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*MARIAN KRÓLAK**), *ZBIGNIEW KOŁAKOWSKI**), *TOMASZ KUBIAK**)

PARAMETRIC ANALYSIS OF THE STABILITY AND LOAD CARRYING CAPACITY OF PRISMATIC SEGMENT SHELLS SUBJECTED TO COMPRESSION

The study is devoted to a parametric analysis of the stability and load carrying capacity of prismatic segment shells built of rectangular sections of cylindrical shells and subjected to compression. Segment shells (columns) with a constant cross-sectional area (weight) have been analysed and all the results obtained have been compared with the results obtained for the cylindrical shell with a radius R and a thickness t_w.

First, an influence of geometrical parameters of the cross-section of single-layer isotropic shells has been analysed and such profiles have been sought for which the load carrying capacity is significantly higher than in the case of the cylindrical shell.

Then, for a selected shape of the shell (apart from higher load carrying capacity, this choice could be influenced by other factors such as, e.g. easiness of manufacturing), an effect of the arrangement and thickness of orthotropic layers of the shell (laminate) on the stability and load carrying capacity has been investigated.

The analysis has shown that one can design a segment shell made of the same orthotropic material and characterised by higher resistance to buckling and load carrying capacity than a single-layer cylindrical orthotropic shell. The results are depicted in the form of plots.

1. Introduction

In the present study, the notion of prismatic segment shells will be referred to the shells that are circumscribed on the cylinder with a radius R, built of n segments (sections) of cylindrical shells with a radius r, whose geometry is determined additionally by angles 2α and 2γ (Fig. 1), and built of

*) *Departament of Strenght of Materials and Structures Technical University of Lodz, Stefanowskiego 1/15, 90-924 Łódź, Poland; E-mail: kola@orion.p.lodz.pl*

n segments of cylindrical shells with a radius R and a central angle δ. For small angles δ, sections of cylindrical shells with a radius R can be replaced by plates with the width $b_R = 2R\sin\frac{\delta}{2} \cong R\delta$.

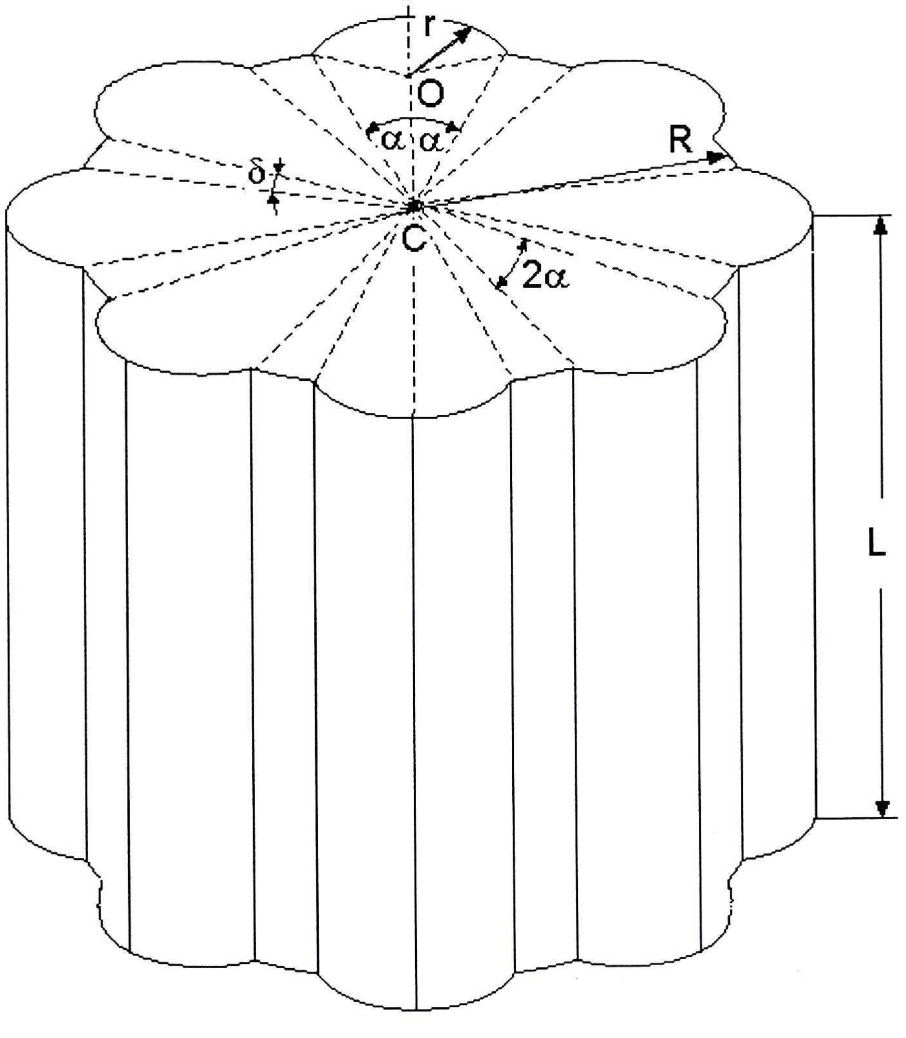


Fig. 1. Segment shell with notation of overall dimensions

The decision to carry out a parametric analysis of the stability and to estimate initially the load carrying capacity of segment shells has been made because of two reasons, namely:

– it has been expected that segment shells, at their certain geometrical parameters, will be characterised by significantly higher resistance to

buckling than cylindrical shells made of the same material and with the same cross-sectional area;
- a computer program has been developed that allows for conducting the above-mentioned analysis in a fast way.

An increase in the resistance to buckling of thin-walled structures that are often subjected to buckling at very low values of effective (reduced) stresses, without increasing their weight, is mostly desirable because in numerous modern structures lightness combined with high load carrying capacity is the fundamental requirement these structures have to satisfy.

In order to make a structure that is light and characterised simultaneously by high load carrying capability, the high strength properties of the material it is made of should be taken advantage of in the best way. These high strength properties can be utilised better in properly profiled load carrying elements of thin-walled structures than in elements with compact cross-sections.

In recent years, mainly thanks to fibrous composites, it has been possible to design strength properties of materials in such a way as to make the load carrying capacity of elements made of these composites very high indeed.

However, the reliability of thin-walled structure operation can be endangered not due to the effort of the structural material but to its buckling (a loss of stability).

In the present study, some suggestions how to enhance the resistance of compressed thin-walled structures to a loss of stability (buckling) without increasing their weight (maintaining the same cross-sectional area) and applying the same material will be presented.

In the case of thin-walled structures made of an isotropic material, one of the basic ways proposed to increase the structure resistance to buckling is a selection of the proper profile (cross-section shape), at which critical stresses are significantly higher than in the profiles that have been used so far.

In thin-walled structures made of an orthotropic material, apart from a selection of the proper profile of the cross-section, it is proposed that a multilayered wall should be made of the same orthotropic material but with such an arrangement of its individual layers that the bending rigidities of these multilayered walls will be approximately equal („isotropic" wall to bending) in two directions perpendicular to each other (that is to say, in the direction of compression and in the direction perpendicular to it) (Królak, Kowal-Michalska, Świniarski, 2002). Among other known ways of increasing the resistance to bending of thin-walled structures, one can mention the following:
- reinforcement of the thin-walled element with appropriate stiffeners (both the position and the dimensions of stiffeners are important) and/or membranes;

– selection of appropriate geometrical ratios (dimensions), provided it is possible in a given structure.

