

**Key words:** *buckling, load carrying capacity, thin-walled structures, composite, intermediate stiffeners*

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## MODAL INTERACTIVE BUCKLING OF COMPOSITE TUBULAR POLE STRUCTURE WITH INTERMEDIATE STIFFENERS

The analysis of buckling, post-buckling behaviour and load carrying capacity of prismatic composite pole structures is conducted. The asymptotic expansion established by Byskov-Hutchinson is used in the second order approximation. The thin-walled tubular columns are simply supported at the ends and subject to the uniform compression. Several types of cross-sections with and without intermediate stiffeners are considered. The present paper is the continuation of a previous paper by the authors (1999) where the modal interaction of thin-walled composite beam-columns was investigated.

### Some notation

$M_{ix}, M_{iy}, M_{ixy}$	bending moments resultants for the $i$ -th wall,
$N_{ix}, N_{iy}, N_{ixy}$	in-plane resultants for the $i$ -th wall,
$u_i, v_i, w_i$	displacement components of middle surface of the $i$ -th wall,
$\epsilon_{ix}, \epsilon_{iy}, \epsilon_{ixy}$	strain tensor components for the middle surface of the $i$ -th wall,
$\eta = E_{iy} / E_i$	ratio of Young's moduli in main directions of orthotropy,
$\kappa_{ix}, \kappa_{iy}, \kappa_{ixy}$	curvature modifications and torsion of the middle surface of the $i$ -th wall,
$\lambda^*$	scalar load parameter,
$\sigma_j^* = \sigma_j 10^{-3} E$	dimensionless stress of the $j$ -th buckling mode,
$\sigma_s^*$	limit dimensionless stress for imperfect structure.

Other notations are given in text.

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## 1. Introduction

Thin-walled tubular columns are very often used in many structures such as: sport halls, market halls, halls of a railway, umbrella roofs, cranes, etc. Such columns are also used as supporting structures: power transmission lines, communications lines, circus tents, highways and squares illuminations and as commercial posts. Until now, the steel columns have been frequently used. In recent years, the tubular poles are more and more often made of fibrous composites, what can be attributed to the following reasons:

- high resistance of some fibrous composites to aggressive chemical compounds and to corrosion,
- high strength-to-weight and stiffness-to-weight ratios.

In designing thin-walled structures, not only has the sufficient strength be ensured but also the global stability of the structure and stability of component elements – so called the local stability. The problems of strength, stiffness and even the problems of buckling loads (global and local) of steel tubular pole structures are very well known (Cannon and LeMaster (1987), Dicleli (1997), Design of Steel Transmission Pole Structures (1990)), whereas the problems of modal interactive buckling that may occur in thin-walled tubular columns of polygonal cross-sections are still not sufficiently investigated. It concerns especially the columns made of composite materials of anisotropic properties.

The determination of the load carrying capacity of thin-walled structures requires consideration of the modal interaction of buckling modes in nonlinear analysis of stability. The problem of interaction of the global mode with the local ones, or between local buckling modes, is of great significance.

The problem of local buckling is of special importance because it causes reduction of stiffness of a section, and consequently lowers its load carrying capacity relative to a non-locally buckled section.

Sometimes, very low values of local buckling load, and in effect very low carrying capacity of analysed columns, show the necessity of reinforcing walls by intermediate stiffeners or membranes.

In this paper, the influence of intermediate stiffeners and membranes on buckling and postbuckling behaviour, and also on the load carrying capacity of analysed structures, is investigated.

## 2. Formulation of the problem and basic equations

The considerations concern stability and modal interactive buckling of composite thin-walled tubular pole structure with or without intermediate stiffeners.

The derived equations and formulas, as well as the elaborated computer program, (Kołakowski and Królak (1995)) concern rather widely comprehended stability analysis of thin-walled beam-columns made of composite (orthotropic)

materials of different shapes of cross-sections, subjected to simultaneous compression and bending. However, in the present work the analysis is restricted to the columns of cross-sections in form of a regular polygon with number of sides  $N=4\div 36$  (Fig. 1) subject to the uniform compression.

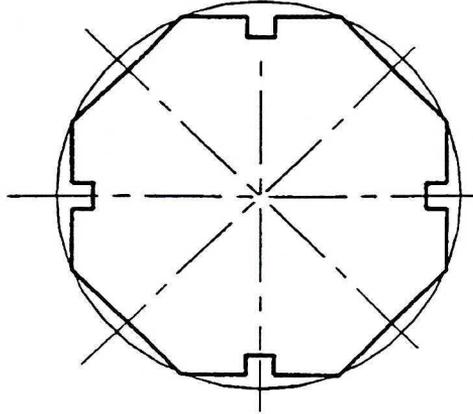


Fig. 1. A type of considered cross-section

The columns of an arbitrary length  $l$  simply supported at the ends are considered. The column walls are made of the composite materials, e.g. fibrous composites, in which the fibres are laid in composite matrices either in one or in two perpendicular directions. In such cases, the fibrous composite is treated as an orthotropic material with a selected orthotropy factor.

The flat walls reinforced by intermediate C-shaped stiffeners are dealt with as rectangular plates of principal axes of orthotropy parallel to their edges. The stiffeners carry a portion of loads and subdivide the structure into smaller elements, thus increasing considerably the load carrying capacity. The introduction of intermediate stiffeners increases the flexural rigidity of structural elements (for more detailed analysis of stiffened plates in the first order approximation see papers by Kořakowski and Teter (1995), Teter and Kořakowski (1996)).

