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Study of Actuarial Characteristics of One-Year and Ultimate Reserve Risk Distributions Based on Market Data

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Abstract

In this work, we perform an analysis of the characteristics of the one-year and ultimate reserve risk distributions commonly used in actuarial science: duration, first development factor, coefficient of variation, skewness coefficient, skewness-to-CoV ratio, emergence factor, emergence pattern, and risk margin run-off patterns. Our study is based on empirical data for two European markets: the Polish and Slovak markets. We provide benchmarks and ranges for the considered characteristics, as well as analyse the relations between them. We study Solvency II lines of business and compare our coefficients of variation to the Standard Formula reserve risk standard deviations. We investigate more deeply the topic of emergence pattern and risk margin run-off patterns.

Keywords: one-year risk, ultimate risk, reserve risk, emergence pattern, risk margin run-off pattern

JEL Classification: G22

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1 Introduction

Insurance companies are exposed, among others, to *reserve risk*. The *ultimate reserve risk* is the risk that the current claims reserves will not be sufficient in the ultimate horizon, i.e. the current claims reserves will not cover all payments made in the future years. The *one-year reserve risk* is the risk that the current claims reserves will not be sufficient after one year, i.e. the current claims reserves will not cover payments made within the next year and the claims reserves set at the end of this year. The presented definition of reserve risk is consistent with the definition presented in Article 105, point 2a of Solvency II Directive (2009) and the view (one-year or ultimate) is dependent on the horizon in which we measure the risk (see e.g. Wüthrich and Merz (2015)). In the one-year view, we predict only the next year payments and evaluate the claims reserve at the end of the next year, i.e. we extend the claims triangle with one diagonal and perform a re-estimation of the claims reserve. In the ultimate view, we predict all the future payments, i.e. we simulate a full run-off of the claims triangle. The one-year view is an important notion introduced by Solvency II Directive, which requires insurance companies to calculate Solvency Capital Requirement from the one-year perspective. This view differs from the ultimate view which is traditionally analysed by actuaries. Analysing the one-year reserve risk is also an important aspect of the business activity of an insurance company, as the deviations from the current reserve position impact directly not only the own funds of the company, but also the financial result. In this regard, observing the one-year risk is especially important as it explains the fluctuations observed in a company's P&L statement, which is usually analysed in yearly time intervals.

The goal of this paper is to provide benchmarks and possible ranges for some characteristics of the reserve risk distributions used in actuarial practice. These types of benchmarks can be found in the literature (see Table A1 in Appendix A.2), however they are scattered and scarce. They usually focus either on a specific characteristic or only on some LoBs. Therefore, it is valuable to perform one comprehensive analysis entailing various aspects of the reserve risk and all LoBs. With this paper, we perform a detailed analysis of the reserve risk by calculating various key actuarial characteristics for the full list of insurance and proportional reinsurance Solvency II LoBs (Lines of Business) for two European countries: Poland and Slovakia. To the best of our knowledge, a complete analysis of this type is not available in the literature and such a study has not been performed for these markets earlier.

Our methodology may be used by insurance companies, e.g. for internal model validation or for the purpose of Own Risk and Solvency Assessment (ORSA) process. The figures and conclusions presented here can be compared with the company's results to perform a plausibility assessment (especially for the reserve risk analysis). The results may be also used as reference values by scientists performing simulation studies. We are aware that our analysis is limited to two European markets (Polish and Slovak). However, in our view, it should also be beneficial for other markets,

as this paper provides benchmarks and general interpretations for common business segmentations (e.g. motor third party liability or property insurance LoBs) and common characteristics (e.g. the possible shapes of emergence pattern or risk margin run-off patterns). This would especially apply to the markets encompassed by the Solvency II regime, since these markets use the same segmentation as in this paper.
In this paper we estimate key actuarial characteristics of the one-year and ultimate reserve risk distributions. These characteristics describe: the structure of the portfolio (the first development factor f_1 and the duration), the second and third moments of the one-year risk distribution (the one-year CoV, the skewness, and the skewness-to-CoV ratio), as well as the second and third moments of the ultimate risk distribution (the ultimate CoV, the skewness, and the skewness-to-CoV ratio). We analyse relations between the characteristics, as well as compare the results for Polish and Slovak markets - both of which provide us with additional insight regarding plausibility checks and interpretation of possible risk metrics. Furthermore, we investigate the relation between the one-year risk and the ultimate risk through the analysis of the following characteristics: the relation between the one-year and the ultimate CoV, the skewness, and the skewness-to-cov ratio, the emergence patterns (used for switching from the ultimate reserve risk to the one-year reserve risk), and the risk margin run-off patterns, where the patterns are based on the run-off of the best estimate reserve or the standard deviation of the future one-year risks. This part also provides additional information regarding the expected shapes and behaviours of the analysed patterns. All these estimates are novel in the literature for the considered markets and can be viewed as the main contribution of this paper.
The paper is structured as follows. In Section 2 we describe the approach used in the study and present an example simulation of one-year and ultimate reserve risk. In Section 3 we define the actuarial characteristics considered in the paper. In Section 4 we describe the analysed datasets and performed calculations. Each subsection in Section 5 is focused on the examination of specific characteristics or relations between them. In Section 6 we conclude and in Appendix we present additional information.

2 Claims development process

In this section we describe the approach to the claims development process, both the general structure and the specific Mack Chain Ladder model used in the paper, and present an example of a single simulation coming from the distributions of the one-year and the ultimate reserve risk.

2.1 General structure

We consider a claims development process for $i, j \in \{1, \dots, n\}$, where $C_{i,j}$ denotes the cumulative payments made for the i-th accident year up to the j-th development period and $X_{i,j}$ denotes the incremental payments made for the i-th accident year in the j-th development period. The values of $C_{i,j}$ and $X_{i,j}$ are usually presented in the form of a triangle, where rows represent years in which the claims occurred (accident year) and the columns represent years, in relation to the accident years, in which the payments were made (development year). We assume that there are n historical accident years and that there is no further development of the claims after development year n. We call $C_{i,n}$ the ultimate loss for the i-th accident year. We also assume that we are at the time $t = i + j = n$, therefore we omit the subscript regarding the moment of calculation. As insurance companies are obliged to estimate the claim reserves, we do it with the best estimate of the outstanding loss at the end of the n-th calendar year defined by

$$BE_i = \mathbb{E}[X_{i,n+2-i} + X_{i,n+3-i} + \ldots + X_{i,n}|C_{i,n+1-i}], \quad BE = \sum_{i=1}^{n} BE_i,$$

which is the expected value of the future payments conditional on the knowledge up to point $t = n$. This position evolves over time. Its change directly impacts the own funds and the financial result of the company. As the value of BE_i is deterministic, we evaluate the reserve risk for two different time horizons: the one-year view (BE^{1Y}) and the ultimate view (BE^{Ult}),

$$\begin{aligned} BE_i^{1Y} &= X_{i,n+2-i} + \mathbb{E}[X_{i,n+3-i} + \ldots + X_{i,n}|C_{i,n+2-i}], \quad BE^{1Y} = \sum_{i=1}^{n} BE_i^{1Y}, \\ BE_i^{Ult} &= X_{i,n+2-i} + X_{i,n+3-i} + \ldots + X_{i,n}, \quad BE^{Ult} = \sum_{i=1}^{n} BE_i^{Ult}. \end{aligned}$$

These two positions are random variables and they may be interpreted as the value of the best estimate of the liabilities that we would set at $t = n$ if we had the information from $t = n + 1$ for the one-year view ($1Y$) or after the full run-off of the liabilities for the ultimate view (Ult). We can easily notice that the expected value does not change as

$$\mathbb{E}[BE_i^{1Y}|C_{i,n+1-i}] = \mathbb{E}[BE_i^{Ult}|C_{i,n+1-i}] = BE_i \text{ for each accident year } i.$$

The financial loss to the company is then defined by the use of the claims development result concept:

$$CDR^{1Y} = BE - BE^{1Y}, \quad CDR^{Ult} = BE - BE^{Ult}.$$

2.2 Mack Chain Ladder Model

For the choice of the claims development model, we follow the methodology presented in Section 2 in Szatkowski and Delong (2021) and in Mack (1993), Mack (1994), Wüthrich and Merz (2008), Wüthrich and Merz (2015). We consider Mack Chain

Ladder model, as it is one of the most popular methods for the estimation of the claims reserve (see ASTIN (2016)) and it played also an important part in the calibration of Standard Formula reserve risk standard deviations (see CEIOPS (2010)). It is based on the following assumptions:

1. $\{C_{i,1},\ldots,C_{i,n}\},\{C_{k,1},\ldots,C_{k,n}\}$ for $i \neq k$ are independent.

2. There exist parameters $f_1,\ldots,f_{n-1}$ that $\mathbb{E}[C_{i,j+1}|C_{i,1},\ldots,C_{i,j}] = f_j C_{i,j}$.

3. There exist parameters $\sigma_1,\ldots,\sigma_{n-1}$ that $Var[C_{i,j+1}|C_{i,1},\ldots,C_{i,j}] = \sigma_j^2 C_{i,j}$.

For the estimation of f_j and σ_j^2 we use the following estimators (in line with Mack (1993)):

$$\hat{f}_j = \frac{\sum_{k=1}^{n-j} C_{k,j+1}}{\sum_{k=1}^{n-j} C_{k,j}}, \quad \hat{\sigma}_j^2 = \frac{1}{n-j-1}\sum_{k=1}^{n-j} C_{k,j}\left(\frac{C_{k,j+1}}{C_{k,j}} - \hat{f}_j\right)^2. \tag{1}$$

The best estimate of the outstanding loss at the end of the n-th calendar year in the Mack Chain Ladder model is calculated as

$$BE_i = C_{i,n-i+1} \cdot \left(\prod_{j=n-i+1}^{n-1} \hat{f}_j - 1\right), \quad BE = \sum_{i=1}^{n} BE_i.$$

Apart from the classical approach, we make additional assumptions:

4. In order to estimate the one-year risk, we follow the re-reserving procedure, which is based on the re-calculation of the development factors and projection of the claims into the future, taking into account the new stochastic diagonal of the triangle . This method is also known as “Actuary-in-the-Box”, see e.g. Ohlsson and Lauzeningks (2009). We follow the decisions that are made at the moment of the claims reserves calculation and if any exclusions are made, see Appendix A.3, we repeat them one year from now in the calculation (the new diagonal is always taken fully into account).

