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USING CONDITIONAL AVERAGING OF DELAYED SIGNALS TO MEASURE PHASE SHIFT ANGLE

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Abstract

A novel measurement method and a brief discussion of basic characteristics of measuring the phase shift angle between two sinusoidal signals of the same frequency are presented in this paper. It contains a mathematical model for using conditional averaging of a delayed signal interfered with noise to measure the phase shift angle. It also provides characteristics of conditional mean values and discusses the effect of random interferences on the accuracy of the phase shift measurement. The way to determine the variance of the conditional mean value, together with the assessment of standard and expanded uncertainty, are described. The uncertainty characteristic shows the complementary properties of the discussed angle measurement principle φ for small absolute values $|\varphi|$ (minimum for $\varphi = 0$) relative to the correlation principle, where the minimum measurement uncertainty is present for $\varphi = \pi/2$.

Keywords: conditional averaging, delayed signal, mean value.

1. Introduction

In phase shift angle measurements, the most common interferences are due to noise, harmonics, and fixed components occurring in signals. In measurement practice, in addition to interference occurring in both channels, there are also experiments where the sinusoidal input (test) signal of the physical system is free from distortion, while the output signal delayed in the physical system is subject to interference.

Electronic phase meters which convert phase shift angle values into time interval values are not immune to random interferences. Random distortions affect the accuracy of determining the zero crossing points for both signals and have a direct effect on the accuracy of the measurement of the time segment corresponding to the phase shift φ between the analyzed signals [1–5]. The accuracy of the phase shift angle measurement for the interfered signals can be improved by using statistical algorithmic methods based on analysing the entire available signal or its pertinent fragments *e.g.* determining the correlation between two signals shifted by the angle φ of the signals and the

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multi-point approximation algorithm. Algorithmic methods can be optimised depending on the assumed criterion. In practical applications, criteria such as *linear minimum variance estimator* (LMVE), *maximum a posteriori probability* (MAP) and the *maximum likelihood* (ML) are the most commonly used.

The correlation method uses statistical information contained in interfered signals. The error of the correlation phase meter increases as do the noise content and higher harmonics. The error module has a maximum value for phase shift angles of 0° and 180° and a minimum value for 90°. The disadvantages of the correlation method include the limitation of the frequency of the analyzed signal (to kHz frequencies), the complexity of the implementation as well as the cosine nature of the processing function and the limited measuring range (0 to 180°) [6, 7].

The multi-point algorithm is designed to operate with sinusoidal variable signals for which the zero point coincides with the maximum point of the signal derivative. Its value can be determined using the regression straight line slope. The multi-point approximation method allows to improve the accuracy of the phase shift angle measurement with broadband noise where the mean value equals zero. However, this method loses its advantages in the case of interference in a frequency band that includes or is close to the frequencies of the analyzed signals shifted in phase.

The accurate models for phase noise in frequency and phase estimation of a jittered sinusoid were introduced in [8]. The authors analyzed the physical aspects and interrelations between these models and their application in the design of a LMVE. A comparison of the usability of these models is supported by computer simulation.

The problem of joint estimation of angular parameters of a single sinusoid with Wiener phase noise of the carrier and white Gaussian noise observed in the additive has been presented in [9]. The theoretical basis for phase-based estimation of the unknown carrier frequency in the time domain using the ML method has been discussed with taking into account the initial carrier phase, with simultaneous a *posteriori maximum likelihood* (MAP) estimation of the time-varying carrier phase noise. It was theoretically demonstrated that the obtained estimates are unbiased, and the performance of the mean squared error as a function of SNR, observation length, and phase noise variance was verified using Monte Carlo simulations [9].

With the criterion of *likelihood function maximum* (ML), it is possible to optimize the estimation of phase and phase shift angle of signals when using quadrature detection. Methods of measuring the phase shift angle based on the determination of components in phase as well as in quadrature processing (followed by combined processing) of these components are orthogonal methods of measuring the phase shift angle. The combined processing of quadrature components has an impact on the main characteristics of the phase shift measurement process: measurement time, resolution, accuracy, *etc*. The accuracy of orthogonal methods for measuring phase shift angle is characterised by random errors and systematic errors. In the event of random disturbance of the main components (sine and cosine), there is an error in the evaluation of the phase shift angle. In special situations, calculating phase shift values from non-linear relationships at small values of the parameters, their relations and differences, may show little accuracy. The optimum measurement principle does not eliminate measurement errors. Instead, it allows them to be theoretically minimised under the measurement conditions assumed. Phase meters which apply orthogonal phase shift angle measurement methods are characterised by complex processing algorithms and a relatively high price. Their distribution in technical applications is not significant. Their advantage lies in the fact that their measuring ability in the case of phase shift angle of signals disturbed by noise is more accurate than in other measurement methods [1, 10].

