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LATENT REACTIONS IN SUSPENSION SYSTEMS OF VIBRATORY MACHINES SUPPORTED BY LEAF SPRINGS

The paper discusses an occurrence of latent reactions in the direction perpendicular to the movement of the suspension systems in vibratory machines supported by parallel leaf springs. These reactions occur despite an apparent lack of any forces acting in that direction and may – for long machine bodies – reach values comparable with the exciting force of the vibrator.

Analytical relationships allowing for estimation of the value of reactions were derived. The obtained results were verified by means of the Geometric Element Method as well as by the performed experiments.

1. Introduction

Bodies of vibratory machines that perform translatory and rectilinear vibratory motions, such as e.g. vibratory conveyors [4], are usually supported by systems of parallel leaf springs in a manner shown schematically in Fig. 1.

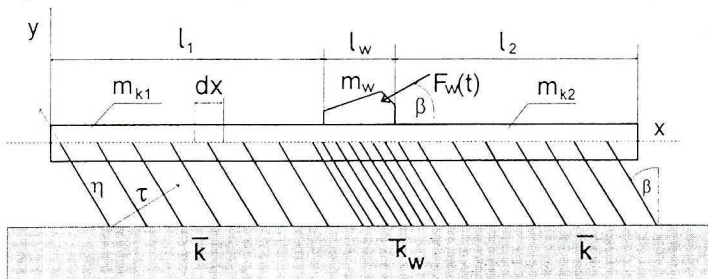


Fig. 1. Continuous model of conveyor

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Fig. 1, illustrates the machine body of a mass m_k with the attached vibrator of a mass m_w . The vibrator produces an exciting force $F_w(t)$ in the τ direction inclined to the level at the β angle.

The results of inclining the leaf springs at the $\beta + \pi/2$ angle is that during the vibration of the body neither its displacement nor the applied forces have components in the longitudinal direction η of the springs. In the case of a massive foundation or a large stiffness of the ground base, when the exciting force of a vibrator is

$$F_w(t) = F_{w0} \sin \omega t \quad (1)$$

the amplitude of the vibrations of the machine body in the τ direction equals

$$A \cong \frac{F_{w0}}{k + k_w - \omega^2(m_k + m_w)} \quad (2)$$

where: k — total elasticity constant in the τ direction of springs supporting the through outside the zone of the attachment of the vibrator,

k_w — as above, but for springs supporting that part of the through which was stiffened for the attachment of the vibrator,

$$m_k = m_{k1} + m_{k2}.$$

Thus, the total value of the reaction of the ground in the direction of vibrations equals [2]

$$F_\tau \cong -(k + k_w)A \sin \omega t \quad (3)$$

Experimental investigations [1], [5] have shown, however, that there are also forces in the longitudinal η direction of springs and that their value may be quite significant.

This phenomenon cannot be explained by means of the analysis of the non-deformable system.

2. Theoretical analysis

Let's consider a case of a machine body of low flexural rigidity and high compression stiffness, outside the zone of the attachment of the vibrator (Fig. 1). We are assuming a high flexural rigidity and compression stiffness of the body in the zone of the attachment of the vibrator – as it is usually the case in typical structures of vibratory machines.

The mass of the body for the unit of length is denoted as

$$\bar{m}_k = \frac{m_{k1}}{l_1} = \frac{m_{k2}}{l_2}$$

while the stiffness of the supporting system for the unit of length in the direction of vibrations is denoted as

$$\bar{k} = \frac{k_1}{l_1} = \frac{k_2}{l_2} \quad \text{— for the body outside the vibrator,}$$

and

$$\bar{k}_w = \frac{k_w}{l_w} \quad \text{— for the body under the vibrator.}$$

The differential equation for the motion of the system is derived on the bases of the analysis of forces applied to the segment of the body, dx , in length (Fig. 2).