5. In order to calculate the third moments of the one-year and the ultimate risk distributions, in line with Szatkowski and Delong (2021) and Appendix 1 in England at al. (2019), we assume that the individual development factors $F_{i,j} = \frac{C_{i,j+1}}{C_{i,j}}$ have a lognormal distribution parametrized by the mean and variance (given by the Mack Chain Ladder assumptions).

6. The parameter estimation error is in line with Equation (4.19) from from Buchwalder et al. (2006).

2.3 Example of one-year and ultimate reserve risk simulation

In this section we present an example triangle and results for the realization of one-year and ultimate reserve risk distributions (for $n = 5$). The formulas for the estimators of f_j and σ_j are given by Equations (1) and an example calculation of BE is shown in Figure 1.

Figure 1: Example calculation of BE (dark grey - historical payments, white - expected payments)

Accident year i	Development year j: 1	2	3	4	5
1	310	450	488	511	529
2	353	523	566	595	616
3	386	572	614	644	667
4	426	614	663	696	720
5	466	682	736	773	800

Accident year i	BE_i
1	0
2	21
3	53
4	106
5	334
BE	514

j	1	2	3	4
f_j	1.464	1.080	1.049	1.035
σ_j	0.408	0.134	0.066	0.033

As defined in Section 1 the one-year and the ultimate reserve risk are related to the sufficiency of the current volumes of claims reserve but measure the risk in different time horizons. In the simulation of the one-year risk, we simulate the next year payments and then apply Chain Ladder method to the new triangle - we calculate new development factors $f_1^{ReRes}, \ldots, f_{n-1}^{ReRes}$ and estimate the claims reserve at the end of the next year (known as a re-reserving procedure). In the simulation of the ultimate risk, we simulate all the remaining payments in the triangle. An example simulation and results are presented in Figure 2, where BE_i^{1Y} denotes the reserves that would be set at $t = n$ if we had the information from $t = n + 1$ and BE_i^{Ult} denotes the reserves that would be set at $t = n$ if we had the information after the full run-off of liabilities.

Figure 2: Example simulation of BE^{1Y} and BE^{Ult} (dark grey - historical payments, light grey - simulated payments, white - expected payments)

One-year view

Accident year i	Development year j: 1	2	3	4	5
1	310	450	488	511	529
2	353	523	566	595	620
3	386	572	614	635	660
4	426	614	700	731	759
5	466	720	790	824	856

Accident year i	BE_i^{1Y}
1	0
2	25
3	46
4	145
5	390
BE^{1Y}	606

j	1	2	3	4
f_j^{ReRes}	1.483	1.097	1.044	1.039

Ultimate view

Accident year i	Development year j: 1	2	3	4	5
1	310	450	488	511	529
2	353	523	566	595	620
3	386	572	614	635	670
4	426	614	700	725	752
5	466	720	810	842	881

Accident year i	BE_i^{ULT}
1	0
2	25
3	56
4	138
5	415
BE^{ULT}	634

We calculate the impact on the financial result of the company by calculating the claims development result (CDR) for the presented simulations,

$$CDR^{1Y} = BE - BE^{1Y} = 514 - 606 = -92,$$
$$CDR^{Ult} = BE - BE^{Ult} = 514 - 634 = -120.$$

As the claims development result is a profit function, we may notice that the company suffered a loss in both time horizons, as the claims reserve turned out to be insufficient to cover the future payments. We may also observe a higher loss in the ultimate view than in the one-year view. The simulations of the distributions of BE^{1Y} and BE^{Ult} are used for the estimation of the key characteristics of, respectively, the one-year and the ultimate reserve risk.

3 Choice of actuarial characteristics

In this paper, we consider the following actuarial characteristics:

i) We analyse the structure of the claims triangle by investigating duration and first development factor f_1. Duration, in this context, tells us how fast the claims in a given claims triangle are settled and whether examined line of business has a short tail (fast settlement) or a long tail (slow settlement). First development factor f_1 provides information about the scale of the development in the second year. In line with Guy Carpenter (2014), we could use the percentage of payments made in the first year instead of f_1, however we decided to go with f_1, as in our view it gives more information on the potential volatility of the claims in the triangle (as usually most of the volatility of the reserve risk realises in the second development year and higher f_1 allows for a higher volatility in this development period). For the f_1 value we use Formula (1), while for the duration we use Formula (A1).

ii) We analyse the second and third moments of the ultimate reserve risk distribution with coefficient of variation CoV_{Ult}, skewness coefficient $Skew_{Ult}$, and skewness-to-CoV ratio SC_{Ult}. CoV_{Ult} is a standard parameter, widely used in the industry, providing the standard deviation of the outstanding claims in relation to the claims reserve (expected value of the outstanding claims). Skewness coefficient $Skew_{Ult}$ provides additional information regarding the shape and the tail of the distribution and is also analysed in the actuarial literature on reserve risk. Skewness-to-CoV ratio SC_{Ult} gives a standardised version of skewness coefficient. This ratio was introduced and described in the paper Dal Moro and Krvavych (2017), where the authors divide the distributions depending on their SC value, e.g. $1.5 < SC < 3$ is denoted as "moderately skewed", while $SC > 4$ is described as "extremely skewed". All of these parameters provide us insight regarding the volatility of the outstanding claims.

For the ultimate standard deviation we use the Mack formula (Equation (2.2) in Wüthrich and Merz (2015)), while the ultimate skewness coefficient is calculated using a simulation approach.

iii) We analyse the second and third moments of the one-year reserve risk distribution with coefficient of variation CoV_{1Y}, skewness coefficient $Skew_{1Y}$, and skewness-to-CoV ratio SC_{1Y}. The measures have the same interpretation as above, but this time we measure the risk in the one-year horizon. The value of CoV_{1Y} can be directly compared to the Standard Formula reserve risk standard deviation parameter. For the one-year standard deviation we use the Merz-Wüthrich formula (Equation (2.3) in Wüthrich and Merz (2015)), while the one-year skewness coefficient is calculated using a simulation approach.

iv) We analyse the risk emergence of payments which constitute the ultimate payment by investigating emergence factors and emergence patterns. The emergence pattern is described with a vector $(\alpha_1, \ldots, \alpha_n)$, where each object is known as an emergence factor. The value of α_i describes how much of the reserve risk for a specific accident year emerges within the next calendar year (assuming that the risk is measured with standard deviation). As the emergence patterns focus on accident years, we may also consider an aggregated number referring to the whole portfolio. We denote this number as an aggregated emergence factor α and it describes how much of the whole portfolio's reserve risk emerges within the next calendar year. The aggregated emergence factor α and the emergence pattern $(\alpha_1, \ldots, \alpha_n)$ are calculated as the ratios of the standard deviations of the one-year distribution and the ultimate distribution (see α and α_i in Section 2.3 in Szatkowski and Delong (2021)). We follow the methodology set in Appendix B in Scarth et al. (2020) and we use the "ultimate conditional risk-decay emergence pattern" version. The relation between the one-year and the ultimate risk is usually modelled with an emergence pattern formula, as proposed in Bird and Cairns (2011), England at al. (2012), Scarth et al. (2020). This approach postulates a linear relationship between these two risks. The aggregated emergence factor is also considered in paper Wüthrich and Merz (2015) and is given by Equation (8.3).

v) We analyse the risk margin run-off pattern, which is another characteristic used in practice that provides us with information about the risk emergence. The risk margin at a given date is defined as the discounted sum of future risk capitals multiplied by a cost of capital rate. The risk margin run-off pattern, as presented in Wüthrich and Merz (2015), allows us to project the future risk margins. It gives information on the run-off of the risk margin over the future calendar years for the whole portfolio and at each moment of time it provides information on the current cost of the future risk capitals for the company to hold. The run-off pattern can be interpreted as the percentage of the risk margin being held at specific moments in the future in relation to the risk margin held at the moment of calculation. The run-off is usually calculated according to one of the two patterns: the standard deviation of the one-year risk or the best estimate reserve observed in the future calendar years. The risk margin run-off patterns are calculated in line with the equations presented in paper Wüthrich and Merz (2015). For the the standard deviation based risk measure we follow Equation (7.4) and for the best estimate reserves based risk measure we use Equation (7.5).

In this paper, when we use the symbol $1Y$ we calculate a characteristic of BE^{1Y} and the symbol Ult is used for BE^{Ult}. In relation to the definitions presented in Section 2.1 for one-year risk, and similarly for ultimate risk, we have the following (we omit the subscript i denoting a single accident year):

$$CoV^{1Y} = \frac{Sd[BE^{1Y}]}{\mathbb{E}[BE^{1Y}]},$$
$$Skew^{1Y} = \frac{\mathbb{E}[(BE^{1Y} - \mathbb{E}[BE^{1Y}])^3]}{Sd[BE^{1Y}]^3},$$
$$SC^{1Y} = \frac{Skew^{1Y}}{CoV^{1Y}}.$$

For the risk emergence, we have the following

$$\alpha_i = \frac{Sd[BE_i^{1Y}]}{Sd[BE_i^{Ult}]}, \text{ for } i = 1, 2, \ldots, n, \quad \alpha = \frac{Sd[BE^{1Y}]}{Sd[BE^{Ult}]}.$$

The presented approach can be also described differently using the claims development result. As the value of BE is deterministic and the claims development results is a loss function we have the following equations for one-year risk, and similarly for the ultimate risk,

$$Sd[BE^{1Y}] = Sd[CDR^{1Y}],$$
$$\mathbb{E}[(BE^{1Y} - \mathbb{E}[BE^{1Y}])^3] = \mathbb{E}[(-CDR^{1Y} - \mathbb{E}[-CDR^{1Y}])^3].$$

We would like to note here, that as standard Mack Chain Ladder assumptions are limited to the first and second moments of the distributions only, there is no unique way to estimate the skewness coefficient. The approach used in this paper is in line with Appendix 1 in England at al. (2019), where we receive the third moment by assuming the distribution of the individual development factors $F_{i,j} = \frac{C_{i,j+1}}{C_{i,j}}$. It allows for the simulation of one-year and ultimate risk distributions, therefore leading to an estimation of the value of skewness coefficient also for one-year risk. A different approach is suggested in the work of Dal Moro (2012b), where an assumption for the structure of the skewness coefficient is added to the standard Mack Chain Ladder assumptions and an estimator for this value is proposed (only in the ultimate view), however this approach does not allow to simulate the run-off of the triangle without further assumptions.