An improvement in the accuracy of the phase shift angle measurement for signals subject to interference can also be achieved by using algorithms based on conditional signal averaging. Such algorithmic methods are less complex than those discussed above.

In papers [11] and [12] application of conditional averaging of signals to measure the phase angle was presented. In these works the noise delayed signal module was averaged. The algorithms presented in the above-mentioned works use nonlinear processing of the delayed signal.

The measurement method and a description of the principle of measuring the phase shift angle called the 'arc sine' algorithm have been proposed. It was assumed that the primary sine signal was free of distortion, while the delayed secondary signal was disturbed by stationary and additive noise with normal distribution. In the proposed method, the conditional averaging of signals was also used, but the nonlinear transformation (absolute value of the signal) was not used. It provides a mathematical model of the measurement, measurement uncertainty assessment, as well as a comparison with the correlation method. Examples of experimental results and a summary were provided.

2. Mathematical model of the measurement

In the measurement of the phase shift angle, the ergodic stationary stochastic signals in a mathematical processing model can be expressed as:

$$x(t) = A_x \cos\left(\omega \cdot t + \varphi_p\right), \tag{1}$$

$$y(t) = A_y \cos\left(\omega \cdot t + \varphi_p + \varphi\right), \tag{2}$$

$$z(t) = y(t) + n(t), \tag{3}$$

where φ_p is random initial phase common to the input $x(t)$ and output $y(t)$ signals with uniform distribution of probability $p(\varphi_p) = \dfrac{1}{2\pi}$ in the domain $[-\pi, \pi]$, φ – constant phase shift angle of $y(t)$ relative to $x(t)$ with the condition that $-\dfrac{\pi}{2} \le \varphi \le \dfrac{\pi}{2}$, $n(t)$ – random interference with additive influence on the output signal $y(t)$ and uncorrelated with signals $x(t)$ and $y(t)$.

Assuming that only the delayed signal $y(t)$ is interfered by normal noise $N(n;\, 0,\, \sigma_n)$ and assuming the values of process instances at times t_1 and t_2 ($\tau = t_2 - t_1$) as:

$$x_1(t) = x\,(t = t_1) = A_x \cos\left(\omega \cdot t_1 + \varphi_p\right), \tag{4}$$

$$y_2(t) = y\,(t = t_2) = A_y \cos\left(\omega \cdot t_2 + \varphi_p + \varphi\right), \tag{5}$$

one can determine the general functional relationship between signals $x(t)$ and $y(t)$ as:

$$y(t+\tau) = A_y \cos\left(\omega \cdot \tau + \varphi \pm \arccos \frac{x(t)}{A_x}\right). \tag{6}$$

The relationship above does not have the initial phase φ_p and it can be further simplified by assuming that phase $\varphi_p = 0$. In the analyzed principle of phase shift angle measurement, conditional averaging of the delayed and disturbed interrupted signal $z(t)$ is used with the condition that $x(t) = 0$. Determining a measurement characteristic, which is the conditional expected value $E\,(z|_{x=0})$, requires calculating the conditional probability density $p\,(z|_{x=0})$.

For the condition that $x(t) = 0$ reached in the interval $(0 \div 2\pi)$ with a negative and positive derivatives respectively, using the general properties of the Dirac delta, the probability density for the delayed signal is:

$$p_1\left(y|_{x=0}\right) = \delta\left[y - A_y \cos\left(\omega \cdot \tau + \varphi + \frac{\pi}{2}\right)\right], \tag{7}$$

$$p_2\left(y|_{x=0}\right) = \delta\left[y - A_y \cos\left(\omega \cdot \tau + \varphi + \frac{3}{2}\pi\right)\right], \tag{8}$$

where $\delta(\ )$ is the Dirac Delta function.

Assuming an additive influence of the independent interference signal $n(t)$, the conditional probability density is not dependent on the condition and is:

$$p\left(n|_{x=0}\right) = p(n) = N\left(n;\ 0,\ \sigma_n\right). \tag{9}$$

Relying on the relationships of probability density conditional functions (7) and (9), with the condition that $x = 0$ in the interval $(0 \div \pi)$, the conditional probability density of $z(t)$ can be determined. For the sum of signals (3), the probability density is expressed by the function convolution:

$$p_{z|x=0}(z) = \int_{-\infty}^{\infty} p_1\left(y|_{x=0}\right) p_n(z-y)\,\mathrm{d}y. \tag{10}$$

