THE ARCHIVE OF MECHANICAL ENGINEERING

Vol. L 2003 Number 3

Key words: *pump, flow characteristics, power consumption characteristics, geometry of flow channels*

ANDRZEJ BŁASZCZYK⁾*

RELATIONSHIPS BETWEEN THE CONDITIONS THAT DETERMINE THE SHAPE OF THE FLOW AND POWER CONSUMPTION CHARACTERISTICS AND THE PUMP DESIGN AND PERFORMANCE PARAMETERS

The relationships between the conditions that describe the shape of the flow and power consumption characteristics and the pump design and performance parameters are described.

These relations concern:
- flow characteristics (throttling curves) described with a fourth-order polynomial,
- non-overloading power consumption characteristics of the pump.

The pumps that have to exhibit such characteristics are these designed to operate in an arbitrary installation. These pumps must also be characterised by cavitation-free operation in the whole range of discharge variability. In the relations presented, the condition of cavitation-free operation is considered as well.

NOMENCLATURE

A – flow cross-sectional area,
b – width,
c – absolute velocity,
D – diameter,
H – total head,
K – value of the function derivative,
$\bar{P}$ – dimensionless power at the pump shaft,
$\bar{P}_t$ – dimensionless friction power at the impeller shrouds,

*⁾ *Institute of Turbomachinery, Technical University of Lodz, 219/223 Wólczanska St. 93-005 Lodz, Poland E-mail: akblaszc@p.lodz.pl*

$\bar{P}_m$ – dimensionless friction power in bearings and glands,
n – rotational speed,
n_q – kinematic specific speed,
u – transport velocity,
Q – discharge, volume flow rate,
w – relative velocity,
β – relative flow angle,
β^* – slope angle of the blade camber line,
β_0 – angle between $\boldsymbol{u}$ and $\boldsymbol{w}$ at the flow around the blade with a zero load discharge,
μ – coefficient of the blocking area,
τ_c – dimensionless factor of the circumferential component of the absolute fluid velocity,
φ – pump discharge coefficient,
φ_2 – impeller discharge coefficient,
Ψ – head coefficient,
Ψ_u – impeller head coefficient.

All quantities are expressed in the fundamental units of the SI system.

Indices

M – point of the maximum power consumption,
max – maximum,
N – nominal,
s – losses,
1 – blade leading edge,
2 – blade trailing edge.

1. Introduction

The development of the design methods of impeller pumps that are based on the one-dimensional theory consists in a more precise expression of empirical formulas that describe the relations between hydraulic and geometrical parameters of centrifugal pump hydraulic components.

In the case of pumps that operate in stationary installations in the neighbourhood of the nominal point (H_N, Q_N), these relations are the relations between hydraulic and geometrical parameters occurring in the nominal point.

Pumps operating in an arbitrary installation are often required to have a specified shape of the flow and power consumption characteristics and to be characterised by cavitation-free operation in the whole range of discharge. Then, the algorithms of design methods should take into account relationships between the conditions that determine the flow and power consumption characteristics and the pump design and hydraulic parameters in the whole range of discharge variability.

The empirical formulas of the design method of pumps characterised by special performance requirements are presented below.

2. Conditions that determine the shape of the flow and power consumption characteristics and cavitation-free operation of the pump

2.1. General remarks

Pumps designed for operation under various installation conditions are required to have the required shape of:

- stable flow characteristics that fulfils the condition $\frac{d\Psi}{d\varphi} < 0$ for $0 \le \varphi \le \varphi_{max}$,
- non-overloading power consumption characteristics that satisfies the conditions $\frac{d\bar{P}}{d\varphi} = 0$, $\bar{P}(\varphi) \le \bar{P}_{SE}$ in the range $0 \le \varphi \le \varphi_{max}$, where $\bar{P}_{SE}$ – electric motor rated power,

and to be characterised by cavitation-free operation in the whole range of the discharge variability.

2.2. Conditions determining the shape of the flow characteristics $\Psi(\varphi)$

On the basis of the results of experimental investigations presented in [3], [7], it has been established that the flow characteristics of pumps with the specific speeds $n_q = 10 \div 60$ can be described in the best way by the following fourth-order polynomial:

$$\Psi = a_0 + a_1\varphi + a_2\varphi^2 + a_3\varphi^3 + a_4\varphi^4 \tag{2.1}$$

where: $a_0 \div a_4$ – constants related to the shape of the flow characteristics.

In order to define the shape of the curve described by Eq. (2.1), it has been assumed that the following conditions are sufficient:

$$\Psi = \Psi_{max} \qquad \text{for } \varphi = 0 \tag{2.2.a}$$

$$\Psi = 0 \qquad \text{for } \varphi = \varphi_{max} \tag{2.2.b}$$

$$\Psi = \Psi_N \qquad \text{for } \varphi = \varphi_N \tag{2.2.c}$$

$$\frac{d\Psi}{d\varphi} = K_N \qquad \text{for } \varphi = \varphi_N \tag{2.2.d}$$

$$\frac{d\Psi}{d\varphi} = K_0 < 0 \qquad \text{for } \varphi = 0 \tag{2.2.e}$$

Conditions (2.2.a)÷(2.2.e) are illustrated graphically in Fig. 1.