4 Technical aspects

The analysis is performed using paid triangles in two market-wide portfolios published by the National Financial Supervisors for: Poland KNF (2020) and and Slovakia NBS (2020). The data is divided into Solvency II lines of business, which are presented in Table A5, and includes both direct insurance business and accepted proportional reinsurance. The data covers accident years 2005–2019, amounting to 15 years of history for each LoB. The adjustments for the considered historical claims triangles are discussed in the Appendix A.3 – our goal is to perform as few adjustments as possible and exclude only the most evident outliers. The data does not include LoB 3 (Workers' Compensation Insurance), as both of the countries do not report figures for this LoB (along with some other European countries, see EIOPA (2020)). The results are also presented in Appendix A.2. The datasets are available in the links provided in the bibliography of the paper: KNF (2020) and and NBS (2020).
The calculations are performed in the R environment. For the estimation of the first and second moment of one-year and ultimate risk distributions, which allow for an analytical calculation, we use the formulas implemented in the *ChainLadder* R package. We use the "MackChainLadder" and "CDR" functions, which calculate: reserves, one-year standard deviation, and ultimate standard deviation - both for single accident years and in an aggregated view. In case exclusions are made (see Appendix A.3), we use the "weights" option in the "MackChainLadder" function. For the estimation of the third moment of one-year and ultimate risk distributions we use algorithms implemented by us in the R environment, which simulate the full triangle run-off (as presented in an example in Section 2.3). The results for the skewness coefficient and the skewness-to-CoV ratio are presented based on simulations. We perform $5 \cdot 10^6$ simulations for each LoB. For the validation purposes, the results for the mean and the standard deviation are compared with the results based on simulations, while the skewness coefficient is compared with the equations presented in Appendix A.1 in Szatkowski and Delong (2021) (these equations are applicable to the case without estimation error, but allow for a partial theoretical validation).
The simulation algorithm implemented in R environment consists of two steps. In the first step, we introduce the parameter uncertainty, following the approach in Equation (4.19) from Buchwalder et al. (2006) - we simulate realizations of the development factors, thus introducing the dependency between accident years. In the second step, we simulate the run-off of the triangle in an iterative fashion - following the assumptions of: conditional expected value, conditional variance, and lognormal distribution, we simulate the value in the next triangle cell $C_{i,j+1}$ conditioned on the value in the previous cell $C_{i,j}$ ($C_{i,j+1}$ is conditional on $C_{i,j}$ only through its first two moments, we fit the lognormal distribution parameters using the method of moments). In the ultimate view, we simulate the full run-off of the triangle, while for the one-year view we take into account only the next diagonal and use a re-reserving algorithm - namely, we re-estimate the reserves conditional on the new information

(in case any exclusions are performed with respect to historical years, we repeat them in re-reserving).

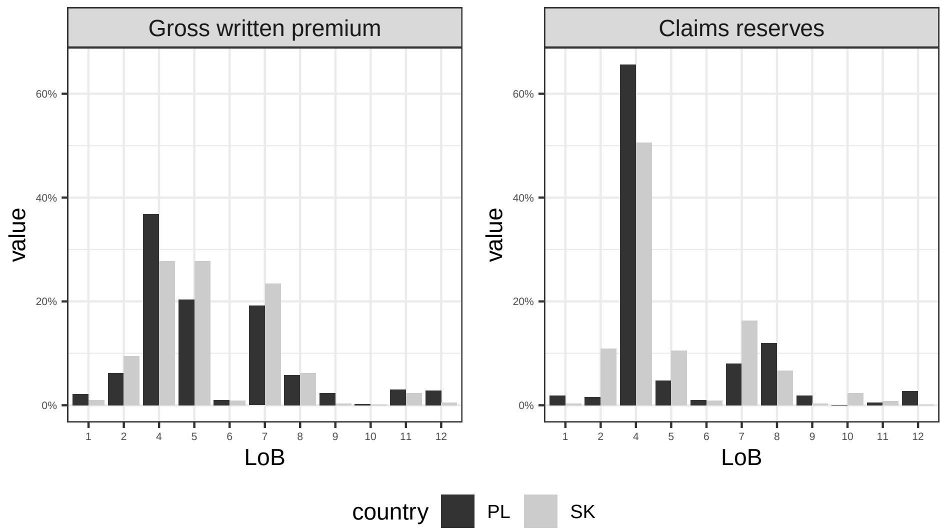
5 Results of the study

In the following subsections, we present an analysis and a comparison of the key characteristics of one-year and ultimate reserve risk distributions for Polish and Slovak market data. When we refer to a range, we present the range with the exclusion of two highest and two lowest values observed in 22 triangles. We would like to note that for some of the LoBs, the observed claim amounts in the triangle are low, for some accident years even less than a million euro (especially for LoBs 10 and 12). However, in order to have a complete study, we present the results for all LoBs. Due to some large values which make the charts illegible, we decide not to plot the results of CoV_{Ult} (142.8%), $Skew_{Ult}$ (1.50), CoV_{1Y} (131.5%), $Skew_{1Y}$ (1.57) for LoB 9 for Slovakia, which are interpreted as outliers. These high values are caused by the low claims volume and the feature of this LoB, as it is characterised by high salvage and subrogation volumes, which usually decrease the expected value of the outstanding claims and increase their volatility, leading to high CoV and $Skew$. We plot other characteristics for LoB 9 for Slovakia on the charts (duration, f_1, SC_{Ult}, SC_{1Y}) and take into account the results for CoV and $Skew$ in every analysis. Additional results are presented in Appendix A.2. We would like to also note here that the choice of Mack Chain Ladder model has an impact on the received benchmarks, other models and assumptions could lead to different results.

5.1 Market comparison

We start with the insurance market comparison between Poland and Slovakia. Based on the data from EIOPA (2020), we may assess that the market structure of the gross written premium for Poland and Slovakia is quite similar (see Figure 3). The majority of the gross written premium (about 56%) for both markets is made up by the motor business (LoBs 4 and 5) with a slightly different allocation - for Slovakia the share of the gross written premium is the same for LoB 4 (Motor Vehicle Liability Insurance) and LoB 5 (Other Motor Insurance) - about 28%, while for Poland, LoB 4 makes up 37% and LoB 5 20% of the total volume. Regarding other LoBs, the structure is similar, with slightly more business in Slovakia in LoB 2 (Income Protection Insurance) and LoB 7 (Fire and Other Damage to Property Insurance), whereas LoB 9 (Credit and Suretyship Insurance) and LoB 12 (Miscellaneous Financial Loss) have higher shares in Poland. The share of the gross written premium is similar for both markets for LoB 8 (General Liability Insurance) - about 6% each and LoB 11 (Assistance) - about 3% each. Other LoBs: LoB 1 (Medical Expense Insurance), LoB 6 (Marine, Aviation and Transport Insurance), and LoB 10 (Legal Expenses Insurance) are low in volume and make up about 3% of the gross written premium for each market. Regarding the difference in the scale of the markets, assessing it by the gross written premium, the Polish market is about 10 times bigger than the Slovak market.

Figure 3: The market share for the gross written premium and the claims reserves

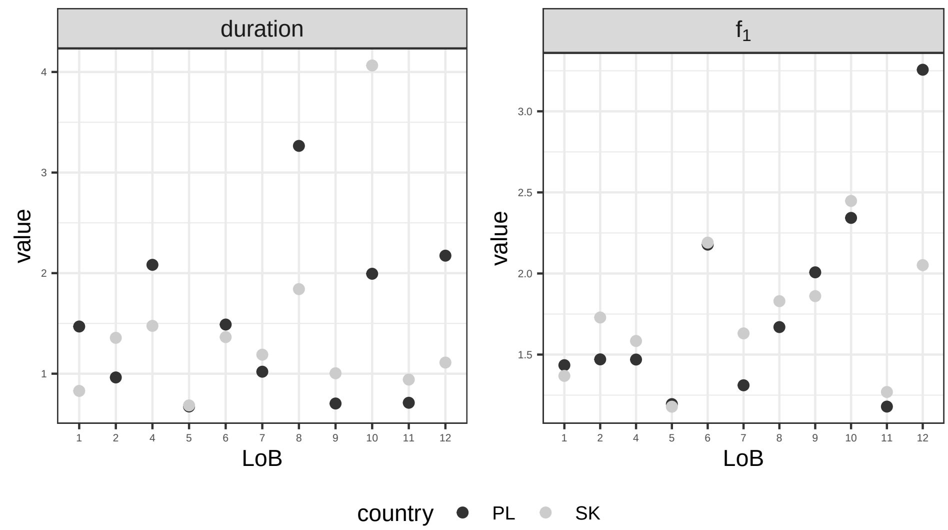


We also compare the claims reserves calculated by us based on the examined triangles - presented in Figure 3. We would like to note here that we present the results of a pure Chain Ladder algorithm with single exclusions of extreme development factors, and this approach may not reflect the current market value of the claims reserves. The aim of the analysis is to perform a general comparison of the reserves structure. The conclusions concerning the reserves structure and the difference in the scale of the markets are similar as for the gross written premium, with the main difference being that long-tailed LoBs increase in their share (LoBs 4 and 8), while short-tailed LoBs decrease in their share (LoBs 5 and 7) for the claims reserves compared to the gross written premium.