Based on the properties of the function convolution $p_n(z-y)$ with the Dirac Delta function (7), expression (10) can be transformed to arrive at the conditional probability density:

$$\begin{aligned} p_{z|x=0}(z) &= \int_{-\infty}^{\infty} \delta\left[y - A_y \cos\left(\omega \cdot \tau + \varphi + \frac{\pi}{2}\right)\right] \cdot \frac{1}{\sqrt{2\pi}\sigma_n} e^{-\frac{(z-y)^2}{2\pi\sigma_n^2}}\,\mathrm{d}y \\ &= \frac{1}{\sqrt{2\pi}\sigma_n} e^{-\frac{\left[z - A_y \cos\left(\omega\cdot\tau+\varphi+\frac{\pi}{2}\right)\right]^2}{2\pi\sigma_n^2}}. \end{aligned} \tag{11}$$

The conditional expected value of the sum (3) after substitution (11) and calculations:

$$E\left[z|_{x=0}\right] = \int_{-\infty}^{\infty} z p_{\,z|x=0}(z) = A_y \cos\left(\omega \cdot \tau + \varphi + \frac{\pi}{2}\right) = -A_y \sin\left(\omega \cdot \tau + \varphi\right). \tag{12}$$

Taking into account the probability density (8) for the condition that $x = 0$ in the interval $(\pi,\ 2\pi)$ and with the appropriate transformation, an expression is arrived at for the conditional expected value:

$$E\left[z|_{x=0}\right] = A_y \cos\left(\omega \cdot \tau + \varphi + \frac{3\pi}{2}\right) = A_y \sin\left(\omega \cdot \tau + \varphi\right). \tag{13}$$

Signal models for averaging according to relationship (12) for $\varphi_p = 0°$ and $\varphi = -90°$ are shown in Fig. 1.

For $\tau = 0$ relationships (12) and (13) are described by the following functions:

$$E\left[z|_{x=0}\right] = -A_y \sin\varphi, \tag{14}$$

$$E\left[z|_{x=0}\right] = A_y \sin\varphi. \tag{15}$$

The two expressions (14) and (15), with the condition that $x = 0$ with a negative or positive derivative value, can be used to measure the phase shift angle value in intervals $-\frac{\pi}{2} \le \varphi \le \frac{\pi}{2}$ and with the characteristics shown in Fig. 2.

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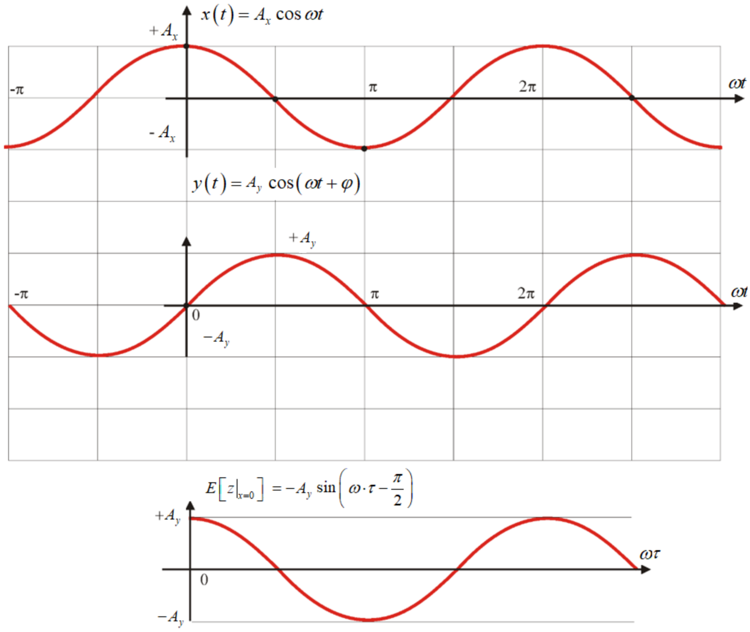

Fig. 1. Models of signals for $\varphi_p = 0°$ and $\varphi = -90°$ and characteristic $E\,[z|_{x=0}]$ for $\varphi = -90°$.

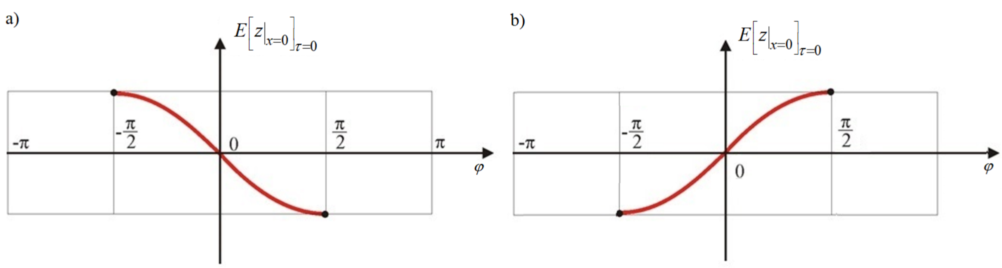

Fig. 2. Characteristics of $E\,[z|_{x=0}]_{\tau=0}$ in the function of angle φ: a) $\left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_{(x=0)} < 0$; b) $\left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_{(x=0)} > 0$.

