THE ARCHIVE OF MECHANICAL ENGINEERING

Vol. XLVII 2000 Number 1

Key words: *residual stress, experimental and numerical methods*

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ON CONTEMPORARY METHODS OF RESIDUAL STRESS ANALYSIS

The residual stress analysis is discussed in the paper. However, the author has not intended to present, even partially, all aspects of this very broad problem. The aim of this work has been limited to a review of contemporarily used experimental, numerical and hybrid methods, and to outline the directions of possible developments.

1. Introduction

The influence of residual stresses on strength and working life of materials and constructions, as well as methods of determining distributions of residual stresses in real structures, have been broadly investigated during last 85 years. Starting from the pioneer's work of Heyne at 1914 [1], tens of scholars and engineers have been creating theories, constructing classifications, developing methods of measurement and computation, conducting hundreds of uphill and expensive tests on small specimen as well as large structures. The residual stress analysis has than long tradition and very rich literature. However, this part of the Mechanics of Solid is still not completed: many problems need more investigations, some of them require fundamental solutions.

From the point of view of Mechanics, if the continuum as a model of the material has been assumed, residual stresses do not differ from general stresses, which are defined in the Strength of Materials. A determined state of residual stress creates in the structure the same effects as an analogous state of stress of different origin. It obeys the superposition principle in the same way and under the same conditions as all others states of stresses. Residual stress can be summed up together with others until the yield point of the material is not exceeded. After yielding, it has to be considered along the principles of the Theory of Plasticity. Residual stress can be defined then as such a stress which

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exists in some region of the structure or in the construction not subjected to an external action. The external action can be revealed in different forms: loading, temperature, physical-chemical factors and others. Such definition of residual stress is not totally sufficient if, closer to reality, we accept not continuum, but discrete model of the material. Considering the size of areas where residual stresses are acting, we can distinguish macrostresses and microstresses.

The first concept comprises these states of stresses, which equilibrate in areas comparable with external dimensions of the body.

The second concept is related to such stress, which appears in the volumes enclosing a few crystal grains, and are created between them due to their nonuniform deformation. This concept covers also residual stresses, which are connected with reactions inside the crystal grain within atomic network. Our further considerations are limited mainly to the macrostresses and, in small range, to the microstresses of the first kind.

Residual stresses can be created in many different ways. The most commonly, they are introduced by:

- differences in dimensions in the process of fitting the elements of the structure (assembly stresses),
- plastic permanent deformations in some areas of the structure due to such processes as rolling, shot-peening, drawing, machining, abrasive working etc.,
- nonuniform solidification and cooling of metals due to casting and welding,
- changes of the volume in some areas of the body due to heat and chemical treatment (surface hardening, carburizing, nitriding etc.).

Residual stresses can be created intentionally to improve strength properties of an object, but also introduced by uncontrolled production processes causing then usually many negative effects, including even the destruction of the structure. Residual stress influences strongly the strength, fatigue limit, deformations of the construction and cracking tendencies of the material.

2. The fundamentals of residual stress analysis

The residual stress analysis leads to the determination, in any point of the unloaded body, the all components of the stress tensor (six independent stress components)

$$\sigma_{ij} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}. \tag{1}$$

Such an analysis can be conducted experimentally by testing real objects and by the computation (nowadays, almost exclusively using numerical methods: Finite Element Method and rarely - Integral Boundary Method). The third way is the hybrid analysis - the mixed method, where some portions of information are determined experimentally, and the total solution is found numerically by substituting experimental data into the computational algorithm. At present, the two first ways are used in parallel. The experimental way has, however, much longer history and tradition and also such advantage that the analysis is conducted on real objects which not always we know how to model numerically with sufficient accuracy.

The problem of measuring all stress components, complex in general, appears to be much more difficult for residual stresses, when the structure is not loaded, and the stresses are balanced within some of structure's portions. The majority of the experimental methods use the strains or deformations to determine the residual stress. Only a few of them apply the change of some others physical or chemical properties of the material as an indicator of residual stresses (e.g. magnetic or ultrasonic methods).

Residual stresses may create the elastic deformations or cause the additional secondary plastic deformations (different from those, which are introduced as an effect of the production process). The secondary yielding can be determined only by the numerical solution or using the hybrid technique. The direct separation of the measured strains into the elastic and plastic portions is not possible. This situation creates many limitations and errors in the quantitative evaluation, because usually we assume a priori the linear relation between stresses and strains (the Hooke's law for izotropic or orthotropic materials). The total strain at the investigated point of the body can be described in the following way

$$\varepsilon_{ij} = k_{ij} + \frac{1}{2G}\left(\sigma_{ij} - \frac{\nu}{1+\nu}\sigma_{mm}\delta_{ij}\right), \qquad (2)$$

where

σ_{ij} – $(i, j = 1,2,3)$ - the residual stress components in the Cartezian coordinates system,

ε_{ij} - the total strain components,

k_{ij} - the components of the strains created by the production process,

ν - Poisson ratio,

G - Kirchhoff modulus,

δ_{ij} - Kronecker delta (for $i = j$ $\delta_{ij} = 1$; for $i \neq j$ $\delta_{ij} = 0$).

It is necessary to emphasise that $\varepsilon_{i,j}$ is the total strain and has to fulfill the compatibility conditions, but k_{ij} does not need to fulfill these conditions. The residual stress σ_{ij} should fulfill the equilibrium equations and boundary conditions. If the k_{ij} is known, we can find the σ_{ij} in the total area of the structure solving the equilibrium equations with the boundary and compatibility conditions and taking into account the eq. (2). We assume that the secondary yielding in this case does not appear.

If the secondary yielding appears, then instead of (2) the flow rule associated with the yield criterion has to be applied. The numerical computation, though using different theoretical approach, is here possible (mainly by the FEM method), although 3-D nonlinear analysis (due to the plastic flow and sometimes due to the large strains) is still limited by the size of the problem.

The basic difficulty, however, is to determine the k_{ij} strain components: they appear usually as a result of the complex physical and chemical changes in the material, and can not be determined without experimental data. In many cases, however, the k_{ij} can be computed numerically using algorithms based on physical models assumed according to experimental observations. Elastic-plastic contact problem and welding process are here good examples: the residual stresses distributions have been determined there as a part of the total solution.

The problem of determination of the k_{ij} can be totally overcome if we conduct measurements on real objects. The strains measured directly (e.g. using the X-ray diffraction method), or after the process of the stress relaxation, are residual strains described by the second term in eq. (2). We assume that the relaxation does not influence the k_{ij} introduced by the production process and that the measured strains increments are created only by the increments of the relaxed residual stresses

$$\Delta\varepsilon_{ij} = \frac{1}{2G}\left(\Delta\sigma_{ij} - \frac{\nu}{1+\nu}\delta_{ij}\Delta\sigma_{mm}\right). \tag{3}$$

Determining the $\Delta\sigma_{ij}$ from eq. (3) we have to assume that the secondary yielding does not exist. The basic idea of this consideration practically can be reduced to the statement that if the applied stress σ creates the strain ε, then the elimination of this stress is accompanied by the disappearance of ε, which means - in the measurable effects - the appearance of the "$-\varepsilon$" introduced by the "$-\sigma$". The similar effect we can get by direct measurement.

Residual stresses distributions have to be self-balanced, because the considered structure is not subjected to any external action. Therefore, for any portion of the body isolated by same section, should be fulfilled the equations which express that the sums of the forces and the moments created in the process of relaxing residual stresses in the considered section are equal zero.

Generally, there is not exist any experimental method by which the all $\Delta\varepsilon_{ij}$ strain components can be determined in the total area of the structure. In order to find the residual stresses distributions from the accessible strain readings, it is necessary to introduce many simplifying assumptions. These include reducing the problem to a two-dimensional or one-dimensional, conducting the measurements by the method of local stress relaxation (e.g. the hole - drilling method, the crack - compliance method), etc.

3. Experimental methods for determining residual stresses

There are many methods for determining residual stresses in real structures. They can be classified in different ways, using different criteria.

We can classify them as follows: methods for measuring the macrostress and the microstress, methods which allow the measurements at the surface of the body and in the interior area, methods which are based on the strains measurements versus the others, by which the change of some physical-chemical properties of the material due to the introduced stresses is determined. The most commonly used, however, is the division into the destructive, semidestructive and nondestructive methods, the division based on the level of destruction of the object due to the tests.

3.1. Destructive methods

As destructive are classified such methods which cause total destruction of the structure. The methods are based exclusively on the measurements of the strains (assumed elastic) created by the relaxation or the change of the residual stress. The relaxation or the change are introduced either by the sectioning of the total body into small pieces, or due to the removing (sometimes growing) of the consecutive thin layers of the material, which causes the measurable deformations of the remaining part of the structure.

The destructive methods usually provide the most complete and exact set of information; very often therefore, they are used for verifying the other methods. It does not mean, however, that it is possible to apply them for the evaluation of the residual stress distribution in any object of an arbitrary shape. The vital problem is the proper way of interpretation of the measured strains, in other words, the way of computing the residual stresses on the basis of the strains.