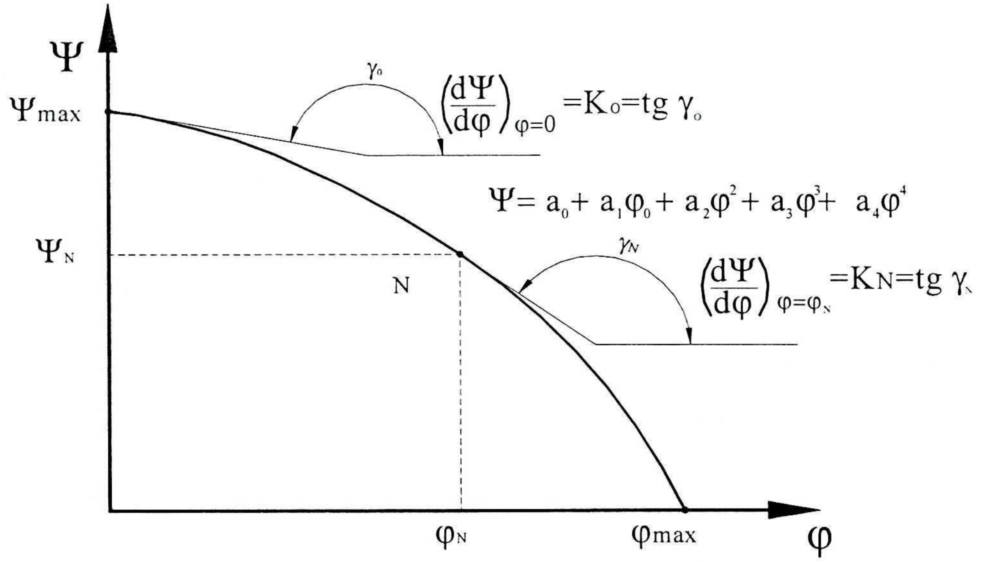


Fig. 1. Conditions describing the pump flow characteristics $\Psi(\varphi)$

The relation between conditions (2.2.a÷2.2.e) and the coefficients of the polynomial $a_0 \div a_4$ are presented in the form of Eqs. (2.3.a÷2.3.e):

$$a_0 = \Psi_{max} \tag{2.3.a}$$

$$a_1 = K_0 \tag{2.3.b}$$

$$a_2 = -3\frac{1}{\varphi_N^2}(\Psi_{max} - \Psi_N) - \frac{1}{\varphi_N}(K_N + K_0) - \frac{1}{\varphi_N}K_0 +$$

$$+\frac{\varphi_N^2}{\varphi_{max}^2(\varphi_N - \varphi_{max})^2}\cdot\left\{\frac{\varphi_{max}^2}{\varphi_N^2}(\Psi_{max} - \Psi_N)\left(3-2\frac{\varphi_{max}}{\varphi_N}\right) - \Psi_{max} + \right. \tag{2.3c}$$

$$\left. -\frac{\varphi_{max}^2}{\varphi_N}(K_N - K_0)\left(1 - \frac{\varphi_{max}}{\varphi_N}\right) + \varphi_{max}\left[\frac{\varphi_{max}}{\varphi_N}\left(3-2\frac{\varphi_{max}}{\varphi_N}\right) - 1\right]K_0\right\}$$

$$a_3 = 2 \cdot \frac{1}{\varphi_N^3} (\Psi_{max} - \Psi_N) + \frac{1}{\varphi_N^2} (K_N + K_0) - \frac{2\varphi_N}{\varphi_{max}^2 (\varphi_N - \varphi_{max})^2} \cdot$$

$$\left\{ \frac{\varphi_{max}^2}{\varphi_N^2} (\Psi_{max} - \Psi_N) \left(3 - 2\frac{\varphi_{max}}{\varphi_N} \right) - \Psi_{max} + \right. \tag{2.3.d}$$

$$\left. - \frac{\varphi_{max}^2}{\varphi_N} (K_N - K_0) \left(1 - \frac{\varphi_{max}}{\varphi_N} \right) + \varphi_{max} \left[\frac{\varphi_{max}}{\varphi_N} \left(3 - 2\frac{\varphi_{max}}{\varphi_N} \right) - 1 \right] K_0 \right\}$$

$$a_4 = \frac{1}{\varphi_{max}^2 (\varphi_N - \varphi_{max})^2} \cdot \left\{ \frac{\varphi_{max}^2}{\varphi_N^2} (\Psi_{max} - \Psi_N) \left(3 - 2\frac{\varphi_{max}}{\varphi_N} \right) - \Psi_{max} + \right.$$

$$\tag{2.3.e}$$

$$\left. - \frac{\varphi_{max}^2}{\varphi_N} (K_N - K_0) \left(1 - \frac{\varphi_{max}}{\varphi_N} \right) + \varphi_{max} \left[\frac{\varphi_{max}}{\varphi_N} \left(3 - 2\frac{\varphi_{max}}{\varphi_N} \right) - 1 \right] K_0 \right\}$$

