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FEM MODELLING OF BUCKLING AND POST-BUCKLING BEHAVIOUR OF PLASTIC STRIPS

Because of clear trend in the world to design lighter plastic products, the phenomenon of creep buckling of visco-elastic solids becomes increasingly important. This paper reports the intermediate results of the research project on loading capability and buckling of plastic containers, carried out in the Laboratory of Mechanical Reliability (LMB) of TU Delft.

Based on the earlier developed non-linear visco-elasticity model for engineering plastics, the FE simulation of the delayed in time buckling of plastic strips have been performed as a first step toward the understanding and predicting creep-buckling behaviour of plastic carriers.

1. Introduction

In order to save material (i.e. production costs), but also due to growing concern of society about environment, there is clear trend to make plastic products lighter. This implies also decreasing wall thickness and, consequently, introduction into design additional stiffness ribs (to enable these lighter products to carry a load). Unlike metals, however, plastic materials feature intense time-dependent behaviour: like visco-elasticity and physical ageing. Thus, if the plastic product is supposed to sustain long term loading, the creep induced buckling (or, in the other words, buckling delayed in time) should be taken into account.

The traditional approaches of structural analysis of plastic products, based on the elasticity or elasto-plasticity models of material behaviour, are not capable to cope with the above phenomenon.

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In this paper, the recently developed model on non-linear visco-elasticity is applied for Finite Element (FE) simulations of buckling and post-buckling behaviour of High Density PolyEthylene (HDPE) strips under compression loading. The aim of this work is to verify the model developed, but also to predict the creep buckling behaviour of visco-elastic solid with simple shape.

2. Main elements of the material model

Foremost, the main elements of the non-linear visco-elasticity model [1], which is used in FE simulation, will be given. This model can be considered as a generalisation of the Schapery model [2]:

$$\varepsilon\left(\sigma(t),t\right)=J_0[\sigma(t)]+\sum_i \phi_i\left[\sigma(t)\right]\int_0^t F_i(t-\zeta)\frac{\partial\left[g_i(\sigma)\right]}{\partial\xi}d\xi \tag{1}$$

Following forms for the functions in equation (1) seem to be appropriate [3] for description of experimental data for many plastics:

$$J_0[\sigma(t)]=A[\sigma+\sigma^{\beta}];$$

$$\phi_i[\sigma]=\exp(\gamma_i\sigma)$$

$$g_i[\sigma]=D_i\sigma^{\alpha_i} \quad \text{or} \quad D_{i,1}\sigma^{\alpha_{i,1}}+D_{i,2}\sigma^{\alpha_{i,2}}$$

$$F_i(t)=\left(1-\exp\left(-\lambda_i t\right)\right). \tag{2}$$

The parameters in these functions should be estimated, based on data from creep and recovery tests. For this the equation (1) can be rewritten as follows:

for creep $$\hat{\varepsilon}\left[\sigma_k,t\right]=A_{[0,k]}+\sum_{i=1}^{n}A_{[i,k]}\left(1-\exp\left(\frac{-t}{10^i}\right)\right) \tag{3}$$

for recovery $$\hat{\varepsilon}\left[\sigma_k,t\right]=\sum_{i=1}^{n}B_{[i,k]}\left(\exp\left(\frac{-t}{10^i}\right)-\exp\left(\frac{-(t+t_1)}{10^i}\right)\right). \tag{4}$$

where σ_k – constant loading level.

There are strong advantages in choosing the time functions $F_i(t)$ in the form of Prony series. Firstly, this choice makes it possible to apply an efficient numerical scheme [4] for calculation of hereditary integrals. Secondly, it gives better possibilities for establishing the parameter identification procedure. This procedure is based on the idea of minimisation of the relative deviation between experimental data and model prediction. The resulting set of material parameters for HDPE is given in Table 1.

Table 1.

The set of model parameters for description of HDPE

Term, i	A				β	
0	$7.852\cdot10^{-4}$				0.0	
	$D_{i,1}$	$\alpha_{i,1}$	$D_{i,2}$	$\alpha_{i,2}$	γ_i	λ_i
1	$.1278\cdot10^{-4}$	2.759	$.2592\cdot10^{-3}$	1.059	$-.2404\cdot10^{-1}$	10^{-1}
2	$.3364\cdot10^{-3}$	1.075	$.5910\cdot10^{-5}$	2.872	$-.1473\cdot10^{-3}$	10^{-2}
3	$.3729\cdot10^{-3}$	1.118	$.9672\cdot10^{-5}$	2.740	$.6727\cdot10^{-1}$	10^{-3}
4	$.5814\cdot10^{-4}$	2.193	$.2842\cdot10^{-9}$	6.254	$.4689\cdot10^{-1}$	10^{-4}
5	$.6187\cdot10^{-3}$	1.547	$.6753\cdot10^{-11}$	9.779	$.9244\cdot10^{-3}$	10^{-5}
6	$.4855\cdot10^{-1}$	1.637	$.2791\cdot10^{-2}$	2.786	-3.377	10^{-6}

The prediction of creep and recovery behaviour of HDPE (based on the equations (1), (2) and parameter set from the Table 1) is presented in Fig. 1.