5.2 Duration and first development factor f_1

In this subsection, we present the analysis of duration and first development factor f_1. The formula for the calculation of duration is (A1) and for the estimate of f_1 is (1). The comparison of the characteristics is presented in Figure 4.
Regarding the duration and the first development factor f_1, values presented for both countries show similar behaviour. We may notice that most of the durations

Figure 4: Results for duration and f_1

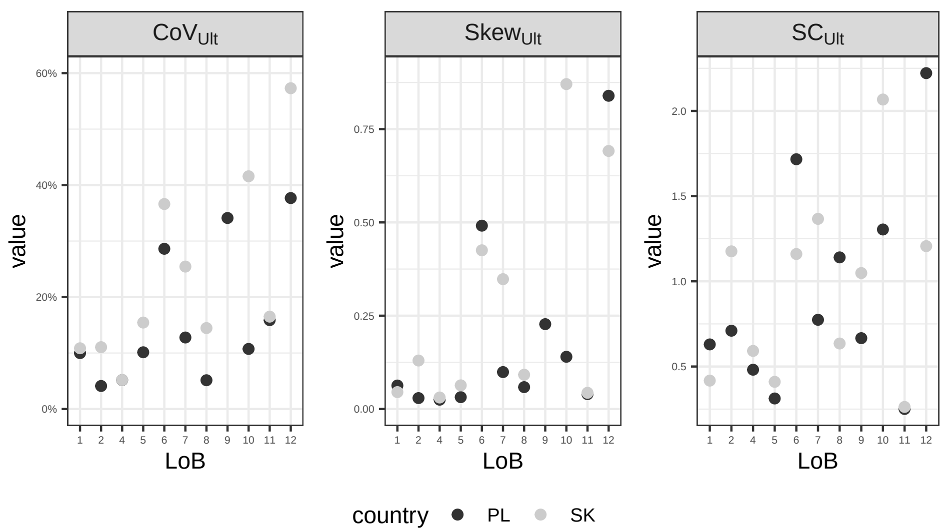


lie in the range 0.70 to 2.17, while most f_1 fall in the range 1.19 to 2.34. For both countries, LoBs 4 and 8 have some of the highest duration values. This is in line with expectations, as both of LoBs are comprised of liability products which tend to have longer tails (see results in EIOPA (2011) or Table 2 in Dal Moro and Krvavych (2017)). We may note however, that the duration for these two LoBs is higher for Poland than Slovakia, and f_1 is lower for Poland. Comparing the products of the development factors for LoB 4, we observe similar values (1.94 for Poland and 1.93 for Slovakia), which might point to the conclusion that in both countries the tail is similar, with claims being handled quicker in Slovakia. For LoB 8, the difference between the product of the development factors is higher (3.04 for Poland and 2.35 for Slovakia), which could be caused by e.g. a different product structure inside this LoB, as general liability insurance is comprised of many different insurance types. For both countries, we observe high values of the duration and f_1 for LoB 10. Additionally, also in line with expectations, LoBs 5 and 11 have lower values of the duration, as claims in these LoBs tend to be handled quickly (this is also seen in very similar f_1 close to 1.2, pointing to a high share of claims handled in the first development year, i.e. within their accident year). Regarding the differences between the characteristics observed for two markets, they are noticeable for LoB 7 and LoB 12. The Polish data demonstrates higher duration, f_1 and market share than the Slovak data for LoB 12, while the reverse takes place for LoB 7. This could be caused by a potential different classification of a volatile and prone to revaluations business interruption portfolio, which is dependent on the terms of policies (see EIOPA (2019)), and the fact that due to its varied nature LoB 12 itself could have a significantly different structure. LoB 6 has a similar between countries duration (1.49) and f_1 (2.18), which is in line with expectations as this line of business is also prone to high revaluations (especially for large claims). Notably, we do not observe very high durations (above 5) or very high f_1 (above 4) in the data, as in Bird and Cairns (2011) or Guy Carpenter (2014). However, in the mentioned papers, high values of the characteristics are observed for specific liability business (e.g. medical professional liability or product liability), which are included here in LoB 8 together with more typical, short-tailed business (see also comments in Section 8.1 of Wüthrich and Merz (2015)).

5.3 Second and third moments of the ultimate distribution

In this subsection, we present the analysis of coefficient of variation CoV_{Ult}, skewness coefficient $Skew_{Ult}$, and skewness-to-CoV ratio SC_{Ult} of the distribution of the ultimate reserve risk. The comparison is presented in Figure 5.

Figure 5: Results for CoV_{Ult}, $Skew_{Ult}$, and SC_{Ult}



Regarding CoV_{Ult}, the estimated values for both countries present a similar behaviour. We may notice that most of the values lie in the range 5.2% to 41.6%, which is in line with Dal Moro and Krvavych (2017). LoBs 6, 9 and 12 are LoBs with the highest CoV_{Ult}, which is in accordance with expectations, as these LoBs are highly volatile (see e.g. EIOPA (2011)). We may also see that the property insurance (LoB 7) tends to have a higher CoV_{Ult} than the motor insurance (LoBs 4 and 5), which is also consistent with Dal Moro and Krvavych (2017) and may point to a low share of bodily injury claims in LoB 4. The results for LoB 4 are very close for both countries (CoV_{Ult} is about 5.2%) and are similar to the results from Wüthrich and Merz (2015), where CoV_{Ult} close to 4% was estimated for similar portfolios. Additionally, the low volatility for LoB 8 might be surprising, however the most volatile LoBs tend to be specific products of liability insurance, while here they are mixed with other business, and the estimated values (Poland: 5.1%, Slovakia: 14.5%) are similar to the range of results for liability from England at al. (2012). This topic is further discussed in Section 5.4.

Regarding $Skew_{Ult}$ and SC_{Ult}, we focus on the observed ranges of the estimates, as the relations between different characteristics are explained in more detail in Section 5.6. As far as $Skew_{Ult}$ is concerned, we observe the highest values for LoBs 6, 9 and 12, similarly as for CoV_{Ult}. Some LoBs have values of $Skew_{Ult}$ close to 0 for both countries (LoBs 1, 4, 5, 11), which points to a symmetric risk distribution. We could expect that this result implies low volatility of the ultimate reserve risk (possibly a shape of the distribution close to a normal distribution, see also comment in Appendix 1 in England at al. (2019)). Additionally, the assessment of skewness (either by investigating $Skew_{Ult}$ or SC_{Ult}) gives us additional information on the risk - e.g. LoBs 4 and 8 for Poland have the same CoV_{Ult}, but LoB 8 has a slightly higher $Skew_{Ult}$ pointing to a more skewed distribution in this LoB (in line with Annex 1 in EIOPA (2011)). The range observed for $Skew_{Ult}$ is 0.03 to 0.84 and is similar to the range observed in Dal Moro (2012a). The SC_{Ult} coefficient, on the other hand, varies less between different LoBs and its typical range is 0.31 to 1.72. This range is slightly lower than the conclusions presented in Dal Moro and Krvavych (2017). In general, as mentioned in Appendix 1 in England at al. (2019), we would usually not expect significant skewness.

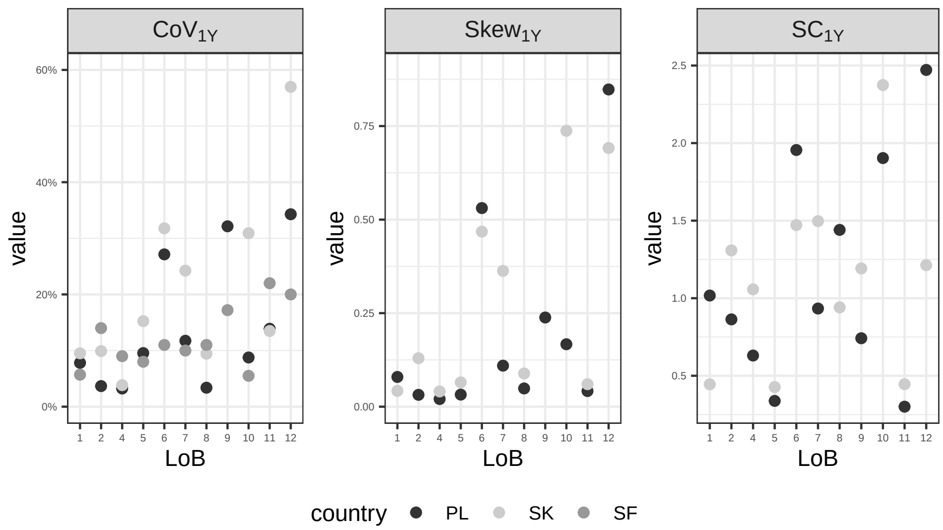
We would like to stress that our estimates of $Skew_{Ult}$ and SC_{Ult} are based on the assumption of lognormal distribution of individual development factors, which is added to the classical Mack Chain Ladder model assumptions.

5.4 Second and third moments of the one-year distribution

In this subsection, we present the analysis of one-year coefficient of variation CoV_{1Y}, skewness coefficient $Skew_{1Y}$, and skewness-to-CoV ratio SC_{1Y} of the distribution of the one-year risk. The comparison is presented in Figure 6 together with the Standard Formula (SF) reserve risk standard deviations.

Regarding CoV_{1Y}, we focus on the comparison of the results between the LoBs, the countries and the Standard Formula (SF) reserve risk standard deviations. A comparison of different characteristics is also performed in Section 5.6. We may notice that in the case of Polish and Slovak market data, most of the standard deviations are either lower or comparable to the Standard Formula standard deviations. For the Polish data, the results for the volatile LoBs 6, 9 and 12 are the only which are significantly higher than the SF parameters. Similar situation occurs for the Slovak

Figure 6: Results for CoV_{1Y}, $Skew_{1Y}$, and SC_{1Y}.



data, but here we also observe significant differences for LoBs 5 and 7. For LoB 5, there are many low revaluations in later development years, which do not impact the expected value of reserves significantly (as the development factors are close to 1), but increase the standard deviation (see also Section 5.8). For LoB 7, we refer to the comment presented in Section 5.2. Regarding LoB 4 with the highest market share (as presented in Section 3), the results for both markets are significantly lower than the SF standard deviation (Poland: 3.2% and Slovakia: 3.8% vs the Standard Formula: 9.0%) and are similar to the ones presented in ISVAP (2006) and Wüthrich, Merz (2015).