In the case when the values of five coefficients $a_0 \div a_4$ of polynomial (2.1) are the input data in the designing process, the conditions (2.2.a÷2.2.e) assume the form of the following equations:

$$\Psi_{max} = a_0 + a_1 \cdot 0 + a_2 \cdot 0 + a_3 \cdot 0 + a_4 \cdot 0 \tag{2.4.a}$$

$$0 = a_0 + a_1 \cdot \varphi_{max} + a_2 \cdot \varphi_{max}^2 + a_3 \cdot \varphi_{max}^3 + a_4 \cdot \varphi_{max}^4 \tag{2.4.b}$$

$$\Psi_N = a_0 + a_1 \cdot \varphi_N + a_2 \cdot \varphi_N^2 + a_3 \cdot \varphi_N^3 + a_4 \cdot \varphi_N^4 \tag{2.4.c}$$

$$K_N = a_1 + 2a_2 \cdot \varphi_N + 3a_3 \cdot \varphi_N^2 + 4a_4 \cdot \varphi_N^3 \tag{2.4.d}$$

$$K_0 = a_1 + 2a_2 \cdot 0 + 3a_3 \cdot 0 + 4a_4 \cdot 0 \tag{2.4.e}$$

The known quantities $a_0 \div a_4$ and the assumed quantity φ_N allow for calculating the values of conditions (2.2.a÷2.2.e) from Eqs. (2.4.a÷2.4.e). Design methods applicable to the type of pumps described here are based on the relationships between the values of conditions (2.2.a÷2.2.e) and the hydraulic and geometrical parameters of hydraulic systems of these machines.

2.3. Conditions determining the shape of the pump non-overloading power consumption characteristics

The dimensionless equation of the pump power consumption characteristics expressed as:

$$\bar{P} = \bar{P}_u + \bar{P}_t + \bar{P}_m, \tag{2.5}$$

can be written in the form:

$$\bar{P} = [\Psi_u(\varphi_2)] \cdot \varphi_2 + \bar{P}_t + \bar{P}_m \tag{2.6}$$

Taking into account Eq. (2.6) in (2.7) (relations describing the pump non-overloading power consumption characteristics):

$$\left.\begin{aligned} \left(\frac{d\bar{P}}{d\varphi}\right)_{\varphi = \varphi M} &= 0 \\ [\bar{P}(\varphi)]_{max} &\leq \bar{P}_{SE} \end{aligned}\right\} \tag{2.7}$$

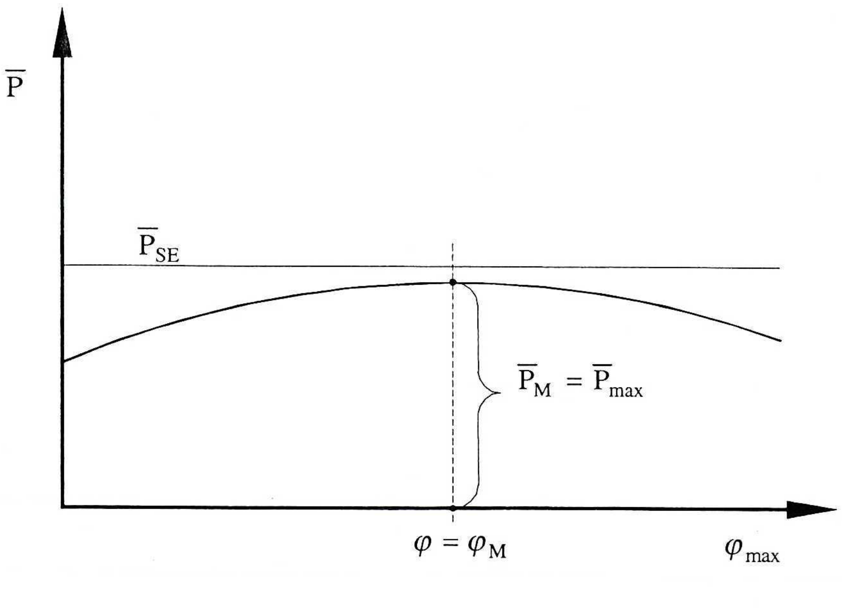


Fig. 2. Pump non-overloading power consumption characteristics

we obtain formula (2.8):

$$\frac{d\bar{P}}{d\varphi_2} = \left(\frac{d[(\Psi_u(\varphi_2) \cdot \varphi_2)]}{d\varphi_2}\right)_{\varphi_2 = \varphi_{2M}} = 0 \tag{2.8}$$

The condition $\frac{d\bar{P}}{d\varphi_2} = 0$ is equivalent to the condition $\frac{d\bar{P}}{d\varphi} = 0$ *).

Figure 2 presents graphically the system of Eqs. (2.7).

2.4. Conditions determining the pump cavitation-free operation

The pumps under analysis should be also characterised by cavitation-free operation in the whole range of discharge variability ($0 < \varphi < \varphi_{max}$). The main parameters that decide about the cavitation at the impeller inlet are the inlet blade diameter D_1 and the blade width b_1.

In [2], the way of determination of the boundary dimensions of the diameter D_1 and the width b_1 for the pump type under consideration has been given. The empirical formulas for determination of the values of D_{1gr} and b_{1gr} are given below:

$$\left.\begin{aligned} D_{1gr} &= \frac{c_D}{\pi \cdot n} \\ b_{1gr} &= \frac{\varphi_{max} \cdot u_2^2 \cdot b_2 \cdot \mu_2}{c_b} \end{aligned}\right\} \tag{2.9}$$

where: $c_D \leq 13.2$ m/s, $c_b \leq 79.2$ m^2/s^2 constant coefficients.