For a model of the delayed signal delayed by the phase shift angle φ and interfered by an additive normal noise $N(0, \sigma_n)$, the relationship determining $\{z(\tau_i)\}_k$ for the k-th instance (time from the start of registering until the time τ_i) is as follows:

$$\{z\,(\tau_i)\}_k = -A_y \sin\,(\omega \cdot \tau_i + \varphi) + n_k\,, \tag{16}$$

where n_k are independent interferences for particular, k-th instances.

For $\tau_i = 0$:

$$z(0)_k = -A_y \sin\varphi + n_k\,. \tag{17}$$

Using maximum likelihood estimation for independent registration results $z(0)_k = z_k$, one can find a favourable estimator of the parameter φ in the form of $\hat{\varphi}_{\mathrm{opt}}$. For the interval $-\frac{\pi}{2} \le \varphi \le \frac{\pi}{2}$, where the function $-A_y \sin\varphi$ is unambiguous and $\cos\varphi \ne 0$, the optimal parameter estimator φ

to maximise the likelihood function is defined by the expression:

$$\hat{\varphi}_{\text{opt}} = -\arcsin\left(\frac{\frac{1}{M}\sum_{k=1}^{M} z_k}{A_y}\right), \tag{18}$$

where M is the number of averaged instances of signal $z(\tau)_k$.

In order to determine the estimated value $\hat{\varphi}$ of the phase shift angle, the arc sine function must be calculated from the quotient of the arithmetic mean of the interfered instance of $z(t)$ and the amplitude A_y of the signal shifted by the angle φ.

Based on equations (14) and (18):

$$\hat{\varphi} = -\arcsin\left(\frac{\hat{E}\left[z|_{x=0}\right]}{\hat{A}_y}\right) = -\arcsin\frac{\overline{z}|_{x=0}}{\hat{A}_y} = -\arcsin\frac{\overline{z}|_{x=0}}{\hat{A}_{yi}}, \tag{19}$$

where $\hat{E}\left[z|_{x=0}\right] = \overline{z}|_{x=0}$ is the conditional value of the arithmetic mean as an experimental evaluation for the condition that $x(t) = 0$ expected value of the interfered signal $y(t)$ shifted by the angle φ relative to the signal $x(t)$, $\hat{A}_{yi}$ is the arithmetic mean value as an experimental evaluation of the amplitude A_y of the delayed signal $y(t)$.

If experimental estimators $\hat{\sigma}_z^2$ and $\hat{\sigma}_n^2$ of the variance of signals $z(t)$ and $n(t)$ are available, the following relationship may be used to calculate the estimate $\hat{A}_y$:

$$\hat{A}_y = \sqrt{2\left(\hat{\sigma}_z^2 - \hat{\sigma}_n^2\right)} = \hat{\sigma}_z\sqrt{2\left[1 - \left(\frac{\hat{\sigma}_n}{\hat{\sigma}_z}\right)^2\right]}. \tag{20}$$

The block diagram of signal processing for evaluation $\hat{\varphi}$ is shown in Fig. 3.

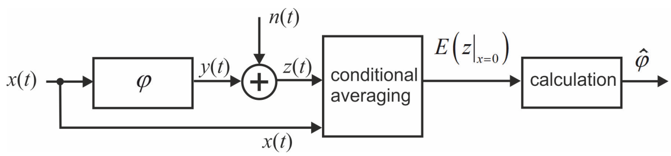

Fig. 3. Scheme of signal processing model in phase shift measurement.