Modern technologies allow for manufacturing composite materials and laminates with required characteristics, as well as for making thin-walled elements with complex shapes of the cross-section and small geometrical imperfections.

The present study is aimed mainly at the determination of such parameters of thin-walled segment shells made of isotropic or orthotropic materials, including shells with multilayered walls made of orthotropic layers, at which values of critical loads under axial compression are significantly higher than those for cylindrical shells made of the same material, with the same cross-sectional area and the identical length.

The computations will be performed with a computer program developed on the basis of the equations of stability and the formulas for calculations of stability and initial post-buckling equilibrium paths of thin-walled shell/plate structures with complex profiles that have been generated earlier. The computational results, especially those referring to critical loads of segment shells under axial compression ,will be compared with the results obtained for the ideal cylindrical shell. We will show a possibility of achieving a significant increase in the resistance to a stability loss for prismatic segment shells under compression, at their certain geometrical parameters, in comparison with the ideal cylindrical shell with a radius R and the same cross-sectional area, mainly through a modification of the cross-section shape, will be shown.

2. Formulation of the problem

Let us consider a thin-walled structure circumscribed on the cylinder with a radius R and a length L, built of n segments (sections) of cylindrical shells with a radius r, whose geometry is determined additionally by angles α and γ and built of n sections of cylindrical shells with a radius R and a central angle δ(Fig. 1). For small angles δ, sections of cylindrical shells with a radius R can be replaced by plates with the width $b_R = 2R\sin\frac{\delta}{2}$. For the angle $\alpha = 0$, we obtain the ideal cylindrical shell with a radius R. The wall thickness of this shell will be denoted by t_w. In turn, for the angle $\delta = 0$, we obtain a thin-walled segment structure built of n identical sections of cylindrical shells with a radius r and a thickness t, whose geometry depends on angles α and γ.

The cross-section through 3 segments of the segment shell presented in Fig. 1 is shown in Fig. 2.

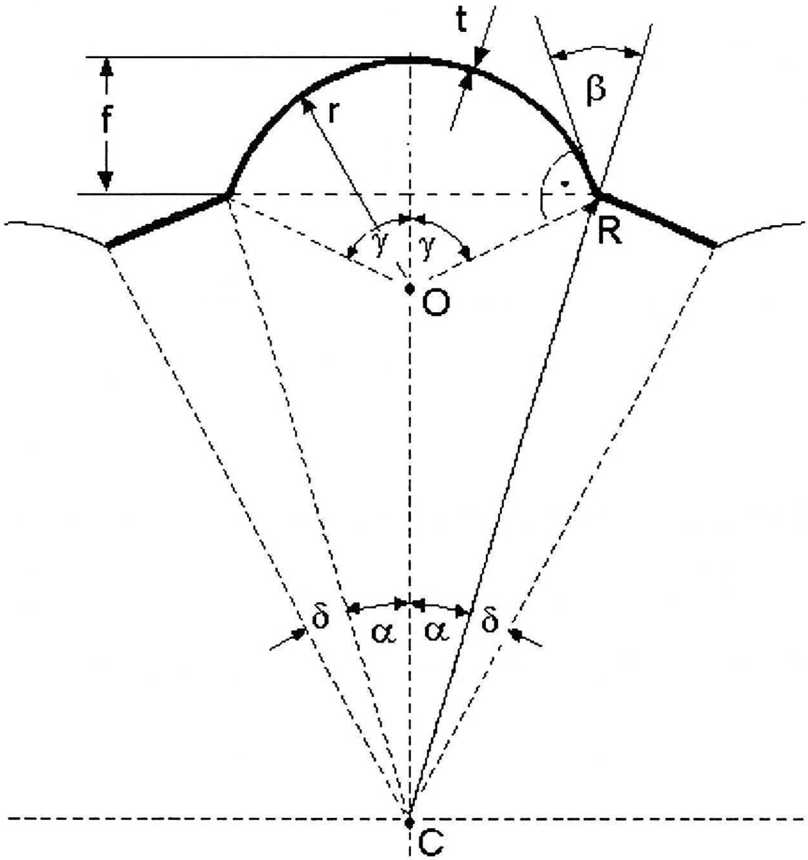


Fig. 2. Overall dimensions of analysed segment shell

The middle segment is described by radii R and r, angles α, β and γ and a shell thickness t. In the general case of the shell depicted in Fig. 1, the angle

$$\alpha = \frac{\pi}{n} - \frac{\delta}{2}. \tag{1}$$

The angle β between the radius R going through the segment edge and the tangent to the shell segment with the radius r on the same edge, depends on a rise f of the shell section and can vary within the range $0 \le \beta \le \frac{\pi}{2}$ (Fig. 2) The quantities n, R, L, t_w, δ and β will be treated as the known (given) ones. The quantities γ, r, and f can be calculated from the following formulas:

$$
\gamma = \frac{\pi}{2} + \alpha - \beta,
$$
$$
r = \frac{\sin\alpha}{\cos(\alpha-\beta)}R = \frac{\sin\alpha}{\sin\gamma}R, \tag{2}
$$
$$
f = r(1-\cos\gamma).
$$

From the condition saying that cross-sectional areas of the ideal cylindrical shell with a radius R and a wall thickness t_w and of the segment shell shown in Fig. 1 are identical, the following formula for the constant thickness of the segment shells under consideration has been obtained:

$$
t = \frac{2\alpha + \delta}{\delta + 2\gamma\dfrac{\sin\alpha}{\sin\gamma}} \cdot t_w . \tag{3}
$$

In the further part of this study, the stability (critical loads) of segment shells made of isotropic, orthotropic (composite) and multilayered (orthotropic laminates) materials subjected to axial compression will be analysed and their load carrying capacity will be estimated.

Owing to this analysis, it will be possible to determine under what geometrical parameters these segment shells will be characterised by significantly enhanced resistance to buckling (i.e. by higher values of critical stresses) and thus by higher load carrying capacity than the ideal cylindrical shell with a radius R and the identical cross-sectional area.

3. Basic equations and a description of capabilities of the computer program developed

The equations of stability for a section of the multilayered cylindrical shell have been generated by means of the variational method (Królak, Kołakowski, 2002), employing the asymptotic theory of conservative systems (Koiter, 1976). The rigidity of multilayered shells has been described with formulas of the classical theory of multilayered plates (Jones, 1975).

The geometrical equations have been assumed in the following form:

$$
\begin{aligned}
&\varepsilon_1 = u_{1,1} + 0.5u_{m,1}u_{m,1},\\
&\varepsilon_2 = u_{2,2} + 0.5u_{m,2}u_{m,2} - ku_3,\\
&\varepsilon_2 = 0.5(u_{1,2} + u_{2,1}) + 0.5u_{m,1}u_{m,2},\\
&\kappa_1 = -u_{3,11},\ \kappa_2 = -u_{3,22} - ku_{2,2},\ \kappa_3 = 2u_{3,12} - ku_{2,1}
\end{aligned} \tag{4}
$$

where k is the curvature of the cylindrical shell section ($k_r = \frac{1}{r}$ or $k_R = \frac{1}{R}$), and the summation with respect to m is from 1 to 3 ($m = 1, 2, 3$). It is necessary to take into account the additional terms in the expressions concerning variations in curvatures in order to expand the range of application of the obtained equations of stability to shells characterised by larger rises (than, for instance, in Volmir, 1967) and at analysing interactive buckling (e.g. accounting for an influence of the global buckling on the local one).