A plate model is adapted for the structure (Fig. 2). For the  $i$ -th plate component more precise geometrical relationships are assumed in order to enable the consideration of both out-of-plane and in-plane bending of each plate:

$$\begin{aligned}
 \varepsilon_{ix} &= u_{i,x} + \frac{1}{2}(w_{i,x}^2 + v_{i,x}^2), \\
 \varepsilon_{iy} &= v_{i,y} + \frac{1}{2}(w_{i,y}^2 + u_{i,y}^2), \\
 2\varepsilon_{ixy} &= \gamma_{ixy} = u_{i,y} + v_{i,x} + w_{i,x} w_{i,y},
 \end{aligned}
 \tag{1}$$

$$\kappa_{ix} = -w_{i,xx}, \quad \kappa_{iy} = -w_{i,yy}, \quad \kappa_{ixy} = -w_{i,xy}.$$

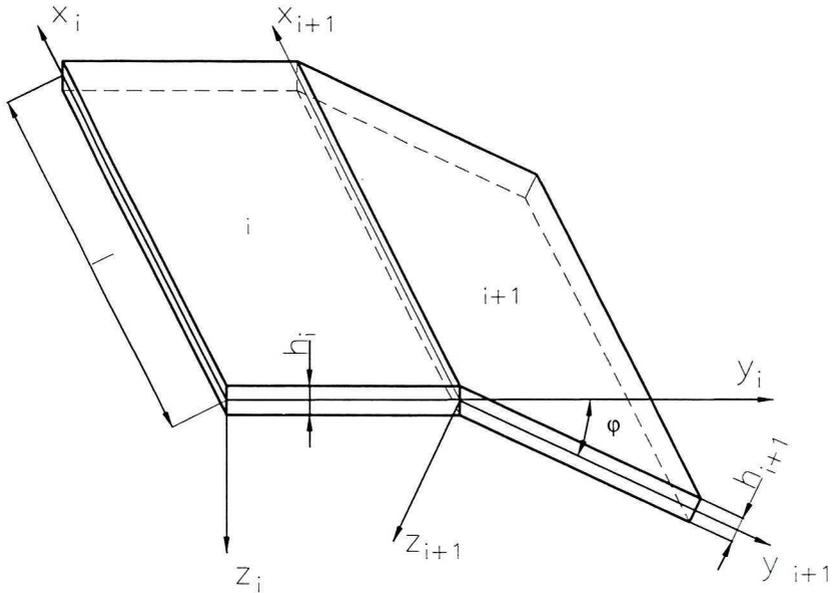


Fig. 2. A segment of considered structure with co-ordinate axes

For the  $i$ -th wall, physical relationships are formulated in the classical form applied for linearly elastic orthotropic plates (Karman and Tsien (1941)).

The differential equilibrium equations resulting from the Principle of Virtual Work and corresponding to expressions (1) for the  $i$ -th plate can be written as follows:

$$\begin{aligned}
 N_{ix,x} + N_{ixy,y} + (N_{iy}u_{i,y})_{,y} &= 0, \\
 N_{ixy,x} + N_{iy,y} + (N_{ix}v_{i,x})_{,x} &= 0, \\
 (N_{ix}w_{i,x})_{,x} + (N_{iy}w_{i,y})_{,y} + (N_{ixy}w_{i,x})_{,y} + (N_{ixy}w_{i,y})_{,x} + \\
 + M_{ix,xx} + M_{iy,yy} + 2M_{ixy,xy} &= 0.
 \end{aligned} \tag{2}$$

The composite material is assumed as a homogenous in a macro-scale.

On the basis of the derived formulas, the computer program was elaborated. The program made it possible to analyse buckling and postbuckling behaviour of thin-walled structures regarding modal interactive buckling, and also to find the load carrying capacity of structures made of composite (orthotropic) materials.

### 3. Solution of the problem

The problem is solved by Byskov and Hutchinson asymptotic method in the second non-linear approximation (1977). Displacement fields  $\bar{U}$ , and sectional force fields  $\bar{N}$ , are expanded into power series in the buckling mode

amplitudes,  $\zeta_j$  ( $\zeta_j$  is the amplitude of  $j$ -th buckling mode divided by thickness of the first component plate,  $h_1$ ):

$$\begin{aligned}\bar{\mathbf{U}} &= \lambda \bar{\mathbf{U}}_i^{(0)} + \zeta_j \bar{\mathbf{U}}_i^{(j)} + \zeta_j \zeta_k \bar{\mathbf{U}}_i^{(jk)} + \dots \\ \bar{\mathbf{N}} &= \lambda \bar{\mathbf{N}}_i^{(0)} + \zeta_j \bar{\mathbf{N}}_i^{(j)} + \zeta_j \zeta_k \bar{\mathbf{N}}_i^{(jk)} + \dots\end{aligned}\quad (3)$$

where: the upper indices (0), (j), (jk) note respectively: prebuckling, buckling and post-buckling fields.

By substituting the expansion (3) into equations of equilibrium (2), junction conditions and boundary conditions, the boundary value problems of zero, first and second order can be obtained. The zero approximation describes the pre-buckling state, while the first approximation that is the linear problem of stability, enables us to determine the critical loads of global and local value and their buckling modes. This question can be reduced to a homogeneous system of differential equilibrium equations. The second order boundary problem can be reduced to a linear system of non-homogeneous equations, which describes postbuckling equilibrium paths.

Numerical aspects of the problem being solved for the first and the second order fields (like in the paper by Kołakowski et al. (1999)), resulted in the introduction of new orthogonal functions in the sense of boundary conditions for two longitudinal edges.

The system of ordinary differential equilibrium equations (2) for the first and the second order approximation is solved by the modified transition matrices method in which the state vector of the final edge is derived from the state vector of the initial edge by numerical integration of the differential equations in the transverse direction using the Runge-Kutta formula by means of the Godunov orthogonalization method (Bidermann, (1977)).

The detailed description of the method of solution and of junctions conditions for the considered problem is given in the work by Kołakowski et al. (1999).

The most important advantage of this method is that it makes it possible to describe a complete range of behaviour of the thin-walled structures from all global (flexural, flexural-torsional, lateral, distortional and their combinations) to local stability (local distortional, local symmetric and antisymmetric modes) for intermediate stiffeners of different shapes and flexural rigidities.