The equation (1) was extended to 3-D form, based on the assumptions that:

- Material is compressible and initially isotropic;
- The processes of change of volume and shape are uncoupled;
- The rate of viscous flow is proportional to the effective stress $\hat{\sigma}$;

As a result, the equation (1) can be rewritten as follows:

$$\bar{\varepsilon}=\left[(1+\nu_0)\mathbf{M}_D+(1-2\nu_0)\mathbf{M}_H\right]\tilde{\varphi}(\hat{\sigma})\bar{\sigma}+$$
$$+\left[(1+\nu_1)\mathbf{M}_D+(1-2\nu_1)\mathbf{M}_H\right]\sum_{i=1}^{n}\tilde{\phi}_i(\hat{\sigma})\bar{\sigma}\int_0^t\left(1-\exp\left(-\lambda_i(t-\xi)\right)\right)\frac{\partial\left[\tilde{g}_i(\hat{\sigma})\bar{\sigma}\right]}{\partial\xi}d\xi. \tag{5}$$

Where $J_0[\bar{\sigma}]=\tilde{\varphi}(\hat{\sigma})\bar{\sigma},\quad g_i[\bar{\sigma}]=\tilde{g}_i(\hat{\sigma})\bar{\sigma}.$ (6)

The following matrix and vector notations are used:

$$\bar{\sigma}=\left\{\sigma_{xx}\sigma_{yy}\sigma_{zz}\tau_{xy}\tau_{yz}\tau_{zx}\right\}^T;\qquad \bar{\varepsilon}=\left\{\varepsilon_{xx}\varepsilon_{yy}\varepsilon_{zz}\gamma_{xy}\gamma_{yz}\gamma_{zx}\right\}^T. \tag{7}$$

$$\mathbf{M}_D=\begin{bmatrix}\frac{2}{3}&-\frac{1}{3}&-\frac{1}{3}&0&0&0\\-\frac{1}{3}&\frac{2}{3}&-\frac{1}{3}&0&0&0\\-\frac{1}{3}&-\frac{1}{3}&\frac{2}{3}&0&0&0\\0&0&0&2&0&0\\0&0&0&0&2&0\\0&0&0&0&0&2\end{bmatrix}\qquad \mathbf{M}_H=\begin{bmatrix}\frac{1}{3}&\frac{1}{3}&\frac{1}{3}\\\frac{1}{3}&\frac{1}{3}&\frac{1}{3}\\\frac{1}{3}&\frac{1}{3}&\frac{1}{3}\end{bmatrix}. \tag{8}$$