We recognize that the standard deviations for Slovakia are always higher than the results for Poland. This is consistent with the observations made during the parametrization of the Standard Formula reserve risk standard deviations - it was noticed that there exists a diversification effect in reserve risk, e.g. higher volume portfolios tend to have lower coefficients of variation (see CEIOPS (2010) or EIOPA (2011)). During the Standard Formula parameters calibration, the parametrization of the reserve risk standard deviations included a special adjustment in order to achieve the parameters valid for an average-size entity ("median portfolio size at market level" as stated in EIOPA (2011)). This fact also explains why most of the parameters obtained for Polish and Slovak market are lower than the SF standard deviations (as these two markets should be bigger in size in comparison to an average European size entity). This topic was also noticed and commented in Chan and Ramyar (2016) - the authors propose in this work "a few possible ways to adjust the

CoV for diversification effect" (Section 3.2.5 and Appendix B). Another justification for higher parameters in the Standard Formula is that, as it was noted in CEIOPS (2010), the calculation of the standard deviation parameters was performed using a mix of Merz-Wüthrich formula and Premium Risk like methods - and the pure Merz-Wüthrich formula usually provided lower results.
We quantify the impact of the differences in the reserve risk standard deviations parameters based on the market data and the Standard Formula taking into account the market claims reserves structure for Poland and Slovakia, in order to have a higher impact of the LoBs which are more important for the market, as presented in Section 5.1. This comparison can be also interpreted as a calculation of a company reserve risk standard deviation in a situation where the structure of the claims reserve of the company is the same as of the market. We follow the steps:

1. For each LoB we set the percentage of the share in the total claims reserves, presented in Figure 3, as the basis for the calculation.

2. We take the CoV_{1Y} estimated for the markets and the Standard Formula parameters as presented in Figure 6 and multiply them by the percentage from step 1. We receive a capital requirement for each LoB.

3. We hierarchically aggregate the results using the Standard Formula square root formula - firstly on the LoB level and then on the risk category level, aggregating Health Underwriting Risk (LoBs 1 and 2) with Non-Life Underwriting Risk (LoBs 4 - 12).

The results are presented in Table 1. The numbers presented there can be understood as a market reserve risk standard deviation, being a percentage of the current claims reserve. The column "Estimated parameters" denotes the situation where we use the parameters estimated for a specific country based on the market data, while the column "SF parameters" denotes the Standard Formula reserve risk standard deviations.

Table 1: Results for a market reserve risk standard deviation

	Estimated parameters	SF parameters
Poland	4.2%	7.7%
Slovakia	6.4%	6.5%

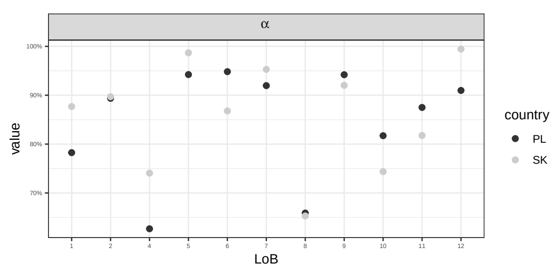
We note that the result for Polish market is significantly different if the parameters are taken based on the data (4.2%) or straight from the SF (7.7%). This is due to the fact that the Polish market consists mainly of LoB 4 (66% share) and LoB 8 (12% share) and both of these LoBs have significantly lower standard deviations based on the data (LoB 4: 3.2% (PL) vs 9.0% (SF), LoB 8: 3.4% (PL) vs 11.0% (SF)). For the Slovak market, the estimated market reserve risk standard deviation based on the

data is similar to that based on the Standard Formula standard deviations. Despite the fact that we have a decrease in standard deviation from 9.0% (SF) to 3.8% (SK) for the main part of the portfolio (LoB 4 - 51% share), we have a significant increase from 10.0% (SF) to 24.2% (SK) for LoB 7 (16% share) and from 8.0% (SF) to 15.2% (SK) for LoB 5 (11% share), which lead us to similar market reserve risk standard deviations (SK: 6.4% vs SF: 6.5%).
Summarizing, we may come to the conclusion that the parameters used in the Standard Formula calculation are conservative in comparison to these based on the market data for Poland and Slovakia.
Regarding $Skew_{1Y}$ and SC_{1Y} values, the conclusions are similar as for the ultimate values, discussed in Section 5.3. Most of the presented values lie in the range 0.03 to 0.74 for $Skew_{1Y}$ and 0.43 to 1.96 for SC_{1Y}. The ranges for $Skew_{1Y}$ are narrower, while for SC_{1Y} are wider than the ultimate ones - it is discussed in more detail in Section 5.7.

5.5 Aggregated emergence factor α

We present the analysis of the aggregated emergence factor α. The comparison is presented in Figure 7. As described in Section 3, the emergence factors can be calculated either for every accident year (presented in Section 5.8) or in an aggregated way as one number for all accident years. In this section, we follow the latter approach. The value of the aggregated emergence factor α is important from practical point of view as it provides the relation of the standard deviations for the one-year and the ultimate risk, and can serve as an approximation of the relation of the risk capitals for the one-year and the ultimate risk.

Figure 7: Results for the aggregated emergence factor α.



Once again, we recognize that the results are similar between the countries and, based on them, we may divide the observed LoBs into three groups:

i) Short risk emergence - LoBs: 5, 7, 9, 12; $\alpha \geq 90\%$ for both countries.

ii) Medium risk emergence - LoBs: 1, 2, 6, 10, 11; $90\% > \alpha \geq 75\%$ for at least one of the countries.

iii) Long risk emergence - LoBs: 4, 8; $\alpha < 75\%$ for both countries.

These figures denote how much of the remaining reserve risk, measured with the standard deviation, is expected to emerge in the next calendar year. If the ratio is close to 100%, then almost all of the volatility of the reserve risk realizes in the next year, while for ratios closer to 0% on the contrary - there is very low volatility in the reserve risk in the next year and the volatility emerges later.
Our classification based on estimation for the market is generally in accordance with Annex 1 in EIOPA (2011), with the exception of LoB 1 and 11 where we would expect a short risk emergence. For LoB 1, we have $\alpha \leq 90\%$ for both Poland (78.3%) and Slovakia (87.7%), while for LoB 11 we observe some historical significant development factors in development years 2 and 3, leading to 87.5% for Poland and 81.8% for Slovakia.
The overall range of observed α parameters is 65.9% to 95.3%, consistent with the observations in Wüthrich and Merz (2015). In the mentioned paper some lower values (closer to 50%) were observed. However, this only took place in some dedicated liability portfolios.

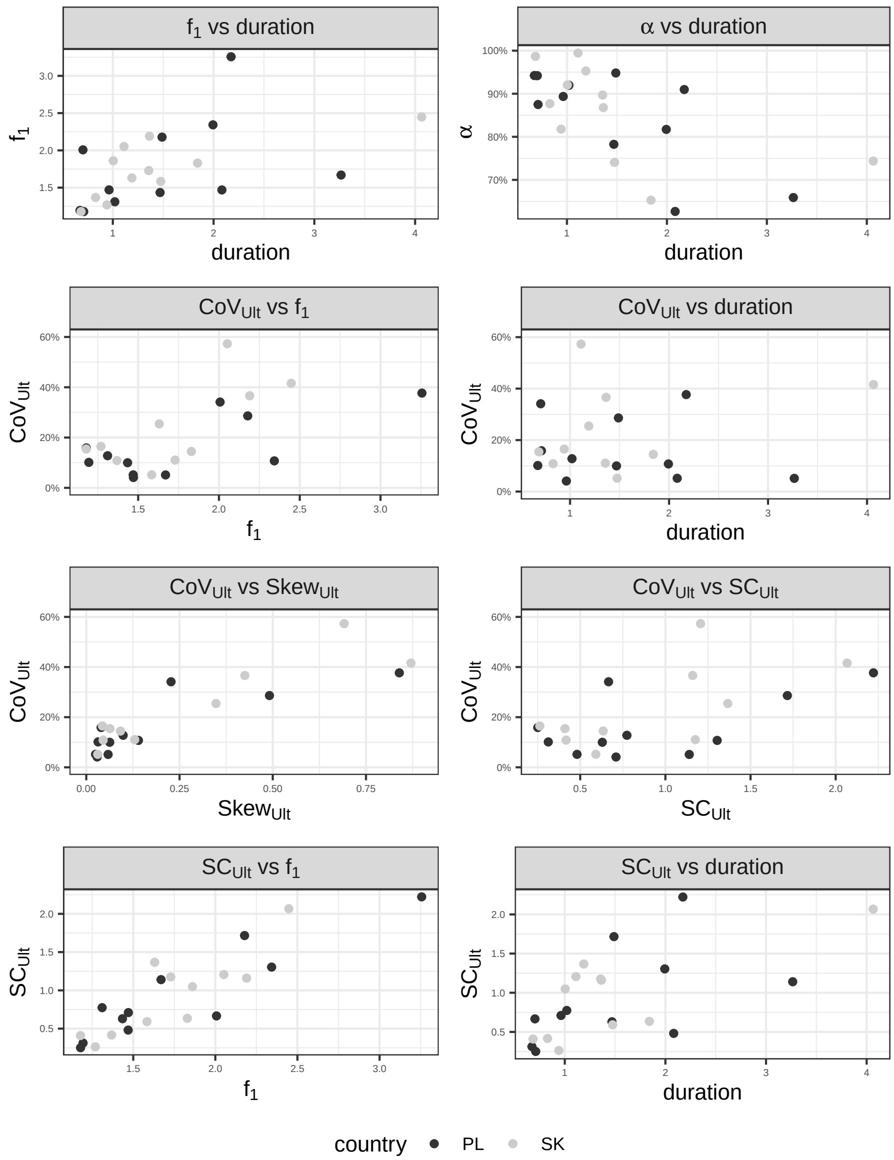
5.6 Relations between reserve risk characteristics

Based on the results presented in Section 5, we can make conclusions regarding relations between the considered reserve risk characteristics. These conclusions are based on the graphical assessment of the behaviour and the Spearman's rank correlation coefficient of the characteristics. The characteristics are plotted in Figure 8 and the values of the correlation coefficients are presented in Table A4. We focus here on the characteristics of the ultimate reserve risk distribution. The conclusions for the one-year distribution are the same. The relations between one-year and ultimate characteristics are discussed in Section 5.7.

1. The duration is positively correlated with the first development factor f_1 and negatively correlated with the emergence factor α. This observation is natural as f_1 is included in the calculation of duration, see Equation (A1), and α may be interpreted as a form of a tail measure. The conclusion regarding duration and f_1 is also confirmed in Guy Carpenter (2014).