#### 4. Aim of the work

The aims of the work are as follows:

1) analysis of the influence of the following parameters of the structure:

- number  $N$  of a regular polyhedron walls,
- number  $n$  of intermediate stiffeners,
- stiffness of intermediate stiffeners,
- parameter of orthotropy  $\eta$  of the material (materials) of column walls,

- column length  $l$   
on the stability (buckling load), the postbuckling behaviour (postbuckling equilibrium paths) and on the load carrying capacity of tubular poles;
- 2) determination of minimum number of walls at which the tubular column behaves (with regard of stability) in the same way as a cylindrical shell (tube);
- 3) calculation of different buckling modes (global and local) and the analysis of the interaction of these buckling modes provided that the interaction between them occurs.

## 5. Analysis of numerical results

The elaborated computer program allows us to analyse the behaviour of thin-walled columns of cross-sections with at least one symmetry axis.

The walls of a column may be of different thickness and may be made of materials of different properties (e.g. different orthotropy factor  $\eta$ ). The pole structure may be subjected to axial or eccentric compression.

During the analysis of local buckling, the segment of a column of a length  $l$  is considered. In this work, detailed calculations are conducted for thin-walled columns of a tubular cross-section subjected to axial compression.

The numerical calculations are conducted for columns of cross-sections in a form of a regular polygon of different number  $N$  of sides but of the same circumference length. The length of the circumference, measured along the middle line of section walls, is equal to the circumference of a cylindrical shell of radius  $R$ , thus:

$$Nb = 2\pi R$$

where  $b$  is the width of a polyhedron (column, pole) wall or the length of a side of a regular polygon. It means that for a column (pole) without stiffeners, or with the same number of stiffeners, the area of the cross-section is the same.

It is assumed that the thickness  $h$  of all walls is constant (such columns are most often produced).

Geometrical parameters of the considered column (segment of a column) are assumed as follows:

$$l/R=2; \quad b/R=2\pi/N; \quad h/R=0.02; \quad R=50 \text{ mm.}$$

### 5.1. Analysis of upper buckling loads

At the very beginning, on the basis of the first order approximation, the critical loads for columns without stiffeners of number of walls  $N= 4\div 36$  are found. The calculations are performed for structures made of orthotropic materials with parameter of orthotropy  $\eta=0.3031$  and  $\eta=3.2992$  and of isotropic material ( $\eta=1$ ). Material constants for these three cases are given in Table 1 (Chandra and Raju (1973)).

Table 1.

Elastic constants for the various cases of composite beam-columns

Spec. No	$\eta = E_y/E$	$\nu$	$G/E$
1	0.3031	0.09093	0.4002
2	1.0000	0.3	0.3846
3	3.2992	0.3	0.1213

It was assumed that all walls are made of the same material, and the following notation is introduced:

- $E$  – Young's modulus in the longitudinal direction (the axis  $x$  of the column),
- $E_y$  – Young's modulus in the circumferential direction,
- $\nu$  – Poisson's ratio describing the material deformation in the circumferential direction during tension (compression) in the direction of a column axis,
- $G$  – shear modulus.

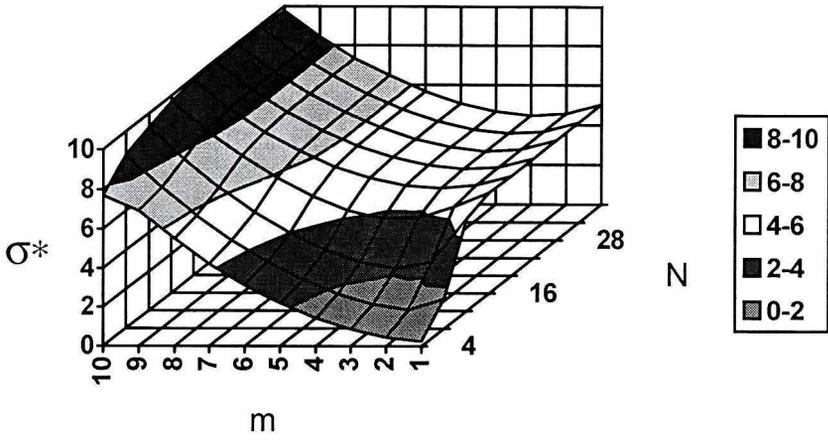
In Fig. 3, the diagrams of the dimensionless buckling stress  $\sigma^* = \sigma_{cr} 10^3/E$  are shown for three columns of material parameters given in Table 1. These diagrams are presented as function of two variables – number of axial half-waves  $m$  and number of polyhedron walls  $N$ . It can be seen that, for columns of number of walls  $N \geq 20$ , when the number of axial half-waves  $m$  is constant, the values of the dimensionless critical stress  $\sigma^*$  do not depend on the number of walls  $N$ , and practically are equal to the upper critical stress of a cylindrical shell of radius  $R$ , length  $l$  and of wall thickness  $h$ , made of the same material as the analysed polygonal column. For  $N < 20$ , the values of the dimensionless critical stress  $\sigma^*$  decrease with the decreasing number of column walls. In the presented diagrams, one can also see that the values of  $\sigma^*$  depend on the number of axial half-waves  $m$ , and change in a different way with the number of walls of the polyhedron and with the orthotropy factor  $\eta$  of the column material.

In the case of non-stiffened tubular pole, it can be seen that the dimensionless critical stress increases with increasing value of orthotropy factor  $\eta$  and with increasing number  $N$  of component walls. For a given value of  $\eta$  and  $N$ , the minimum value of  $\sigma^*$  is achieved at a different value of axial half-waves  $m$ .