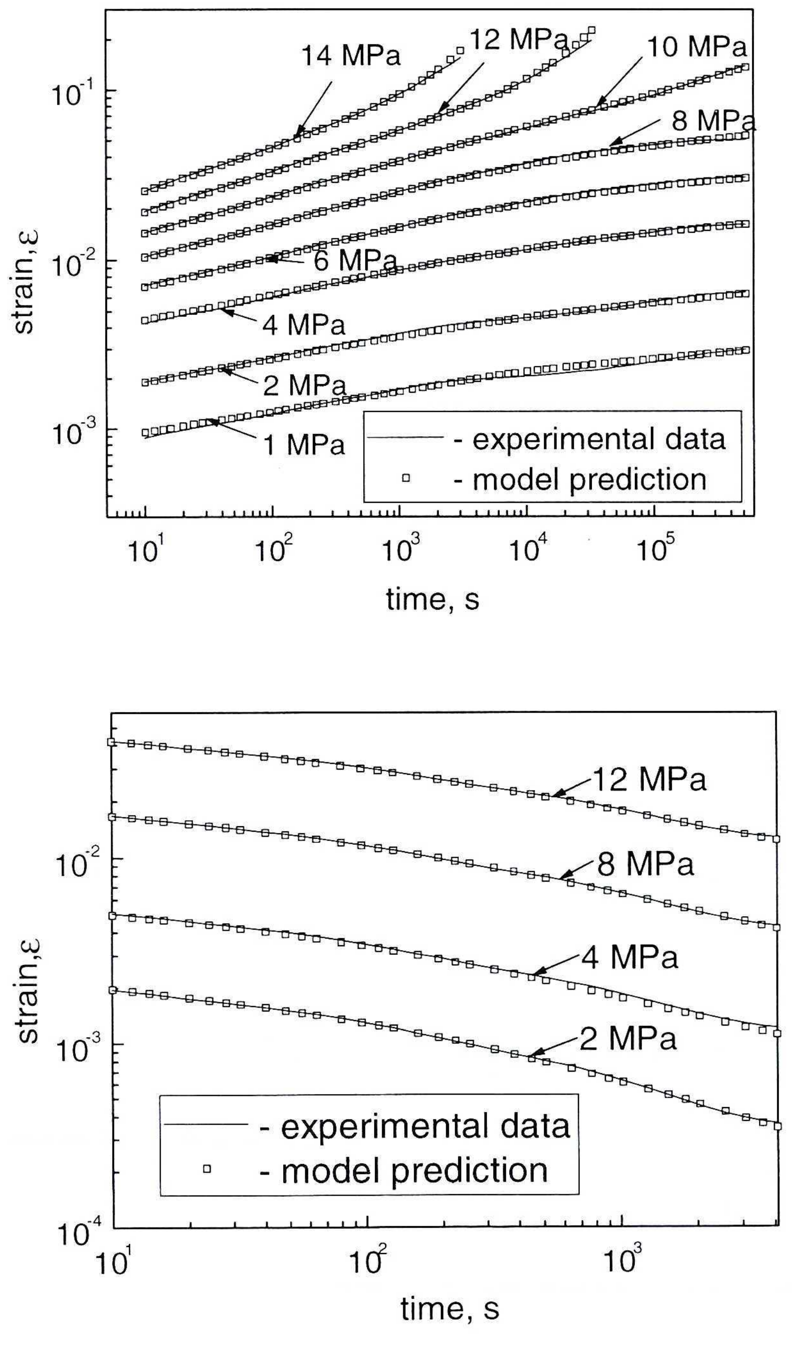

Fig. 1 Experimental data [6] and model prediction for the creep and recovery behaviour of HDPE

3. Implementation of material model into FE package

Since most of the FE packages are based on the displacement formulation, the visco-elasticity model (1) of Kelvin-Voight type should be inverted and rewritten in the incremental form as follows:

$$\Delta\bar{\sigma} = \mathbf{L}(\Delta\bar{\varepsilon}, \Delta\ t\ldots) \tag{9}$$

in order to be implemented into a FEA package.

Further the main elements, necessary to derive the equation (9), are given. For enough regular functions $g_i(\sigma)$, such that $\partial^2[g_i(\sigma)]/\partial\ t^2 << 1$, the hereditary integral with the exponential kernel function can be transformed to finite form, following Henriksen scheme [4]:

$$\int_0^t (1-\exp(-\lambda_i(t-\xi)))\frac{\partial[g_i(\sigma)]}{\partial\ \xi}d\xi = g_i(\sigma) + \theta_i(t). \tag{10}$$

The hereditary integral functions $\theta_i(t)$:

$$\theta_i(t) = -\int_0^t \exp(-\lambda_i(t-\xi))\frac{\partial[g_i(\sigma)]}{\partial\ \xi}d\xi \tag{11}$$

can be calculated recurrently as follows:

$$\theta_i(t) = \exp(-\lambda_i\Delta t)\theta_i(t-\Delta t) - \Delta[g_i(\sigma)]\Gamma_i(\Delta t). \tag{12}$$

Here, the following notation has been introduced for convenience:

$$\Gamma_i(\Delta t) = 1-\exp(-\lambda_i\Delta t)/\lambda_i\Delta t. \tag{13}$$

Further, the total differential of the equation (5) has to be derived and inverted to the form (9). Unfortunately, the numerical scheme, based on the total differential, shows low convergence ability and often becomes unstable [6]. Therefore, similarly to [7], it has been assumed that the pre-integral functions $\phi_i(\sigma)$ do not vary during the time increment. In addition, only partial factorisation has been used, while inverting the incremental stress-strain relation (i.e. scheme is neither explicit nor implicit, but a mixed one). As a result, the following incremental relation has been derived [6]:

$$\begin{aligned}\Delta\bar{\sigma} = &\{[(1+v_0)\mathbf{M}_D + (1-2v_0)\mathbf{M}_H]\tilde{\varphi}(\hat{\sigma})\}\times\\ &\times\{\Delta\bar{\varepsilon} - [(1-v_1)\mathbf{M}_D + (1-2v_1)\mathbf{M}_H]\times\\ &\times\sum\begin{bmatrix}[\phi_i(\hat{\sigma})\tilde{g}_i(\hat{\sigma})(1-\Gamma_i(\Delta t))]\Delta\bar{\sigma}(t-\Delta t)+\\ +\phi_i(\hat{\sigma})(\exp(-\lambda_i\Delta t)-1)\bar{\theta}_i(t-\Delta t)\end{bmatrix}\Bigg\}\end{aligned} \tag{14}$$

where: $$\tilde{\theta}_i(t) = \exp(-\lambda_i\Delta t)\tilde{\theta}_i(t-\Delta t) - \Delta[\tilde{g}_i(\hat{\sigma})\bar{\sigma}]\Gamma_i(\Delta t). \tag{15}$$

While deriving this relation, it is implied that the loading history always starts from $\sigma(0) = 0$.

The above derived scheme is recurrent. To calculate the viscous strains increment for the current moment t, only the data for the stresses field σ and internal parameters $\widetilde{\theta}_i$ from the previous step are necessary. Although for the models with large number of elements this numerical scheme can require large amounts of computer memory, the advantage in decreasing of calculation time is obvious.