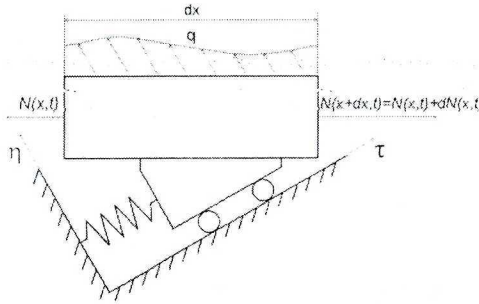


Fig. 2. Graphical presentation of forces applied to a segment of the body

The following descriptions are used:

- $N(x,t)$ — longitudinal force in the body (sign plus for stretching while sign minus for compression),
- $q(x,t)$ — continuous load produced by the reaction of the supporting system in the η direction longitudinal to springs.

The high value of compression stiffness of the leaf springs and of the body causes that each segment of the body makes, in an agreement with constrains, identical movement in the η direction:

$$\tau(t) = A \sin \omega t . \tag{4}$$

Dynamic equations of motion of the body segment, dx , along the τ co-ordinate and in the η direction are as follows:

$$\bar{m}_k dx \cdot \ddot{\tau}(t) = -\bar{k} dx \cdot \tau(t) + dN(x,t) \cdot \cos \beta , \tag{5}$$

$$\bar{m}_k dx \cdot 0 = q(x,t) \cdot dx - dN(x,t) \cdot \sin \beta = 0 . \tag{6}$$

In the above formulations there are no moments of bending and no cutting forces since previous assumptions do not allow for any relative displacements of body segments.

Taking into account that

$$dN(x,t) = \frac{\partial N(x,t)}{\partial x} dx , \tag{7}$$

and

$$\ddot{\tau}(t) = -\omega^2 A \sin \omega t . \tag{8}$$

Equations (5) and (6) become:

$$(\bar{k} - \omega^2 \bar{m}_k) A \sin \omega t = \frac{\partial N(x,t)}{\partial x} \cos \beta, \quad (9)$$

$$q(x,t) = \frac{\partial N(x,t)}{\partial x} \sin \beta. \quad (10)$$

Separating variables by substituting in Equation (9)

$$N(x,t) = N_x(x) \cdot N_t(t), \quad (11)$$

and reintroducing the result into Equation (9) we receive

$$(\bar{k} - \omega^2 \bar{m}_k) A \sin \omega t = \frac{\partial N_x(x)}{\partial x} N_t(t) \cdot \cos \beta, \quad (12)$$

where from we have directly:

$$N_t(t) = A \sin \omega t, \quad (13)$$

$$\frac{\partial N_x(x)}{\partial x} = \frac{\bar{k} - \omega^2 \bar{m}_k}{\cos \beta}. \quad (14)$$

By integrating Equation (14) we arrive at

$$N_x(x) = \int \frac{\bar{k} - \omega^2 \bar{m}_k}{\cos \beta} dx = \frac{\bar{k} - \omega^2 \bar{m}_k}{\cos \beta} x + c \quad (15)$$

Utilising the boundary condition, we receive - for the left side of the body:

$$N_{1x}(x) = \frac{\bar{k} - \omega^2 \bar{m}_k}{\cos \beta} x, \quad (16)$$

$$N_1(x,t) = \frac{\bar{k} - \omega^2 \bar{m}_k}{\cos \beta} x \cdot A \sin \omega t \quad (17)$$

(factors "1" and "2" denote the left and the right side of the body, respectively).

Then, from Equation (10) we have

$$q(x,t) = (\bar{k} - \omega^2 \bar{m}_k) t g \beta \cdot A \sin \omega t. \quad (18)$$

Axial reactions of the left side of the body, l_1 , on the middle part, l_w , is equal to

$$N_{1w} = -N_1(l_1, t) = -\frac{\bar{k} - \omega^2 \bar{m}_k}{\cos \beta} l_1 A \sin \omega t. \quad (19)$$

Similar reasoning concerning the right side of the body, l_2 , gives the following dependency

$$N_{2w} = -\frac{\bar{k} - \omega^2 \bar{m}_k}{\cos \beta} l_2 A \sin \omega t \quad (20)$$

Thus, we are able to write the equation of motion for the middle part of the machine, l_w , projected on the τ and η axes:

$$m_w \ddot{\tau} = F_w(t) + N_{1w} \cos \beta + N_{2w} \cos \beta - \bar{k}_w \cdot l_w \cdot \tau(t) \quad (21)$$

$$m_w \cdot 0 = -N_{1w} \sin \beta - N_{2w} \sin \beta + R_{w\eta}(t) \quad (22)$$

where

$R_{w\eta}(t)$ — force in the η direction produced by axial forces in springs, applied to the body of the machine at the middle segment l_w , it means at the zone of the attachment of the vibrator.