The physical equations have been written in such a way as to make it possible to use them for multilayered walls made of linearly elastic orthotropic materials as well.

The equations of equilibrium have been obtained in the following form:

$$
\begin{aligned}
&[N_1(1 + u_{1,1}) + N_3 u_{1,2}]_{,1} + [N_2 u_{1,2} + N_3(1 + u_{1,1})]_{,2} = 0\,, \\
&[N_1 u_{2,1} + N_3(1 + u_{2,2}) - ktN_6]_{,1} + [N_2(1 + u_{2,2}) + N_3 u_{2,1} - ktN_5]_{,2} = 0\,, \qquad (5) \\
&(tN_{4,1} + N_1 u_{3,1} + N_3 u_{3,2})_{,1} + (tN_{5,1} + 2tN_{6,1} + N_2 u_{3,2} + N_3 u_{3,1})_{,2} + kN_2 = 0\,.
\end{aligned}
$$

In the above-mentioned equations, N_1, N_2, N_3 are the dimensionless sectional forces, N_4, N_5, N_6 – the dimensionless sectional moments, u_1, u_2, u_3 – the components of the displacement vector, t – the shell thickness, and k – its curvature.

As it has been mentioned above, after expanding the fields of displacements $\bar{U}_k$ and the fields of sectional forces $\bar{N}_k$ into power series with respect to the buckling mode amplitudes ζ_n (the amplitude of the n-th buckling mode divided by the thickness t of the wall assumed to be the first one), Koiter's asymptotic theory has been employed.

$$
\begin{aligned}
\bar{U}_k &= \lambda \bar{U}_k^{(0)} + \zeta_n \bar{U}_k^{(n)} + \ldots \\
\bar{N}_k &= \lambda \bar{N}_k^{(0)} + \zeta_n \bar{N}_k^{(n)} + \ldots
\end{aligned} \qquad (6)
$$

where $\bar{U}_k^{(0)}$, $\bar{N}_k^{(0)}$ are the pre-buckling state fields, and $\bar{U}_k^{(n)}$, $\bar{N}_k^{(n)}$ the n-th buckling mode fields.

After the substitution of the expansions (Koiter, 1976) into the equilibrium equations, the continuity conditions and the boundary conditions (corresponding to the jointed support at the girder ends), the boundary problem of the zero and first order for the case of uniform compression along the generating lines of the cylindrical shell segment has been obtained.

In the case of the load that varies along the shell perimeter in the pre-critical state (e.g. at bending), such a load can be modelled with a step-like varying load (constant in individual strips the shell is divided into). The obtained system of homogeneous ordinary differential equations, with the corresponding conditions of the interaction of segments, has been solved by the transition matrix method, having integrated numerically the equilibrium equations along the circumferential direction in order to obtain the relationships between the state vectors on two longitudinal edges. During the integration of the equations, Godunov's orthogonalization method is employed.

On the basis of the obtained solution to the task, a computer program has been developed. Thanks to this computational code, critical loads and initial post-buckling equilibrium paths can be calculated and the load carrying capacity of thin-walled shell/plate structures (shells, beams, columns) with an arbitrary prismatic cross-section, with at least one axis of symmetry, subjected to axial compression, eccentric compression or bending in the symmetry plane of the structure, can be estimated. Structure walls can be made of isotropic and/or orthotropic (composite) materials that are single- or multilayered (laminates).

The structure can be reinforced with longitudinal edge stiffeners (open profiles) or/and intermediate stiffeners. Different modes of buckling, i.e. global (flexural, flexural-torsional and torsional), distorsional, local and interactive buckling, can be analysed.

4. Numerical calculation results and their analysis

The numerical calculations have been performed for segment shells with a constant thickness of walls, subjected to uniform axial compression. The shell walls made of isotropic, orthotropic and multilayered materials with orthotropic layers have been analysed.

The calculations have been aimed at the determination of such geometrical parameters of segment shells and such an arrangement of orthotropic layers in multilayered shell walls at which critical loads of the local or global buckling are significantly higher than for the ideal cylindrical shell with a radius R and the identical cross-sectional area. In thin-walled structures (that are subject to buckling at low values of effective stresses), an increase in values of critical stresses is equivalent to an increase in their load carrying capacity.

4.1. Calculations of isotropic shells built of n segments of cylindrical shells ($\delta = 0$) subjected to compression

The calculations have been carried out for shells with 4–18 segments, whose cross-sections are shown in Fig. 3.

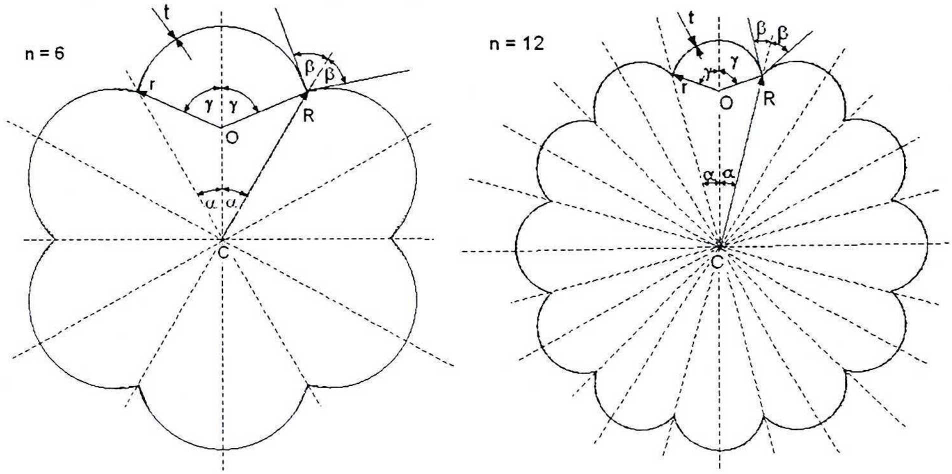


Fig. 3. Cross-section of shells with 6 and 12 segments

The calculations have been made for the following data: R = 400 mm, L = 800 mm, n = 4÷18, t ≤ 1 mm, β = 0÷90°, $E = 2 \times 10^5$ MPa, v = 0.3. Some selected results of these calculations are presented in Table 1 and plotted in Figs. 4 and 5.

Table 1.