The condition for cavitation-free operation of the pump in the whole range of discharge variability is as follows:

$$\left.\begin{aligned} D_1 &\leq D_{1gr} \\ b_1 &\leq b_{1gr} \end{aligned}\right\} \tag{2.10}$$

*) $\frac{d\bar{P}}{d\varphi} = \frac{d\bar{P}}{d\varphi_2} \cdot \frac{d\varphi_2}{d\varphi}$; $\frac{d\varphi_2}{d\varphi} = \eta_v + \varphi \frac{d\eta_v}{d\varphi} > 0$, as $\frac{d\eta_v}{d\varphi} > 0$; thus, if $\frac{d\bar{P}}{d\varphi_2} = 0$, then also $\frac{d\bar{P}}{d\varphi} = 0$ where η_v is pump volumetric efficiency.

3. Relationships between the conditions that determine the shape of the flow characteristics $\Psi(\varphi)$ and the power consumption characteristics $\bar{P}(\varphi)$ of the pump and the geometrical and hydraulic parameters of the impeller

3.1. General remarks

The basic quantities that decide about the shapes of the characteristics $\Psi(\varphi)$ and $\bar{P}(\varphi)$ are the geometrical and hydraulic parameters of the impeller.

The flow of the working medium through the pump hydraulic system is of the three-dimensional nature. In the calculations of flow parameters, CFD methods are used to investigate the flow structure in the pump channels designed with the methods based on simplified models. A one-dimensional model is commonly used in these methods. It is assumed in it that:

- flow through passages is referred to the mean streamline,
- averaged velocities along the areas of selected cross-sections of passages and related to the central streamline can be depicted in the form of velocity triangles, (Fig. 3),
- equations of mass and energy conservation hold in the conventional control cross-sections,
- displacement of the velocity triangle vertex at the impeller outlet takes place along the line that corresponds to the hypothesis of the impeller operation under variable conditions $\beta_2 = \text{const}$ [5] or $B = \text{const}$ [1], [3], [7], in a wide range of discharge variability.

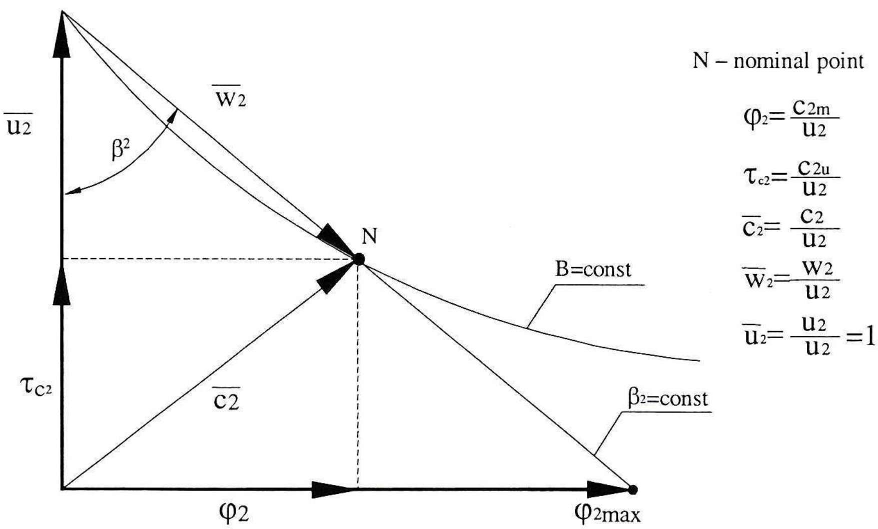


Fig. 3. Changes in the position of the outlet velocity triangle vertex during variations in discharge for the hypothesis assuming that B = const and β_2 = const

The hypothesis that $B = \text{const}$ has been assumed. According to it, the angle between the blade design angle and the medium flow angle at the impeller outlet ϑ_2 is equal to:

$$\vartheta_2 = \beta_2^* - \beta_2 \tag{3.1}$$

where: β_2^* – blade design angle at the impeller outlet,
β_2 – medium flow angle at the impeller outlet,

or

$$\vartheta_2 = B\,(\beta_2^* - \beta_{02}) \tag{3.2}$$

where: β_{02} – medium flow angle at the blade outlet with a zero load discharge (the pump for which $H = 0$),
B – impeller shape coefficient, which is a function of geometrical parameters of the impeller,

$$B = \mathrm{k} \cdot \frac{\mu_2}{\mu_1} \cdot \frac{b_2 D_2}{b_1 D_1} \cdot \frac{b_2}{D_2 - D_1} \cdot \frac{1}{\sqrt{\tau}} \tag{3.3}$$

where: k – coefficient in Kuczewski's method [6],
μ_2 and μ_1 – coefficients of the blocking area at the impeller outlet and inlet, respectively,
D_2 – impeller outlet diameter,
b_2 – impeller width at the outlet,
D_1 – impeller blade inlet diameter,
b_1 – impeller blade inlet width,
τ – density of the impeller blade cascade, solidity.

According to the hypothesis that $B = \text{const}$, the vertex of the medium velocity triangle at the impeller outlet displaces with changes in discharge along the curve shown in Fig. 3.

The relations resulting from the assumption of the hypothesis that $B = \text{const}$, which relate the conditions determining the pump flow and power consumption characteristics to the hydraulic and geometrical parameters, are presented below.

In Fig. 4, the main geometrical dimensions of the impeller are presented schematically. The notations applied are used in the subsequent sections of the present paper.

