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APPROXIMATION OF CYCLES OF PARAMETERS IN FOUR-STROKE INTERNAL COMBUSTION ENGINE BY MEANS OF HARMONICS METHOD

The work presents cycle models of cylinder pressure and models of forces in crank-piston system based on a sample of experimental results. The models make it possible to determine the cycles in an arbitrary state of engine operation. Model limitations and the conditions for model applicability are also discussed. An example simulation of the processes is presented for well identified and verified models pertaining to the engine of Polonez 1,5 GLI automobile.

The method can also be applied to other types of engines after identification of the model parameters based on a sample of at least six indicator courses measured in different states of engine operation.

List of symbols

a_i, b_i, c_i, d_i, e_i, f_i – parameters of approximation model,
α_i, β_i, γ_i, δ_i, ε_i, φ_i – parameters of pressure cycle,
A_{ai}, B_{ai}, C_{ai}, D_{ai}, E_{ai}, F_{ai} – parameters of approximation models of tangential
A_{bi}, B_{bi}, C_{bi}, D_{bi}, E_{bi}, F_{bi} force cycle,
i – successive number of harmonic component,
I_{zodb} – mass moment of inertia of vehicle reduced to flying wheel,
I_{zs} – mass moment of inertia of moving masses in piston-crank system,
k – number of harmonics approximating cycle of engine parameter,
M_i – moment indicated in cylinder,

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M_{op}	– moment of receiver resistance reduced to flying wheel,
M_{sw}	– moment of self resistance of engine,
M_e	– usable moment of engine,
n	– rotational speed,
n_j	– rotational speed of idle run,
n_M	– rotational speed at maximum moment,
n_N	– nominal rotational speed,
n_{max}	– maximum rotational speed,
p	– indicated pressure,
P_{ai}, P_{bi}	– harmonic amplitudes of gas force,
Q	– engine degree of admission,
T	– tangential force,
T_{AV}	– cycle average of tangential force,
T_{ai}, T_{bai}, T_{bai}	– harmonic components of tangential force,
T_{bi}, T_{bbi}, T_{bbi}	– total, gas and inertial force, respectively,
α	– rotation angle of crankshaft,
ε	– angular acceleration of crankshaft,
η_m	– mechanical efficiency of engine,
ν	– relative angular velocity of crankshaft,
ω	– angular velocity of crankshaft.

1. Introduction

The knowledge of pressure cycles $p_i(\alpha)$; $\alpha \in \langle 0;4\pi)$ of different, arbitrary states of operation of combustion engine is the condition for solving many research problems concerning the virtual engine. The latter is an abstract product of modelling of actual engine, whose operation is examined by computer simulation.

Obtaining reliable data related to the pressure function by indicator measurements in cylinders of the engine is costly, and it is especially difficult when the engine operates with varying operational state parameters (rotational speed, supply rate, load), as it is in unsteady state of operation (the transient processes).

It is expedient to substitute the set of functions of actual pressure cycles, or forces in the piston-crank system, by the cycles found by approximation. This is due to the phenomena that take place evsteady operational conditions, like nonuniform run of actual engine (fluctuations of instantaneous crankshaft rotational speed referred to a multi-cycle average), uneven supply in individual cylinders of a multicylinder engine, and lack of repeatability of engine cycles. A relevant approximation could not significantly deteriorate adequacy of the applied models.

There exist engine models of high adequacy that make it possible to determine actual pressure cycles. However, identification of parameters of such models for a particular engine needs many procedures and experimental research.

The concept presented by the Author is based on an input-output model ("black-box" approach) that makes it possible to predict the course of a cycle (cylinder pressure, forces in selected nodes of crank-piston system, forces and moments in multi-cylinder engine) in an arbitrary state of engine operation.

In the model of approximation of cycle function one assumes that for the purposes of parameter identification one needs a sample of a small number of cycles ($p \geq 6$), obtained experimentally, each one measured in substantially different conditions of engine operation.