It is necessary to emphasise that the sufficient level of accuracy of the solution needs the sectioning of the body into enough small elements. On the other hand, the elements have to be enough large in order to allow the measurements of their deformations, and to be enough accessible before the sectioning to allow the determination of their dimensions as the basis to the measurements. This is the reason that, very often instead of the sectioning, the layer removing technique has to be applied, especially in such cases where the sharp gradients of the residual stresses in the subsurface area due to the surface treatment (mechanical, heat, chemical) is investigated.

The destructive methods are carrying the errors due to the relaxation treatment along with the errors caused by the many simplifying assumptions which have to be accepted in order to open the way for stress computation from the strains measurements. The process of sectioning can introduce by itself plastic deformations and create an additional state of the residual stresses.

A good example of the method of sectioning provides the measurement conducted in 1983 in Batelle Columbus Laboratories (USA) [2]. The aim of the tests was the determination of the residual stresses created in rails by the process of straightening and by the plastic deformations due to wheel passages.

It was assumed that the stresses are uniformly distributed along the length of the investigated portion of the rail. The two specimen were cut out from the portion: (a) the large, of the length l and (b) - the thin slice of the thickness f (Fig. 1a). The specimen (a) was sectioned into long bars as shown in Fig. 1b: the relative elongation (or shortening) of these bars with respect to the original l were equal to the residual strains components ($\Delta\varepsilon_x = \Delta\varepsilon_{11}$) in the consecutive points of the rail cross-section. The x axis was taken in the one of the residual stress principal directions. On the face of the thin slice (b) where, by the assumption, the state of stress before the sectioning was the same as in the portion (a), the tens of the strain - gage rosettes were sticked (Fig. 1c) in order to measure the strain components in the y,z directions in different points of the cross-section. The sectioning of the slice into small cubes (relaxing the residual stresses) allowed for the determination the $\Delta\varepsilon_y'$ and $\Delta\varepsilon_z'$ components in the cross section plane. They were not, however, the wanted strains created by the original residual stresses, but the quantities modified by the process of cutting out the slice. The original $\Delta\varepsilon_y$ and $\Delta\varepsilon_z$ were determined by an additional numerical computation (using FEM), taking $\Delta\varepsilon_y'$ and $\Delta\varepsilon_z'$ as the input data. The $\Delta\varepsilon_y$ and $\Delta\varepsilon_z$ found in this way, as well as $\Delta\varepsilon_x$ measured using the specimen (a) have allowed for the determination of the residual stress components $\Delta\sigma_x, \Delta\sigma_y, \Delta\sigma_z$ (eq. 3).

The example demonstrates the difficulties of the measurement of the all strain components. Despite of the fact that the portion of the rail was cut into the small elements, it was not possible to determine these components without the additional numerical analysis. For this reason, the Batelle method has to be considered as a hybrid (mixed) one, but with the dominance of the experimental part. The numerical computation can be avoided by performing a few additional measurements, introducing some simplifying assumptions and resigning from the formal, theoretical exactness of the results, Ref. [2].

The strains in the cross-section plane of the rail (thin slice) can be measured in different ways. In the most recent investigations by the optical method of the moiré interferometry [3], the in-plane cross-sectional strains have been evaluated after the relaxing "the thin slice" by the annealing. The typical fringe pattern for displacements in y direction is shown in Fig. 2.

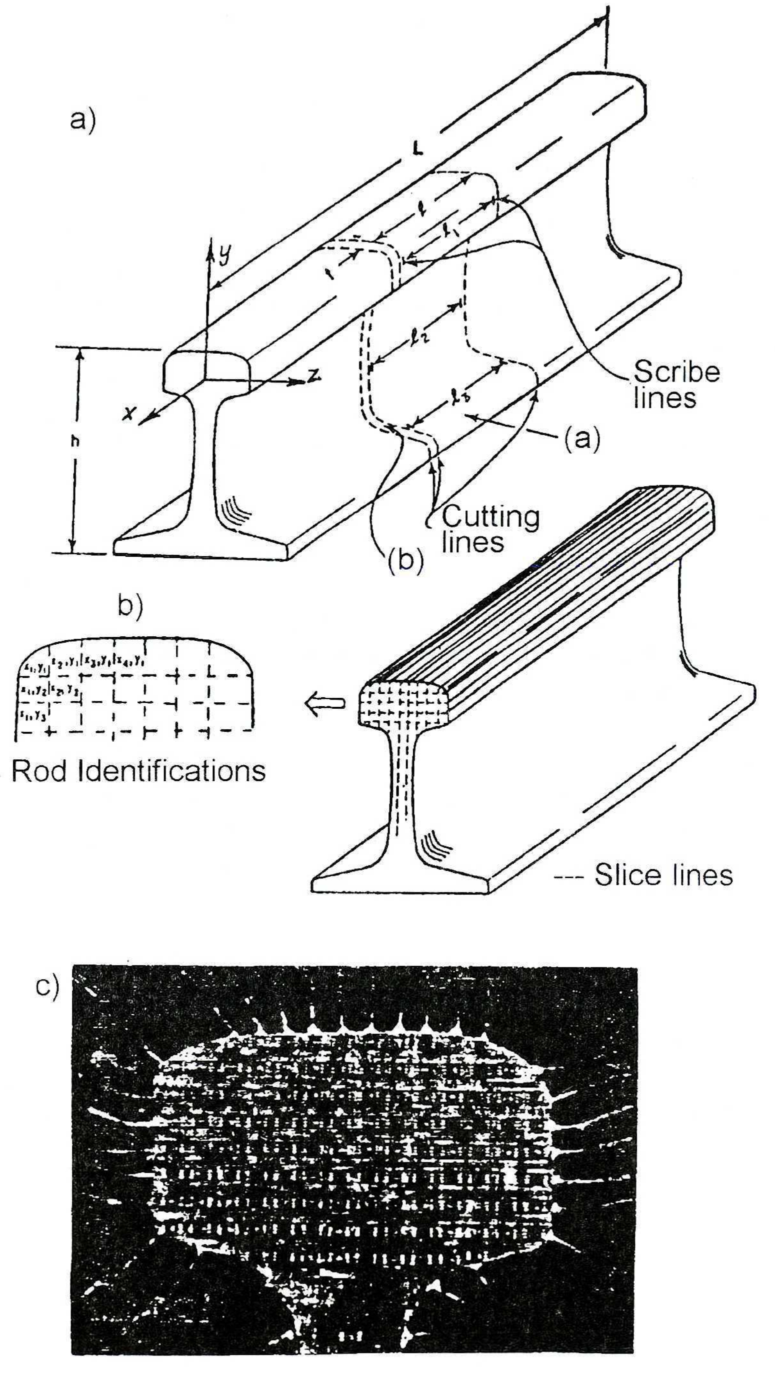

Fig. 1 The sectioning method for determining residual stress in rails (according to Ref. [2]), a) two specimen cut out from a rail portion, b) sectioning of the specimen (a) into long bars, c) rosettes of strain gages at the face of the thin slice

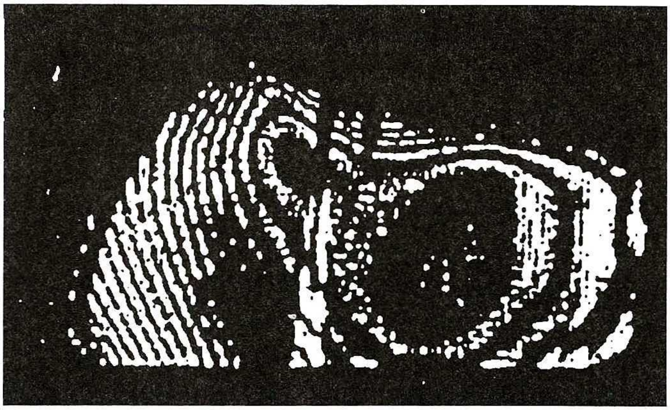

Fig. 2 The displacement $w(y,z)$ contour map of the rail cross-sectional slice obtained by the moiré interferometry (according to Ref. [3])

The Batelle method allows determining the residual stresses distribution in the total area of the rail. The large dimensions of the cut out cubes (4×4×4 mm - Fig. 1c), as well as the difficulties of the replicating the moiré granting in the close vicinity of the running surface on the cross-section cause, however, it is not possible to measure with sufficient accuracy the sharp gradient of the residual stress in the subsurface area. At the depth of a few millimetres σ_x increases from the significant compression (~100 MPa) at the running surface to the high tension: ~+200 MPa (after many wheel passages).

Such measurement was carried out by the method which combines the sectioning with the thin layer removing, developed in [4] (Fig. 3a,b,c). The consecutive steps of the method are the following: cutting out the rail head of the tested portion, sectioning the head into the slices (Fig. 3a), removing the thick layers of the material from the slice up to the state when a thin bar remains (Fig. 3b), removing the thin layers of the material (approx. 0.2 mm in thickness) from the running surface of the bar (Fig. 3c). In the each operation the respective strain measurements by the strain gages were taken. Several simplifying assumptions had to be accepted; the method allowed for the determination only the longitudinal (σ_x) - the most important in rails - residual stress component. The example of the σ_x distribution from the running surface inside the material is shown in Fig. 3d.