2. CoV_{Ult} is positively correlated with f_1, although not with the duration. This is caused by the fact that we observe high CoV_{Ult} due to high risk realizing in early development years only (and therefore high f_1 value and medium duration - e.g. LoB 6). The conclusion regarding CoV_{Ult} and duration is also confirmed in England at al. (2012).

Figure 8: Relations between characteristics of the ultimate distribution



3. $Skew_{Ult}$ is positively correlated with CoV_{Ult}. We also observe a lower, but still positive, correlation between SC_{Ult} and CoV_{Ult}. This is confirmed by the analysis of results from Dal Moro (2012a). This conclusion also provides an additional advantage for using SC_{Ult} - we observe that due to a lower correlation between SC_{Ult} and CoV_{Ult} than between $Skew_{Ult}$ and CoV_{Ult}, SC_{Ult} gives us additional insight into the risk assessment without repeating the information already taken into account in CoV_{Ult}. This is especially visible for LoB 9 in Slovakia, which has the highest CoV_{Ult} and $Skew_{Ult}$ among all considered LoBs, but medium SC_{Ult}.
4. The duration and the first development factor f_1 are positively correlated with SC_{Ult}. This could point to the fact that a long tail and a higher development in the second year have a higher impact on the skewness than on the coefficient of variation resulting in higher skewness-to-cov ratio.

The above observations may serve as the basis for plausibility checks during the internal model results validation and claims reserve calculation. Additionally, they also provide us with the information on the characteristics of the most common claims triangles encountered in practice.

5.7 Relations between one-year and ultimate characteristics

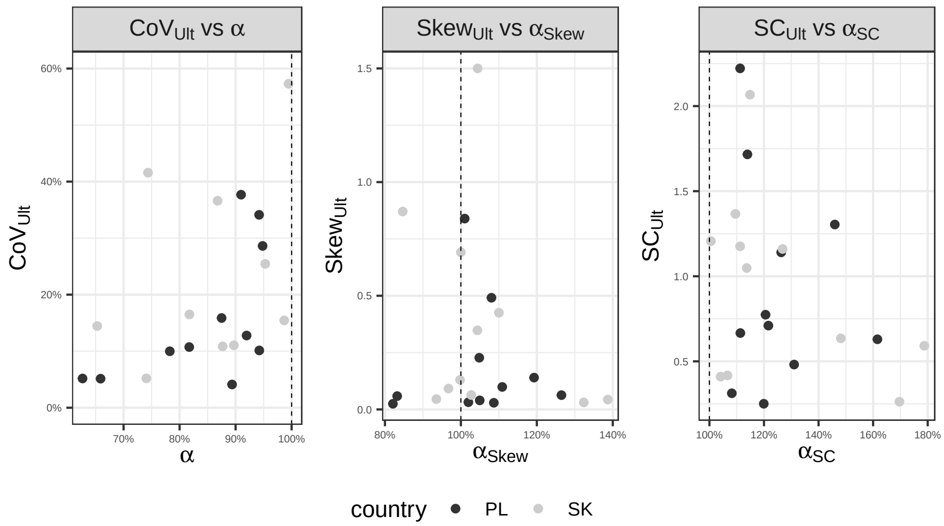
In this subsection we investigate relations between the one-year and the ultimate characteristics. We focus on the second and third moment of the distributions by investigating CoV, $Skew$, SC. The correlation between the one-year and the ultimate characteristic is naturally very high, with Spearman's rank correlation coefficients close to 100% as presented in Table A4. Therefore, we perform this comparison by analysing the correlation between the ultimate characteristic and ratio of the one-year to the ultimate characteristic, where we consider the ratios:

$$\alpha = \frac{CoV_{1Y}}{CoV_{Ult}}, \quad \alpha_{Skew} = \frac{Skew_{1Y}}{Skew_{Ult}}, \quad \alpha_{SC} = \frac{SC_{1Y}}{SC_{Ult}}.$$

The value of α denotes the aggregated emergence factor, described in Section 5.5, and due to this fact we do not use the lower subscript CoV. The results are presented in Figure 9 and the values of Spearman's rank correlation coefficients are presented in Table A4. We can observe three different types of behaviour.

1. For all α, we have that $\alpha < 100\%$, which means that $CoV_{1Y} < CoV_{Ult}$ and when we measure the risk with standard deviation, the one-year risk is lower than the ultimate risk. This is in line with the common belief among actuaries, see e.g. AISAM-ACME (2007). Additionally, α is positively correlated with CoV_{Ult} - the higher the α, the higher the CoV_{Ult}. This is caused by the fact that we observe volatile claims developments due to high risk emerging only in the early years. Most of the estimated values of α lie in the range 65.9% to 95.3%.

Figure 9: Relations between the one-year and the ultimate view for CoV, $Skew$, and SC



2. For α_{Skew}, we do not observe a consistent relation between the one-year and the ultimate risk - for some of the values (7 out of 22) we have that $Skew_{1Y} < Skew_{Ult}$, while for the others (15 out of 22) we have that $Skew_{1Y} > Skew_{Ult}$. Additionally, we note a low negative dependence between α_{Skew} and $Skew_{Ult}$. Most of the estimated α_{Skew} lie in the range 84.7% to 126.5% and we do not obtain any negative values (which is possible for skewness coefficient).

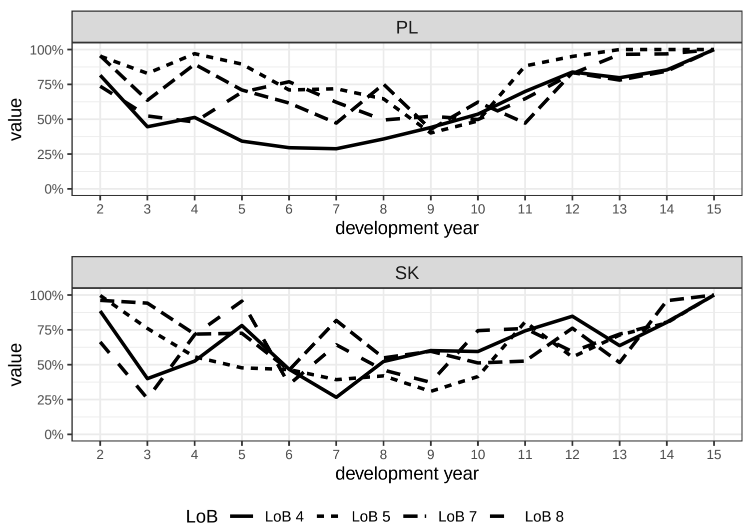
3. For all α_{SC}, we have that $\alpha_{SC} > 100\%$, which means that $SC_{1Y} > SC_{Ult}$. This in turn means that $\frac{Skew_{1Y}}{Skew_{Ult}} > \frac{CoV_{1Y}}{CoV_{Ult}}$, so the reduction in the risk due to switching from the one-year view to the ultimate view is higher for CoV than for $Skew$ (for $Skew$ it is not even always a reduction, as noted above). This is a different result than for CoV-based α values and it is in line with the observations made in Szatkowski and Delong (2021). Additionally, we note a low negative dependence between α_{SC} and $Skew_{SC}$. Most of the estimated α_{SC} lie in the range 106.6% to 161.6%.

Based on the above observations, we note that even though the standard deviation, and therefore CoV, of the distribution increases if we consider the ultimate view of the claims, the skewness measured by SC decreases. This could be understood that the extension of the time horizon in which we measure the risk, increases deviations regarding the claims payments, although a longer time horizon also has a stabilizing effect on the skewness relative to the coefficient of variation. We would like to point out here that the third moment is estimated with an additional assumption that the individual development factors have a lognormal distribution - a different assumption could lead to other results.

5.8 Emergence pattern $(\alpha_1, \ldots, \alpha_n)$

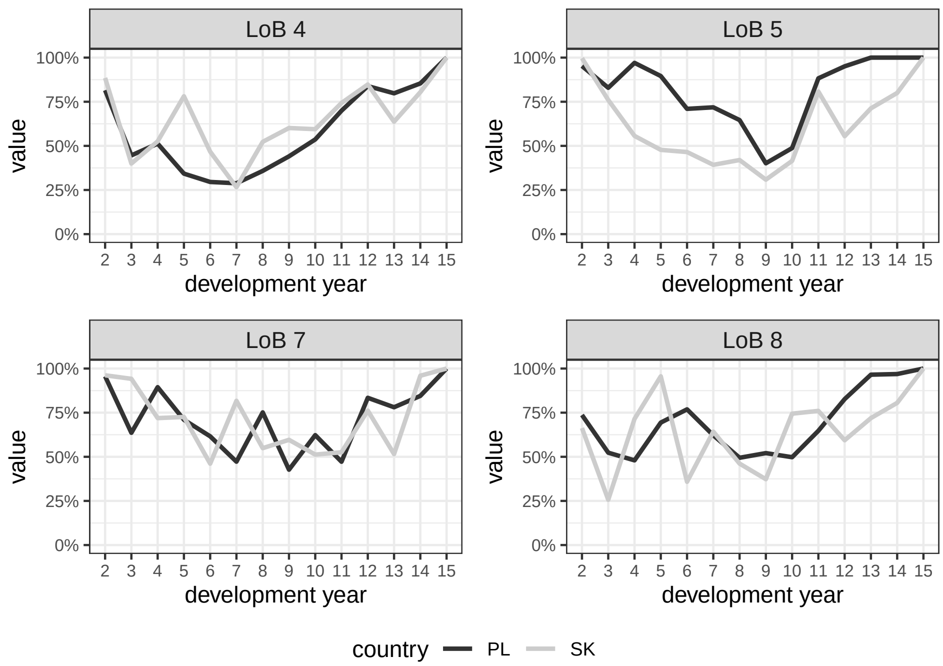
In this subsection we present the analysis of emergence patterns $(\alpha_1, \ldots, \alpha_n)$. The comparison between LoBs is presented in Figure 10, while the comparison between the markets is shown in Figure 11. We follow the approach presented in England at al. (2012), Scarth et al. (2020) and we use the unscaled approach, as we focus here on the results from the accident year perspective. In this part of the paper, we focus on the four most important LoBs for both countries, both from the gross written premium and claims reserves perspectives (see Section 3), namely LoBs: 4, 5, 7, 8 (for Poland 82% and for Slovakia 85% of the total gross written premium).