When taking into account Equations (19) and (20) the following functions for the exciting force $F_w(t)$ of the vibrator and for the reaction $R_{w\eta}$ are received:

$$F_w(t) = [k + k_w - \omega^2(m_k + m_w)]A \sin \omega t \quad (23)$$

$$R_{w\eta}(t) = -(k - \omega^2 m_k) t g \beta \cdot A \sin \omega t \quad (24)$$

where: $k = \bar{k}(l_1 + l_2)$,

$$k_w = \bar{k}_w \cdot l_w,$$

$$m_k = m_{k1} + m_{k2}.$$

The comparison of Equation (23) with Equations (1) and (2) shows that the amplitude of the exciting force necessary to secure a vibration of the body with the amplitude A is identical for both systems (non-deformable and deformable).

Reactions of the ground base due to the axial forces in springs in the η direction outside of the vibrator can be calculated from the formula

$$\begin{aligned} R_{k\eta}(t) &= q(x, t) \cdot (l_1 + l_2) = (k - \omega^2 \bar{m}_k) t g \beta \cdot A \sin \omega t \cdot (l_1 + l_2) = \\ &= (k - \omega^2 m_k) t g \beta \cdot A \sin \omega t \end{aligned} \quad (25)$$

The total value of the reactions of the ground in the τ direction, caused by the action of the whole supporting system in the τ direction, equals

$$R_\tau(t) = -(k + k_w) A \sin \omega t \quad (26)$$

while the total value of the reactions of the ground in the η direction, caused by the action of the whole supporting system, equals

$$R_\eta(t) = R_{w\eta}(t) + R_{k\eta}(t) = 0 \quad (27)$$

what results directly from Equations (24) and (25).

The above formulae show that apart from the reaction of the supporting system at the τ direction of vibrations (Eq. 26) also longitudinal axial reactions, of values comparable to the exciting force of the vibrator, occur in the system. Those reactions being of equal values for the zone of the attachment of the vibrator and for the zone of the support of the body are applied in the opposite directions.

Thus, if the foundation (or the supporting frame) in the zones are characterised by high enough stiffness, and the ground base is soft, those reactions make system equal to zero. However when the stiffness of the foundation (or the frame) is not sufficient, significant dynamic loads are applied to the ground in the axial direction of springs.

In addition, the presented analysis indicates the necessity of securing the adequate load capacity of springs under the vibrator to prevent the loss of stability due to the dynamic force.

3. Numerical verification

The theoretical analysis was performed with several simplifications. The body of the machine was assumed having small flexural rigidity and very high compression stiffness in the axial direction. A very high compression stiffness of leaf springs in the axial direction was also assumed. Therefore, the range of applicability of the presented considerations needed verification by the method which would not require such assumptions.

Values of the forces transmitted to the ground by a typical, long, vibratory conveyor were estimated by means of the Finite Element Method and compared with the analytically found values.

The ALWER - 1.1 conveyor produced by the OFAMA Plant, and characterised by the following parameters, was chosen for experiments:

$$k = 140000 \text{ [N/m]},$$

$$k_w = 15100 \text{ [N/m]},$$

$$m_w = 800 \text{ [kg]},$$

$$m_r = 2900 \text{ [kg]},$$

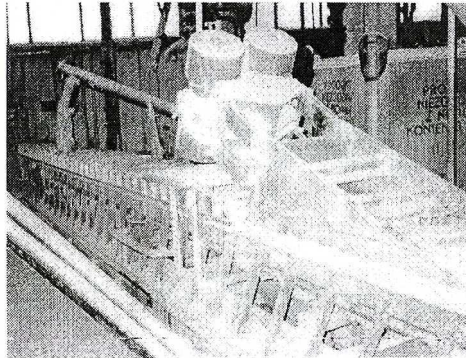
$$\omega = 104 \text{ [rad/s]},$$

$$\beta = 30^\circ,$$

$$F_{w0} = 90 \text{ [kN]},$$

$$l = 17 \text{ [m]} \quad - \quad \text{length of the conveyor},$$

$$J_r = 48128 \text{ [kg} \cdot \text{m}^2\text{]}.$$



Phot. 1. ALWER 1.1 conveyor

The ALWER 1.1 conveyor consists of a trough – on which bulk material is transported – supported by 220 leaf springs connected to the foundation. The leaf springs are attached to the trough in 55 points (in groups of four) equally distributed along its length.