2. Requirements concerning input data

The equation of motion of the engine can be formulated as

$$M_i - M_{SW} - M_{op} = (I_{zodb} + I_{ZS})\,\varepsilon \tag{1}$$

where: M_i – indicated moment of engine,

M_{SW} – moment of engine self resistance (losses) reduced to flying wheel,

M_{ob} – moment of receiver resistance reduced to flying wheel,

$I_{z\,odb}$, I_{ZS} – equivalent moments of inertia of masses reduced to flying wheel, for the receiver and for moving masses in the engine power transmission system (piston-crank system), respectively,

ε – angular acceleration of crankshaft.

It follows on Equation (1) that in unsteady state ($\varepsilon \neq 0$) the engine moment is not equal to the loading moment

$$M_i - M_{Str} \neq M_{ob.,} \tag{2}$$

while the net moment M_e generated in the engine (also called the usable moment) is

$$M_e = M_i - M_{Str} \approx \eta_m \cdot M_i \tag{3}$$

η_m – mechanical efficiency.

The authors of publications on engines are accustomed to treat the engine in a quasi-static way (ω=const, ε=0), and wrongly consider load (M_{ob}) and engine moment (M_e) identical.

The usable moment of engine M_e is a function of engine angular velocity ω and the degree of admission Q that depend on the settings of controls determining the amount of fuel and air supplied to the engine.

The sample of experimentally determined courses (cycles), for example cycles of pressure in the cylinder, whose minimal number was arbitrarily assumed by the Author as $p \geq 6$, should cover various different states of engine operation, that is different rotational velocities $n \in (n_j; n_N)$ and different values of degree of admission $Q \in \langle 0;1 \rangle$.

3. Engine degree of admission Q

In self-ignition engine, the amount of fuel and engine speed are determined by absence of the air throttle in the inlet duct, and in the case of continuous-operation control unit (misleadingly named the multi-range one) by the position of the delivery rod rack. The speed consequently depends on the load (moment of receiver load). It is illustrated in Fig. 1, where control graphs 1, 2, ... refer to consecutive positions of the rack link, pertaining to the degree of admission Q_i.

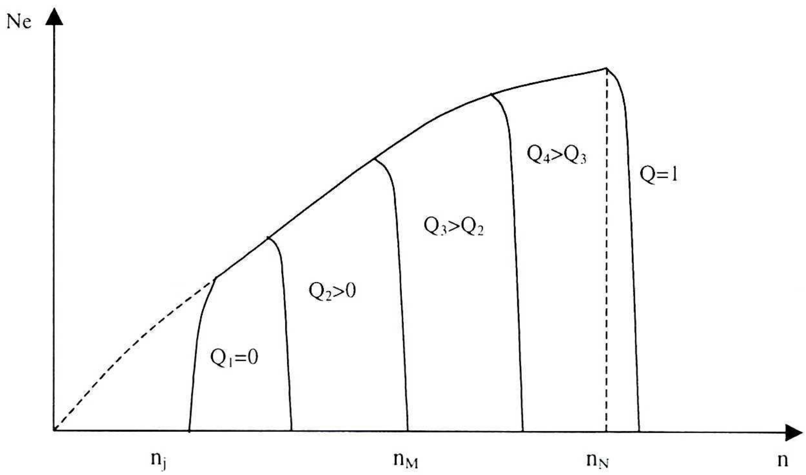

Fig. 1. Transient characteristics of power of self-ignition engine with continuous-operation control unit

In the car equipped with self-ignition engine, it refers to consecutive positions of the accelerator pedal, starting from idle run (pedal released, Q=0) up to the limit position, Q=1.

In spark ignition engine, the operator can work on both the external characteristic with full opening of the throttle, Q=1, and the partial characteristics $Q\in(0;1)$ as well as obtain idle run Q=0; $n=n_j$ with released accelerator pedal and disengaged receiver. Running light refers to Q=0 and engine operating with disengaged receiver at $n\in(n_j;n_{max})$.