Critical stress value for different β angle and number of segments n

β [deg] \ n	4	6	8	10	12	16	18
0	98.805	217.795	324.885	411.5775	512.161	699.033	657.336
7.5	113.293	263.092	379.941	495.237	529.07	635.849	688.793
15	164.767	298.682	401.596	480.227	628.147	638.128	794.184
22.5	198.048	319.733	448.753	500.114	644.263	691.131	856.01
30	226.088	359.779	425.767	556.405	493.89	733.184	663.917
45	260.974	353.283	401.371	378.631	405.927	422.489	334.302
60	232.8	236.4	245.3	136.1	261	216	169.4
75	91.1	79.9	88.9	68.2	76.4	77.9	204.1
82.5	131.0	97.5	161.3	181.0	146.4	197.8	78.9
90	298.0	298.0	298.0	298.0	298.0	298.0	298.0

The calculation results for shells with $n \geq 20$ are not included in the present study as for these shells (especially at smaller values of the angle β), the ratio L/r has not fallen within the range

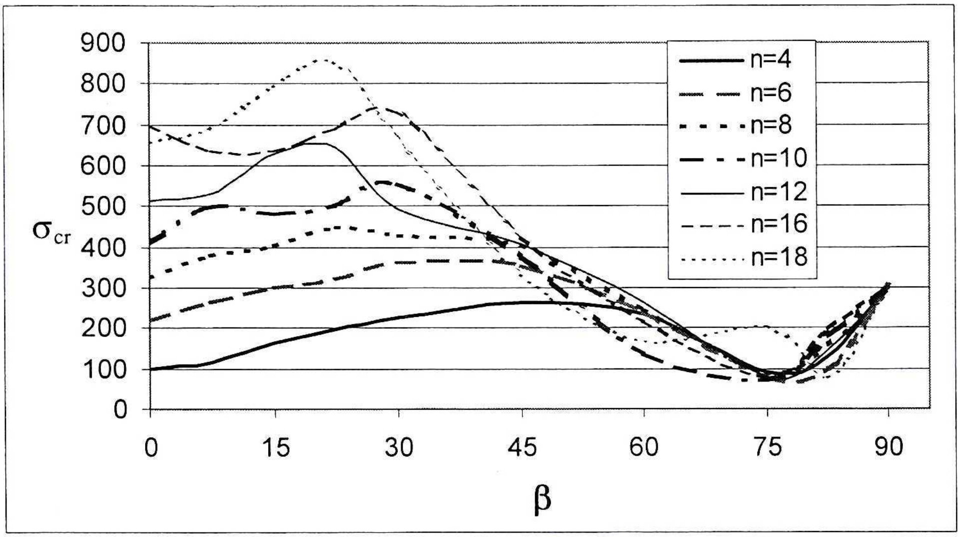


Fig. 4. Critical stress vs. β angle for shells with 4-18 segments

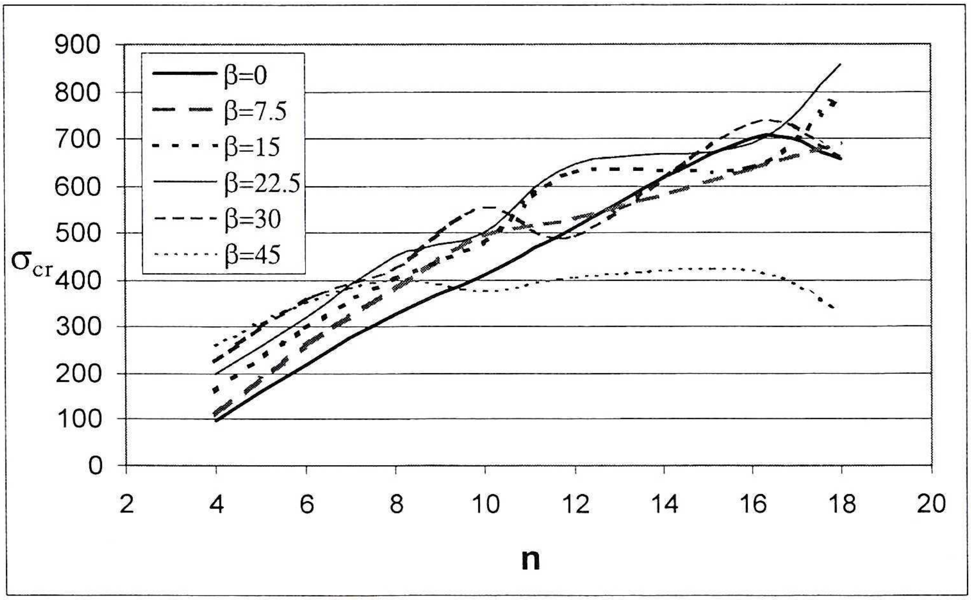


Fig. 5. Critical stress vs. number of segments shells

$$1.38\sqrt{\frac{t}{r}} < \frac{L}{r} < 0.57\sqrt{\frac{r}{t}}$$

that holds for the theory of cylindrical shells with a small rise (Volmir, 1967).

It follows from the conducted computations that for angles $\beta > 55°$ (independent of the number of shell segments), values of critical stresses in the segment shells under analysis are lower or at most equal to the value of critical stress σ^*_{cr} = 297.9 MPa in the ideal cylindrical shell with the radius R = 400 mm and the thickness t_w = 1 mm (with the same cross-sectional area). Also in the four-segment shell, the values of critical stresses are lower than in the whole range of changes in the angle (Fig. 4). Maximum values of critical stresses for shells with $n \geq 6$ increase with an increase in the number of shell segments and in practice occur for angles β that fall within the range $20° \leq \beta \leq 30°$. A sharp increase in these stresses, in comparison to stresses for the ideal cylindrical shell, occurs already for shells with n = 10, for which maximum critical stresses are more than twice as high (≈ 650 MPa) as in the ideal cylindrical shell (≈ 300 MPa).

Taking into account the fact that the lower critical stress in the cylindrical shells under compression is approximately three times lower than the upper critical stress, and that the lower critical stress in the cylindrical panel under compression is equal to more than 1/2 of the upper critical stress, it should be expected that the load carrying capacity will increase more than 3 times in the 10-segment shell under analysis in comparison to the ideal cylindrical shell. Such a high increase in the load carrying capacity is caused by refractions at the angle 2β that occur where segments contact each other. It can be seen in Fig. 3 that, in particular, refractions at the angle $30° \leq 2\beta \leq 60°$ play a role of longitudinal stiffeners and cause an increase both in the critical stresses and the load carrying capacity.

4.2 Analysis of the influence of the angle δ on the values of critical stresses in shell segments

The computations have been carried out for 10-segment shells characterised by the following data: R = 400 mm, L = 800 mm, n = 10, β = 30°, δ = 1–10 [deg], t = 0.73–0.82 mm. The results of calculations are shown in Fig. 6.

It follows from this diagram that with an increase in the angle δ, the critical stress first decreases slowly (in the range $0 \leq \delta \leq 5.5°$) and next (for $\delta > 5.5°$) decreases quite sharply. Moreover, with a change in the angle δ, a number of half-waves of the buckling mode changes along the shell length.

If we treat shell elements with a radius R and a central angle δ as plate elements, then the values of critical stresses vary slightly, and for small angles δ, they do not change in practice.