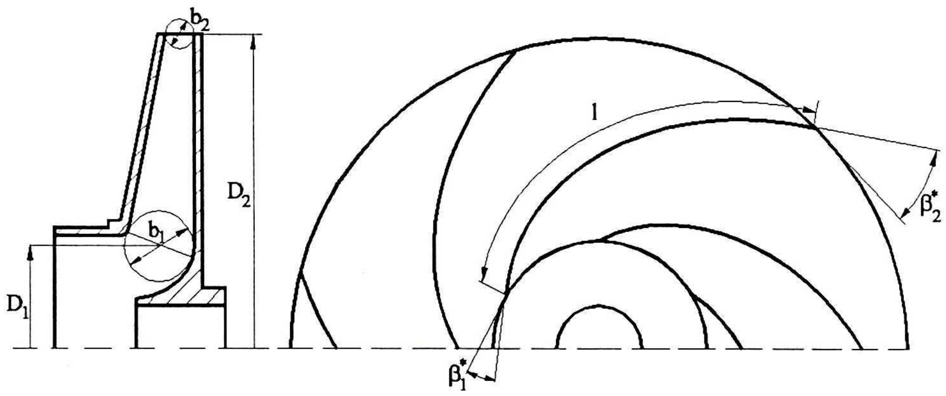


Fig. 4. Main geometrical dimensions of the impeller

3.2. Head coefficient Ψ_{max} for $\varphi = 0$

As it can be seen in Fig. 1, the maximum value of the head coefficient occurs at $\varphi = 0$. According to [1], [3], it is calculated from the following relationship:

$$\Psi_{max} = a_0 = 2 \cdot k_H \tag{3.4}$$

where: k_H – coefficient of the relative head, $k_H = \dfrac{\Psi_{max}}{\Psi_{u\ max}}$.

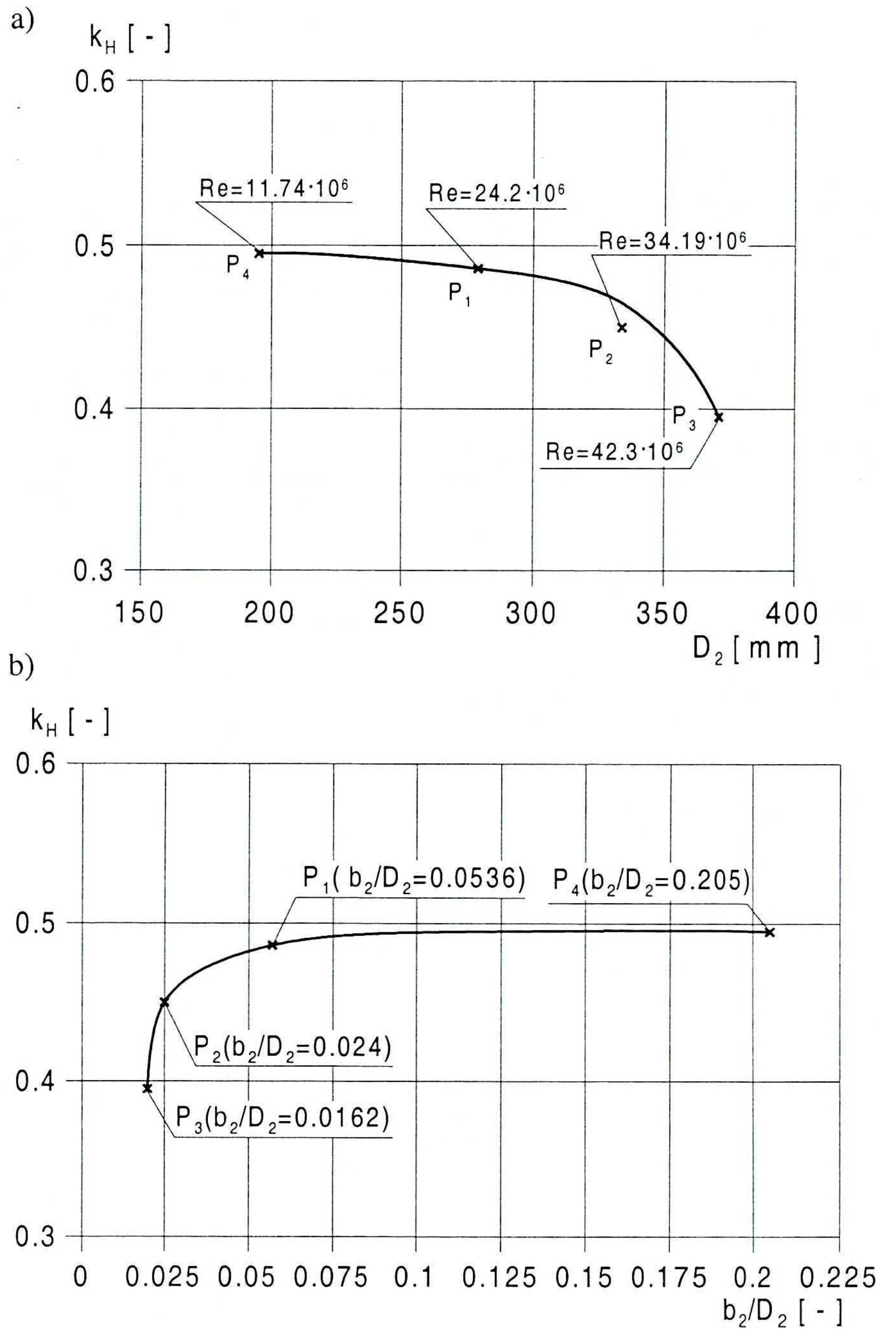
For the value of k_H known from the diagram $k_H(D_2)$ shown in Fig. 5a, we can read the value of the diameter D_2, and from the diagram $k_H(b_2/D_2)$, Fig. 5b, the value of the ratio b_2/D^2, from which b_2 is determined for the known diameter D_2.