When considering spark ignition fuel-injection engine, one can determine the degree of admission in the following way. At a selected rotational speed of the engine (for instance, according to the Polish Norm it is $n_x=0.75n_N$) one determines the relation between injection time T_{wtr} and the degree of admission Q basing on two different states:

$Q(T_{wtr}\ min)=0$, and $Q(T_{wtr}\ max)=1$.

The graph in Fig. 2 shows this relation for a single-injector (SPI-type) engine of the POLONEZ 1,5 GLI automobile.

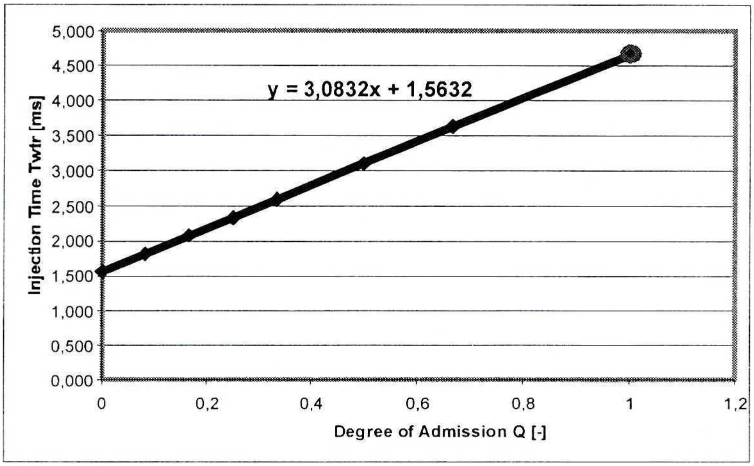

Fig. 2. Relation between injection time T_{wtr} and the degree of admission Q of the POLONEZ 1,5 GLI engine

The relation is quite clear, as the amount of fuel supplied to the engine is proportional to the time of injection. Invariable degree of admission Q_x refers to a stable position of air throttle in the whole range of usable engine speeds, and the value of n is determined by engine load M_{ob}. In running light, $M_{ob}=0$, the position of the throttle is chosen to maintain a desirable engine speed $n \in \langle n_j, n_{max} \rangle$.

Table 1 contains the data of indicator courses measured in different states of engine operation [5]. These data were used for identification of model parameters and for verification of the models.

Table 1.

Measured indicator courses in the area of operation states of the Polonez 1,5GLI engine

Q \ n	850	950	1100	1200	1500	2000	2700	3500	4300	4600	5000
0	A1		C1		E1	F1	G1		J1		M1
1/12			C2								
1/6	A3	B3	C3		E3				J3		M3
1/4			C4	D4						K4	
1/3			C5		E5	F5	G5	I5	J5		
1/2	A6		C6	D6		F6	G6			K6	M6
2/3	A7			D7			G7	I7		K7	M7

denotations:

Q – engine degree of admission (Q=0 – idle run, Q=1 – full supply),

n – crankshaft rotational speed [RPM].

Tests denoted A, B, ... I are based on experimental data of cylinder pressure function.

Tests J, K, M are based on extrapolation of pressure functions determined by the velocity characteristics of engine rotational moment.

Fig. 3 shows illustrative power characteristics of spark ignition engine.

