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Hilbert Space

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Length, width, depth… Knowing these parameters allows us to judge which table will fit into our dining room, which bed is right for our bedroom. We use three measures to describe the space around us, but we rarely ask ourselves whether reality is, in fact, three-dimensional.

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When the full theory describing classical physics of electromagnetic phenomena (such as the flow of electric current in a copper wire or the working of a compass) was formulated back in the second half of the nineteenth century, it became clear that light propagates in the form of a wave. This theory predicted the speed of light to be approx. 300,000 km per second – but it did not specify with respect to what this speed should be measured. When we think about a car travelling at 100 km per hour, it is implied that the speed is relative to the Earth's surface, to the towns and villages being passed by. Why, then, does the theory of electromagnetism not specify what the speed of light is relative to?

The problem was initially solved by inventing the concept of the "luminiferous ether" – an enigmatic substance filling the universe and serving as an absolute reference point. It was argued that all motion was described relative to the ether, including the speed of light. However, the trouble was that experiments (in particular the famous Michelson–Morley experiment) conclusively refuted the ether hypothesis. The question of a reference point for the speed of light therefore remained unanswered.

In the early twentieth century, Albert Einstein posited the following theory: regardless of the (constant) speed at which we chase or run away from a light wave, the speed we measure it to be travelling at will always be the same, being equal the value predicted by the theory of electromagnetism. In other words, in all systems which move relative to one another at a constant speed (known as "inertial" systems), the speed of light is the same constant. This may seem counterintuitive; after all, when someone is running away from you, if you increase the speed of your chase, the distance between you and your quarry will decrease; if you run in the opposite direction, the other person will seem to be running away even faster. Einstein posited that the speed of light is a constant and showed that this has serious consequences for how we perceive physical space and the passing of time.

Frame of reference

In order for the speed of light to be a constant in all inertial systems, however, the three dimensions of space must be linked to a fourth – the passage of time. This brings us to the concept of four-dimensional *spacetime*. The sizes of physical objects and the time they exist in are relative to one another and to the speed of the observer. Time and space are not absolute concepts; it is necessary to specify in each case the system they are being measured *relative* to – hence the name for Einstein's (special) theory of *relativity*.

The revolutionary predictions of the special theory of relativity have been confirmed through numerous experiments, and Einstein continued working to eventually formulate the general theory of relativity. It concludes not only that spacetime is four-dimensional; in fact it has a highly complex structure and its shape is affected by massive objects such as planets and stars. The closer a given object is to such a massive gravitational object in space, the more slowly time passes there relative to a system a considerable distance away from the gravitational object. Comparisons of clocks on the Earth's surface with those onboard the satellites forming the GPS network (which are farther away from the Earth) reveal discrepancies which, while miniscule, are still large enough that had they not been adjusted for, the satellite navigation system would be so inaccurate as to be useless. Making this adjustment would not be possible were it not for the general theory of relativity. The deformation of spacetime can be so severe near a sufficiently massive object as to create a region "excluded" from the surrounding universe – a region nothing can escape from. Such objects are known as black holes, and their mysterious properties are a continual source of inspiration for scientists as well as authors and artists.

The field of quantum mechanics emerged around the same time as both theories of relativity. Experiments carried out at the turn of the nineteenth century revealed that the classical theory of electromagnetism was inaccurate and insufficient in its descriptions of certain phenomena, failing to provide answers to questions such as why atoms – understood as positively-charged nuclei orbited by negatively-charged electrons – do not simply collapse in a fraction of a second. Had this been the case, the matter which we are made of and which surrounds us would be unstable and it would be impossible for the universe as we know it to exist. This clear contradiction between theoretical predictions and reality required an explanation which could not be provided by the "old" physics.

Schrödinger's cat

The development of quantum mechanics, solving the problem of atomic stability and myriad other phenomena observed on the atomic and subatomic scale,

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Schrödinger's cat – both dead and alive

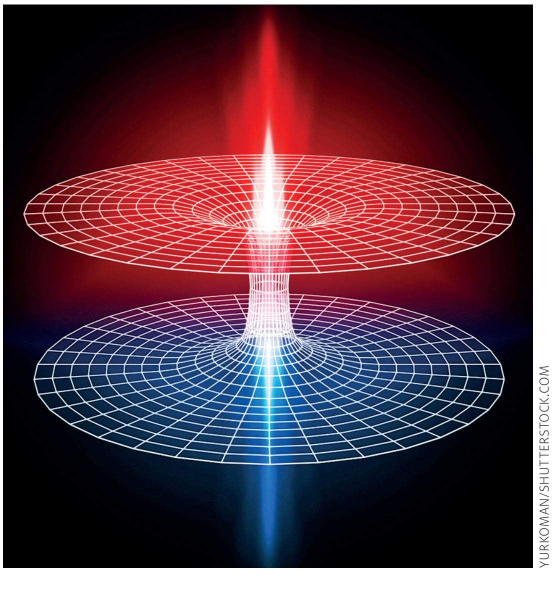
was spread out over time. In the 1920s, the theory appeared to be ready; Erwin Schrödinger formulated the fundamental equation and proposed interpretations of its solutions. Unfortunately this led to conclusions so unlike the observable reality around us that its validity is being questioned even today.

In quantum mechanics, a description of the universe as a collection of objects existing in three-dimensional space with the additional parameter of time (traditional Newtonian physics) or a four-dimensional spacetime (Einstein's theories) is replaced by an abstract concept of a state (an object comprising complete information about a system) which has evolved in a multidimensional (perhaps even infinitely dimensional) space known as Hilbert space. The difference has major consequences, and it is not just a theoretical issue relevant to physicists only. In Hilbert space, even an individual atom can exist in an infinite number of opposing states at the same time; for example, it can exist in several places at once. This is known as superposition. The problem is, however, that no one has ever observed an electron or any other object existing in several places at the same time. So what's going on?

The founders of quantum mechanics interpreted superposition as a coexistence of physical systems in opposite states being an inevitable property of quantum objects. The sheer act of taking measurements – activating cameras, sensors, or even just making an observation – must in and of itself obliterate this superposition. The object "chooses" a single state from all the available options. The process is based on pure chance; there is no way of predicting the outcome of a single observation – we can only say which outcomes are more probable than others. According to this interpretation, observers have no access to a "pure" object existing in Hilbert space with its myriad possibilities. The act of observation causes the system to collapse into a single state. We can then briefly assign it classical properties, such as three coordinates describing its position in the "traditional" space we occupy. Yet left to its own devices, a quantum object instantaneously returns to its natural state, where it exists in a superposition.

A common example of superposition is the famous "Schrödinger's cat" thought experiment: if we apply the principles of quantum mechanics, we could have a superposition of a cat which is simultaneously dead and alive. We don't know what state it is in until we open the box – but in that very moment the enigmatic process of chance selects one of the options and we can never know what the state was before that point.

We now have *two* ways of answering the question posed in the opening paragraph. In classical (non-quantum) physics, the concept of space is replaced by four-dimensional spacetime, as required by Einstein's theories of relativity. In quantum mechanics, systems evolve in a multidimensional Hilbert space where they can coexist in many opposing states known as superpositions. All that remains is to answer the final question: What, then, is space itself? The answer is simple, but alas, it will not fit into the margins of this article. ■



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Einstein-Rosen bridge in a space-time tunnel

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