The body is excited to vibrations by the set of two counter running vibrators which produce the exciting force $F_{w0} = 90 \text{ [kN]}$ acting in the τ direction and

crossing the center of gravity. Vibrators are fitted to the trough through the frame which stiffens the short segment of the trough.

Introducing the parameters of the conveyor into Equations (24) and (26) and doing analytical calculations we are getting the following values of forces:

$$R_{\tau \max} = 3,8 \text{ [kN]}$$

$$R_{w\eta \max} = 44 \text{ [kN]}$$

The verification by means of the Finite Element Method was done with the application of the Pro/MECHANICA software. It is a group of modules utilising GEM/GEA (Geometric Element Modelling and Geometric Element Analysis) technology based on the discretisation of the object performed by the Geometric Element Method.

The aim of the analysis was the simulation of the ALWER 1.1 vibratory conveyor and the estimation of axial forces occurring along leaf springs and caused by dynamic phenomena in a continuous system. Thus, we wanted to estimate forces transmitted to the ground when typical values of flexural rigidity and compression stiffness of the trough and the leaf springs were assumed. To simplify the model, the trough of the conveyor was assumed to be the beam whose flexural rigidity and compression stiffness comply with the ones of the real conveyor. Instead of 220 leaf springs located at 55 cross sections only 21 substituting springs placed uniformly along the length of the trough were assumed. The substitute springs were selected in such a way that the sum of their flexural rigidity and the sum of their compression axial stiffness were relevant to the sum of stiffness of 220 real springs.

The simulated model is shown in Fig. 3.

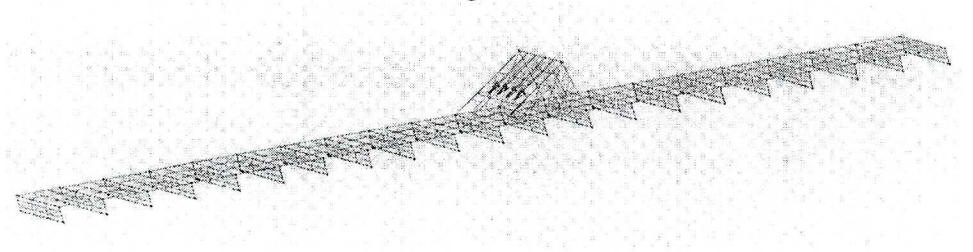
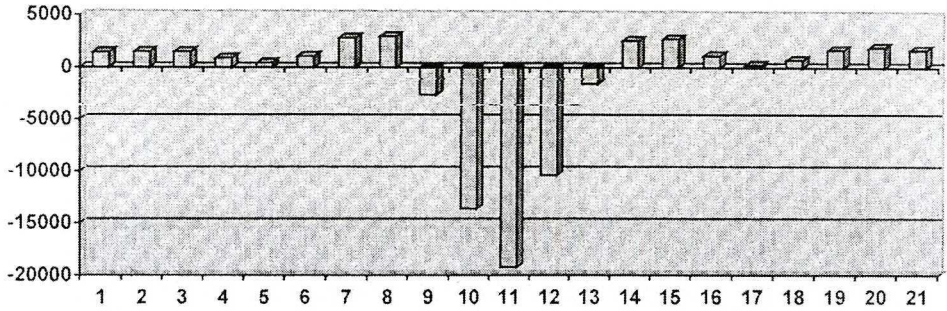


Fig. 3. Model of the real conveyor simulated by the Pro/MECHANICA software

The obtained results are presented by means of the bar charts representing the axial force of each of 21 springs. Leaf springs 8, 9, 10, and 11 illustrate the supporting system of the vibrator (Fig.4).