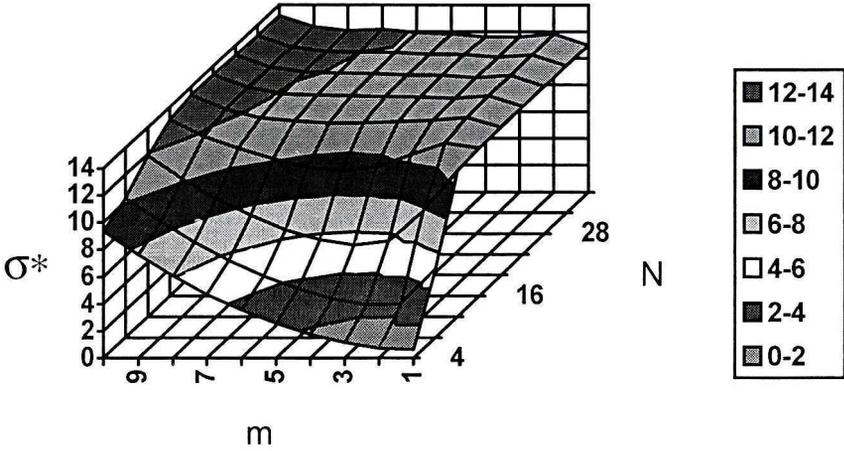
Let us consider now the critical stresses for columns reinforced by intermediate stiffeners of C-shaped cross-section and of dimensions shown in Fig. 4.

The diagrams in Fig. 5 show the influence of intermediate stiffeners on the values of  $\sigma^*$  for a thin-walled isotropic column of a square cross-section with four intermediate stiffeners placed in the middle of the width of each wall and with eight stiffeners (two stiffeners per each column wall dividing it on three strips of equal width). The calculations were conducted for stiffeners of a height  $H = 4$  mm and  $H = 6$  mm.

a)  $\eta=0.3031$



b)  $\eta=1$



c)  $\eta=3.2992$

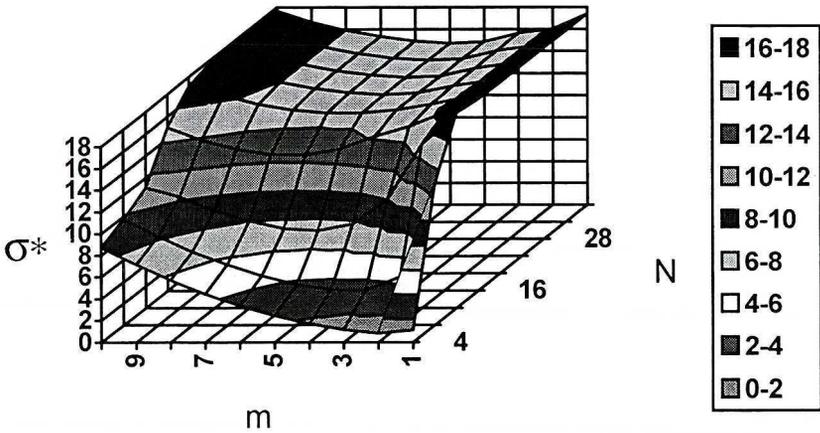


Fig. 3. Dimensionless critical stress for non-stiffened tubular columns

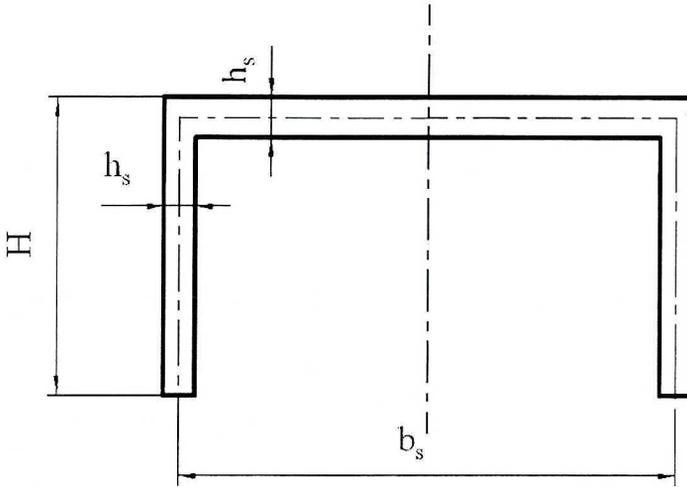


Fig. 4. Shape of stiffener cross-section -  $h_s=1$  mm,  $b_s=4$  mm,  $H=4$  or  $6$  mm

It follows from the diagrams shown in Fig. 5 that:

- the intermediate stiffeners (especially their quantity) increase the values of  $\sigma^*$  in a significant way,
- the stiffeners of assumed dimensions have a large slenderness, so the one half-wave buckling mode ( $m=1$ ) occurs at rather small critical stresses,
- for number of half-waves  $m \geq 4$ , the significant influence of intermediate stiffeners number  $n$  on values of  $\sigma^*$  is observed, while the influence of stiffeners height  $H$  can be practically neglected (above some minimal value of  $H$ ).

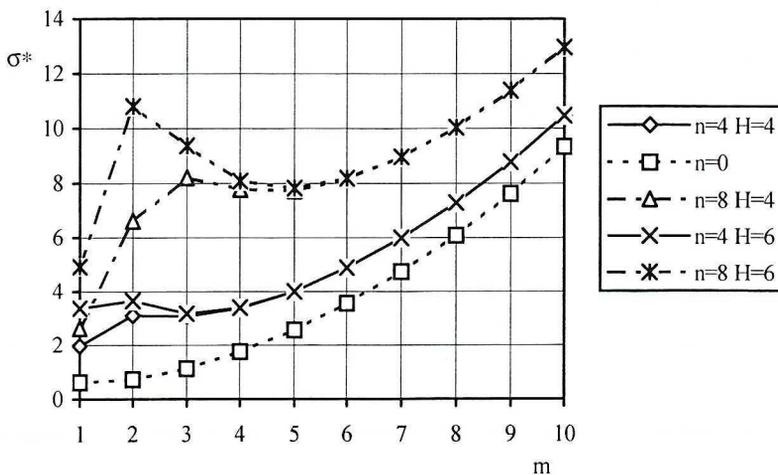


Fig. 5. The influence of intermediate stiffeners on values of critical stress for isotropic column of a square cross-section

In Fig. 6, the influence of stiffeners height  $H$  and the number  $N$  of polyhedron walls on values of  $\sigma^*$  is presented for thin-walled isotropic columns of a cross-section in the form of square, regular octahedron, dodecahedron and sextodecimohedron, in which each wall has one central intermediate stiffener. Proportionally small influence of stiffeners on values of  $\sigma^*$  for  $m=1$  indicates large stiffeners slenderness, assumed in calculations, and in some cases the buckling nodal lines in the circumferential direction coincide with longitudinal stiffeners (then the stiffeners do not buckle).