3.3. Maximum discharge coefficient

The second boundary parameter of the flow characteristics that corresponds to $\Psi = 0$ is the maximum discharge coefficient φ_{max} (condition 2.2.b) determined from Eq. (2.4.b). On the basis of experimental investigations [1], [3], [7], it has been established that for the type of pumps under consideration, the following relationship holds:

$$\varphi_{max} = \mathrm{tg}\beta_{2N} \rightarrow \beta_{2N} = \mathrm{arc\ tg}\,\varphi_{max} \tag{3.5}$$

where: β_{2N} – medium outlet angle at the impeller outlet for the nominal flow through the impeller.



Figs 5a and b. Coefficients of the relative head $k_H(D_2)$ and $k_H(b_2/D_2)$

3.4. Impeller shape coefficient B

The subsequent relationships between the impeller geometry and hydraulic parameters result from Eq. (2.8). This equation can be also written as follows:

$$\frac{d\bar{P}}{d\varphi_2} = \frac{d(2\tau_{c2} \cdot \varphi_2)}{d\varphi_2} \tag{3.6}$$

where: τ_{c2} – dimensionless circumferential component of the absolute fluid velocity, $\tau_{c2} = \frac{1}{2}\Psi_u$.

From the velocity triangle shown in Fig. 3, the following relation results:

$$\tau_{c2} = 1 - \varphi_2 \operatorname{ctg} \beta_2 \tag{3.7}$$

According to the hypothesis assumed B = const

$$\beta_2 = (1 - \beta)\beta_2^* + B \operatorname{arc\,tg} \varphi_2 \tag{3.8}$$

Taking into account relation (3.8) in (3.7), we obtain:

$$\tau_{c2} = 1 - \varphi_2 \operatorname{ctg} \left[\beta_2^*(1 - B) + B \operatorname{arc\,tg} \varphi_2\right] \tag{3.9}$$

If we take into account formula (3.9) in Eq. (3.6) and assume that the first derivative equals zero for the discharge φ_{2M}, then we obtain the following relation:

$$B = \frac{(1 + \varphi_{2M}^2)(1 - 2\tau_{c2M})}{\varphi_{2M}^2 + (1 - \tau_{c2M})^2} \tag{3.10}$$

When we consider (3.3) in (3.10), the following equality can be written:

$$\frac{(1 + \varphi_{2M}^2)(1 - 2\tau_{c2M})}{\varphi_{2M}^2 + (1 - 2\tau_{c2M})^2} = k \cdot \frac{\mu_2 b_2 D_2}{\mu_1 b_1 D_1} \cdot \frac{b_2}{D_2 - D_1} \cdot \frac{1}{\sqrt{\tau}} \tag{3.11}$$

Relation (3.11) relates the geometrical parameters of the impeller to the hydraulic parameters in the maximum power consumption point M.

3.5. Hydraulic parameters of the maximum power consumption point M

The hydraulic parameters that determine the position of the maximum power consumption point are, according to [1, 3], φ_{2M} and τ_{c2M}, described by the equations:

$$\varphi_{2M} = \frac{(Q_W)_M}{A_2 u_2} \tag{3.12}$$

$$\tau_{c2M} = 1 - \varphi_{2M} ctg\left[\beta_2^*(1 - B) + B \operatorname{arc\,tg}\varphi_{2M}\right] \tag{3.13}$$

On the basis of the investigations carried out [1, 3, 7], it is suggested to assume the values of φ_{2M} from the range:

$$\operatorname{tg}\frac{\beta_{2N}}{2} \leq \varphi_{2M} \leq \operatorname{tg}\beta_{2N} \tag{3.14}$$

For the assumed φ_{2M} and the determined angle β_{2N} (3.5), the value of τ_{c2M} is calculated from the formula:

$$\tau_{c2M} = \frac{\operatorname{tg}\beta_{2N}}{4\varphi_{2M}} \tag{3.15}$$

The known values of the co-ordinates of the maximum power consumption φ_{2M} and τ_{c2M} allow for calculating the flow angle β_{2M} on the peripheral diameter D_2:

$$\beta_{2M} = \operatorname{arc\,tg}\frac{\varphi_{2M}}{1 - \tau_{c2M}} \tag{3.16}$$

The known value of the angle β_{2M} is employed to determine the design angle β_2^*. From the transformation of relation (3.8) for $\varphi_2 = \varphi_{2M}$, we obtain:

$$\beta_2^* = \frac{\beta_{2M} - B \operatorname{arc\,tg}\varphi_{2M}}{1 - B} \tag{3.17}$$