The layer removing/growing method, well known for many years, has been recently often and commonly used (e.g. Refs. [5], [6], [7]) either as an independent technique, or in combination with the X-ray diffraction method.

The new production processes (e.g. coating the surface of the structures by the thin films which creates the residual stresses between the layers of different materials), have revitalised the old method and opened it for the new areas of application.

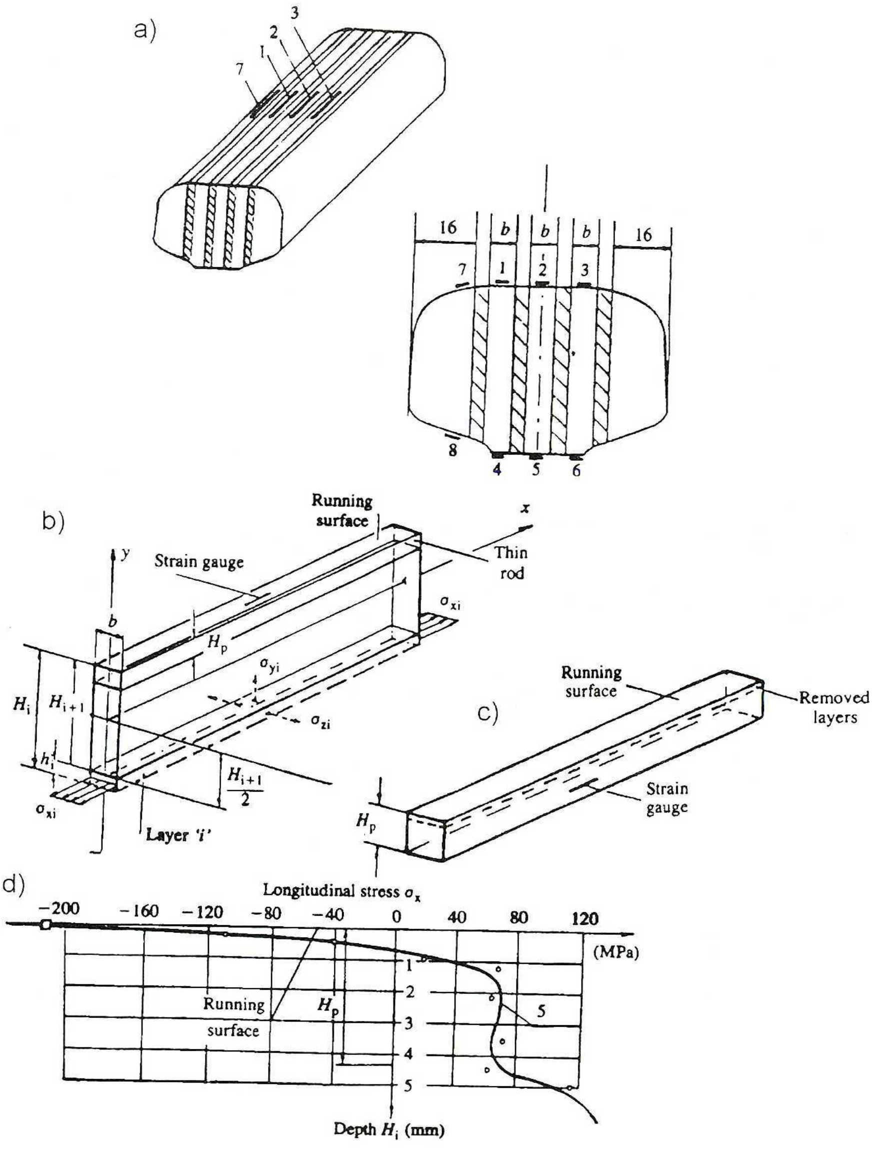

Fig. 3 The sectioning / layer removing method for determining residual stress in a rail head (according to ref. [4]): a) cutting the rail head into plates; positions of strain gages, b) process of plate sectioning: removing layers, c) process of removing layers from the thin bar, d) distribution of longitudinal (σ_x) stresses along the centreline if the rail head cross section in the vicinity of the running surface

3.2. Semidestructive methods

Semidestructive methods cause only small damages: the tested structure after the repair, or sometimes even without, can be used again according to its application. The semidestructive methods usually allow for the determination of residual stresses in small areas of the object. They use measurements of strains created by local stress relaxations. Therefore they carry the fundamental theoretical error caused by the fact that, in the elastic region, the deformation of the body at any point depends on its total (not the local) loading. The de Saint Venant principle indicates, however, that the mentioned error is small, practically much smaller than that created by the applied relaxing process and measuring techniques. The main advantages of the semidestructive methods are the following:

- the evaluation of residual stress in small areas close to the "point type" measurement,
- the method can be practically applied to an object of an arbitrary shape.

These advantages have caused that the one of the semidestructive methods has reached the position of the normalized industrial technique and it is commonly used to date. It is the hole drilling method.

The hole drilling method, applied first time by Mathar [8], is based on the measurements of displacements introduced in the structure by the stress relaxation due to the hole drilling process. These measurements, performed first by mechanical devices, then by strain gages, now often by optical methods, allow for determination of residual stress in thin discs, then in thick plates and, at the end, stress distributions inside the material up to the depth equal approximately to the hole diameter. The solution of the last problem was determined first in [9], [10], [11], later independently and in a little different way in [12] many years ago. However, the works in order to improve the method are still continued e.g. [13], [14], [15], [16].

The principle of the incremental hole drilling method for determining stress distributions inside the material is presented in Fig. 4. Fig. 4a shows the distribution of residual stress which exist in the material, Fig. 4b - the reactions which should be applied at the surface of the drilled hole to create the equivalent state of stress in the surrounding material, Fig. 4c - the effect caused by the fact that the hole surface is stress-free (which implies the liquidation of the stress from Fig. 4b by applying the identical stress but with the opposite sign). Fig. 4d - approximation of the "real" distribution by step distribution. If at the step "i" of the hole surface the reactions uniformly distributed along h_i are described as

$$\sigma_{hi} = \frac{1}{2}(\sigma_{1hi} + \sigma_{2hi}) + \frac{1}{2}\left(\sigma_{1hi}\cos 2\theta_{1\alpha i} + \sigma_{2hi}\cos 2\left(\theta_{\alpha i} + \frac{\pi}{2}\right)\right),$$
$$\tau_{r\theta i} = -\frac{1}{2}\left[\sigma_{1hi}\sin 2\theta_{\alpha i} + \sigma_{2hi}\sin 2\left(\theta_{\alpha i} + \frac{\pi}{2}\right)\right], \qquad (4)$$

they create, at the free surface of the body around the hole, the radial strains equal to:

$$\varepsilon_{in}(\theta_{\alpha i}) = A_{in}\Delta h_i(\sigma_{1hi} + \sigma_{2hi}) + B_{in}\Delta h_i\left[\sigma_{1hi}\cos 2\theta_{\alpha i} + \sigma_{2hi}\cos 2\left(\theta_{\alpha i} + \frac{\pi}{2}\right)\right] \qquad (5)$$

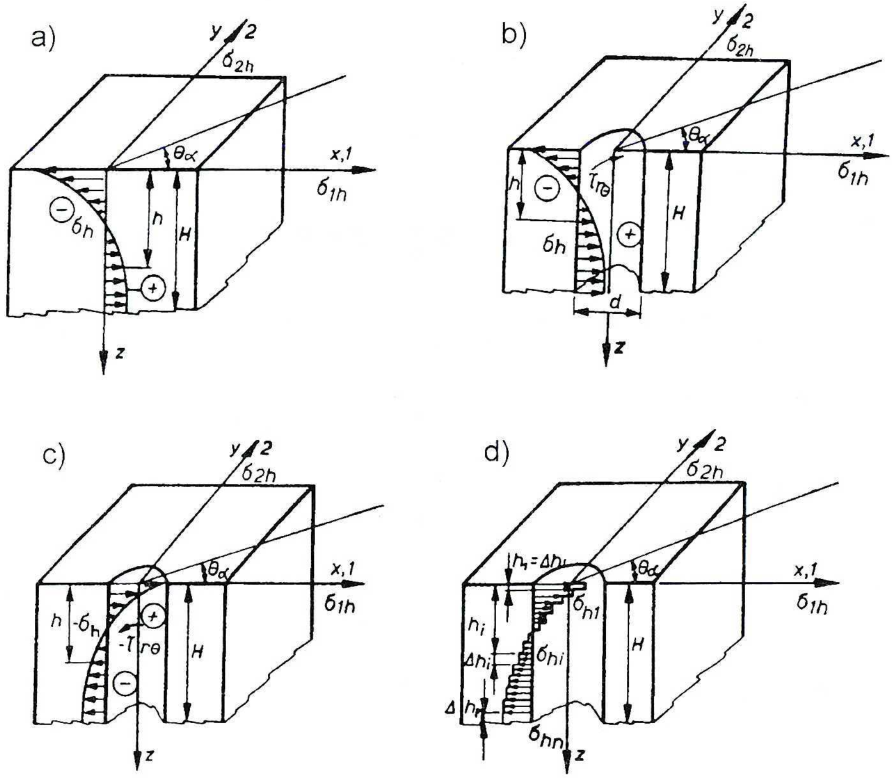

Fig. 4 The principle of the hole-drilling method (according to Ref. [9]); a) residual stress existing in the material, b) the reactions which should be applied at the surface of the drilled hole, c) applying the stresses with opposite sign, d) approximation of the real stress distribution

In the eqs. (4), (5) σ_{1hi}, σ_{2hi} denote the average values of the principal residual stresses existing on Δh_i; σ_{hi}, $\tau_{r\theta i}$ - the normal and the shear stresses equivalently applied at the i-step of the hole surface; A_{in} and B_{in} - the coefficients determined numerically (by the FEM method). The nondimensional values of these coefficients (for an arbitrary hole diameter) were computed solving the 56 problems presented schematically in Fig. 5b. The hole was drilled by the consecutive increments shown in Fig. 5a.