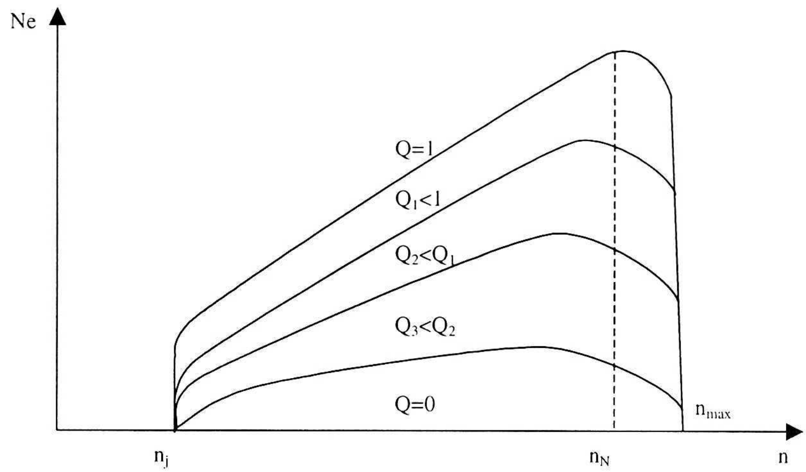

Fig. 3. Characteristics of power vs. rotational speed for spark ignition engine

4. Transient processes (unsteady states)

The approach presented in Section 3 is characteristic for quasi-static analysis of the engine. Each state of operation is there determined by three invariable parameters (Q, M_{ob}, n), and the duration of transients between the states is sufficiently long to allow the engine to reach thermal equilibrium in each state, T_s=const, where T_s is engine temperature. In this way, one determines quasi-static characteristics of the engine.

The transient states characterised by fast changes of parameter values must be treated in a different way. Fast change means that the transient from state A $((Q^A; M^A_{ob}) \Rightarrow n_A)$ to state B $((Q^B; M^B_{ob}) \Rightarrow n_B)$ depends on such factors as range, course and speed of change of the degree of admission $Q^A \Rightarrow Q^B$, as well as range, course and speed of change of engine load $M^A_{ob.} \Rightarrow M^B_{ob}$. At the same time, the change of speed $n_A \Rightarrow n_B$ depends on flexibility of engine moment, inertia of movable masses, thermal inertia of the engine, etc., and it also depends on the properties of engine control algorithm.

Instantaneous values of engine working characteristics are then difficult to determine during transient processes, and are influenced by the sensitivity of the engine to the changes of state parameters. This sensitivity can be substantially different even for individual engines of the same type that differ, for example, in the duration of exploitation period.

Therefore, the considered problem needs the examination to be carried out in standard conditions [7] that, unfortunately, have not been specified yet. Relaying on quasistatic characteristics, when one considers transient processes, can give only a rough approximation of the phenomena with errors that may be difficult to assess.

5. Model of cycles of indicated pressure changes

If we assume that after a sufficiently long warm-up the engine operates in conditions close to thermal equilibrium, when engine temperature does not change much, then the instantaneous pressure in the cylinder can be expressed by a function

$$p=p(\alpha, Q, n) \tag{4}$$

where α – crankshaft rotation angle $\alpha \in \langle 0; 4\pi \rangle$

$Q \in \langle 0;1 \rangle$: $n \in \langle n_j; n_{max} \rangle$.

Let us denote

$$\nu = \frac{n}{n_N} \tag{5}$$

where n_N – engine nominal speed,

ν – dimensionless speed of engine,

then $\nu \in \langle \nu_j; \nu_{max} \rangle$, where $\nu_{max} = \frac{n_{max}}{n_N}$; $\nu_j = \frac{n_j}{n_N}$.

The speed of engine equals

$$n = \nu \cdot n_N \quad \text{or} \quad \omega = \nu \cdot \omega_N \; \omega = \nu \; \omega_N \tag{6}$$

The cylinder pressure function can be expanded into a harmonic series

$$p(\alpha,Q,\nu)=p(\alpha,Q,\nu)=\sum_{i=0}^{i=k}\left[Pa_i(Q,\nu)\cdot\cos\frac{i\alpha}{2}+Pb_i(Q,\nu)\cdot\sin\frac{i\alpha}{2}\right] \tag{7}$$

According to [4] one can assume that the number of harmonics sufficient for accurate approximation of the actual pressure function is $k \geq 20$.