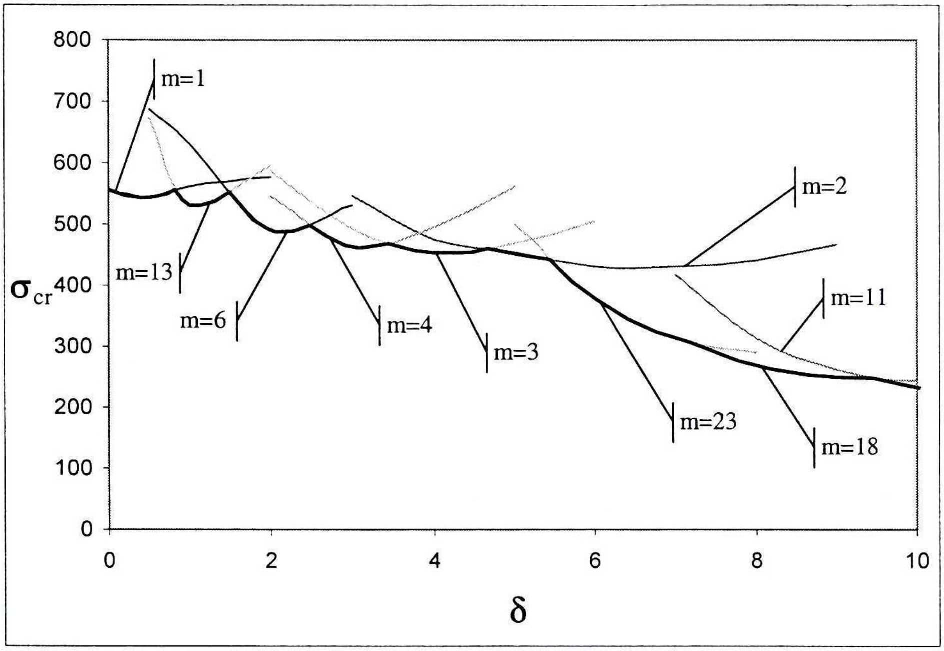


Fig. 6. Envelope of critical stresses for 10 segment shells vs. δ angle for different buckling modes

4.3 Influence of the material orthotropy on the stability of shell segments

The influence of the material orthotropy on the stability of segment shells has been analysed for shells characterised by the following dimensions: R = 400 mm, L = 800 mm, α = 30°, 22.5°, 18°, 15° and 11.25°, i.e. for n = 6, 8, 10, 12 and 16 and for β = 0°, 15°, 30° and 45°. The remaining quantities that describe the geometry of shells have been calculated from the formulas:

$$\gamma = \frac{\pi}{2} + \alpha - \beta$$

$$r = \frac{\sin\alpha}{\sin\gamma} \cdot R$$

$$t = \frac{\alpha}{\gamma} \frac{\sin\gamma}{\sin\alpha} \cdot t$$

where t_w = 1 mm is the thickness of the cylindrical shell of the same cross-sectional area as the segment shells under analysis. The calculations have been carried out for shells made of an isotropic material and four orthotropic materials, whose coefficient of orthotropy is $\eta = E_1/E_2$ and whose other material properties (Poisson's ratios and the Young modulus) are presented in Table 2.

Table 2.

Poisson's ratios and the Young the Young modulus for orthotropic materials

	$\eta = E_1/E_2$	ν_{12}	ν_{21}	G/E_2
1	1	0.3	0.3	0.3846
2	1.9797	0.3	0.15192	0.39375
3	3.2992	0.3	0.09093	0.40019
4	7.6045	0.3	0.03945	0.40912
5	13.7362	0.3	0.02184	0.40659

The properties of orthotropic materials are quoted after Chandra and Raju (1973).

The calculated dimensionless values of the critical stresses $\sigma_{cr} = \frac{\sigma_{cr}}{E_2}$ are shown in Table 3 (3a-3e).

Table 3.

a) n=6

	β = 0		β = 15		β = 30		β = 45		β = 90	
	m	σ	m	σ	m	σ	m	σ	m	σ
1	22	11	1	15	1	18	1	18	2	15
2	17	12	18	18	16	23	1	24	10	17
3	16	13	15	20	14	25	13	28	8	19
4	12	16	12	24	11	31	10	34	7	23
5	10	18	10	28	9	35	8	39	5	26

b) n=8

	β = 0		β = 15		β = 30		β = 45		β = 90	
	m	σ	m	σ	m	σ	m	σ	m	σ
1	1	16	1	20	1	21	1	20	2	15
2	20	19	19	26	18	31	1	27	10	17
3	17	21	17	30	16	35	1	35	8	19
4	13	25	13	36	12	42	11	44	7	23
5	11	28	11	41	10	48	10	51	5	26

c) n=10

	β = 0		β = 15		β = 30		β = 45	
	m	σ	m	σ	m	σ	m	σ
1	1	21	1	24	1	28	1	19
2	17	26	22	34	1	35	1	31
3	18	28	19	38	17	44	16	45
4	14	34	14	46	13	53	13	55
5	12	38	12	53	11	60	10	62

d) n=12

	β = 0		β = 15		β = 30		β = 45	
	m	σ	m	σ	m	σ	m	σ
1	1	26	1	31	1	25	1	20
2	23	32	1	39	1	40	1	28
3	20	35	20	47	19	53	1	39
4	15	42	15	57	14	64	14	65
5	13	47	13	65	12	72	10	73

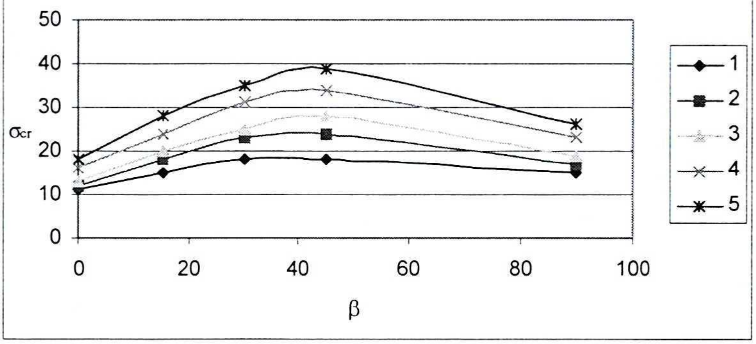
e) n=16

	β = 0		β = 15		β = 30		β = 45	
	m	σ	m	σ	m	σ	m	σ
1	1	35	1	32	1	37	1	21
2	26	45	1	46	1	46	1	39
3	23	49	1	64	1	56	1	53
4	17	59	19	79	17	85	1	73
5	15	67	14	89	14	98	12	95

For an isotropic material (η =1), at $E_2 = 2 \times 10^5$ MPa, the results of calculations are the same as the results included in section 4.1.