4. Model verification

The MARC FEA package was chosen for the computer simulations, because of its open structure that allows for easy access to the variables (like stresses, strains, time increment), but also due to its extended facilities to handle the geometrically and physically non-linear problems.

To verify the material model and numerical scheme developed, the ramp tensile and compressive tests[1)] were simulated. The tensile specimen, described in specification ISO R 527, was modelled by three-dimensional isoparametric brick elements of type 7 [8]. One end of the specimen was "clamped", while a time-dependent displacement was applied to another end, until 20% of deformation were reached. The displacement was modelled by step-wise function (200 steps) using MARC options DISP CHANGE and AUTO CREEP [9] to simulate instantaneous variations of displacement and time-dependent changes during the time step respectively. Three different strain rates (6 % per minute, 0.6 % per minute, and 0.06 % per minute) were modelled. The results of FE simulations of tensile ramp loading are given in Fig. 2. The maximum deviation between experimental data and numerical prediction is less then 5%.

Simulation of the ramp compressive tests indicates whether a model based on the tensile creep experiments can be used for description of material behaviour under compressive loading.

As a compressive specimen, very short strip (10×10×3.9mm) was taken, to avoid a buckling onset. To model such a specimen, only one brick element (type 7) has been used. The type of boundary conditions and loading history were the same as for simulations of tensile tests. Total strain of 10% was reached in these computer experiments.

The results of these simulations are shown in Fig. 3. The point that the deviation between FE predictions and experimental data is less then 4.5%, but the accuracy of these FE calculations was set to 5%, confirms the possibility to use the model, based on tensile creep tests, to simulate situations with compressive loading.

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1) The tensile and compressive experiments with constant strain rate for HDPE were carried out at the LMB by Ph.D. student Johan Beijer.

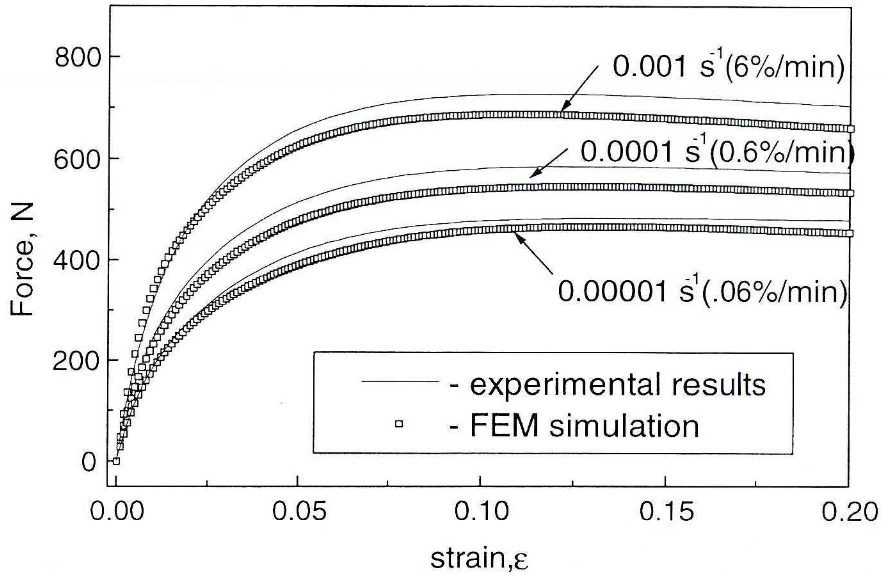

Fig. 2 FE simulation of ramp tensile tests on HDPE

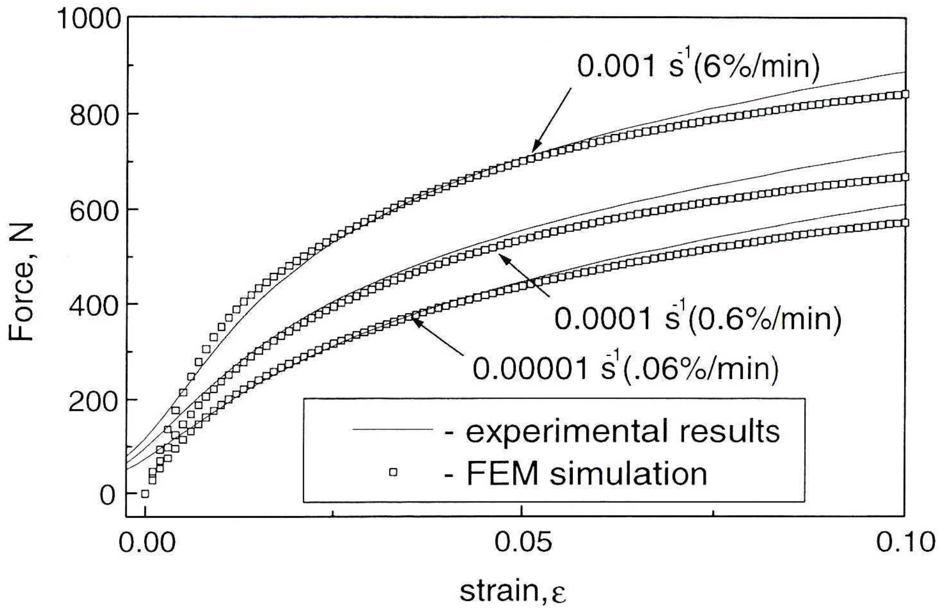

Fig. 3 FE simulation and experimental results of ramp compression tests of HDPE for three different strain rates

5. Experiments on buckling of HDPE strips

The specimens (strips) were cut out of the extruded HDPE plates and finished by milling to assure high precision and surface quality of specimens. The cross-section of all specimens was 14.95×3.1 mm. The distance between clamps of testing device will be referenced further on as the length of specimen.