Harmonic amplitudes P_{ai} and P_{bi} are functions of two variables Q and ν, and the following approximation can be used to represent them

$$P_{ai}= a_{pi}+\nu\cdot b_{pi}+Q\cdot e_{pi}+Q^2\cdot d_{pi}+\nu^2\cdot e_{pi}+Q\cdot\nu\cdot f_{pi} \tag{8}$$

Similarly

$$P_{bi}= \alpha_{pi}+\nu\cdot\beta_{pi}+Q\cdot\gamma_{pi}+Q^2\cdot\delta_{pi}+\nu^2\cdot\varepsilon_{pi}+Q\nu\cdot\varphi_{pi} \tag{9}$$

The set of model parameters $p(\alpha, Q, \nu)$:

$$a_{pi}\ ;\ b_{pi}\ ;\ c_{pi}\ ;\ d_{pi}\ ;\ e_{pi}\ ;\ f_{pi} \tag{10}$$

$$\alpha_{pi};\ \beta_{pi}\ ;\ \gamma_{pi}\ ;\ \delta_{pi}\ ;\ \varepsilon_{pi};\ \varphi_{pi}$$

can be determined based on experimental results, for example the cycles taken from Table 1. The identification of parameters must be based on at least six indicator courses that significantly differ in supply ratings and engine speed.

6. Forces in crank system

Similarly as it was in the case of pressure cycles, the model of forces in crank system can be represented in the form of harmonic series. Example relationships defining the model of the tangential force T are presented below:

$$T(\alpha,Q,\nu) = T_g(\alpha,Q,\nu) + T_b(\alpha,\nu) \qquad (11)$$

where T_g – tangential force component resulting from pressure of working medium in cylinder (tangential gas force)

$T_b(\alpha, \nu)$ – tangential force component resulting from inertia of moving masses of the engine power transmission system,

$$T(\alpha,Q,\nu) = \sum_{i=1}^{k}\left[T_{ai}\cos\frac{i\alpha}{2} + T_{bi}\sin\frac{i\alpha}{2}\right] \qquad (12)$$

Harmonic amplitudes are the following functions:

$$T_{ai}(Q,\nu) = T_{gai} + T_{bai} = A_{ai}+\nu\cdot B_{ai} + Q\cdot C_{ai}+ \nu^2\cdot D_{ai}+ Q^2\cdot Ea_i + \nu\cdot Q\cdot F_{ai} \qquad (13a)$$

$$T_{bi}(Q,\nu) = T_{gbi}+T_{bai}= A_{bi}+\nu\cdot B_{bi}+Q\cdot C_{bi}+ \nu^2 D_{bi}+Q^2 E_{bi}+\nu\cdot Q\cdot F_{bi} \qquad (13b)$$

The relationship between the set of parameters (10) and the set of parameters of Equations (13a) and (13b) results from geometrical properties of the crank-piston system.

In the work [2], a postgraduate student supervised by the Author performed identification of parameters of tangential and radial force models for the Polonez 1,5GLI engine.

Figures 4 and 5 present example values of harmonic amplitudes T_{ai} found from Equation (13a) in selected states of engine operation, and Fig. 6 shows the waveforms of tangential force determined from Eq. (12).