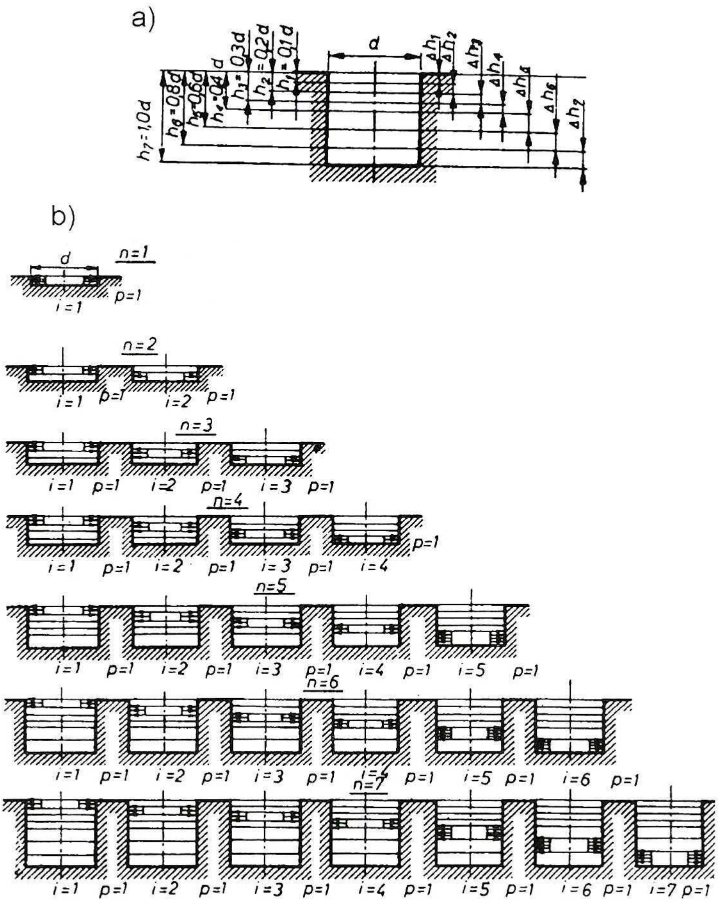

Fig. 5 Determining the computational coefficients for the hole-drilling method (according to Refs [9] and [10]); a) the assumed increments of the drilled hole, b) the problems solved to obtain coefficients A_{in} and B_{in} for each step of the hole deepening

The strains ε_{in} at the surface of the tested object can be measured in different ways. Along with the commonly used strain gages rosette with 3 gages sticked radially around the hole, the holographic and moiré interferometry are often applied nowadays. This last method is presented in Fig. 6. Fig. 6a shows the principle of the measurement, Fig. 6b the examples of the surface displacements contour maps after the consecutive hole's depth increments, Fig. 6c - the residual stress distributions derived from the surface readings. The tested specimen was shot peened, thus compressive residual stresses with almost equal values of both principal components were created in a thin surface layer.

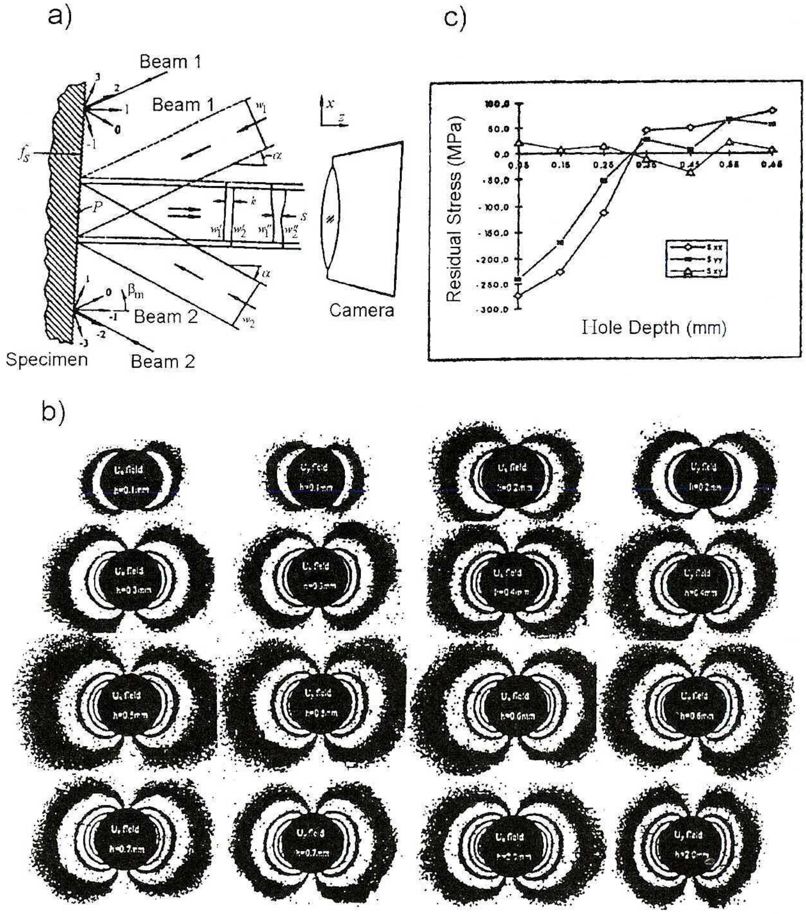

Fig. 6 The hole drilling method: surface displacement measurements provided by moiré interferometry (according to Ref. [15]); a) the principle of the measurement, b) displacements contour maps, c) the residual stress distribution inside the material

The hole drilling method can be used for determining residual stresses in objects of different shapes, dimensions, constructed from different materials: in rails, pipelines, crankcases, turbine blades and disc and others. These stresses are created by many kinds of production processes: casting, welding, heat and chemical treatments, cold work etc. The accuracy of the method depends on the technique of strains measurement, the accuracy of the cylindrical hole shape and position, the magnitude of plastic deformations introduced by the drilling process. It is difficult then to present some general evaluation of the level of accuracy.

Along with the hole drilling, *the crack compliance technique* is recently often used and broadly discussed as an another semi-destructive method [18], [19], [20]. Residual stress determination is based on the measurement of strains in the vicinity of the surface crack cut in the tested object. It is hard to admit, however, that the crack compliance technique is a new development: the idea of the method has been known for last 45 years and the applications were presented already in 1963 [21].

3.3. Nondestructive methods

Nondestructive methods do not injure the structure of the tested object. They can be divided into two groups: these which use strains or displacements measurements for residual stresses determination (e.g. the X-ray difraction method, the penetration method, the neutron dfifraction method), and those, which are based on the relation between stress and some physical or chemical properties of the material (e.g. the ultrasonic method, the Barkhausen magnetic noise method).

Generally, if the strain/displacement measurements are undertaken, the tested area is very small. Residual stresses determined in this way can be microstress or macrostress evaluated - practically - at the point. This situation carries evident advantages but also the limitations, although, e.g. in the X-ray and the neutron diffraction methods, it is possible to separate the effects caused by the stresses acting within a single grain and by the macrostresses.

<u>Penetration method</u> [22], [23] is based on the experimentally and numerically determined phenomenon that the permanent plastics surface deformation around an indentation of an axisymmetrical punch (a ball, a cone etc.) is strongly dependent on the state of stress in the surface layer of the material. The out of plane displacement distribution allows for the evaluation of the values and directions of principal residual stresses in the thin subsurface layer. The principle of the method is presented in Fig. 7, and the normal displacements contour maps determined around the indentations of different conical punches using optical interference technique are shown in Fig. 8. The pictures are shown for the states: "without stress", 1-D tension and 1-D compression, respectively.

The method can be very successfully applied for determining residual stress distributions in the thin surface layers hardened by heat and chemical surface treatments (up to 62 HRC hardnes number). In order to evaluate the approximate residual stress distribution inside the material, a few indentations by the punches of the different sizes must be performed: it allows for the different depths of the penetration.

The main disadvantages of the method are: not enough high level of accuracy and the need for separate calibrations of each investigated material.