In Figs. 7a-7e, a change in values of the dimensionless critical stresses $\bar{\sigma}_{cr} = \sigma_{cr} / E_2$ for the shells under analysis as a function of the angle β can be seen. It is easy to read the values of the angle β at which maximum values of critical stresses occur in shells made of a given material. For a shell with the defined number of segments, critical stresses grow with an increase in the coefficient of orthotropy η (E_1 – is the modulus of elasticity of the material along the axial direction of the shell, that is to say, along the direction of compression). However, more than a 13-times increase in the coefficient of orthotropy (modulus E_1) causes high but not larger than a 5-times increase in critical stresses.

a) n = 6


σcr
β
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10
20
30
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60
80
100
1
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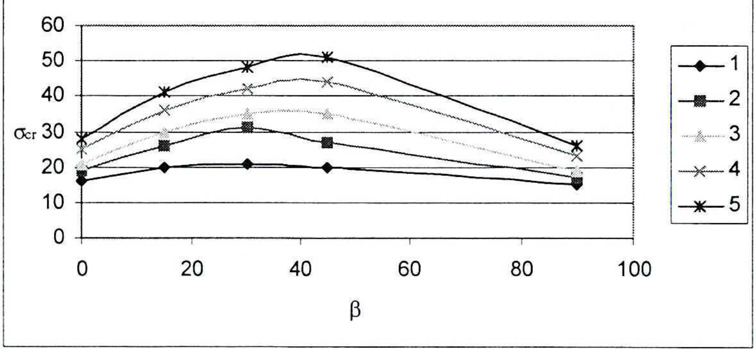
3
4
5

b) n = 8


σcr
β
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100
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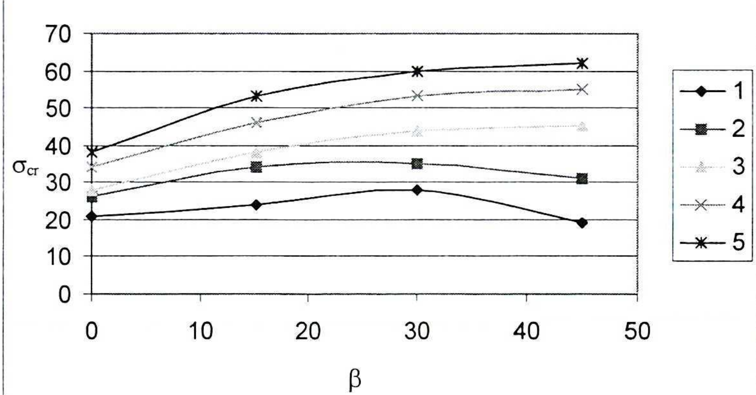
4
5

c) n = 10


σcr
β
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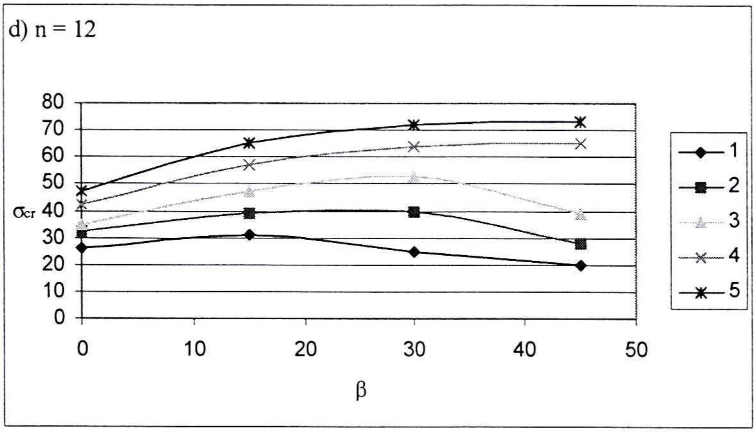

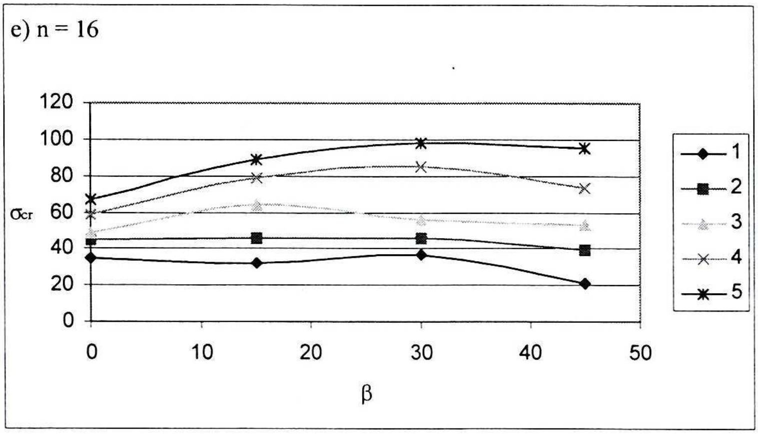

Fig. 7. Critical stress vs. β angle for orthotropic shells (material properties given in Table 2) with 6, 8, 10, 12 and 16 segments

It can be seen from the diagrams presented in Fig. 8 that critical stresses for orthotropic shells grow with an increase in the number of shell segments, in the range of $n = 6 \div 16$ under analysis.

4.4. Multilayered segment shell

The calculations have been made for a segment shell built in the following way: 10 cylindrical segments with the radius $r = 123.2$ mm and the obtuse angle $2\gamma = 155°$ and 10 segments with the radius $R = 400$ mm and the central angle $\delta = 1°$. The remaining dimensions have been assumed as follows: $L = 800$ mm, $\alpha = 17.5°$, $\beta = 30°$. The shell wall is built of five layers with the respective thicknesses: $t_1 = t_5 = 0.06$ mm, $t_2 = t_4 = 0.13$ mm and the middle layer $t_3 = 0.37$ mm. All the layers are made of the same orthotropic material with the coefficient of orthotropy $\eta = 13.7362$, $\nu_{12} = 0.3$, $\nu_{21} = 0.02184$ and

$G/E_2 = 0.40659$, where E_1 is the modulus in the axial direction of the shell (the direction along which compressive loading acts) in layers 1, 3 and 5, whereas E_1 is the modulus in the circumferential direction and E_2 is the modulus in the axial direction in layers 2 and 4. Thus, this is a shell with the symmetrical arrangement of layers, in which the material of layers 2 and 4 is turned through an angle of 90° with respect to the material of layers 1, 3 and 5. The thickness and arrangement of layers have been selected in such a way as to make the shell bending rigidity in the axial and circumferential direction approximately equal, and to make the compression rigidity of the shell in the axial direction higher than in the circumferential direction (Królak, Kowal-Michalska, Świniarski, 2002).