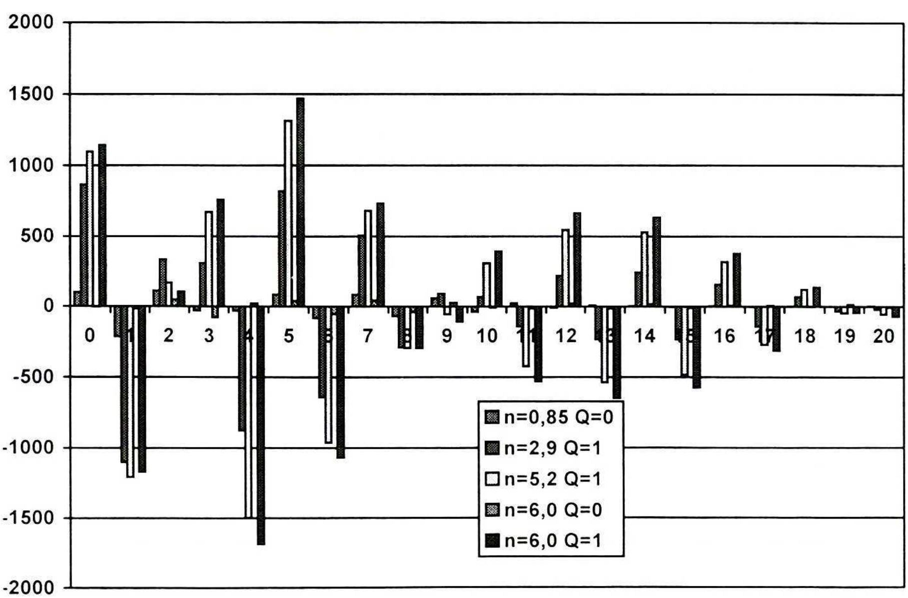

Fig. 4 Values of parameters Ta_i of equation describing tangential force (13a) in selected states of engine operation (rotational speed n in [10^3 RPM])

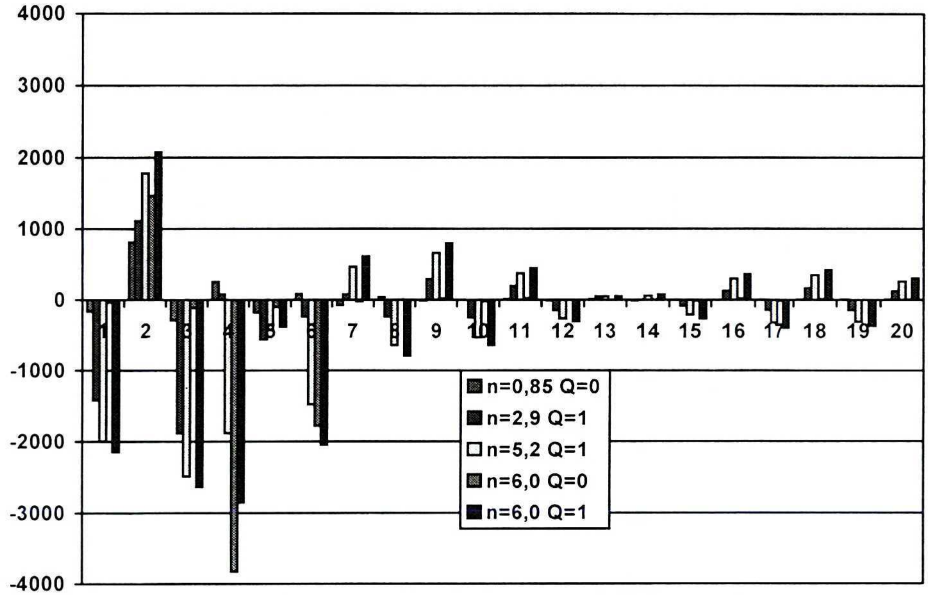

Fig. 5 Values of parameters Tb_i of equation describing tangential force (13b) in the states of engine operation as in Fig. 4