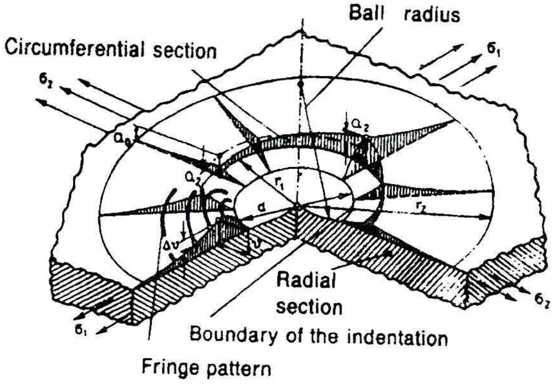

Fig. 7 The nondestructive penetration method (according to Ref. [23]): a typical picture of plastic deformation around an indentation

X-ray diffraction method exploits the fact that when a material is under stress, applied or residual, the resulting elastic strains cause the atomic planes in the crystal structure to change their spacings. By this method, one can directly measure this interplanar atomic spacings which are indicative of the macroelastic strain in the specimen. Stress values can be determined from elastic strains in the crystals as one knows the elastic constants of the material and assuming that stress, when below the yield point, is proportional to strain.

The basic equation relating X-ray diffraction principles to stress-strain relation can be written then as

$$\varepsilon_{\phi\psi} = \frac{d_D}{D} = \frac{1+v}{E}\sigma_\phi \sin^2\psi - \frac{v}{E}(\sigma_1 + \sigma_2), \tag{6}$$

where v and E are elastic constants and, according to Fig. 9: σ_ϕ - stress in the plane of the surface of the specimen at an angle of ϕ with a principal stress direction in the specimen surface, ψ -angle between the surface normal and the normal to the crystallographic planes from which an X-ray peak is diffracted, $\varepsilon_{\phi\psi} = \Delta D / D$ - strain in the direction defined by the angles ϕ and ψ, σ_1, σ_2 - principal stresses in the surface plane of the specimen. The value of d_D / D can be found from Bragg's law

$$n\lambda = 2D\sin\theta, \tag{7}$$

as

$$\frac{dD}{D} = -\text{ctg}\,\theta d\theta, \tag{8}$$

where n is an integer λ - the X-ray wavelenght, D - the lattice spacing.

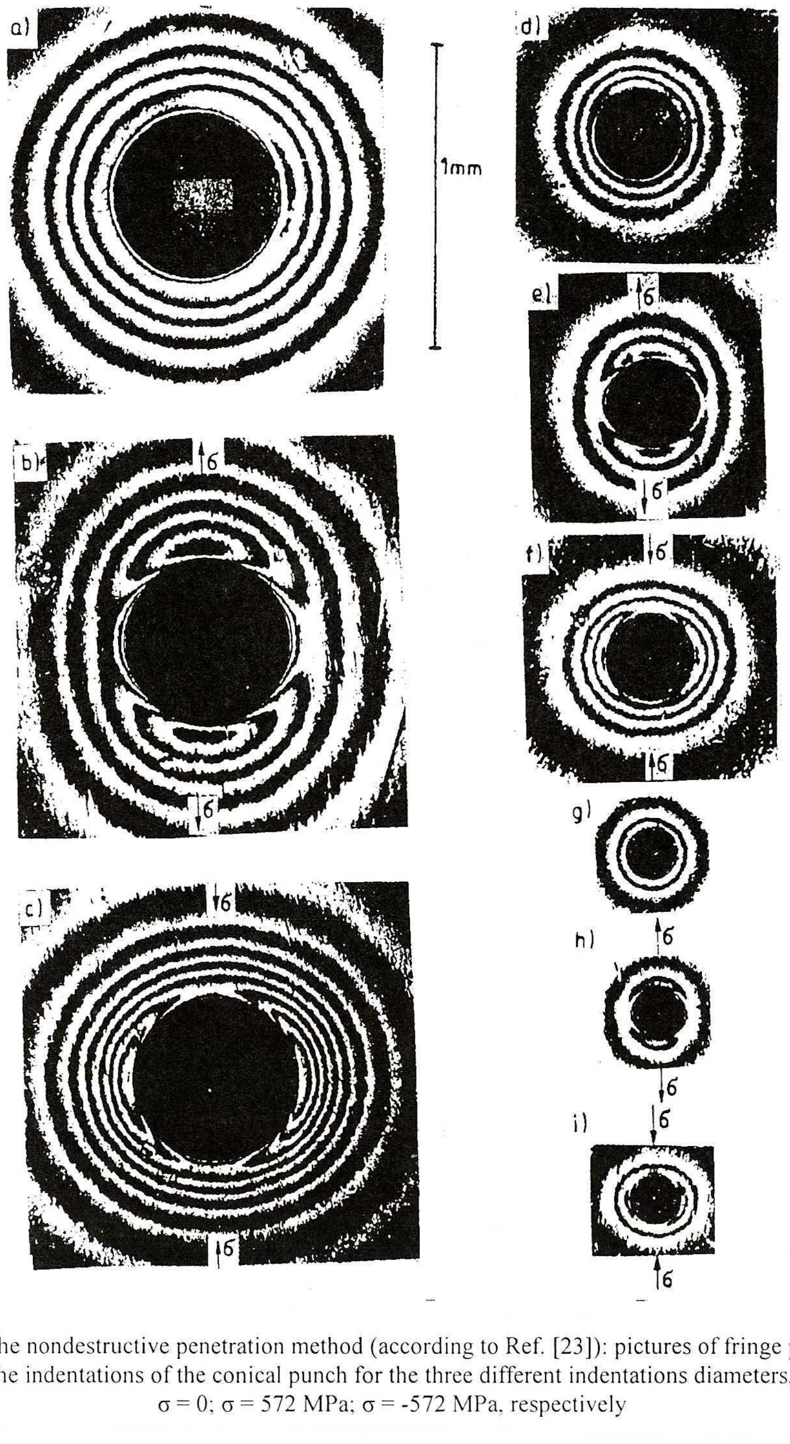

Fig. 8 The nondestructive penetration method (according to Ref. [23]): pictures of fringe patterns around the indentations of the conical punch for the three different indentations diameters, and for $\sigma = 0$; $\sigma = 572$ MPa; $\sigma = -572$ MPa, respectively

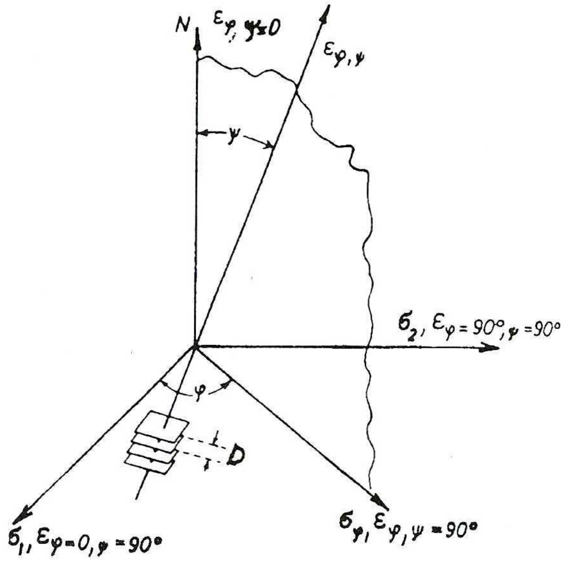

Fig. 9 Symbols used in X-ray strain measurement

Consequently, every lattice strain d_D / D leads to a change $d\theta$ in the Bragg angle, i.e., the X-ray diffraction lines undergo changes in their peak positions. To determine the lattice strains we measure these line shifts.

The eq. (6) is valid for a biaxial state of stress at the very surface of the specimen subjected to a state of multiaxial residual stresses where the normal stress component is inherently zero. The assumption usually made is that the stress component perpendicular to the surface does not influence the X-ray stress determination because of small depth of effective penetration of the X-rays into metallic specimen. In other cases, however, where penetration is deeper or large stress gradients exist near the surface, the third stress component must be taken into account. Standard methods have been developed to evaluate this influence [26].

There are three basic tehniques for obtaining stress (strain) readings from a polycrystalline material by the X-ray method. These are called sin-square-psi ($\sin^2\psi$) technique, double exposure technique and single exposure technique; all assume the plane stress condition in the shallow of X-ray penetration. In order for the X-ray method to be used as a measure of the residual or applied stress (without a need of foregoing measurements in a zero-stress condition for comparison) the D spacing of planes must be found for at least two different orientations with the metal surface, e.g. ψ_1 and ψ_2.

In the $\sin^2\psi$ technique the several X-ray readings at a series of different ψ values contribute to its statistically very high accuracy. The obtained interatomic spacings D_1, D_2, D_3 etc. with a wide range of ψ values serve in order to plot D against $\sin^2\psi$; the plot falls usually on a straight line. The intercept with the axis at $\sin^2\psi = 0$ is equal to $\frac{v}{E}(\sigma_1 + \sigma_2)$ and σ_ϕ is proportional to the slope of the line. The direction of the measured surface stress σ_ϕ is the line of coincidence of the surface plane and the plane containing the incident and diffracted beam (kept the same for all ψ angles).

The double exposure technique requires that the interplanar spacings of planes with two orientations with the specimen surface are obtained. Usually, one at $\psi = 0$ and one at $\psi = 45^0$ are selected as shown in Fig 10a.

