_{hp} when head h^* changes. Determination of a change of rotational speed gain δn^*_{hp} involves the correction coefficient k_{hp} in the following dependence:

$$\delta n^*_{hp} = \delta n^*_{hp=1} \cdot k_{hp} \tag{38}$$

where:

$\delta n^*_{hp=1}$ – increase of rotational speed determined in the case of head $h^* = 1$.



a) b)

Fig. 9. Maximum changes of rotational speed $\Delta n^*_{max\,p}$ in function of head h^* for a) pump 20A40 of specific speed $n_q \cong 30$, b) pump shown in [11] ($n_q \cong 42$)

The increase of rotational speed δn^*_{hp} depends on the changes of maximum rotational speed $\Delta n^*_{max\,p}$ as defined in the equation (26). Given the distribution of rotational speed $\Delta n^*_{max\,p}$ characteristic curve in function of head

h^*, one can define the coefficient of efficiency correction k_{hp}. Sample characteristics of maximum speed changes $\Delta n^*_{max\,p} = f(h^*)$ are shown in Fig.9 as curves with dots. Fig.9.a presents a characteristic of speed changes $\Delta n^*_{max\,p}$ based on the characteristic curve of a sample one-grade, one-stream pump type 20A40 (Warsaw Pump and Fixtures Company, $P^n = 90$ kW, $n_n = 1480$ rpm). Fig.9.b presents a characteristic of speed changes $dn^*_{max\,p} = g(h^*)$ based on the characteristic curve according to [11].

Approximating functions of characteristic curve of rotational speed changes $\Delta n^*_{max\,p}$ in the form of a dot-free course are, with a negligible error, equivalent to the original characteristic curve including dots. The characteristic curve of speed changes $\Delta n^*_{max\,p}$ and its approximating line are thus marked in the same way – $\Delta n^*_{max\,p}$. Approximating functions of characteristic curve of speed changes $\Delta n^*_{max\,p}$ in the domain X on the basis of Fig.9 are defined as follows:

$$\Delta n^*_{max\,p} = 0{,}082x + 0{,}0518 \quad \text{Fig.9.a} \tag{39}$$

$$\Delta n^*_{max\,p} = 0{,}023x + 0{,}1751 \quad \text{Fig.9.b} \tag{40}$$

Characteristic curves of speed changes $\Delta n^*_{max\,p}$ dependent on head h^*, as based on Figure 9, become linear functions in the following form:

$$\Delta n^*_{max\,p}(h^*) = a_{hp}h^* + b_{hp} \tag{41}$$

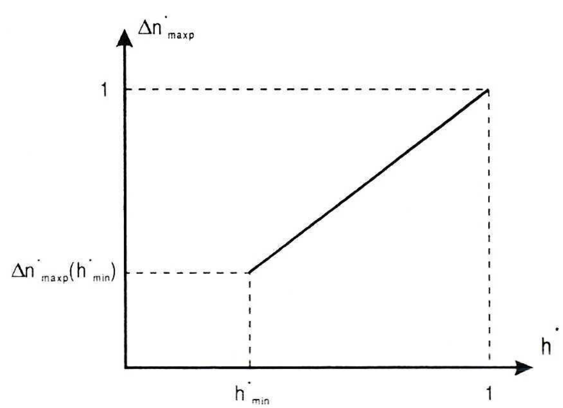


Fig. 10. Characteristic curve of maximum rotational speed changes

The diagram of maximum rotational speed changes is shown in Fig.10. Two known points of rotational speed changes as per Fig 10 are assumed. For any two heads h^*_1 and h^*_2, the following point pairs $(\Delta n^*_{max\,p1}, h^*_1)$, $(\Delta n^*_{max\,p2}, h^*_2)$

result. Assuming that $h_1^* = h_{min}^*$ and $h_2^* = h_{max}^* = 1$, one obtains the following dependence of maximum rotational speed changes in function of head h^*, according to Fig. 10:

$$\Delta n_{max\,p}^*(h^*) = a_{hp}(h^* - h_{min}^*) + \Delta n_{max\,p}^*(h_{min}^*) \tag{42}$$

where:

$$a_{hp} = \frac{\Delta n_{max\,p}^*(1) - \Delta n_{max\,p}^*(h_{min}^*)}{1 - h_{min}^*}$$

Assuming known nature of speed changes $\Delta n_{max\,p}^*(h^*)$ (42), and determining the maximum value of rotational speed changes $\Delta n_{max\,p}^*(1)$ at head $h^* = 1$, we define the value of correction coefficient k_{hp} as follows:

$$k_{hp} = \frac{\Delta n_{max\,p}^*(h^* \neq 1)}{\Delta n_{max\,p}^*(1)}\,, \tag{43}$$

On the basis of equations (42), (43), the ultimate form of correction coefficient k_{hp} is obtained:

$$k_{hp} = \frac{1 - \alpha_p}{1 - h_{min}^*}(h^* - h_{min}^*) + \alpha_p \tag{44}$$

where:

$$\alpha_p = \frac{\Delta n_{max\,p}^*(h_{min}^*)}{dn_{max\,p}^*(1)}$$

The coefficient k_{hp} becomes an equivalent of the function defining $\Delta n_{max\,p}^*$ as dependent on head h^*.

4. Conclusion

In the process of controlling operation of a pumping unit, the pump efficiency η^* has a significant impact on the overall efficiency of the unit η_g. Available literature does not present any analytical characteristics of pump efficiency. Based on a sample characteristic curve, the efficiency characteristic curves at constant head of pump were determined. A generalized form of the lower slope of the efficiency characteristic curve was defined as a quadratic function. A method of calculating the quadratic function parameters on the basis of characteristic curve for any head h^* was presented. On the basis of the quadratic function and assuming a permissible efficiency deviation from the maximum value, the changes of rotational speed in relation to the optimum speed can be estimated.

Ultimately, the resulting conditions of estimating the rotational speed can reduce the value of unit energy consumption e of water pumping. Advantages of the method include its simple results, which can be easily implemented in commonly used PLC controllers. The control method discussed here allows for application of the well-known PI regulator.

The results are error-prone due to the scarcity of available characteristic curves covering pump capacities Q over and above its nominal value. If capacity meets the condition: $Q > 1{,}3\,Q_n$, cavitation is more likely to occur and the drive system may suffer from overload. Therefore, applications include maximum pump capacities that correspond to the frequency of the motor drive voltage $f = 51 \div 52$Hz.

The overarching analytical criterion of the discussion presented in the article is generalisation of the pump efficiency characteristic curves. All negative phenomena that may occur in the field of the lower slope of the pump characteristic curve at capacities $Q > 1{,}3\,Q_n$ were therefore ignored.

Manuscript received by Editorial Board, December 12, 2003;
final version, April 20, 2004.

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Wyznaczenie uogólnionej postaci opadającej części charakterystyki sprawności pompy odśrodkowej

Streszczenie

W artykule przedstawiono kryterium oceny energochłonności pompowania wody. Wyznaczono charakterystykę opadającej części sprawności pompy dla przypadku: stałej wartości przyrostu prędkości obrotowej, zmiennej wartości przyrostu prędkości obrotowej. Zdefiniowano ujednoliconą postać opadającej części charakterystyki sprawności pompy dla zmiennej wysokości podnoszenia. Zdefiniowano prawostronny współczynnik korekcji sprawności pompy.