3.6. Hydraulic parameters of the impeller nominal point

The dimensionless co-ordinates that describe the position of the impeller nominal point are φ_{2N} and Ψ_{uN}. The relationship defining the impeller nominal discharge coefficient as a function of its hydraulic and geometrical parameters, generated in [1], has the following form:

$$\varphi_{2N} = \frac{\mathrm{tg}\beta_{2N} - \mathrm{tg}\,[\beta_2^*(1 - B)]}{B\{1 + \mathrm{tg}\beta_{2N} \cdot \mathrm{tg}\,[\beta_2^*(1 - B)]\}} \tag{3.18}$$

On the basis of the relation $\tau_{c2N} = \frac{1}{2}\Psi_u$ and taking into account formula (3.9) for the nominal discharge, we can write:

$$\Psi_{uN} = 2\left\{1 - \varphi_{2N}\,\mathrm{ctg}\left[(1 - B)\,\beta_2^* + B\,\mathrm{arc\,tg}\varphi_{2N}\right]\right\} \tag{3.19}$$

3.7. Relation between the nominal hydraulic parameters of the pump and the impeller

Both the pairs of parameters are related to each other by efficiencies. The head coefficients Ψ_N and Ψ_{uN} are related to each other by the hydraulic efficiency $(\eta_h)_N$ (formula (3.20)), and the coefficients φ_N and φ_{2N} by the volumetric efficiency $(\eta_\upsilon)_N$ (formula (3.21)).

$$(\eta_h)_N = \frac{\Psi_N}{\Psi_{uN}} \tag{3.20}$$

$$(\eta_\upsilon)_N = \frac{\varphi_N}{\varphi_{2N}} \tag{3.21}$$

3.8. Derivatives of the flow characteristics of the pump $\Psi(\varphi)$ and the impeller $\Psi_u(\varphi_2)$

The value of the derivative of the flow characteristics $\Psi(\varphi)$, K_N, (formula (2.2.d)) is determined from Eq. (2.4.d) for the know value of φ_N. In the light of the fact that in the vicinity of the nominal point of the pump operation, the total flow losses are the least:

$$\left(\sum \Psi_s\right)_{\varphi = \varphi_N} = \left(\sum \Psi_s\right)_{\min} \tag{3.22}$$

the first derivatives of the flow characteristics of the pump $\Psi(\varphi)$ and the impeller $\Psi_u(\varphi_2)$ for this efficiency should satisfy the equation *):

$$\left.\begin{aligned} &\frac{(\eta_v)_N}{(\eta_h)_N}\cdot\left(\frac{d\Psi}{d\varphi}\right)_{\varphi=\varphi_N}=\left(\frac{d\Psi_u}{d\varphi_2}\right)_{\varphi_2=\varphi_{2N}}=K_N' \\ &\frac{(\eta_v)_N}{(\eta_h)_N}\cdot K_N=K_N' \end{aligned}\right\} \tag{3.23}$$

The graphical illustration of condition (3.23) is presented in Fig. 6.

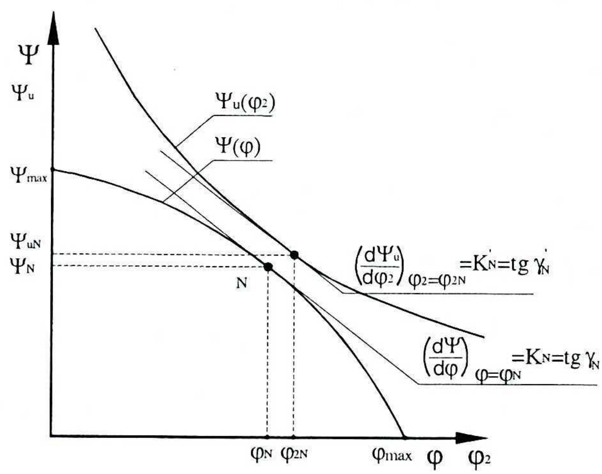


Fig. 6. Pump flow characteristics $\Psi(\varphi)$, impeller flow characteristics $\Psi_u(\varphi_2)$

*) As $\Psi_u = \dfrac{\Psi}{\eta_h}$; $\varphi_2 = \dfrac{\varphi}{\eta_v}$; $\varphi = \varphi_2 \cdot \eta_v$, due to the fact that:

– $\dfrac{d\varphi}{d\varphi_2} = \eta_v + \varphi_2 \dfrac{d\eta_v}{d\varphi_2} \cong \eta_v$,

– $\dfrac{d\Psi_u}{d\varphi_2} = \dfrac{d}{d\varphi_2}\left(\dfrac{\Psi}{\eta_h}\right) = \dfrac{d}{d\varphi}\left(\dfrac{\Psi}{\eta_h}\right)\cdot\dfrac{d\varphi}{d\varphi_2} = \eta_v \cdot \dfrac{d}{d\varphi}\left(\dfrac{\Psi}{\eta_h}\right)$

– $\dfrac{d}{d\varphi}\left(\dfrac{\Psi}{\eta_h}\right) = \dfrac{1}{\eta_h}\cdot\dfrac{d\Psi}{d\varphi} + \Psi\left(\dfrac{-1}{\eta_h^2}\cdot\dfrac{d\eta_h}{d\varphi}\right)$; for the maximum of the function $\eta_h(\varphi)$,