The single exposure technique is based on the fact that a single incident X-ray beam is diffracted in an entire cone of reflections (Fig. 10b). A plane perpendicular to the cone axis intercepts the cone in a circle when the specimen is unstressed, but if the specimen is stressed the circle becomes an ellipse, and the deviation of its shape is indicative of the stress (strain) in the sample. To read this deviation, detectors are placed 180 degrees apart, as shown in Fig. 10c.

The X-ray stress measurements are performed using film cameras, CCD cameras, or special detectors.

The most common sources of errors and misapplications in residual stress measurements by X-ray are: selection of elastic constants (the crystal are anizotropic), diffracted peak location, microstress, grain size, microstructure and surface conditions. Despite the fact that X-ray provides stress readings only to a depth of less then 0.025 mm and that the error sources listed above must be considered, the non-contact X-ray diffraction method is presently the one of the two generally applicable, truly non-destructive methods for measuring residual stress. Its reliability has been proved and documented by hundreds of engineers and scientists over the past two decades e.g. [24], [25], [26]. At the Fifth International Conference on Residual Stress held in Linköping (Sweden) in 1997, during the session titled "Measurement of Residual Stresses", between 55 delivered papers, 22 were devoted fully or partially to the X-ray method: e.g. [27], [28], [29].

Neutron diffraction e.g. [31] though based on the same principle as the X-ray method, does not carry many errors and limitation of the previous one. The neutron diffraction is at present the second commonly used non-destructive method, despite the complex and expensive equipment needed. At the mentioned previously session of the Residual Stress Conference, the 18 from 55 papers presented the method, e.g. [30], [32], [33], [34].

X-rays interact electromagnetically with orbiting electrons in atoms of the tested material and are strongly absorbed after penetrating less than 100 μm in most metals. In contrast, the interaction of the uncharged neutrons with electrons is weak and they penetrate several centimetres. In iron for example, the linear absorption coefficient for neutrons is approximately 20000 times smaller than this for X-rays, similarly resulting in relative depth of penetration. This significant depth of penetration causes that strains, being measured distantly from the surface, are influenced by the six stress components and, in general, the principal stress/strain directions are not known. Therefore, differently than in the X-ray method for plane states of stresses, the strain in an arbitrary diffraction vector $\phi\psi$ direction in expressed as

$$\varepsilon_{\phi\psi} = \frac{dD}{D} = \varepsilon_{xx}\cos^2\phi\sin^2\psi + \varepsilon_{yy}\sin^2\phi\sin^2\psi + \varepsilon_{zz}\cos^2\psi + \varepsilon_{xy}\sin 2\phi\sin^2\psi + \varepsilon_{xz}\cos\phi\sin 2\psi + \varepsilon_{yz}\sin\phi\sin 2\psi\,, \tag{9}$$

where x, y, z are the directions of the specimen Cartezian coordinate system.

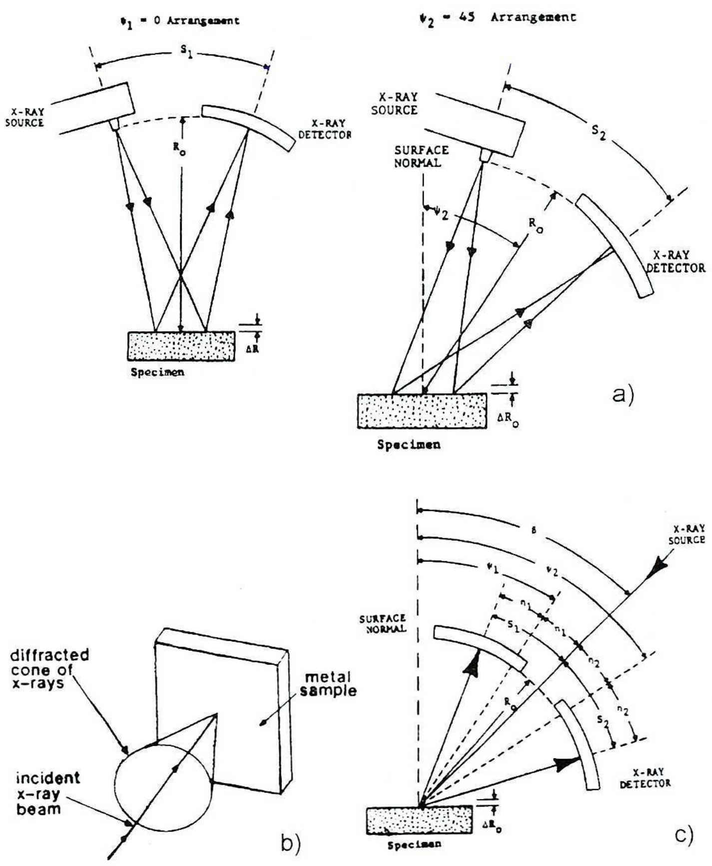

Fig 10 Schematic diagrams of X-ray technique arrangements a) double-exposure arrangement; b) cone of diffracted beams produced by a single X-ray beam; c) single-exposure arrangement, R_0 the specimen - to - detector radius

By making strain measurements at six or more orientations, the components of the strain tensor and position in the specimen can be determinated, and the stress can be computed using the Hooke's law. Sometimes the principal axes 1,2,3 are known from the symmetry of the specimen, frequently they are assumed in order to simplify measurements, then

$$\varepsilon_{\phi\psi} = \varepsilon_1 \cos^2 \phi \sin^2 \psi + \varepsilon_2 \sin^2 \phi \sin^2 \psi + \varepsilon_3 \cos^2 \psi \tag{10}$$

and only three strain readings are needed.

The results of measurements provided by the neutron diffraction (changes in the lattice spacings) are averaged over volumes of a few cubic millimetres, much greater than individual grain seizes, so that many grains contribute to the diffraction peaks. This eliminates, to some extent, the errors created by anizotropy of crystal structure, although, in many cases the results are here not satisfactory [31].

In Fig. 11a, the outline of the Japan Ressearch Reactor - 3M is shown. Many neutron diffraction and small angle scattering apparatus are located along the beam lines. One of them is shown schematically in Fig. 11b [30].

In Fig. 12 neutron diffraction residual stress measurement results are exemplified. A plastically bent welded tube (left three plots) and a bent seamless tube (right three plots) had been investigated. The weld location is indicated by the filled triangles. The stress level is proportional to the gray scale such that white corresponds to - 400 MPa, and black to 400 MPa [34].

Ultrasonic method is based on the phenomenon of an approximately linear change in ultrasound velocity with applied or residual stress. Stress is measured by inducing a sound wave into the tested object and measuring the time of its passage through some distance, or determining some other velocity related parameter. There are many types of wave modes which can be used for measurements: these include bulk waves such as longitudinal and shear, and surface waves.

As it has been told, the primary effect of stress-induced strain on ultrasonic propagation in metals is on velocity. However, there are other characteristics of metals, which affect the ultrasonic velocity to the same degree as stress. These include microstresses, multiply phases, crystallographic texture, grain size, dislocations density and distribution. This situation is the source of significant errors, and also causes that ultrasonic stress measurement should be calibrated relative to the particular material being investigated in order to acquire the proper elastic-acoustic constants.

The time of the ultra sound wave passage (needed to find the velocity value) has to be measured along some distance, usually of at least few centimetres, to provide the sufficient level of accuracy. As a result, an average value of stress is determined along this distance, which is the additional limitation of the method.

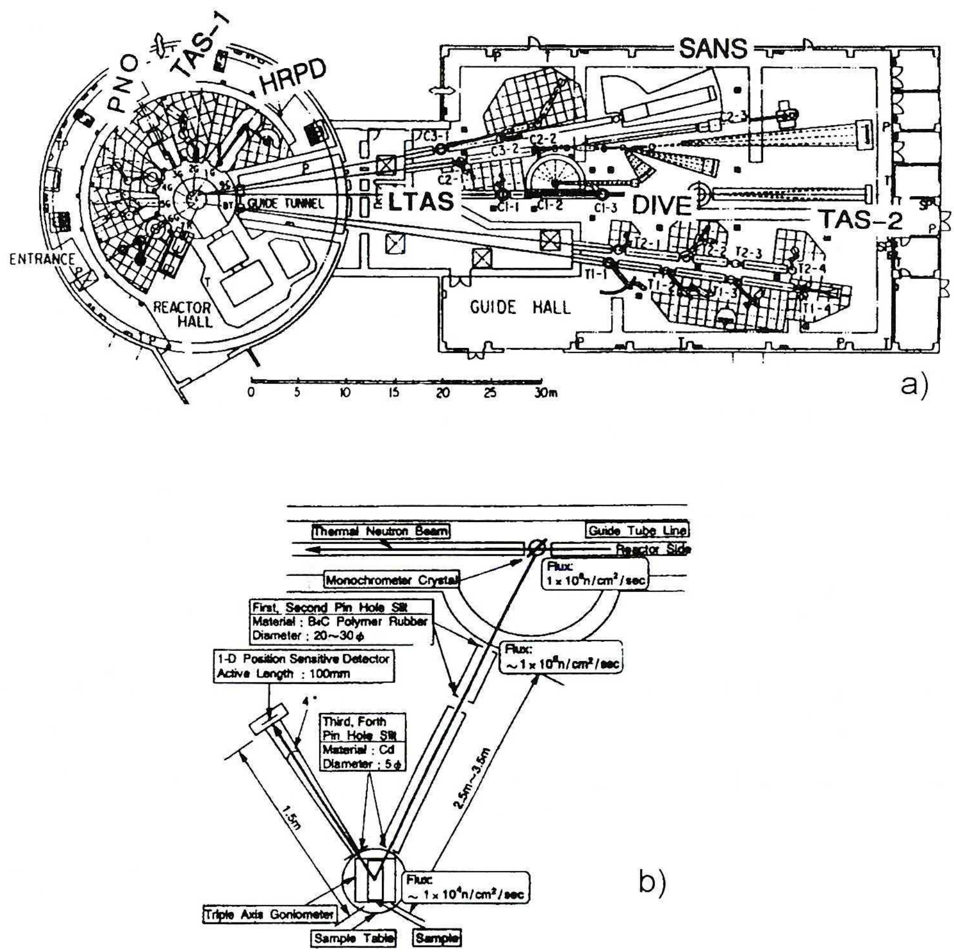

Fig. 11 Neutron diffraction arrangements in Tokai Establishment of Japan Atomic Energy Research Institute (according to Ref. [30]); a) outline of Japan Research Reactor - 3M; b) outline of neutron diffraction apparatus