The Table under the bar chart gives exact values of forces transmitted into the ground by each leaf spring. Two last columns of the Table summarise the force values of the same sense.

In the analytical part of this paper it was stated that the reactions caused by the dynamic phenomena in the trough of the conveyor and transmitted to the ground make system equal to zero whereas the bar chart shows that the value is not zero.



1	2	3	4	5	6	7	8	9	10	11
1440	1480	1480	880	400	1040	2800	2900	-	-	-

12	13	14	15	16	17	18	19	20	21	-	+
-10400	-1600	2560	2720	1120	240	720	1680	1920	1680	47[kN]	25[kN]

Fig. 4. Force [N] transmitted to the ground by the conveyor

The above discrepancy can be explained by the fact that – in practice – neither the compression stiffness of springs and the trough reaches infinity nor the flexural rigidity of the trough equals zero.

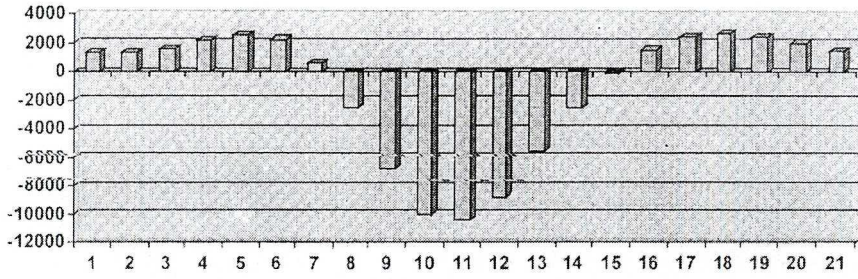
Nevertheless, in an agreement with the theoretical considerations, the vector of the force under the zone of attachment of the vibrator is directed oppositely to the vector of the force under the other parts of the conveyor.

The sum of the forces transmitted to the ground measured in the part of the trough under the vibrator equals 47 [kN], and is very close (107%) to the value estimated in the analytical way. The sum of the forces in the remaining part of the trough equals 25 [kN].

In the subsequent part of the research, the influence of the flexural rigidity of the trough as well as the axial stiffness of the springs and the trough on the force transmitted to the foundation was investigated.

The bar chart in Fig.5 illustrates forces transmitted to the ground when the flexural rigidity of the trough was increased by three orders of magnitude comparing to the initially tested conveyor, while the compression stiffness of the trough and leaf springs was left without any changes. As it can be seen from Figs. 4 and 5, the stiffening of the trough not only influences the distribution of the forces transmitted to the ground, but also significantly lowers their value.

Afterwards, we estimated the influence of axial stiffness of leaf springs on the force transmitted to the ground when the flexural rigidity of those springs was the same as in the initially investigated conveyor. (There are in practice conveyors in which the trough is suspended on rockers mounted by means of a joint instead of leaf springs, while the elasticity of the suspension system is effected by springs).

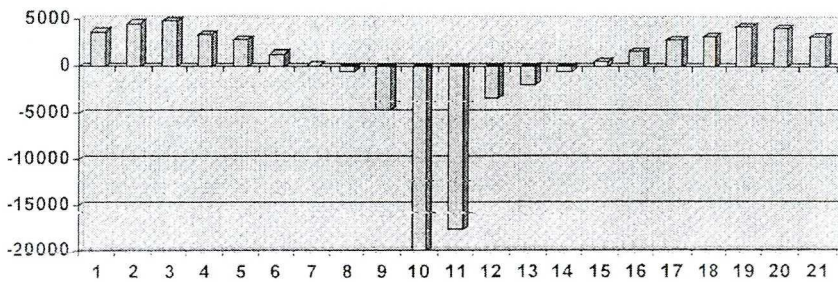


1	2	3	4	5	6	7	8	9	10	11
1360	1320	1600	2160	2560	2240	640	-	-	-	-
-	-	-	-	-	-	-	2560	6800	10000	10400
12	13	14	15	16	17	18	19	20	21	-
8800	5600	2500	100	1520	2480	2720	2480	2000	1440	31[kN]
-	-	-	-	-	-	-	-	-	-	25[kN]