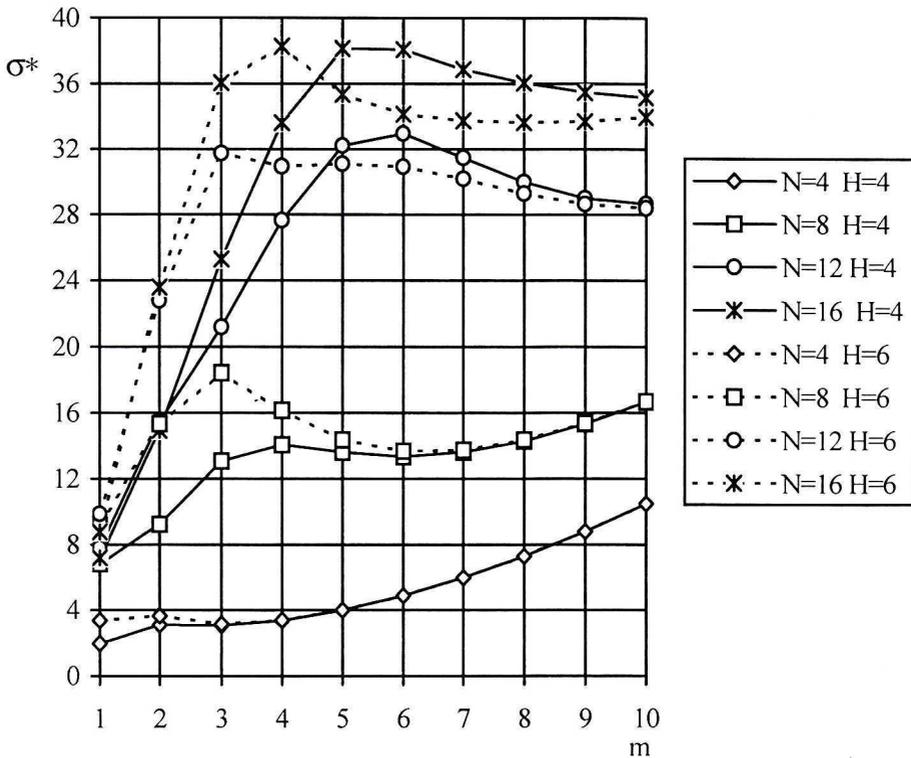


Fig. 6. Dimensionless critical stress versus number of half-waves for isotropic tubular poles of walls number  $N$  with one central stiffener placed on each wall ( $N=n$ )

The shapes of buckling modes of walls of isotropic columns with a cross-section in a form of regular 16-gon (non-stiffened and with four intermediate stiffeners placed every fourth wall) are presented in Fig. 7. The symmetrical and anti-symmetrical buckling modes of column walls are shown for the number of half-waves  $m=1$  and  $m=5$ .

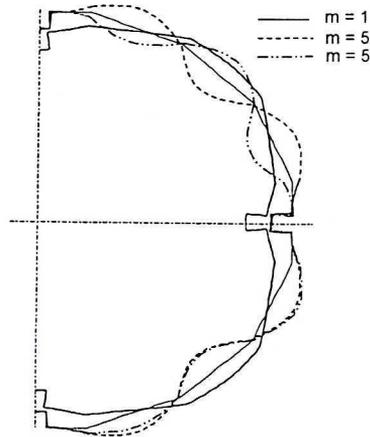


Fig. 7a. Shapes of buckling modes for isotropic column with four intermediate stiffeners (number of walls  $N=16$ )

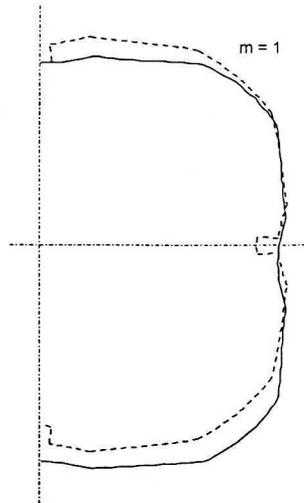


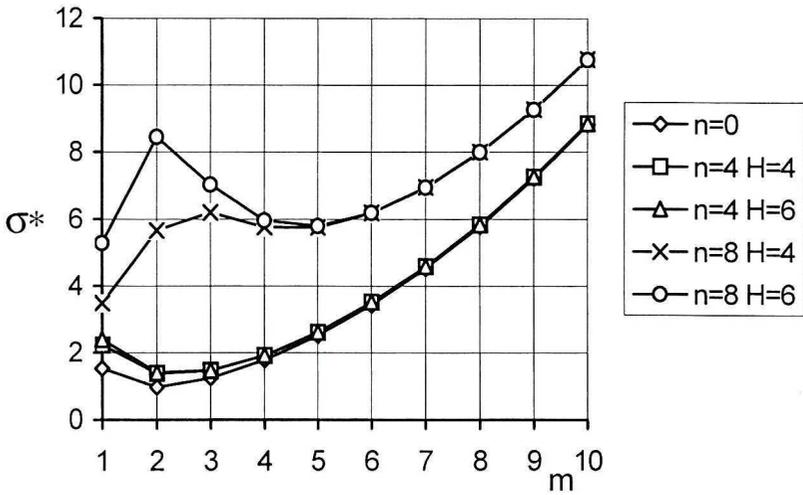
Fig. 7b. Comparison of buckling mode shapes ( $m=1$ ) for isotropic column with four intermediate stiffeners (dotted line) and without stiffeners (full line) - number of walls  $N=16$

The diagrams of dimensionless critical loads for columns of a cross-section in a form of regular octagon, made of composite materials of orthotropy factor  $\eta$  as given in Table 1, are shown in Fig. 8.