The ultrasonic technique has been offering the promise of three-dimensional, nondestructive residual stress measurements. However, the mentioned above misadvantages cause that it is seldom applied nowadays, and only to some specific tests. Such application developed in the Polish Academy of Science is presented in [35], [36]: the method has been very successfully used for determining residual stresses in rails and train wheels, and has become the one of main standard procedures in this field (Figures 13, 14).

In Fig. 13 the ultrasonic measurements of residual stresses in train wheels are presented. Fig. 13a shows the application of surface waves: the positions of the transducer, the measuring head and the travel of the ultrasonic wave beam are indicated. In Fig. 13b the application of bulk shear waves for the same test is shown. The results of the measurements are exemplified in Fig. 14: the change in the circumferential (hoop) residual stress component in the train wheel head due to multiple brakings of different power is demonstrated. The results of the ultrasonic and the X-ray methods are compared.

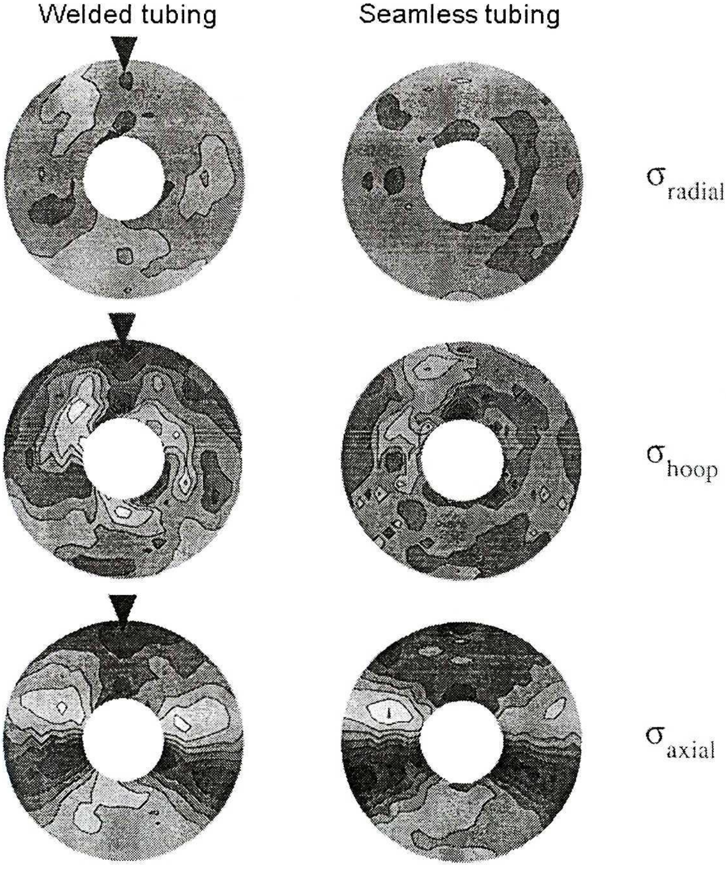

Fig. 12 Neutron diffraction residual stress measurement results for a bent welded tube and a bent seamless tube; white = - 400 MPa, black = 400 MPa (according Ref. [34])

Magnetic methods are based on the relations between stress and magnetic or dielectric quantities such as dielectric and magnetic permittivity, magnetic anisotropy, Barkhausen noise and others. The *Barkhausen noise technique* is at present most commonly used. It is connected with measuring the number and magnitude of abrupt magnetic reorientations made by the magnetic domains in a ferromagnetic material. These reorientations, random in amplitude, duration, and temporal separation, are described as noise. The appearance of magnetic noise, and its parameters in the process of magnetization depend on stress level in the structure [37].

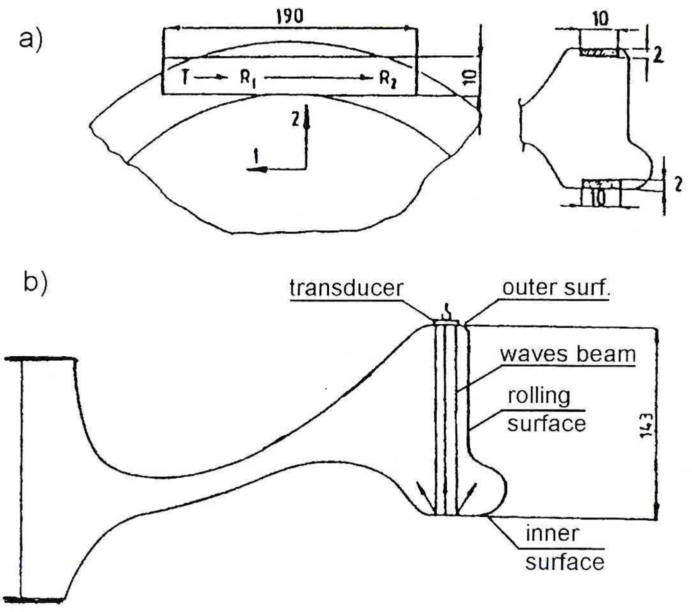

Fig. 13 Ultrasonic techniques for residual stress measurements in train wheels a) surface waves arrangement; b) bulk shear waves arrangement

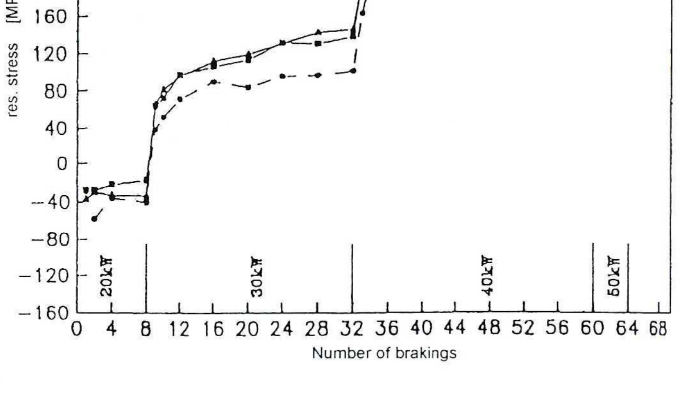

Fig. 14 The change in the circumferential residual stress component in the train wheel head due to multiple brakings of different power

The Barkhausen noise method has been used mainly for determining one-dimensional, uniform states of stresses. The two-dimensional measurements have been seldom conducted; they need uphill calibrations.

There are many significant limitations of the method; these include low accuracy (± 10 MPa), small depth of penetration, applicability only to ferromegnetic materials, the need for separate calibrations with a slightly different specimen. The examples of the measurements performed using the magnetic noise method are presented in [38], [39].

There are *other nondestructive methods* for measuring residual stress in different materials. For instance, the technique called "TERSA" [40] exploits the phenomenon that absorption of heat and, as an effect, a rise of the temperature of the heated object depends, in some degree, on stresses. Although the authors claim that the method can be applied for ferrous and non-ferrous metals as well as for plastics, the "TERSA" has not gained further development or applications. Usually, the same goes for very specific techniques suited for determining residual stress in special materials (glass, composites, ceramics) or structures.

4. Numerical and hybrid methods for residual stress analysis

The numerical methods are used nowadays as often as the experimental techniques. Many experimental investigations which have been performed during the last decades, as well as measurements broadly conducted at present, provide sufficient number of information in order to model numerically the regular work of the structure, as well as the production and assembly processes which create residual stress. The numerical investigations are easier, cheaper and allow for a much faster analysis of different aspects of the already programmed phenomenon. They have this additional advantage that the boundary and initial conditions, as well as the course of the modelled process assumed at the beginning (even with some errors) remain identical for all analysed cases of the same model. Experiments do not assure such possibilities. The real objects, however, can be investigated only experimentally, and the analysis of new processes and phenomena can not be developed without providing enough experimental data.

The material nonlinearity (plastic flow) and often the geometric nonlinearity (large strains) must be taken into account in the residual stress analysis. This leads to dimensionally large and difficult computational problems (e.g. the convergence of the iteration processes). Therefore, the numerical analysis is broadly applied to two-dimensional problems, seldom and with many limitations in order to solve 3-D problems.