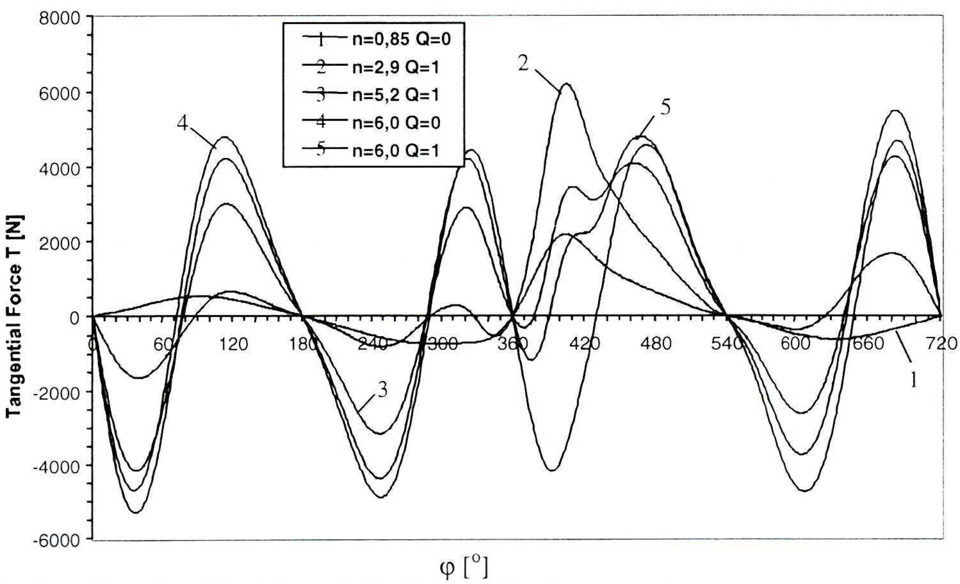

Fig. 6. Functions of tangential force determined from Eq. (12) for selected states of engine operation (rotational speed in [10^3 RPM])