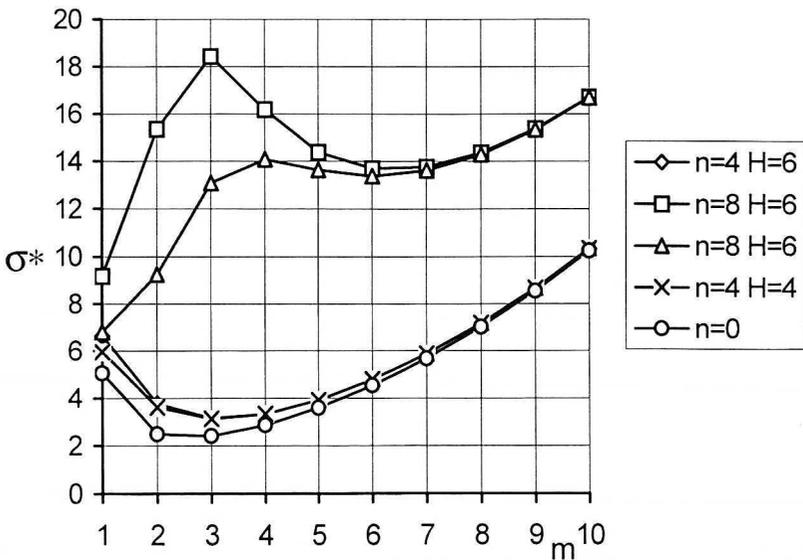
Similarly as for non-stiffened columns (Fig. 3), the values of dimensionless critical stress for columns having the same number of stiffeners of the same dimensions increase with increasing value of orthotropy factor  $\eta$ . Together with the increase of  $m$  the values of  $\sigma^*$  in an insignificant degree depend on the stiffeners height  $H$  (provided that the stiffeners are of some minimal height  $H$  to be able to accomplish the task of stiffening of the structure).

The increase of values of  $\sigma^*$  with increasing number and height of stiffeners is logical. Such dependence can be seen in all diagrams except of the value of  $\sigma^*$  in Fig. 8c for  $m=1$ . These “strange” (unexpected) values of  $\sigma^*$  for  $m=1$ , lower for stiffened columns than for non-stiffened ones, obtained for columns of octagonal cross-section with material orthotropy factor  $\eta=3.2992$ , can be explained by a large slenderness of stiffeners (the slender stiffeners of a length  $l$  loosing global stability cause a forward buckling of column walls). Greater number ( $n=8$ ) of slender stiffeners ( $H=4$  mm) cause the earliest (at the lowest value of stresses) buckling of walls at  $m=1$ .

(a)  $\eta=0.3031$



(b)  $\eta=1.0$



(c)  $\eta=3.2992$

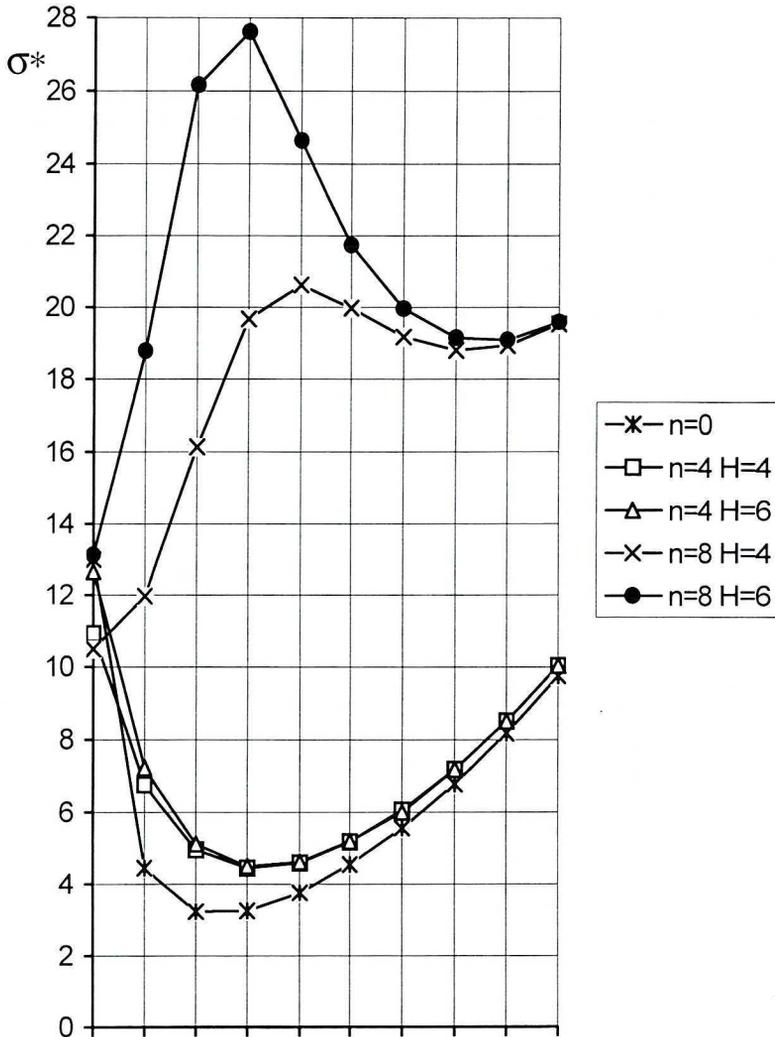


Fig. 8. The influence of intermediate stiffeners on critical stress values for columns with octagonal cross-section (for three values of orthotropy factor  $\eta$ )

**5.2. Analysis of postbuckling equilibrium paths**

The analysis of postbuckling equilibrium paths is restricted to the columns of walls number  $N=4\div 12$ . At  $N>12$ , the postbuckling behaviour of a column is closer to the behaviour of a cylindrical shell of radius  $R=Nb/2\pi$ . Therefore, the analysis of the postbuckling state of columns with cross-sections close to the circular one ( $N>20$ ) should be conducted on the basis of stability equations of cylindrical shells (with a term  $w/R$  in geometrical relations). More detailed analysis should be applied to the postbuckling behaviour of polygonal columns