Fig. 15 presents the 2-D residual stress distribution created by the frictional elastic-plastic rolling process of the steel strip [41]. The plane-strain conditions are assumed and the Finite Element Method applied. The value of the residual stress component in the rolling direction (σ_x) as a function of depth measured from the rolled surface is shown for the free rolling ①, the rolling with braking ② and the rolling with acceleration ③.

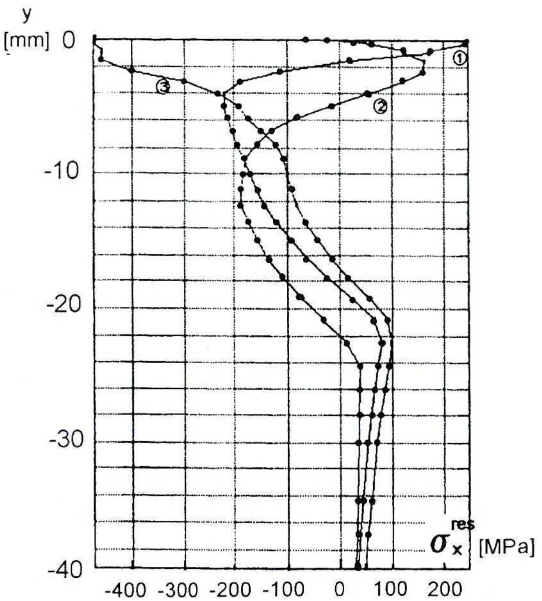

Fig. 15 The 2-D residual stress distribution due to the friction elastic-plastic rolling process of the steel strip, obtained by the FEM computation (according to Ref. [41]): ① - free rolling, ② - rolling with braking (75%), ③ - rolling with acceleration (75%)

On the other hand, Fig. 16 [42], demonstrates the way of determining 3-D solution of the residual stress distribution in a rail head. The applied procedure has been developed in order to diminish the numerical size of the problem. The large portion of the rail is modelled as an elastic body with the mesh of elements of the relativelly small density. The small portion of the rail head in the vicinity of the rolling contact surface modelled as an elastic-plastic geometrically nonlinear structure is divided into great number of small elements, allowing the effective residual stress computation in this area. Both problems have been computed separately briging respective boundary conditions from the "large portion" to the "small" one.

The *hybrid method* applies both numerical and experimental means in order to provide the residual stress analysis. Good examples of such solutions can be found in [43], [44] and [45]. The results of measurement brought from this last paper, obtained using the crack compliance/moiré interferometry experimental method, provide the residual stress distribution in the laser welded structure when combined with FEM computations. In Fig. 17a, the contour map of the displacement around the longitudinal cut of the laser weld is shown. The fringe pattern was obtained by the moiré interferometry. The spacial shape of the displacement field is presented in Fig. 17b.

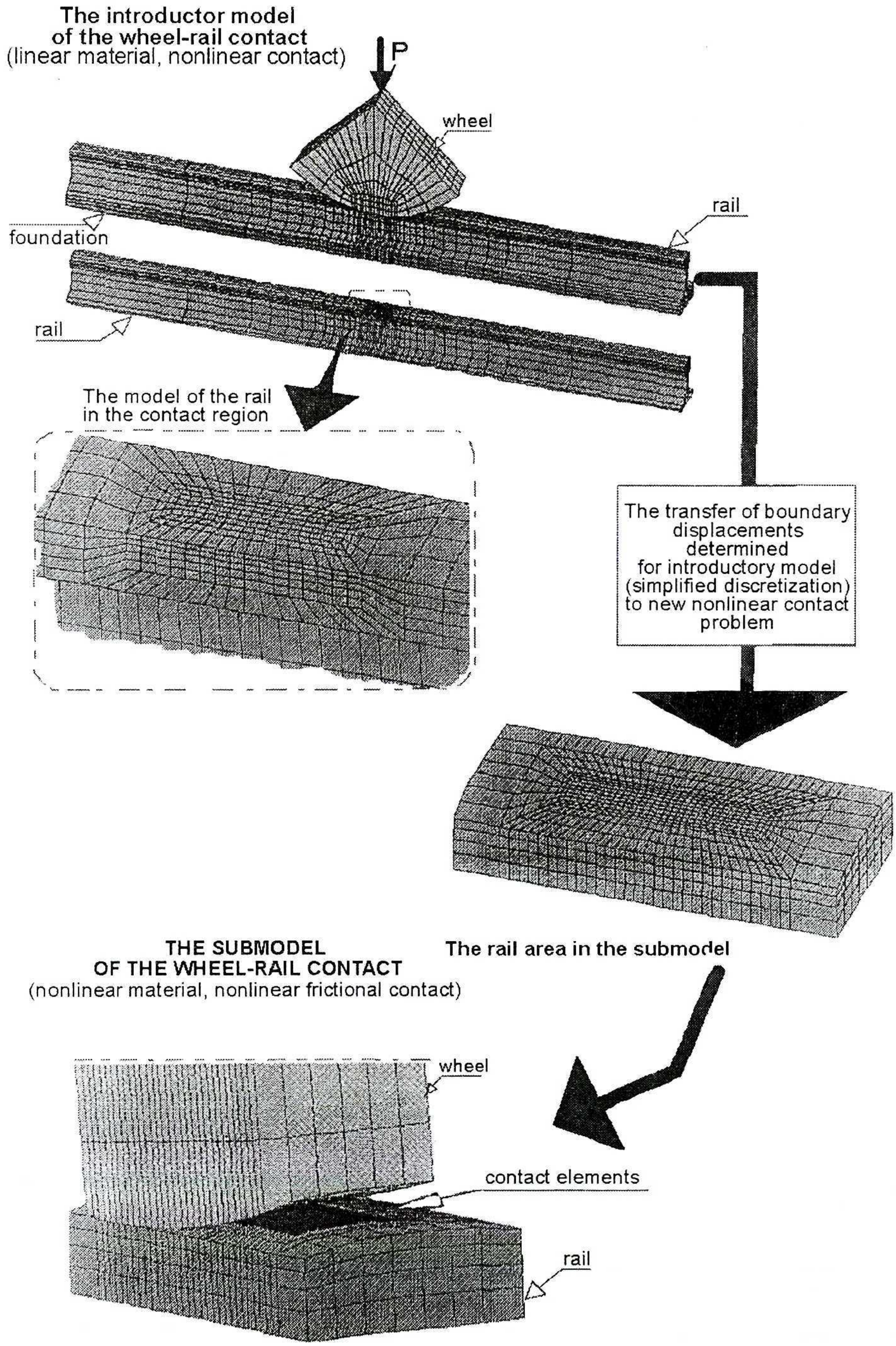

Fig. 16 The way of searching for the 3D numerical solution of the elastic-plastic rolling contact problem: (according to Ref. [42])

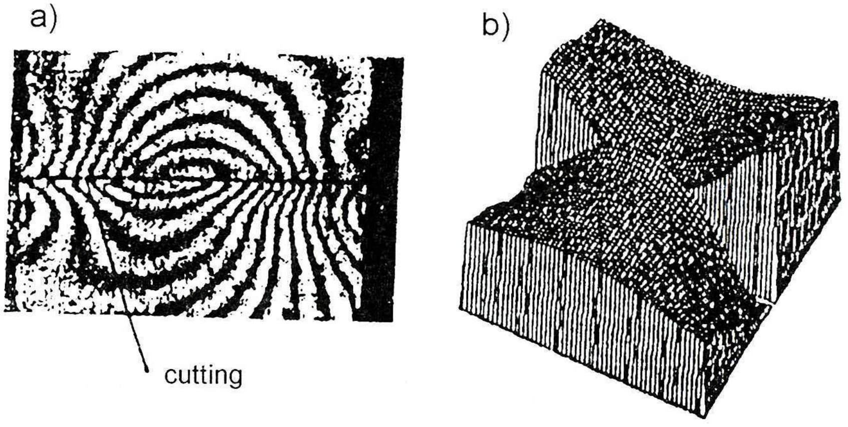

Fig. 17 The hybrid method for determining residual stress in the laser weld (according to Ref. [45]); a) the contour map of the displacement obtained by the moiré interferometry, b) the spacial shape of the displacement field

Although the hybrid method seems to be promising, the necessity of the time consuming, complex investigations, as well as not to high accuracy of the obtained results cause that the method is still not commonly used in engineering practice.

The author does not intend to present - even partially - the all aspects of this very broad problem called "Residual stress analysis". The aim of this work is limited to a review of the contemporarily used methods and to outline the directions of possible developments.

Manuscript received by Editorial Board, January 07, 2000;
final version, March 03, 2000.

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Współczesne metody analizy naprężeń własnych

Streszczenie

W artykule są omawiane problemy analizy naprężeń własnych. Intencją autora nie jest jednakże przedstawienie, nawet częściowe, wszystkich aspektów tego bardzo szerokiego zagadnienia. Cel tej pracy jest ograniczony do przeglądu współczesnych eksperymentalnych, numerycznych i hybrydowych metod badawczych i wskazanie tendencji dalszego rozwoju.