3. Variance of the mean conditional value and the signal-to-noise ratio

For the relationship:

$$\{z(\tau_i)\}_k = -A_y \sin(\omega \cdot \tau_i + \varphi) + n_k(\tau_i), \tag{21}$$

and independent interferences n_k for k-th instances, the variance of the averaged signal is:

$$V_z(\tau_i) = V\left[-A_y \sin(\omega \cdot \tau_i + \varphi)\right] + V\left[n_k(\tau_i)\right] = V_n = \sigma_n^2. \tag{22}$$

The variance of the conditional mean value characteristic is described by the relationship:

$$V\left[E\left(z|_{x=0}\right)\right] = \frac{\sigma_n^2}{M}. \tag{23}$$

The increase in $\left(\frac{S}{N}\right)$ due to averaging will take place in accordance with the relationship:

$$\frac{\left[\dfrac{M\left(-A_y \sin\left(\omega\cdot\tau_i+\varphi\right)\right)}{\sigma_n\sqrt{M}}\right]^2}{\left[\dfrac{-A_y \sin\left(\omega\cdot\tau_i+\varphi\right)}{\sigma_n}\right]^2}=M. \tag{24}$$

4. Evaluation of standard and expanded uncertainties

For the estimator of the phase shift angle of realization of $y(t)$ relative to $x(t)$ in the interval from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ according to the proposed measurement principle, designation $\hat{\varphi}_{xy}$ using the following transformation is used:

$$\hat{\varphi}_{xy}=-\arcsin\left(\frac{\hat{E}_z(0)}{A_y}\right)=f\left(\frac{\hat{E}_z}{A_y}\right). \tag{25}$$

For the evaluation standard uncertainty $u_{\hat{\varphi}_{xy}}$ the following relationship may be used [13]:

$$u_{\hat{\varphi}_{xy}}=\left|\frac{\mathrm{d}\hat{\varphi}_{xy}}{\mathrm{d}\left(\frac{E_z}{A_y}\right)}\right|\cdot u_{\frac{\hat{E}_z}{A_y}}=-\frac{u_{\frac{\hat{E}_z}{A_y}}}{\sqrt{1-\left(\frac{\hat{E}_z}{A_y}\right)^2}}=\frac{\sigma_n}{\sqrt{M}A_y\sqrt{\left(\frac{\hat{E}_z}{A_y}\right)^2}}=\frac{\sigma_n}{\sqrt{M}A_y\cos\hat{\varphi}_{xy}}. \tag{26}$$

Relationship (26) is correct for the assumption that the amplitude A_y is known and its graph is illustrated in Fig. 4.

The uncertainty characteristic $u_{\hat{\varphi}_{xy}}$ shows that large measurement uncertainties occur near the points $\hat{\varphi}_{xy}=\pm 90^\circ$ and the minimum value for $\varphi_{xy}=0^\circ$ is:

$$\left(u_{\hat{\varphi}_{xy}}\right)_{\min}=\frac{\sigma_n}{\sqrt{M}A_y}. \tag{27}$$

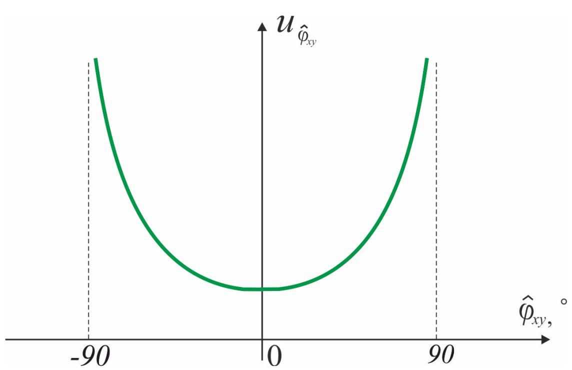

Fig. 4. Relationship between standard uncertainty $u_{\hat{\varphi}_{xy}}$ and $\hat{\varphi}_{xy}$ according to the measurement principle in formula (26).

If the amplitude A_y is not known, then one must calculate its estimate $\hat{A}_y$ and use it in formulae (25) and (26).

When averaging M sequences of a delayed signal interfered by correlated noise samples (low-bandwidth noise with an exponential autocorrelation function), standard uncertainty $u_{\hat{\varphi}_{xy}}$ will increase and for a sufficient number of averaging instances will be:

$$u_{\hat{\varphi}_{xy}} = \frac{\sigma_n}{\sqrt{M} A_y \cos \hat{\varphi}_{xy}} \sqrt{\frac{1+\rho_1}{1-\rho_1}}, \tag{28}$$

where ρ_1 is the value of the normalised autocorrelation function of noise $\rho_1 = \rho(\Delta t)$, while Δt is the sampling interval for processing the signals $x(t)$ and $z(t)$.

The expanded uncertainty for relationships (26) and (28) is:

$$U = k_r \cdot u_{\hat{\varphi}_{xy}}, \tag{29}$$

where k_r is the expansion coefficient which depends on the adopted probability distribution model used for the interference and the confidence level assumed α .