In order to study the buckling phenomenon, the strips with four different lengths (35, 45, 70, and 80 mm) were tested. For the strips shorter than 35 mm, a small clamps misalignment drastically affected the test results. The specimens longer than 80 mm buckled at very low loads. Latter makes the tests with longer strips useless due to the accuracy of testing equipment. All the tests were performed on the 10 kN Zwick machine with facilities for automatic control of applied force or displacement. The strips were clamped in the testing device, after what the clamps were moved automatically to approach zero stresses in specimen before the test starts.

The tests of two different types have been performed. Firstly, the tests on the ramp compression for three different strain rates (5% per minute, 0.5% per minute and 0.1% per minute) were performed. Maximum strain reached in these tests was -10 %. The second set of experiments were tests with the creep compressive loading. After certain loading level (which was slightly lower than one necessary for instant onset of buckling) have been reached, the specimen was kept under constant loading until creep buckling occurs. The time to buckling onset was registered.

As it was mentioned above, even slight misalignment of clamps can drastically influence the results of buckling tests. For instance, the small shifting (0.7 mm) of one clamp to a side, while testing 80 mm long strip, subjected to the compressive load with constant strain rate of 5% per minute, leads to a drop of buckling force by 30% and deviation in post-buckling behaviour by 70...80%. Therefore, special attention was paid to the alignment of clamps in testing device. As a result, good reproducibility of the experimental results was reached: for the tests with similar loading conditions and specimen length, but in different tests series, the deviation of buckling force was less than 5%.

6. FE simulation of the experiments on plastic strips

An example of mesh and boundary conditions of 70 mm long specimen is given in Fig. 4. The mesh shown consists of 42 quadrilateral, three-dimensional shell elements of type 75 [8]. The small finite strain formulation [10] was applied. To simulate clamping of the strip in testing device, all the degrees of freedom were restricted for the nodes at the top and bottom of specimen (Fig. 4).

The deformation history was modelled by step-wise function (100 steps). At each step the nodes at the top of the mesh were displaced to the distance, which causes -0.1% of additional deformation in the strip. To model the loading in time, the specimen was allowed to relax (AUTO CREEP option [9]) during certain time increment, which related to the strain rate, prescribed for test simulated.

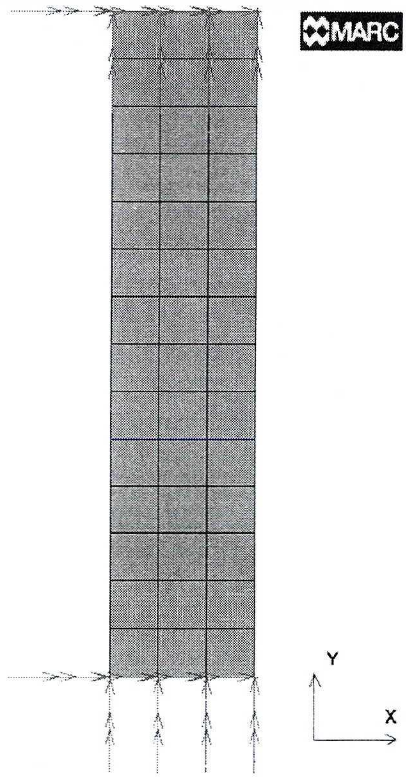

Fig. 4 The scheme of mesh and boundary conditions of the 70 mm long specimen

The buckling of loaded strip was simulated using the MARC facilities for buckling analysis (inverse power sweep method) [10]. In the case when buckling problem involves material non-linearity (e.g. visco-elasticity in our case), the problem must be solved using a perturbation analysis of the structure. It means that at certain moment of a loading history (at certain increment), the linear buckling analysis is performed to estimate the eigenvector ϕ of node displacements for requested buckling mode. At the next increment, the node coordinates are modified to account for the fraction of eigenvector:

$$X = X + \mathbf{f}^{*}\phi / |\phi| \tag{16}$$

The introduced perturbation $\{\mathbf{f}^{*}\phi / |\phi|\}$ will grow (or diminish) in time, if current buckling mode is stable (or unstable) under current loading conditions. Factor **f** was found empirically. For the described model it is equal 0.5. The perturbation analysis for first buckling mode was performed by invoking BUCKLE INCREMENT option [9] right after first "deformation step" (Fig. 5).