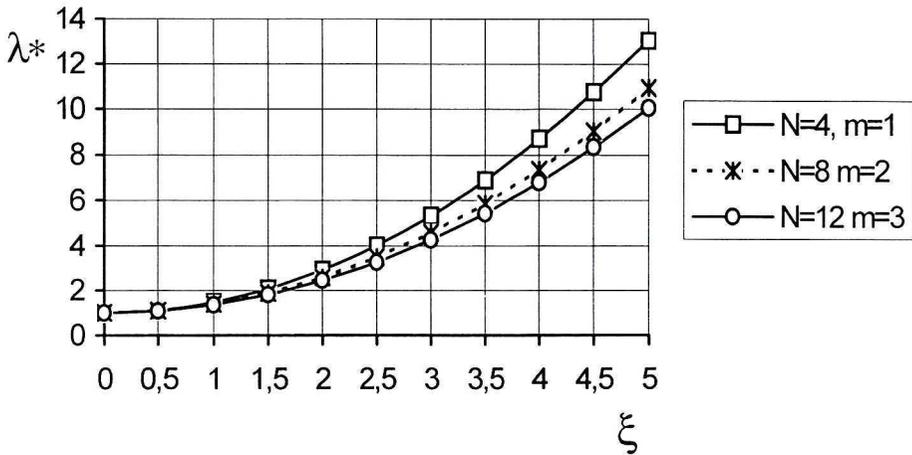
of  $12 < N \leq 20$ , when the angle between the neighbouring walls is in the range of  $150^\circ < \varphi < 162^\circ$  (see Fig. 2).

In Fig. 9, the curves are graphs of the ratio of load parameter to the minimal critical stress ( $\lambda^*$ ) as a function of buckling mode amplitude for non-stiffened tubes with the number of component walls  $N=4, 8$  and  $12$ . In each case, the tubular pole structure behaves as a plated structure – the postbuckling paths are ascending.

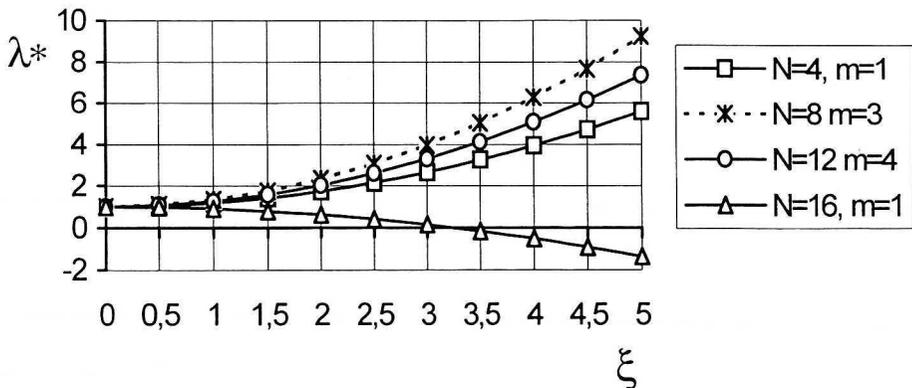
The load parameter  $\lambda^*$  is assumed as the ratio  $\sigma/\sigma_{cr}$  (where  $\sigma_{cr}$  is the critical stress value) then  $\lambda_{cr}^*=1$ .

The same relations are drawn for isotropic columns of a cross-section in form of a regular octagon: without stiffeners and with one central stiffener on each wall (for  $H=4$  mm and  $H=6$  mm) – Fig. 10.

a)  $\eta=0.3031$



b)  $\eta=1.0$



c)  $\eta=3.2992$

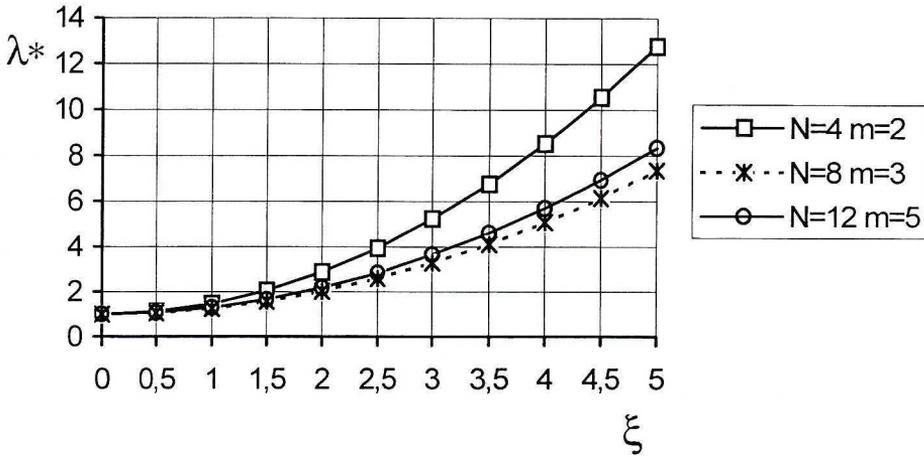


Fig. 9. Postbuckling equilibrium paths for columns with non-stiffened walls (uncoupled stability)