5. Correlation method of measuring phase shift angle

For signal processing models (1–3), assuming that φ_p =, the correlation function $R_{xz}(\tau)$ may be expressed by the following relationship:

$$R_{xz}(\tau) = R_{xy}(\tau) + R_{xn}(\tau) = R_{xy}(\tau), \tag{30}$$

where $R_{xy}(\tau)$ is the correlation function of signals $x(t)$ and $y(t)$; $R_{xn}(\tau) = R_{yn}(\tau) = 0$ are values of the correlation function with no correlation between signals $x(t)$ and $y(t)$ and the disturbance $n(t)$.

Expansion of the function $R_{xy}(\tau)$ for the T – period signal models used:

$$R_{xy}(\tau) = \frac{1}{T}\int_0^T A_x \cos(\omega \cdot t) \cdot A_y \cos\left(\omega \cdot t + \omega \cdot \tau + \varphi_{xy}\right) \mathrm{d}t = \frac{A_x A_y}{2} \cos\left(\omega \cdot \tau + \varphi_{xy}\right). \tag{31}$$

Figure 5 shows the function $R_{xy}(\tau)$ for $\varphi = \frac{\pi}{2}$.

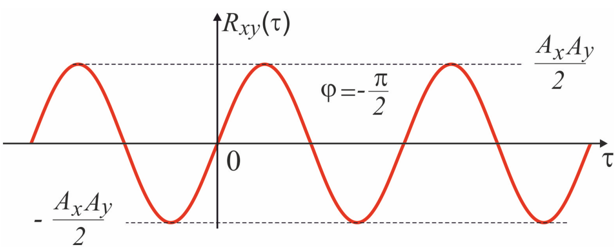

Fig. 5. Correlation function $R_{xy}(\tau)$ for $\varphi = -\frac{\pi}{2}$.

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From the expression (31) for $\tau = 0$, the following relationship is obtained:

$$\cos\varphi_{xy} = \frac{2R_{xy}(0)}{A_x A_y} = \frac{R_{xy}(0)}{A_{xs}A_{ys}}, \tag{32}$$

where A_{xs}, A_{ys} are effective values of signals $x(t)$ and $y(t)$.

The phase shift angle estimation value can be determined from the following relationship:

$$\hat{\varphi}_{xy} = \arccos\frac{\frac{A_x A_y}{2}\cos\varphi_{xy}}{\sqrt{\frac{A_x^2}{2}\left(\frac{A_y^2}{2}+\sigma_n^2\right)}} = \arccos\frac{\cos\varphi_{xy}}{\sqrt{1+\left(\frac{\sigma_n}{\frac{A_y}{\sqrt{2}}}\right)^2}}, \tag{33}$$

where: $\left(\frac{\sigma_n}{\frac{A_y}{\sqrt{2}}}\right)^2$ stands for the noise-to-signal ratio (N/S)

For uniform sampling and recording of n samples of signals $x_i(t_i)$ and $z_i(t_i)$ during the period T, the value of the phase shift angle estimation $\hat{\varphi}_{xy}$ can be calculated from:

$$\hat{\varphi}_{xy} = \arccos\frac{\sum_{i=1}^{n} x_i z_i}{\sqrt{\sum_{i=1}^{n} x_i^2 \sum_{i=1}^{n} z_i^2}}. \tag{34}$$

The absolute measurement error is defined by the expression:

$$\Delta\varphi_{xy} = \hat{\varphi}_{xy} - \varphi_{xy}. \tag{35}$$

Error (35) is a bias error that depends on the signal-to-noise ratio (S/N). The highest error values occur for angles φ_{xy} of 0° and 180°. For a known value of (S/N), one can introduce a correction term into the measurement result.

Figure 6 shows a graph for relationship (35) for three different values of (S/N) for 100, 50 and 20, respectively.

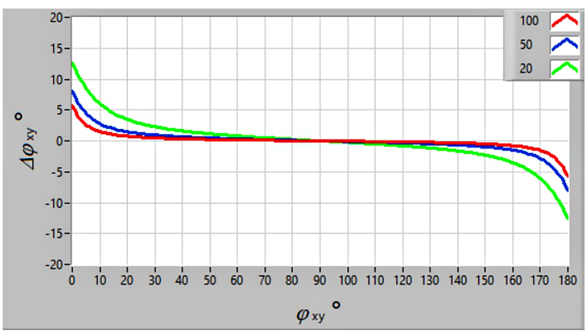

Fig. 6. Relationship between the error $\Delta\varphi_{xy}$ and the value of (S/N).