In order to verify the accuracy of FE modelling, additional calculations with different number of elements in mesh (Fig. 5) and different number of layers [10] in shell element (Fig. 6) were performed for the case of 70 mm long strip loaded by strain rate 5% per minute. It can be seen that for most cases (except of variant with 1 layer, which obviously is not functional) experimental data and results of simulation for post-buckling behaviour is less than 10%. Therefore, for further modelling the mesh with 48 shell elements (5 layer) was chosen.

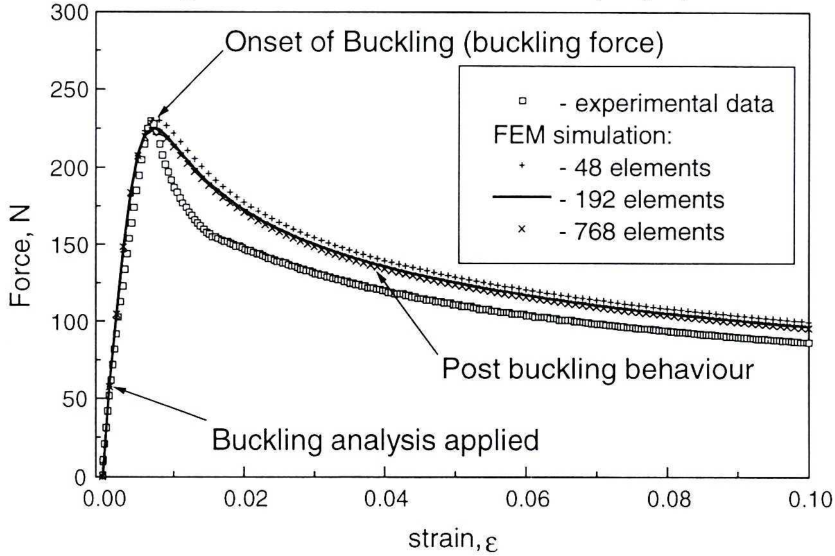

Fig. 5 Simulation of buckling and post buckling behaviour of 70 mm long strip (strain rate of 5% per minute), using mesh with different number of elements

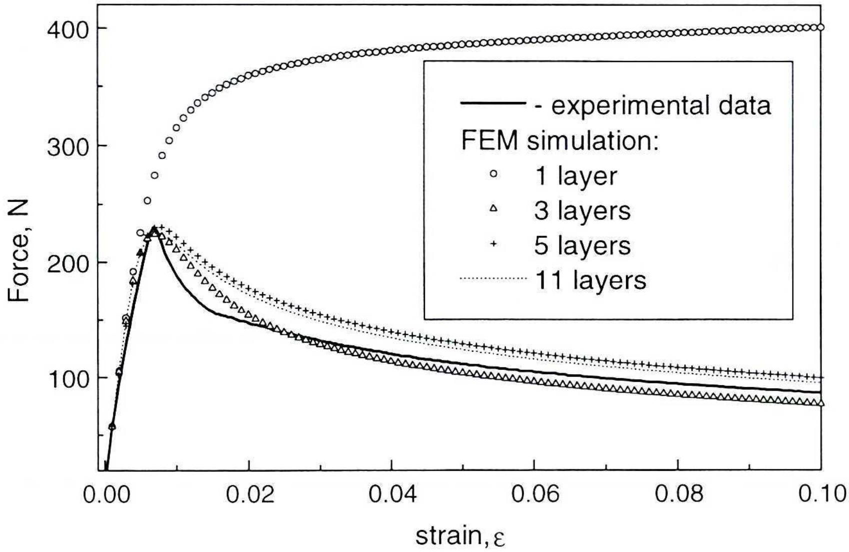

Fig. 6 Simulation of buckling and post buckling behaviour of HDPE strip, using shell elements with different number of layers

7. Results of computer simulations

The results of FE simulations of ramp compression tests with the strain rate of 5% per minute are given in Fig. 7. The maximum deviation between experimental data and computer prediction is less than 10% for all the strip lengths modelled.

In Fig. 8 the results of FE simulations for different strain rates are compared with the experimental data for the case of 70-mm long strip. The predicted buckling force differs less than 1% from the one observed in experiments. Deviation between tests data and numerical prediction for post-buckling development was within 15 %.

The achieved accuracy can be considered as satisfactory, since it is well known that the behaviour of structural elements after buckling or other perturbations (like snap-through effect) is more difficult to predict, especially when non-linearity is involved. Latter is due to possibility of multiple solutions to the non-linear problem. The existence of such multiple solutions can be easily proven by theoretical analysis [11], although theory very rarely can give these solutions itself. If these solutions are close to each other (especially at the moment of perturbation), the numerical errors in computer calculations, which are inevitable due to round-ups, can cause the numerical solution to "jump" from one "theoretical" solution to another one.