Fig. 5. Force [N] transmitted to the ground when flexural rigidity of the trough is increased

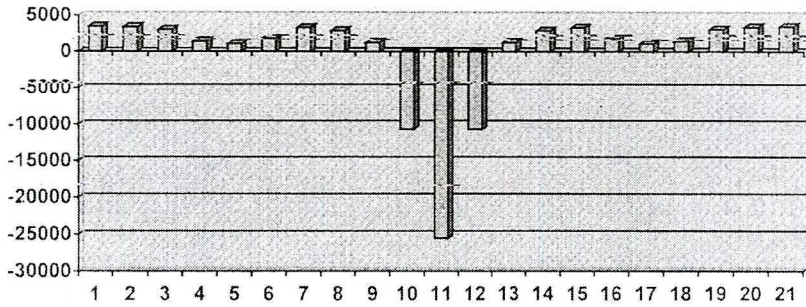
Such case is very similar to the one investigated analytically, in which the axial stiffness of leaf springs was assumed as approaching infinity.

Fig. 6 presents the force transmitted to the ground by the conveyor in which the compression stiffness of leaf springs is several orders of magnitude higher than in the conveyor of typical parameters, while the typical flexural rigidity is maintained without a change. The parameters of the trough of the conveyor in the simulated model are typical parameters of the ALWER 1.1 conveyor. As it can be seen from Fig. 6, when such manner of supporting of the trough is applied the difference between forces below the vibrator, 48 kN, and forces in the remaining part of the trough, 40 kN, is much smaller.



1	2	3	4	5	6	7	8	9	10	11
3600	4480	4800	3360	2800	1280	56	-	-	-	-
-	-	-	-	-	-	-	640	4640	20000	17600
12	13	14	15	16	17	18	19	20	21	-
-1440	-2000	-560	560	1600	2800	3200	4240	4000	3200	48[kN]
-	-	-	-	-	-	-	-	-	-	40[kN]

Fig. 6. Force [N] transmitted to the ground when compression stiffness of the leaf springs is approaching infinity



1	2	3	4	5	6	7	8	9	10	11
1485	3490	3060	1445	1105	1785	3315	2890	1275	-10710	-25300

12	13	14	15	16	17	18	19	20	21	Σ
-10710	1275	2890	3315	1785	1105	1445	3060	3490	1485	46420

Fig. 7. Force [N] transmitted to the ground when compression stiffness of the leaf springs tends to infinity and flexural rigidity of the trough is very low

In order to verify Equations (24) to (27) for systems relevant to the assumptions made for deriving these formulae, the simulation of the supporting system with the increased compression stiffness and the very low flexural rigidity of the trough was performed. Results presented in Fig. 7. show that in this case the sum of forces perpendicular to the direction of motion is close to zero while the sum of forces of the same sense equals 46 kN what comprises 104% of the analytically found value.

4. Experimental verification

The aim of experimental tests performed with the ALWER 1.1. conveyor installed in the Alwernia Chemical Plant was to confirm the presence of forces of reaction along the leaf springs. The ALWER 1.1 conveyor was chosen for experiments because the type of its foundation (non continuous foundation consisting of dilated blocks) facilitated indirect measurements of loads transmitted along the conveyor longitudinally to leaf springs.

The indirect measurements were done by means of a registration of amplitudes of acceleration of foundation blocks in the axial direction of springs, what represented a measure of forces transmitted to the foundation by the supporting system.

The slotted line consisted of the two ICP single axial sensors of acceleration, HP 353B34 and HP 353B16 models, attached to metal elements by means of a magnet or wax and connected with the HP 3560A double channel Dynamic Signal Analyzer. The analyses of vibration signal was performed with the help of the MATLAB software installed on the PC type of computer.

Measurements of the vibration amplitudes of the foundation were done in axial directions of leaf springs along the whole length of the trough. In order to be able to estimate the phase dependencies along the trough, the two channel slotted line was applied. One accelerometer was always installed under the vibrator, while with the second one the vibrations of the foundation along the trough at 18 measuring points were measured.