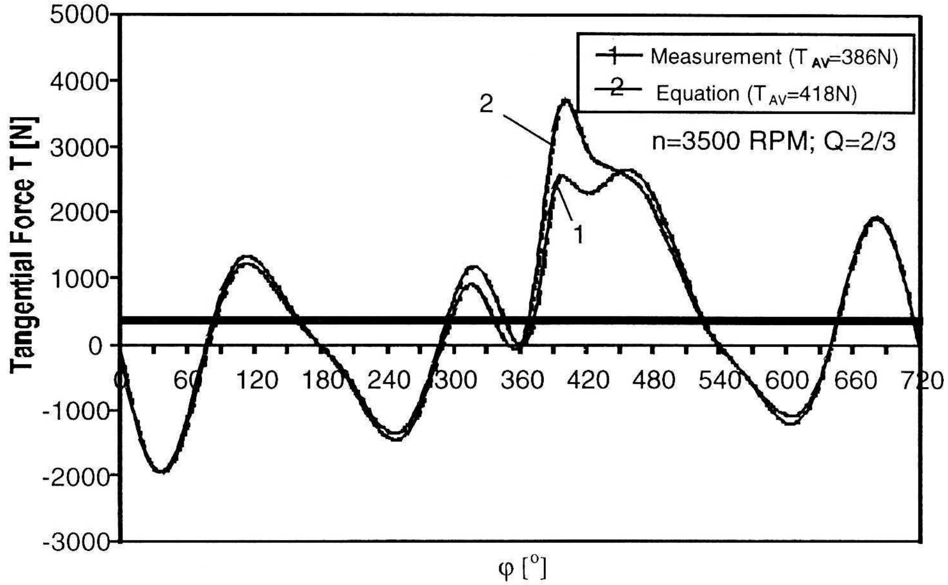

Fig. 7. Comparison of tangential force functions for Q=2/3 and n=3500 RPM

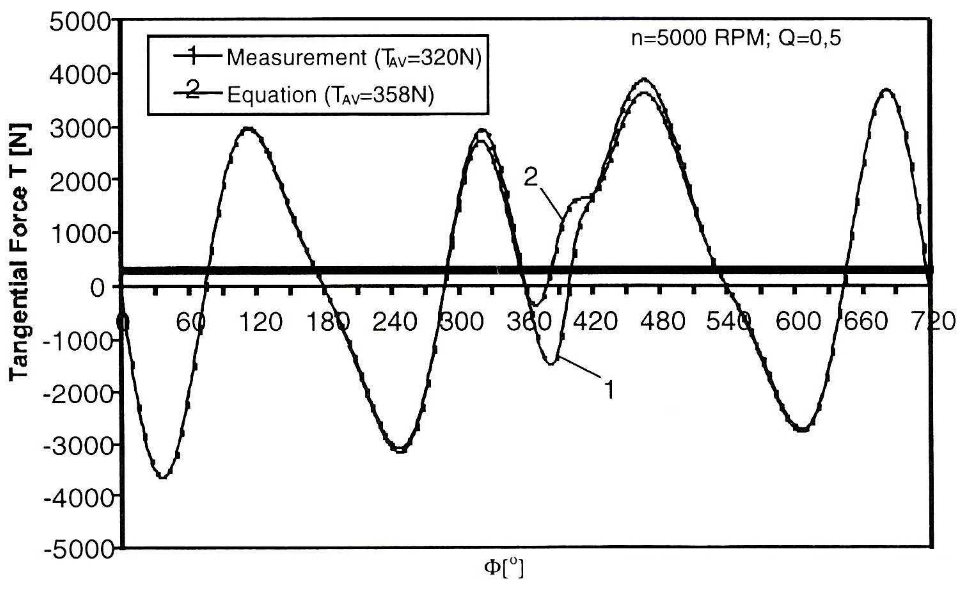

Fig. 8. Comparison of tangential force functions for Q=1/2 and n=5000 RPM

7. Conclusion

Figures 7 and 8 show examples of comparison between tangential force functions determined by measuring pressure cycle and those calculated from Eq. (12).

The functions were selected for a fragmentary verification of the model proposed by the Author. The functions calculated on the basis of harmonics method show evident similarity to the cycles determined from the measured indicator graphs. The differences between cycle average values do not exceed 11%. It shows that adequacy of the model is sufficient, irrespective of poor repeatability of pressure cycles in the engine cylinders.

The developed method of harmonic amplitude functions can be applied to create the model of cycles of radial and tangential force, as well as the model of moment generated in the inside crank and the moment produced by a multicylinder engine.

In the case of two-stroke engine, the period of cycle is $\alpha \in \langle 0; 2\pi \rangle$, and the form of series (7) and (12) should be appropriately changed.

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Aproksymacja cykli parametrów silnika spalinowego metodą harmonik

S t r e s z c z e n i e

Znajomość cykli ciśnienia $p_i(\alpha)$; $\alpha \in \langle 0;4\pi)$ w różnych, dowolnych stanach działania silnika spalinowego warunkuje rozwiązanie wielu problemów badawczych silnika wirtualnego, a więc abstrakcyjnego produktu modelowania silnika spalinowego, którego działanie badamy drogą symulacji komputerowej.

Uzyskanie jednoznacznych danych o przebiegu ciśnienia poprzez indykowanie cylindrów silnika jest drogie i zwłaszcza w warunkach funkcjonowania silnika przy zmiennych parametrach jego stanu działania (prędkości obrotowej, stopniu zasilania, obciążeniu), a więc podczas stanów nieustalonych (procesów przejściowych), szczególnie trudne.

Niejednostajność biegu silnika (wahania wartości chwilowej prędkości wału korbowego względem wartości średniej wielocyklowej), nierównomierność zasilania poszczególnych cylindrów silnika wielocylindrowego, wreszcie niepowtarzalność obiegów silnika nawet w ustalonych warunkach jego działania, upoważniają do zastąpienia zbioru cykli rzeczywistych przebiegu ciśnienia w cylindrze bądź sił w układzie tłokowo-korbowym cyklami uzyskanymi na drodze aproksymacji, bez obawy istotnego pogorszenia adekwatności zastosowanych modeli.

Istnieją wprawdzie modele silnika o dużej adekwatności pozwalające wyznaczać cykle ciśnienia, jednak identyfikacja ich parametrów w odniesieniu do konkretnego silnika wymaga wielu zabiegów i badań eksperymentalnych.