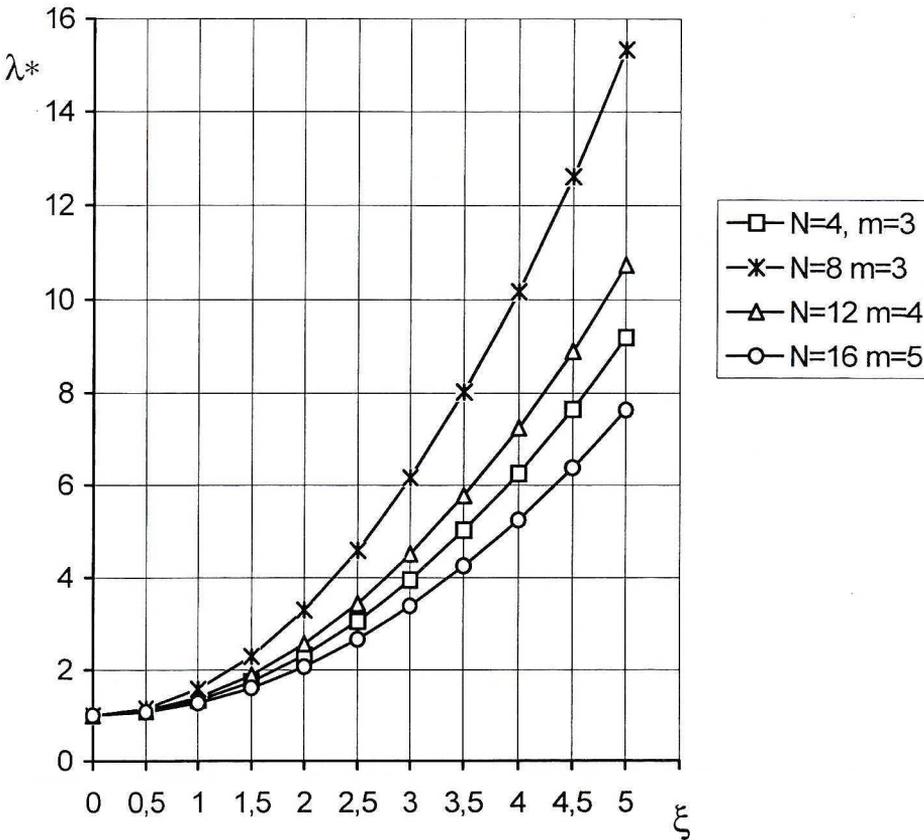


Fig. 10. Postbuckling equilibrium paths for stiffened (4 intermediate stiffeners) columns (uncoupled stability)

### 5.3. Non-linear analysis of the influence of column lengths

The analysis of interactive buckling conducted for non-stiffened columns (with  $N=4,8,12$  walls) of a length  $l=100\div 2000$  mm shows that:

- minimal values of local buckling stress do not depend on the column length;
- values of global buckling stress in a significant way depend on the material constants of the column;
- global buckling stresses ( $m=1$ ) for columns of a length  $l\geq 1600$  mm are directly proportional to the elastic modulus  $E$  in the longitudinal direction and decrease with the increase of a column length;
- there is no interaction between the global flexural buckling mode ( $m=1$ ) and local modes ( $m>1$ ) for  $l=100$ mm;
- the interaction between the global flexural buckling mode and local modes becomes more significant with increasing value of  $l$  and has a practical meaning for columns of  $l=1200\div 2000$  mm.

More detailed analysis of the interaction of three buckling modes is conducted for the octagonal tube of the length  $l=1600$  mm. The interaction of three buckling modes is analysed: global buckling mode ( $\sigma_1^*$ ) at  $m=1$ , local mode (primary) of the lowest value of critical stress ( $\sigma_2^*$ ) and local buckling mode (secondary) of the same number of half-waves as primary local mode ( $\sigma_3^*$ ).

On the grounds of this analysis, the load carrying capacity for the second order approximation ( $\sigma_s^*/\sigma_m^*$ ) is found for columns of orthotropy factor  $\eta$  as given in Table 1. The value of  $\sigma_m^*$  is the minimum of values of  $\sigma_1^*, \sigma_2^*, \sigma_3^*$  for imperfections  $\bar{\xi}_i = |1.0|$  ( $i=1,2,3$ ) (the imperfections are always equal to the wall thickness  $h$ ). The values of load carrying capacity  $\sigma_s^*/\sigma_m^*$  appear to be approximately 30÷40% lower than the minimum of critical stress value  $\sigma_m^* = \sigma_2^*$ . The results of calculations are given in Tabl. 2

Table 2.

Load carrying capacity for octagonal tubes

$\eta = E_y/E$	$\sigma_1^*$	$\sigma_2^*$	$\sigma_3^*$	$\sigma_1^*/\sigma_2^*$	$\sigma_s^*/\sigma_m^*$
0.3031	4.24	0.962	1.08	4.41	0.669
1.0000	4.47	2.34	2.58	1.91	0.709
3.2992	4.47	3.17	3.55	1.41	0.614

The interaction between global buckling modes and local ones at large ratio  $\sigma_1^*/\sigma_2^* = 4.41$  for a column of orthotropy factor  $\eta=0.3031$  indicates high sensitivity of this column on imperfections, in particular, it means that its load

capacity  $\sigma_s^*/\sigma_m^*$  is equal 0.669 and is comparable with the load capacity of columns with orthotropy factors  $\eta=1.0$  and  $\eta=3.2992$ .

Therefore, it is necessary to take in particular consideration the interactive buckling of orthotropic or composite structures when a plate model is adapted in global buckling analysis.

## 6. Final remarks

The results of numerical analysis show that:

- Taking into consideration the strain tensor in an extended form (1) allows us to analyse all possible buckling modes (global and local) of a structure with intermediate stiffeners, including the distortional buckling (because the displacements of stiffeners and corners are possible).
- The interaction of buckling modes occurs only for longer columns  $l/2\pi R > 1$  and then the load carrying capacity of a structure can only be determined by non-linear analysis in second order approximation.
- When the number of polygon sides is greater than 20, the pole structure behaves as a cylindrical shell. Because the term  $w/R$  is neglected in expressions (1) in non-linear analysis, it is impossible to find the lower bound of buckling stress (Schilling (1965)).

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### **Modalne wyboczenie interakcyjne kompozytowych rur cienkościennych z żebrami pośrednimi**

#### **Streszczenie**

W pracy rozpatrzono wyboczenie, stan zakrytyczny i nośność graniczną pryzmatycznych kompozytowych rur cienkościennych. Zastosowano asymptotyczną metodę Byskov'a-Hutchinson'a w ramach drugiego rzędu przybliżenia. Założono, że ściskane cienkościenne słupy o przekroju zamkniętym są swobodnie podparte na obu końcach. Rozpatrzono kilka typów przekrojów poprzecznych z i bez żeber pośrednich. Prezentowana praca jest kontynuacją wcześniejszej pracy tych samych autorów dotyczącą interakcyjnego wyboczenia modalnego kompozytowych belek-słupów.