Figure 10: Emergence patterns for key LoBs for each country



Figures 10 and 11 present emergence patterns, which contain the ratios of the standard deviations of the one-year and the ultimate risk distributions for each accident year. We may identify the accident year with the development years uniquely. The latest accident year is in development year 2 and therefore it is the starting point on the left hand side of the chart. The last value presented is for the oldest accident year and therefore for the latest development year (for development year 15 we assume 100%

Figure 11: Emergence patterns for the countries inside one LoB



as we do not consider further run-off of the triangle). Summarizing, starting from the left hand side, the charts present the ratios of the remaining standard deviation that will emerge in the next year starting from the latest accident year (and therefore the earliest development year).

Analysing the results in Figure 11, we observe that for early development years, the emergence patterns for long-tailed LoBs 4 and 8 tend to be below the emergence patterns for short-tailed LoBs 5 and 7, which is in line with expectations. The behaviour might be different for later development years, however in this case it is caused mainly by the volatility of the estimations. Secondly, from the perspective of the whole portfolio, the emergence factor for the second development year is usually the most important, as the highest share of the risk usually emerges for the latest accident year. This is especially visible for LoBs 5 and 7 in Poland, where the emergence pattern for the latest accident year is similar to the emergence pattern for the whole portfolio (95.3% for the last accident year vs 94.2% for the whole portfolio for LoB 5 and, respectively, 95.6% vs 92.0% for LoB 7). This is slightly different for long-tailed LoBs for the Polish portfolio: 81.5% for the last accident year vs 62.7% for the whole portfolio for LoB 4 and, respectively, 73.7% vs 65.9% for LoB 8. We may also notice a similar ordering between the countries by looking only at the first emergence factor - the highest values are observed for LoBs 5 and 7, then we have LoB 4 with the lowest value for LoB 8 (we have a similar ordering for the aggregated emergence factor α, see Section 5.5). Thirdly, we may notice that generally the emergence patterns are not very smooth and do not present very clear relations, which is also a conclusion in England at al. (2012). However, they provide us with valuable information on the behaviour of the emergence of risk for a specific line of business. We observe more smooth relations for Polish than for Slovak emergence patterns, as Polish market has a higher volumes of claims. Additionally, the smoothest curves are observed for LoB 4, which has the highest share of the claims reserves for both countries. Fourthly, we observe non-monotonic behaviour of the emergence patterns, which is also in line with observations in England at al. (2012).

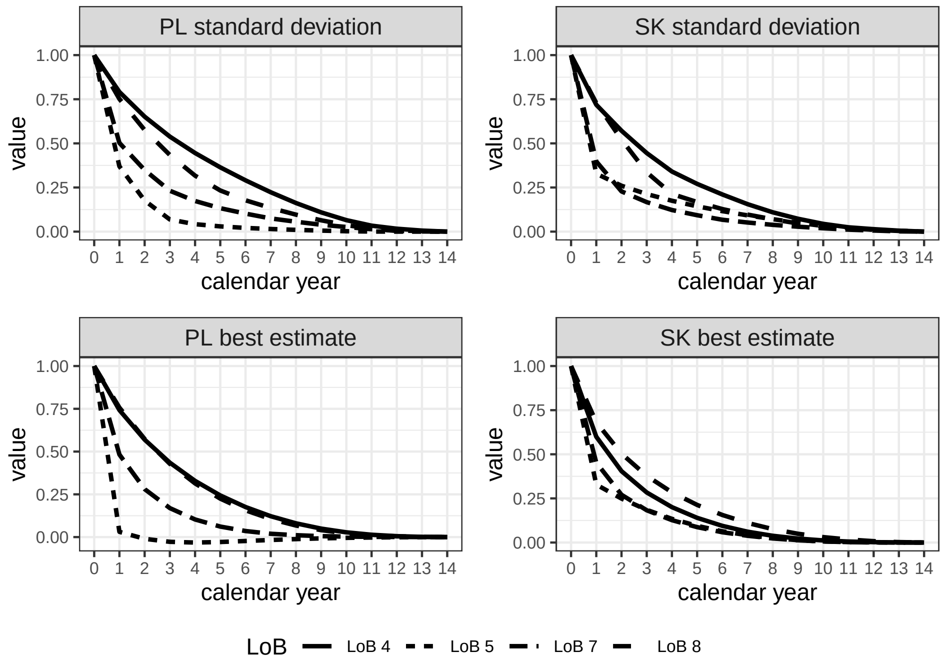
Analysing the results from the LoB perspective, we may notice that the results for LoBs 4, 7, and 8 look similar for both countries. For LoB 4 and LoB 8 we observe a sharp decrease in first development years with a slower increase in later years, which usually means a fast settlement of small, simple claims in first development years followed by a slow development of larger and more involved claims or claims reported with a delay e.g. bodily injuries (see comment in Section 8.1 in Wüthrich and Merz (2015)). For LoB 7, the results are symmetrical with respect to the development years and create a "smile curve" with a non-monotonic behaviour, similarly as noted in Scarth et al. (2020). We can also see that the decrease and the later increase in the curve is less steep for LoB 7 (minimum value 42.7%) than for LoB 4 (minimum value 26.6%), which points to shorter tails of LoB 7 and faster claim reporting, as well as settlement. For LoB 5, there is more risk emerging, in Slovak than in Polish data, in later development years. It is due to many small revaluations late in the triangle and also due to salvage and subrogation as some individual development factors are lower than 1. Once again, we observe that the curves are not smooth and are similar in their behaviour to the ones presented in England at al. (2012) and Scarth et al. (2020). However, we are able to compare the results between the countries and come to similar conclusions on the emergence of risk for most of the presented LoBs.

5.9 Risk margin run-off patterns

In this subsection, we present an analysis of the risk margin run-off patterns. The calculation of the risk margin run-off pattern is performed in line with paper Wüthrich and Merz (2015) and we consider two methods: based on standard deviations of future one-year risks and based on future best estimate reserves. The comparison for each market is presented in Figure 12 and between LoBs in Figure 13. Similarly as for the emergence patterns, we focus on the results for the four most important LoBs for each country, namely LoBs: 4, 5, 7, 8.

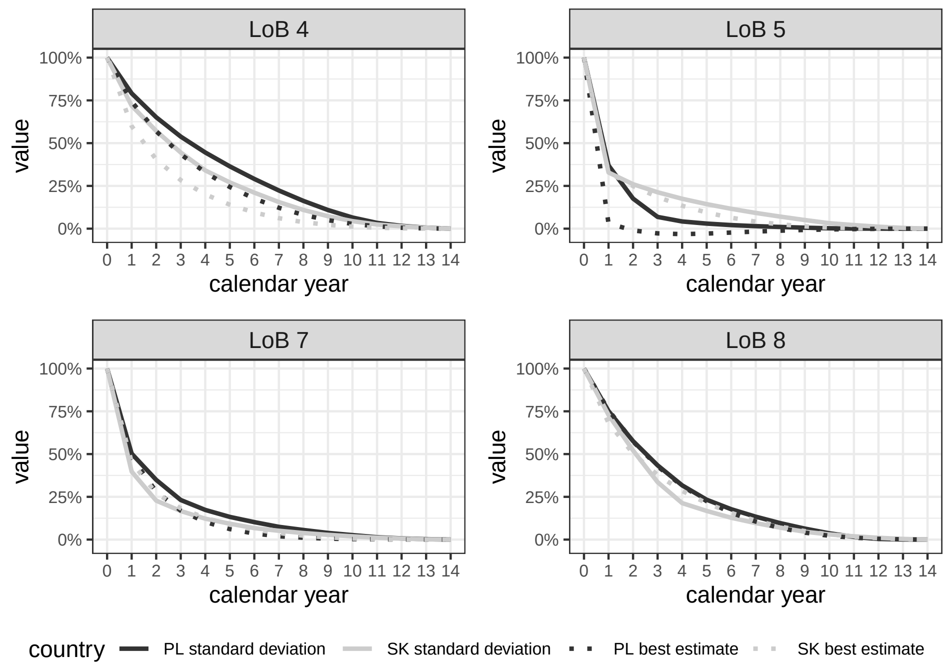
The plots present the run-offs of the risk margin, which is defined as the discounted sum of future risk capitals multiplied by a cost of capital rate. They present how high the risk margin in the future calendar years will be in relation to the current value. We start with the value of 100% for the calendar year 0, understood as the current

Figure 12: Risk margin run-off patterns for key LoBs for each country



date of the risk margin calculation, and forecast the future risk margins in the next 14 calendar years to the complete run-off of the insurance liability.
The results are in line with expectations and earlier observations - a slower risk run-off is observed for long-tailed LoBs 4 and 8, while a faster run-off is observed for short-tailed LoBs 5 and 7. A different observation, compared with emergence patterns (Figure 11) and aggregated emergence factors (Figure 7), is that the risk run-off based on standard deviations of future one-year risks for LoB 4 is always slower than for LoB 8, while for the emergence patterns for some of the development years, especially the early ones, we observe a faster risk emergence for LoB 4. This is due to the fact that the emergence patterns take into account only the emergence of risk in the next year for a specific accident year, while the risk margin run-off patterns take into account the whole lifetime of the liability and all accident years. As the risk margin for Solvency II Best Estimate calculation is the discounted sum of future risk capitals multiplied by a cost of capital rate, a slow emergence of the risk results in the risk capital being held for a longer period of time. Let us also note that our observations do not mean that the risk margin for LoB 4 should be always higher than for LoB 8. The key component in the risk margin is the volume of the risk capital itself at the date of valuation, and we focus here only on the run-off of

Figure 13: Risk margin run-off patterns for the countries inside one LoB



the risk margin. Additionally, and also differently than in Section 5.8, the run-off patterns are quite smooth and lead to clear conclusions - for most of the presented cases we see a clear ordering with LoB 4 having the slowest run-off, followed by LoB 8 and LoB 7 (in that order) and, finally, LoB 5 having the fastest run-off. Analysing the results from LoB perspective, we note that the results for LoBs 4, 7 and 8 look similar for both countries. In most of the presented cases, the risk run-off is slower for the Polish market data than for the Slovak market data, which once again is a different conclusion than in the case of emergence patterns, and could point to a lower amount of claims being handled late in the tail for these LoBs in Slovakia.
We can also choose different drivers for the run-off of risk margin - we choose standard deviations of the future one-year risks or best estimate reserves in the future years, and the choice impacts the results. This fact has already been observed in the papers Gisler (2019) and Wüthrich and Merz (2015). Similarly as in the mentioned papers, we observe that the run-off pattern based on the best estimate reserves is usually faster than the run-off pattern based on the standard deviations.