6. Results of calculations and experimental verification

The experiment involved averaging of the delayed signal with the frequency $f_y = 1$ kHz interfered by broadband noise (Figure 7). Broadband noise affects local distortions of sinusoidal waveform fragments. For given and calculated phase shifts φ_0, the values of φ_e (experimentally determined) and $u_{\hat{\varphi}_{xy}}$ (calculated from the relation (26)) are summarized in Table 1. The φ_0 shifts were determined with an SD1000 Powertek phasemeter with an accuracy of ±0.04° and ±0.005°/kHz for signal frequencies in the range 100÷1 kHz.

a) b)

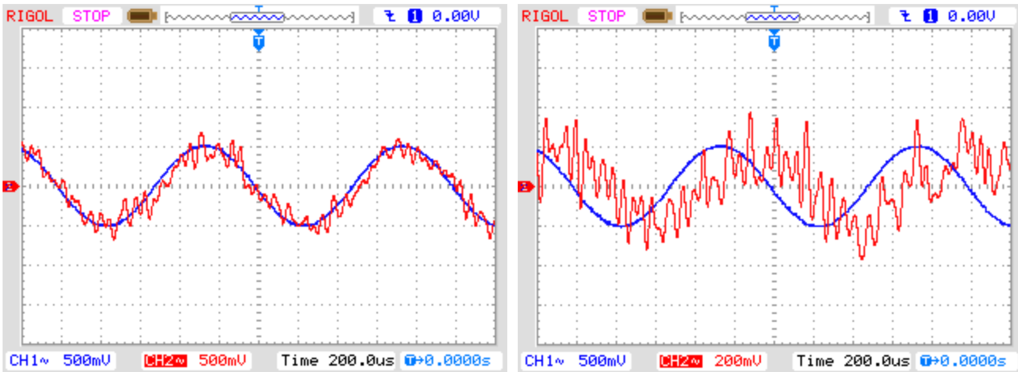

Fig. 7. Waveforms of signals $x(t)$ and $z(t)$: a) $f_x = f_y = 1$ kHz; $\varphi_0 = 4.1°$; b) $f_x = f_y = 1$ kHz; $\varphi_0 = 71.5°$; broadband noise $N(0, 100$ mV$)$

Table 1. Parameters of used sensors.

Shift φ_0 (reference), °	4.1	13.88	30.4	71.5
Shift φ_e (experimental), °	4.07	13.7	30.05	69.60
Absolute error $\varphi_e - \varphi_0$, °	–0.03	–0.18	–0.35	–2.3
Uncertainty $u_{\hat{\varphi}_{xy}}$, °	0.0125	0.0128	0.0145	0.0394

The φ_e shifts were calculated using a digital oscilloscope based on averaging 256 characteristics of $\overline{z}|_{x=0}$.

The delayed signal waveforms interfered by low-band noise are presented in Figure 8. Low-band noise affects the integral distortion of the entire sine wave and the correlation of the values averaged.

The fragments of conditional averaging of the output signal $y(t)$ at the frequency $f_y = 1$ kHz with an additive low-band interference imposed are illustrated in Figure 9. Free components of interference $n(t)$ with the same sign are added successively in time to the high-speed fragments of the sinusoidal signal $y(t)$. Averaging of correlated components results in a k-fold increase in the standard uncertainty value $u_{\hat{\varphi}_{xy}}$ according to relationship (28).

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a) b)

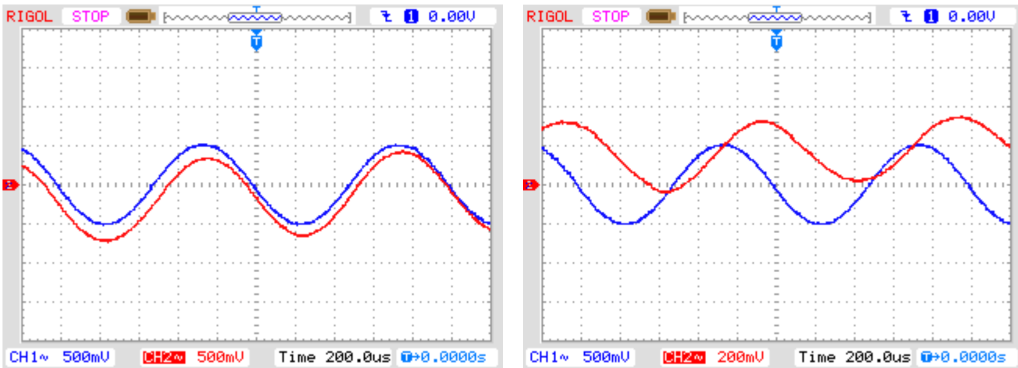

Fig. 8. Graphs of signals $x(t)$ and $z(t)$: a) $f_x = f_y = 1$ kHz; $\varphi_0 = 4, 1°$; b) $f_x = f_y = 1$ kHz; $\varphi_0 = 71.5°$; broadband noise N (0,100 mV).