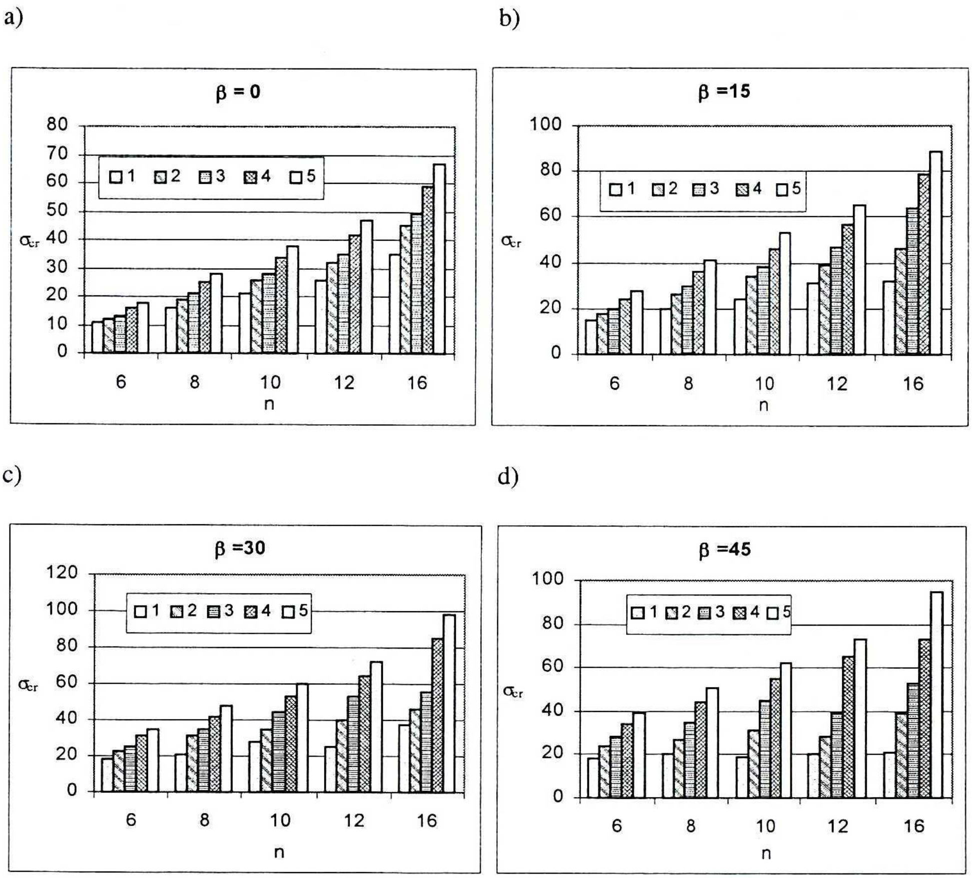

Fig. 8. Bar charts of critical stress for orthotropic shells (see Table 2) with 6, 8, 10, 12 and 16 segments and angle β equal to 0°, 15°, 30° and 45°

As a result of these calculations, the following values of dimensionless critical stresses have been obtained:

a) $\bar{\sigma}_{cr} = \dfrac{\sigma_{cr}}{E_2} = 2.667 \times 10^{-3}$ for the isotropic shell (at the number of half-waves along the shell length $m = 13$, $E_1 = E_2 = E$);

b) $\bar{\sigma}_{cr} = \dfrac{\sigma_{cr}}{E_2} = 5.564 \times 10^{-3}$ for the orthotropic single-layer shell with $\eta = E_1/E_2 = 13.7362$ (at $m = 8$);

c) $\bar{\sigma}_{cr} = \dfrac{\sigma_{cr}}{E_1} = 5.556 \times 10^{-3}$ for the orthotropic single-layer shell with $\eta = E_2/E_1 = 13.7362$ (at $m = 31$);

d) $\bar{\sigma}_{cr} = \dfrac{\sigma_{cr}}{E_2} = 7.969 \times 10^{-3}$ for the five-layer shell described above (at $m = 17$).

The results presented in points b) and c) refer to single-layer shells made of the same material turned through an angle of 90°. The values of the critical stresses in these shells are practically equal.

On the other hand, the critical stress in the five-layer shell is nearly three times higher than the critical stress in the isotropic shell (at the same value of the modulus E_2) and nearly 1.5 times higher than the critical stresses in the orthotropic single-layer shells. In the isotropic shells (see section 4.1), it has been assumed that $E_2 = E_1 = E = 2 \times 10^5$ MPa. For this value of E_2, the critical stresses in the five-layer shell would be equal to:

$$\sigma_{cr} = 15.938 \cdot 10^2 \text{ MPa}.$$

4.5 Shell with a complex cross-section

The cross-section of the shell under consideration is shown in Fig. 9. The shell is composed of three shells combined with each other (for instance, by welding), namely:

- an inner cylindrical shell with the radius $R_w \approx R$ and the thickness $t_1 = 0.3$ mm;
- a segment (middle) shell with the same dimensions as the shell calculated in section 4.4, but with the different wall thickness $t_2 = 0.2$ mm;
- an outer cylindrical shell with the radius R_2 and the wall thickness $t_3 = 0.36$ mm.

The thicknesses t_1, t_2 and t_3 have been selected in such a way that the ratio of the circumference of the respective shell between welds to its thickness is approximately equal, i.e. b_i/t_i = const, and the shell cross-sectional area and its length L = 800 mm are the same as in the case of all the shells calculated so far.

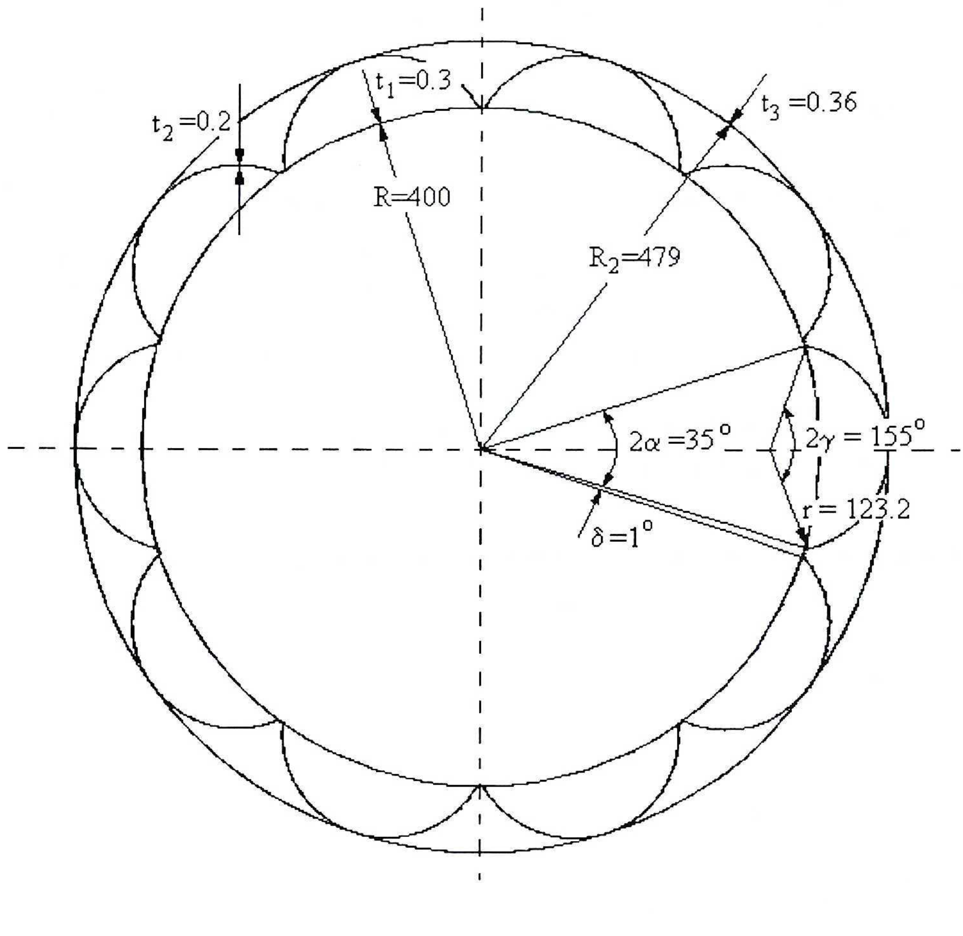

Fig. 9. Cross-section of a complex shell

The calculations of critical stresses for the shell under analysis have been performed with the MES ANSYS version 5.7 software package. Very low values of the critical stresses $\sigma_{cr} = 1.19 \cdot 10^2$ MPa (the stresses that are 2.5 times lower than for the cylindrical shell with the radius R = 400 mm and t_w = 1 mm) have been obtained. As far as buckling is concerned, the outer shell was the weakest one. The buckling modes of the whole shell and of its individual elements are shown in Fig. 10.