6 Conclusions

In this paper we estimate key actuarial characteristics of the one-year and the ultimate reserve risk distributions for Polish and Slovak market data. Our results provide benchmarks and ranges for the parameters of the distributions to be observed in practice. The one-year CoV_{1Y} parameters are also compared to the Standard Formula reserve risk standard deviations. The results generally show that the Standard Formula reserve risk standard deviations are conservative in comparison to the parameters estimated from the market data. We also analyse and interpret the relations between examined characteristics. Finally, we present and assess the emergence patterns and risk margin run-off patterns. Once again, the results provide benchmarks for reserve risk capital and risk margin calculations. As the calculations are based on market triangles, for future research one could consider adjusting the results for diversification effect in order to receive them on a single company level.

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Appendix A

Appendix A.1 Duration approach

In line with Szatkowski and Delong (2021) and Guy Carpenter (2014), we use the duration of the claims development process for a single accident year and we calculate a version of the Macaulay duration:

$$duration = \sum_{j=1}^{n} \frac{\mathbb{E}[X_{i,j}]}{\mathbb{E}[C_{i,n}]} \cdot \left(j - \frac{1}{2}\right) = \frac{1/2}{\prod_{j=1}^{n-1} f_j} + \sum_{j=2}^{n} \frac{(f_{j-1} - 1) \cdot (j - 1/2)}{\prod_{k=j-1}^{n-1} f_k}. \quad \text{(A1)}$$

There is also a different approach taking into account all accident years, however we decide to use the above formula as it allows for a more flexible approach which is insensitive to the changes in exposure between accident years.

Appendix A.2 Results and available benchmarks

In this section we present our results - for Poland in Table A2, for Slovakia in Table A3. The symbols follow the denotations presented in Section 3 with additionally σ_{SF} being the Standard Formula standard deviation for reserve risk. In Table A4 we present Spearman's rank correlation coefficients between the characteristics investigated in this paper. Benchmarks available in the literature are listed in Table A1. The reader can also find additional information dedicated to the Standard Formula reserve risk standard deviations in CEIOPS (2010), EIOPA (2011), and a wide analysis of the Italian MTPL insurance market in ISVAP (2006).

Table A1: Benchmarks available in the literature

Characteristic	Literature
duration	England at al. (2012), Guy Carpenter (2014)
f_1	Guy Carpenter (2014)
CoV_{Ult}	Dal Moro and Krvavych (2017), Dal Moro (2012a), Dal Moro (2012b), England at al. (2012), Wüthrich and Merz (2015)
$Skew_{Ult}$	Dal Moro (2012a), Dal Moro (2012b)
SC_{Ult}	Dal Moro and Krvavych (2017), Dal Moro (2012a)
CoV_{1Y}	Wüthrich and Merz (2015)
$Skew_{1Y}$	-
SC_{1Y}	-
α	AISAM-ACME (2007), Wüthrich and Merz (2015)
emergence patterns	England at al. (2012), Scarth et al. (2020)
risk margin run-off patterns	England at al. (2019), Gisler (2019), Wüthrich and Merz (2015)

Table A2: The results for the Polish market data

LoB	1	2	4	5	6	7	8	9	10	11	12
duration	1.47	0.96	2.08	0.67	1.49	1.02	3.27	0.70	1.99	0.71	2.17
f_1	1.43	1.47	1.47	1.19	2.18	1.31	1.67	2.01	2.34	1.18	3.26
CoV_{Ult}	10.0%	4.1%	5.2%	10.1%	28.6%	12.8%	5.1%	34.1%	10.7%	15.9%	37.7%
$Skew_{Ult}$	0.06	0.03	0.02	0.03	0.49	0.10	0.06	0.23	0.14	0.04	0.84
SC_{Ult}	0.63	0.71	0.48	0.31	1.72	0.77	1.14	0.67	1.30	0.25	2.22
CoV_{1Y}	7.8%	3.7%	3.2%	9.5%	27.1%	11.7%	3.4%	32.1%	8.8%	13.9%	34.3%
$Skew_{1Y}$	0.08	0.03	0.02	0.03	0.53	0.11	0.05	0.24	0.17	0.04	0.85
SC_{1Y}	1.02	0.86	0.63	0.34	1.96	0.93	1.44	0.74	1.90	0.3	2.47
α	78.3%	89.4%	62.7%	94.2%	94.8%	92.0%	65.9%	94.2%	81.7%	87.5%	91.0%
α_{Skew}	126.5%	108.7%	82.1%	102.0%	108.1%	110.9%	83.2%	104.8%	119.3%	105.0%	101.0%
α_{SC}	161.6%	121.6%	131.0%	108.2%	113.9%	120.6%	126.3%	111.3%	146.0%	119.9%	111.2%
σ_{SF}	5.7%	14.0%	9.0%	8.0%	11.0%	10.0%	11.0%	17.2%	5.5%	22.0%	20.0%

Table A3: The results for the Slovak market data

LoB	1	2	4	5	6	7	8	9	10	11	12
duration	0.83	1.36	1.48	0.68	1.36	1.19	1.84	1.00	4.06	0.94	1.11
f_1	1.37	1.73	1.58	1.18	2.19	1.63	1.83	1.86	2.45	1.27	2.05
CoV_{Ult}	10.8%	11.0%	5.2%	15.4%	36.6%	25.4%	14.5%	142.8%	41.6%	16.5%	57.3%
$Skew_{Ult}$	0.05	0.13	0.03	0.06	0.43	0.35	0.09	1.50	0.87	0.04	0.69
SC_{Ult}	0.42	1.18	0.59	0.41	1.16	1.37	0.63	1.05	2.07	0.26	1.21
CoV_{1Y}	9.5%	9.9%	3.8%	15.2%	31.8%	24.2%	9.4%	131.5%	30.9%	13.5%	57.0%
$Skew_{1Y}$	0.04	0.13	0.04	0.07	0.47	0.36	0.09	1.57	0.74	0.06	0.69
SC_{1Y}	0.44	1.31	1.06	0.43	1.47	1.50	0.94	1.19	2.37	0.45	1.21
α	87.7%	89.7%	74.1%	98.7%	86.8%	95.3%	65.3%	92.0%	74.4%	81.8%	99.4%
α_{Skew}	93.5%	99.8%	132.4%	102.7%	110.0%	104.4%	96.7%	104.4%	84.7%	138.7%	100.0%
α_{SC}	106.6%	111.3%	178.7%	104.1%	126.8%	109.5%	148.2%	113.7%	114.9%	169.6%	100.6%
σ_{SF}	5.7%	14.0%	9.0%	8.0%	11.0%	10.0%	11.0%	17.2%	5.5%	22.0%	20.0%

Appendix A.3 Solvency II LoBs and exclusions

In Table A5 we present the list of Solvency II LoBs (Lines of Business) with their numbers and names.

When we fit the Mack Chain Ladder model to the available claims triangles, we intend to perform as few adjustments as possible and exclude only evident outliers. The analysis of the claims triangles led us to performing four exclusions of historically observed individual development factors, all for the Slovak market. These factors have a great impact on the characteristics of the triangle and are significantly higher than realizations in earlier development years. These exclusions are as follows:

Table A4: The Spearman's rank correlation coefficients for the investigated characteristics

	duration	f_1	CoV_{Ult}	$Skew_{Ult}$	SC_{Ult}	CoV_{1Y}	$Skew_{1Y}$	SC_{1Y}	α	α_{Skew}	α_{SC}
duration	100%	64%	-5%	27%	62%	-20%	28%	76%	-58%	-26%	42%
f_1	64%	100%	49%	75%	86%	42%	75%	86%	-4%	-15%	0%
CoV_{Ult}	-5%	49%	100%	84%	43%	97%	85%	36%	47%	-3%	-41%
$Skew_{Ult}$	27%	75%	84%	100%	78%	81%	99%	71%	40%	-10%	-37%
SC_{Ult}	62%	86%	43%	78%	100%	40%	77%	95%	18%	-16%	-19%
CoV_{1Y}	-20%	42%	97%	81%	40%	100%	82%	29%	64%	2%	-54%
$Skew_{1Y}$	28%	75%	85%	99%	77%	82%	100%	72%	38%	-2%	-31%
SC_{1Y}	76%	86%	36%	71%	95%	29%	72%	100%	-3%	-6%	4%
α	-58%	-4%	47%	40%	18%	64%	38%	-3%	100%	8%	-80%
α_{Skew}	-26%	-15%	-3%	-10%	-16%	2%	-2%	-6%	8%	100%	46%
α_{SC}	42%	0%	-41%	-37%	-19%	-54%	-31%	4%	-80%	46%	100%

Table A5: Solvency II lines of business (LoBs)

Number of LoB	Name of LoB
1	Medical Expense Insurance
2	Income Protection Insurance
3	Workers' Compensation Insurance
4	Motor Vehicle Liability Insurance
5	Other Motor Insurance
6	Marine, Aviation and Transport Insurance
7	Fire and Other Damage to Property Insurance
8	General Liability Insurance
9	Credit and Suretyship Insurance
10	Legal Expenses Insurance
11	Assistance
12	Miscellaneous Financial Loss

i) SK, LoB 4 - accident year 2014 and development year 6 ($factor = 1.17$).

ii) SK, LoB 6 - accident year 2011 and development year 9 ($factor = 1.23$).

iii) SK, LoB 7 - accident year 2007 and development year 12 ($factor = 1.05$).

iv) SK, LoB 10 - accident years: $2005, 2006, 2007$ and all development years (due to very low volumes of claims).