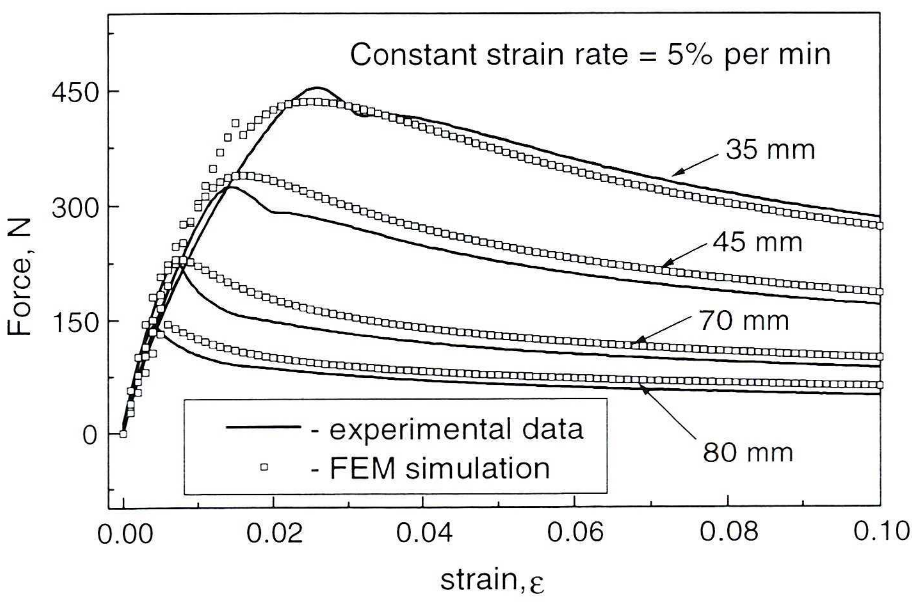

Fig. 7 FE prediction of buckling behaviour of HDPE strips of different length

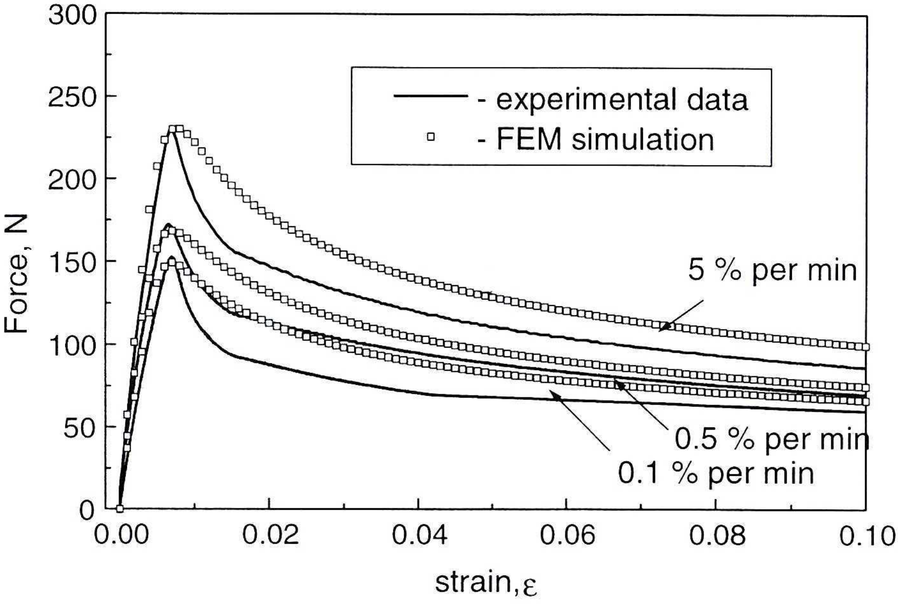

Fig. 8 Prediction of buckling force for the HDPE strip loaded by three different strain rates