a) b)

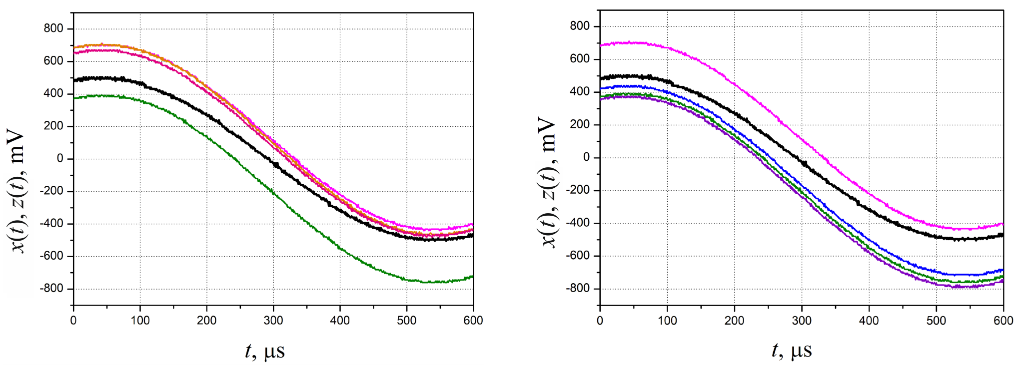

Fig. 9. Waveforms of signals $x(t)$ (black) and $z(t)$ (coloured): $f_x = f_y = 1$ kHz, $\varphi_0 = 0°$; subsequent averaged characteristics $z(t + \tau)|_{x(t)=0}$ with positive values of low-frequency interference (a) and negative values (b).

Example:

Signals with the frequency $f_x = f_y = 1$ kHz. For the interference $n(t) : \rho_1(\Delta t) = 0.8$ with the interval $\Delta t = 0.01 \cdot 10^{-3}$ s between each sample for calculating the arithmetic mean of the summarised instances of $z(t)$.

Increase in uncertainty value $k = \sqrt{\dfrac{1 + 0.8}{1 - 0.8}} = \sqrt{9} = 3$.

7. Conclusions

In phase shift angle measurements of interfered sinusoidal signals carried out according to traditional phase shift processing rules, the information that is used concerns the time shift of only two instantaneous signal values based on a single period of time. This principle of measurement is characterised by sensitivity to interference and a high correlation between the measurement result and the change in the shape of the input signals. To improve the accuracy of the result (reduce variance), two particular values of the measurement results over multiple periods need to be averaged.

The methods of measuring the phase shift angle of sine signals introduced by development studies differ in terms of the complexity of algorithms, the cost of measuring equipment, resistance to interference and accuracy depending on the value of the measured phase shift angle. As regards the improvement of parameters in the selected class of measurement principles, these methods are complementary. Two integral methods are examples of phase shift angle measurement methods for sine signals: the correlation and regression methods.

The advantage of correlation-based phase meters consists in an approximately six-fold reduction of measurement error compared to phase meters which convert phase shift angle values into time interval values based on signal passing through zero. The disadvantages of this measurement method include its non-linear processing and the unequivocal character of phase shift estimation only between $0°\div180°$.

One of regression principles in measuring the phase shift is the use of the "arc sine" algorithm. In order to determine the angle φ, the "arc sine" algorithm uses the inverse sine relationship functions ((14) and (15)) with the condition that $\tau = 0$. The characteristic of the mean average value $E\left[z|_{x=0}\right]$ described in the formulae above can be used to determine the phase shift angle value by determining the arithmetic mean of signal $z(t)$ corresponding to the moment at which signal $x(t)$ passes through zero with the assumed (negative or positive) derivative.

The measurement uncertainty of angle φ_{xy} reaches high values for angles $\pm\frac{\pi}{2}$ and a minimum value for $\varphi_{xy} = 0°$ (Fig. 4). With the disturbance of signal sequences averaged by correlated values of noise, the measurement uncertainty increases.

The uncertainty characteristic reveals complementary features of the measurement principle in relation to the correlation principle, in which large measurement uncertainties occur close to points $\varphi = 0$ and $\varphi = \pi$, and the minimum values for $\varphi = \frac{\pi}{2}$.

The advantages of the proposed measurement method are the simple measurement algorithm, possibility of easy and frequent use with a digital oscilloscope, suitability for applications in technical measurement and in didactics related to signal averaging. The disadvantages include a limited basic measurement range from $-90°$ to $90°$ and a strong nonlinear increase in measurement uncertainty near the limits of this range.

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