$\dfrac{d\eta_h}{d\varphi} = 0$, that is to say $\dfrac{d}{d\varphi}\left(\dfrac{\Psi}{\eta_h}\right) = \dfrac{1}{\eta_h}\cdot\dfrac{d\Psi}{d\varphi}$,

the exact relationship between the slope of the tangents to the function $\Psi(\varphi)_{\varphi=\varphi_N}$ and $\Psi_u(\varphi_2)_{\varphi_2=\varphi_{2N}}$ assumes the form:

$$\left(\frac{d\Psi_u}{d\varphi_2}\right)_{\varphi_2=\varphi_{2N}} = \frac{\eta_v}{\eta_h}\cdot\left(\frac{d\Psi}{d\varphi}\right)_{\varphi=\varphi_N}$$

The relation that describes the derivative of the impeller flow characteristics, which is to be found in [3, 7] for the nominal flow, has the form:

$$\left(\frac{d\Psi_u}{d\varphi_2}\right)_{\varphi_2=\varphi_{2N}} = -2\left\{\frac{1-\tau_{c2N}}{\varphi_{2N}} - \left[\left(\frac{1-\tau_{c2N}}{\varphi_{2N}}\right)^2+1\right]\cdot\frac{B\cdot\varphi_{2N}}{1+\varphi_{2N}^2}\right\} = K_N' \quad (3.24)$$

3.9. Derivatives of the pump flow characteristics $\Psi(\varphi)$ for $\varphi = 0$

When condition (2.2.e) and Eq. (2.4.e) are fulfilled, then the pump stable flow characteristics $\Psi(\varphi)$ is assured.

4. Conclusions

The presented relationships between the conditions that determine the shape of the pump flow and power consumption characteristics should be employed in the algorithm of the design method of the pumps that satisfy the requirements given in section 2.1. The numerical coefficient values in the relationships have been established on the basis of experimental investigations.

Manuscript received by Editorial Board, July 02, 2003;
final version, October 31, 2003

REFERENCES

[1] Błaszczyk A.: Metoda projektowania pomp promieniowych o nieprzeciążalnych charakterystykach poboru mocy. PhD Thesis, Łódź, 1976 (in Polish).

[2] Błaszczyk A.: Sposób wyznaczania parametrów na wlocie do wieńca łopatkowego promieniowego koła wirnikowego. Proc. Scientific Conference HYDROFORUM'85, Gdańsk, September 1985 (in Polish).

[3] Błaszczyk A.: Teoretyczne i doświadczalne badanie struktury przepływu w elementach układów przepływowych pomp przeznaczonych do pracy w różnych warunkach zainstalowania. Report on project no. PB7TO7B04812, Łódź, 2000 (in Polish).

[4] Błaszczyk A.: Method for Designing Pumps Characterised by Special Performance Requirements. Scientific Bulletin of Technical University of Lodz. Cieplne Maszyny Przepływowe – Turbomachinery, no. 122, 2002.

[5] Jędral W.: Pompy wirowe odśrodkowe. Warsaw Technical University Publishers, Warsaw, 1996 (in Polish).

[6] Kuczewski St.: Metoda obliczania kąta wypływu. Reports of the Institute for Heat Technology (ITC), no. 15, IX Warsaw 1961 (in Polish).

[7] Staniszewski J.: Metoda projektowania pomp promieniowych o żądanym kształcie charakterystyki przepływowej. PhD Thesis. Łódź, 1982 (in Polish).

Współzależność warunków określających kształt charakterystyki przepływu i poboru mocy z parametrami konstrukcyjnymi i ruchowymi pompy

Streszczenie

W artykule przedstawiono zależności pomiędzy warunkami opisującymi kształt charakterystyki przepływu i poboru mocy z parametrami konstrukcyjnymi i ruchowymi pompy.

Związki te dotyczą:

- charakterystyki przepływu opisanej wielomianem czwartego stopnia,
- nieprzeciążalnej charakterystyki poboru mocy przez pompę.

Założono, że do określenia kształtu charakterystyki przepływu opisanej równaniem

$$\Psi = a_0 + a_1\varphi + a_2\varphi^2 + a_3\varphi^3 + a_4\varphi^4$$

wystarczające warunki (2.2.a) ÷ (2.2.e) graficznie przedstawione na Rys. 1.

Układ równań (2.4.a) ÷ (2.4.e) dla zadanych wartości $a_0 \div a_4$ i przyjętej φ_N pozwala wyznaczyć wielkości Ψ_{max}, φ_{max}, φ_N, K_N, K_0, które następnie wykorzystuje się w obliczeniach parametrów geometrycznych kanałów hydraulicznych pompy (rozdział 3).

Realizacja nieprzeciążalnej charakterystyki poboru mocy (równanie (2.7), Rys. 3) stawia dodatkowe wymagania w stosunku do geometrii kanałów hydraulicznych uzgodnione z warunkami opisującymi kształt charakterystyki przepływu (rozdział 3).

Realizacja zadanych kształtów charakterystyki przepływu i poboru mocy wymagana jest od pomp przeznaczonych do współpracy z dowolną instalacją, od których wymagana jest też bezkawitacyjna praca w całym zakresie zmian wydajności. Również i ten warunek jest uwzględniony w omawianych zależnościach (rozdział 2.4, wzór (2.9)).

Przedstawione w artykule związki i zależności pomięczy warunkami określającymi kształty charakterystyki przepływu i poboru mocy oraz bezkawitacyjną pracą pompy powinny być wykorzystane w algorytmie projektowania pomp przeznaczonych do współpracy z dowolną instalacją.