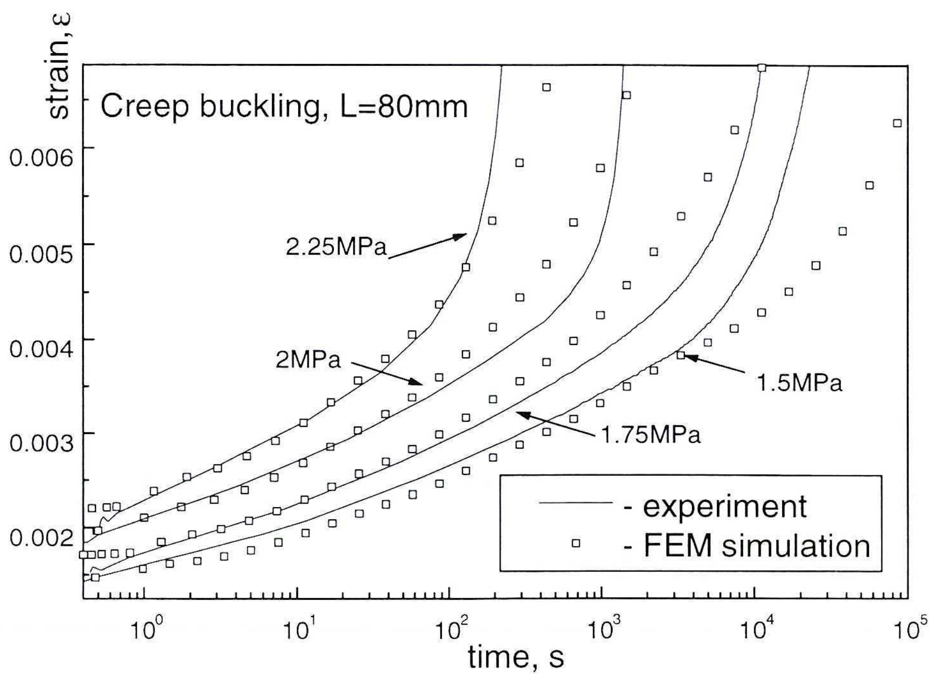

Fig. 9 Experimental data and FE prediction for the compressive creep of HDPE strips

Something similar also happens in experiment. As it was mentioned earlier, the small imperfection in testing set-up can drastically influence the experiment. These disturbances, which are present during both tests and calculations, decrease the accuracy of the prediction of post-buckling behaviour.

This becomes even more pronounced for the results of simulations of creep induced buckling (Fig. 9): the prediction of post-buckling behaviour is rather poor. However, if to consider only periods till buckling onset (Fig. 10), the model developed clearly shows its ability to predict the phenomenon of creep induced buckling.

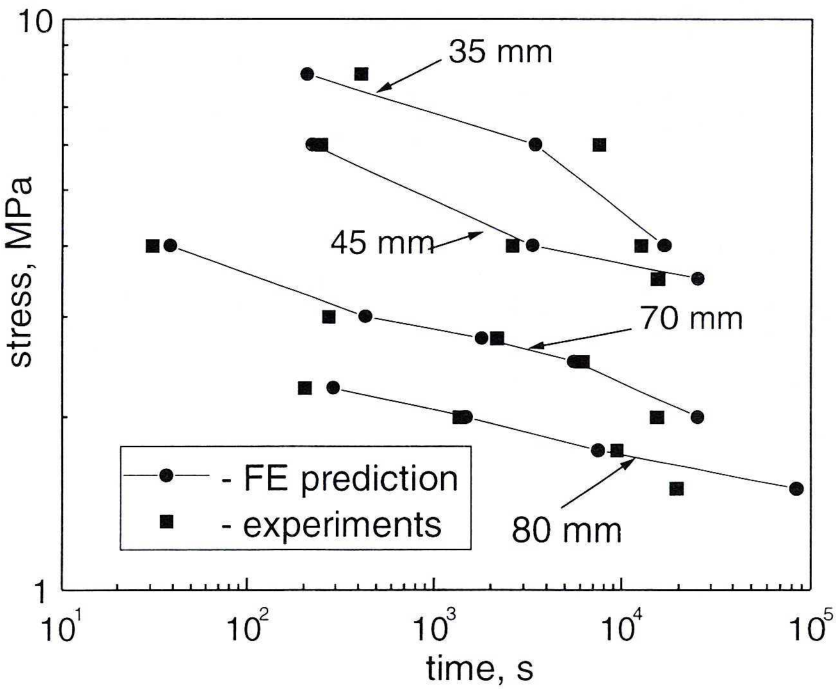

Fig. 10 FE prediction vs. experimental data on the periods till buckling onset in HDPE strips of different lengths under constant compressive loading

8. Conclusions

1. By simulation of ramp tensile and compressive tests on HDPE specimens it was confirmed that the non-linear visco-elasticity model, based on tensile creep-recovery tests, can be used for the FE simulations of the other types of loading of HDPE structural elements.
2. The calculation scheme within FEM packages MARC for modelling of buckling and post-buckling behaviour of the non-linear visco-elastic strips was established. The influence of different parameters of the calculation

model (like number of elements and layers in shell element, etc.) on the accuracy of calculations were studied.

3. It is shown that the established calculation model is able to predict buckling and post-buckling behaviour of HDPE strip under ramp compression loading, as well as the onset of creep induced buckling in clamped HDPE strips.

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Modelowanie metodą elementów skończonych zjawiska wyboczenia i zachowania po wyboczeniu pasków plastykowych

S t r e s z c z e n i e

Wobec oczywistego światowego trendu w kierunku projektowania produktów z lekkich tworzyw, coraz bardziej istotne staje się zjawisko pełzającego wyboczenia brył plastykowych o

właściwościach lepko-sprężystych. Niniejsza praca przedstawia wyniki etapu badań, prowadzonych w Laboratorium Niezawodności Mechanicznej Politechniki w Delft (Holandia), nad obciążalnością i wyboczeniem pojemników plastykowych. Posługując się opracowanym wcześniej nieliniowym modelem lepkościowo-sprężystym dla plastyków konstrukcyjnych, wykonano symulację metodą elementów skończonych zjawiska opóźnionego wyboczenia pasków plastykowych. Stanowiło to pierwszy krok do zrozumienia i przewidywania zjawiska pełzającego wyboczenia nośników plastykowych.