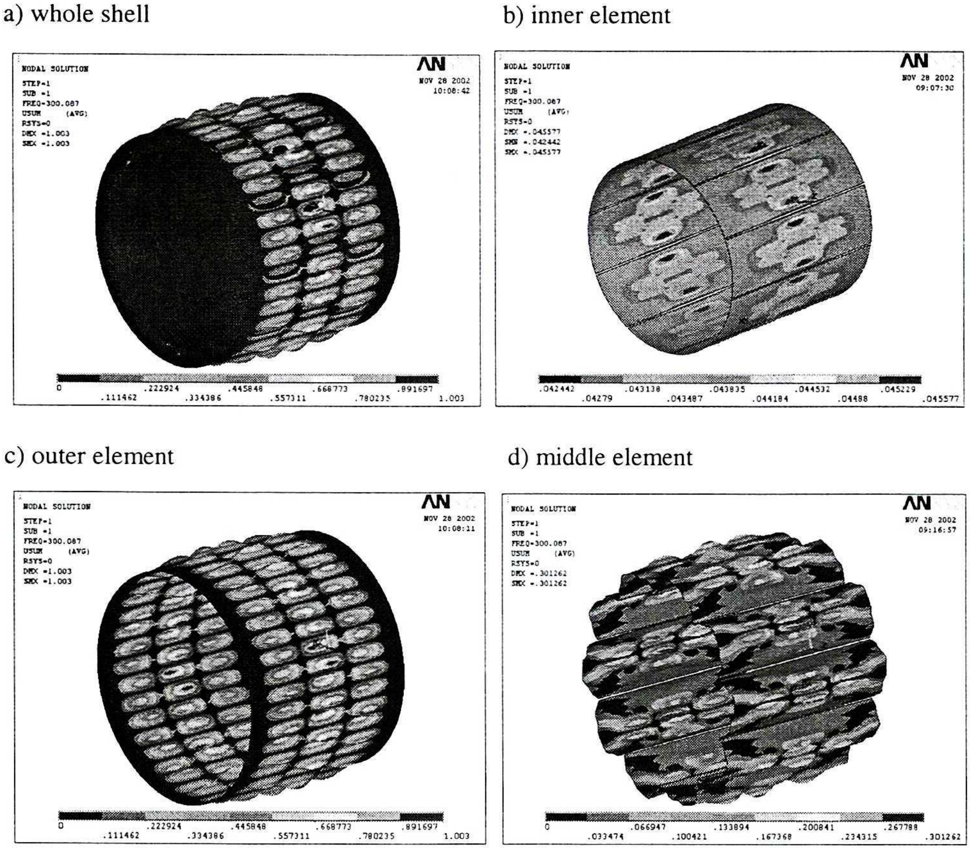

Fig. 10. Buckling modes for elements of complex shell presented in Fig. 9

5. Conclusions

From the parametric analysis that has been conducted, an influence of individual geometrical parameters of the cross-section of multilayered shells on the stability (values of critical stresses) and load carrying capacity of shells under axial compression can be seen easily.

From the diagrams included in this study, one can see at which values of geometrical parameters the resistance of shells to buckling increases and at which values it decreases (at the same cross-sectional area). It is visible that the profile is very significant as regards the stability.

In the segment shells under consideration, the stability has been strongly affected by two parameters, namely: the number of segments (angle α) and the way the segments are combined with each other (angle β). For certain angles β, this combination plays the role of a longitudinal stiffener. A very small rise of segments (for instance for angles β in the range

$70° \leq \beta \leq 80°$) causes that the segment shell behaves like a cylindrical shell with large imperfections, which results in a decrease in the load carrying capacity of this shell.

In this study, the arrangement and thickness of orthotropic layers in multilayered shells (laminates) that is advantageous as regards the stability (value of critical stress) and strength of the structure have been determined as well. The general conclusion to be drawn is that the parametric analysis allows for designing a lightweight multilayered segment shell (column) characterised by high load carrying capacity in such a way as to exploit the strength properties of the material most beneficially.

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REFERENCES

[1] Królak M., Kołakowski Z., Buckling and initial post-buckling behaviour of thin-walled shell and plate structures, Int. J. of Applied Mechanics and Engineering, 2002, vol. 7, No. 2, pp. 491-512
[2] Koiter W. T., General theory of mode interaction in stiffened plate and shell structures, WTHD Report 590, Delft, 1976, 41 p.
[3] Volmir A.S., Stability of deforming systems, Moscow, 1967 (*in Russian*)
[4] Jones R.M., Mechanics of Composite Materials, International Student Edition, Tokyo, McGraw-Hill Kogakusha Ltd., 1975
[5] Pignataro M., Pizzi N., Luongo A., Stability, Bifurcation and Post-critical Behaviour of Elastic Structures, Elsevier, London, 1991
[6] Chandra R., Raju B.B., Post-buckling analysis of rectangular orthotropic plates, Int. J. of Mech. Sci., 1973, Vol. 16, pp. 81-89
[7] Towstik P.E., Stability of thin-walled shells, Moscow, 1995 (*in Russian*)
[8] Królak M., Kowal-Michalska K., Świniarski J., Optimisation regarding local stability and strength of long multilayered plates subjected to compression, Engineering Transaction, 51, 1, 2003.

Analiza parametryczna stateczności i nośności pryzmatycznych segmentów powłokowych poddanych ściskaniu

Streszczenie

Praca dotyczy analizy parametrycznej stateczności i nośności pryzmatycznych powłok segmentowych zbudowanych z prostokątnych wycinków powłok walcowych i poddanych ściskaniu. Analizowane są powłoki segmentowe (słupy) o stałej powierzchni przekroju poprzecznego (ciężarze), przy czym wszystkie uzyskane wyniki porównane są z wynikami uzyskanymi dla walcowej powłoki o promieniu R i grubości t_w.

Najpierw analizowano wpływ parametrów geometrycznych przekroju poprzecznego dla powłok jednowarstwowych izotropowych i poszukiwano takich kształtów przekroju przy których nośność była znacznie wyższa niż dla powłoki walcowej.

W następnej kolejności, dla wybranego kształtu powłoki (na ten wybór oprócz większej nośności mogły mieć wpływ inne czynniki np. łatwość wykonania), analizowano wpływ układu i grubości ortotropowych warstw powłoki (laminatu) również na stateczność i nośność.

Analiza wykazała, że z tego samego materiału ortotropowego można zaprojektować powłokę segmentową warstwową o znacznie większej odporności na wyboczenie i o większej nośności w porównaniu z jednowarstwową powłoką walcową ortotropową. Wyniki analizy przedstawiono w postaci wykresów.