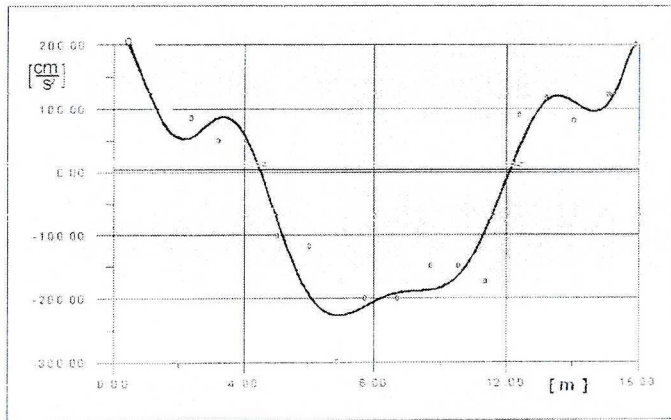


Fig. 8. Acceleration amplitude of the foundation as a function of a co-ordinate of a cross-section along the conveyor

Fig. 8 presents acceleration amplitude of the foundation at 18 measuring points situated along the trough as a function of a co-ordinate of a cross-section along the conveyor.

The amplitude was approximated by the 9th order function. The conveyor was supported at the length of 16 meters. As it can be seen from Fig. 8, the acceleration of the foundation under the vibrator is in anty-phase comparing to the acceleration in the remaining part of the trough. It happens due to the dynamic phenomena occurring along the conveyor.

The ratio of the surface areas under the graph in the part directly below the vibrator and in the remaining part of the trough equals 14: 11. It means that the sum of the forces transmitted to the foundation in the middle part of the conveyor is higher then in the remaining part of the trough, and coincides qualitatively with the distribution of forces determined numerically for the ALWER 1.1 conveyor (Fig. 4). Maximal value of the acceleration amplitude of the foundation under the vibrator in the axial direction of leaf springs amounts to 300 [cm/s²] (measuring point under the vibrator). At the frequency of forcing equal 16.7 [Hz], this value exceeds four times the one permissible for the foundations of vibratory machines (according to the Polish Standard PN-80/B-03040).

Since the structure of the divided foundation under the conveyor is not known precisely, it is not possible to estimate the actual values of forces transmitted to the ground on the bases of the acceleration of individual points of the cross-section. However, it is possible to assess that the distribution of forces along the whole length of the trough is similar to the distribution of acceleration of the foundation blocks (Fig. 8).

Thus, it can be summarised that experimental tests proved:

1. presence of longitudinal forces in leaf springs,
2. differentiation of phases of those forces along the length of the trough – in agreement with the theoretical expectations.

5. Conclusions

1. Forces, occurring due to the dynamic phenomena in the continuous system of the trough, transmitted to the ground in the longitudinal direction of the leaf springs of long conveyors, are of the same order of magnitude as the exciting force of the vibrator and are much higher than the forces connected with the bending of leaf springs in the direction of vibration.
2. Values of forces in the longitudinal direction of the leaf springs for the zones under the trough and under the vibrator can be approximately estimated on the bases of Equations (24) and (25).
3. The accuracy of the estimation of forces under the vibrator is satisfactory for typical structures. The estimation of forces under the trough is more accurate when flexural rigidity of the trough is lower and axial stiffness of the leaf springs is higher.

In real systems, amplitudes of longitudinal forces under the trough are smaller than those found theoretically, and for the investigated cases equal circ. 50% of their value.

4. Experimental investigations of the foundation of the ALWER 1.1 vibratory conveyor fully confirmed the occurrence of forces and the way of their distribution in the longitudinal direction of the leaf springs.

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Ukryte reakcje w układach zawieszenia maszyn wibracyjnych za pomocą listew sprężystych

Streszczenie

W pracy wskazano na występowanie w maszynach wibracyjnych podpartych na układzie równoległych resorów reakcji na kierunku wzdłużnym resorów. Reakcje te występują przy pozornym braku wymuszeń na tym kierunku i dla długich korpusów maszyn osiągać mogą wartości porównywalne z siłą wymuszającą wibratora. Wyprowadzono zależności analityczne dla określenia wartości tych reakcji i przeprowadzono ich weryfikację za pomocą analizy układu z wykorzystaniem metody elementów geometrycznych jak również weryfikacji doświadczalnej.