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Research paper

Optimum railway transition curves for different circular arc radii

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Abstract: This article concerns assessing the dynamical properties and shape optimization of railway transition curves (TCs) for the wide range – 600, 900, 1200, 2000, 3000, and 4000 m – of circular arc radii. The search for the optimum shape means in the current article the evaluation of the curve properties based on chosen dynamical quantities and generation of such shapes with use of a mathematically understood optimization method. As a transition curve in the studies performed, the authors adopted a polynomial of n-th degree, where $n = 9$ and 11. In the study one model of rail vehicle was used. The model represented 2-axle freight car of the average values of parameters. The authors took the so-called standard transition curves of 9$^{\text{th}}$ and 11$^{\text{th}}$ degrees, and 3$^{\text{rd}}$ degree parabola as initial transition curves in the optimization processes. As quality functions (evaluation criteria) the authors used three functions concerning lateral and vertical vehicle dynamics, and creepages in wheel-rail contact. In this work, the results of the optimization – types of the curvatures of the optimum transition curves – were presented and compared.

Keywords: railway transition curves, optimization, railway dynamics, computer simulation

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1. Introduction

Search for the proper (optimum) shape of railway transition curves (TCs) is a problem researchers and engineers deal with for many years. Search for a shape of the TCs are currently not limited only to the TCs themselves [1] and [2], but also include describing the influence of the shape on the dynamics of motion of the vehicle negotiating such a curve [16], [23–25, 27, 29, 30, 32, 33, 37–42]. Detailed study on how the shape of TCs as a factor influences the lateral (and also vertical) dynamics of a rail vehicle has an essential significance also in the current paper. In the context of construction of a modern railway infrastructure for high-speed trains, the ongoing interest in such an approach is clearly visible in last 20 years e.g. [8, 16, 23, 28, 36, 37].

In railway engineering, the most popular transition curve is still a parabolic curve of 3 degree [8], whereas in road engineering, this is the Cornu spiral, termed the clothoid popularly. In railways, while constructing new routes, the clothoid is not often used. Only incomplete clothoid is used in the turnout turning tracks. The reason is the fact, that the parabolic transition curve of 3$^{\text{rd}}$ degree is the same as the first term of the approximate clothoid series expansion. Strictly speaking, the clothoid is not developed in series. This is valid only for functions $\sin\theta(l)$ and $\cos\theta(l)$, where $\theta(l)$ is the slope angle of the tangent in the Cartesian coordinate system. Also, only the clothoid has a linear curvature and the form of the key function $\theta(l)$ follows from this linearity.

A significant increase in the number of publications on the subject of railway and road TCs can be found in the last 15–20 years, e.g. [3–5, 7, 9, 10, 12, 16–18, 22, 23, 26, 29, 30, 32–35, 37–42]. A qualitative change expressed by a resignation from the classical methods of railway TCs properties assessment (a vehicle treated as a point or only as a simple mechanical model) and looking for new shapes is also a fact. At present, the situation has matured to state that the long-established solution in the lights of high-speed trains is not sufficient.

According to the current work authors' opinion, the dynamics of a full vehicle-track system, which takes into consideration a complete description of the vehicle interactions with the track, is not very often examined. In railway engineering, the idea to represent the whole rail vehicle with a mathematical point has ended playing its constructive role. The simplicity of the rail model (the main advantage of the classical approach) makes a full examination of vehicle dynamics, both lateral and vertical, impossible. The applying of advanced vehicle models may give a chance of better assignment of dynamical properties of railway transition curves to freight or passenger trains. Such an approach will include criteria of a general assessment of transition curves other than the passenger comfort. For freight trains fundamental criterion can be wear in wheel-rail contact. For the high-speed trains comfort must be taken into account, preferably combined with other criteria.

The works e.g. [39, 40] have presented that the applying of tools, which are specific to the dynamics of rail vehicles (software for dynamical simulation of vehicle motion) in the shape of the railway transition curves assessment can provide many advantages. First of all, it may provide new informations not available using the classical approach treating all key elements of the system (parts of vehicles) as a point. Secondly, the results of the simulations obtained in this way are not always the same as the results obtained with the classical approach. It can be stated that mentioned works have concluded that the simulation methods, which use the advanced vehicle models, should be at least a complement to the traditional approach. This touches particularly new railway lines, where types of rail vehicles operating are specified. Simulation (numerical) studies may find a justification due to the fact that, there are currently many software packages of AGEM type (automatic generation of the equations of motion) for railway vehicles, including programs by the lead author, for fast construction of mathematical models of these vehicles.

2. Literature survey

An increase in the number of the works, which deal with the problem of the proper shape of railway TCs in the last years is a fact. According to the authors of the current work, there is also a visible division of such works into 4 different groups. These 4 mentioned groups of the works can be represented by example works: [1, 22, 29, 35] respectively. The works from the last three mentioned groups demonstrate, what is the influence of the shape of railway transition curves on their (dynamical) properties. The third group shows that there is a certain number of works, where an approach adopted to the problem of evaluation and formation of railway transition curves is close to that applied by the present authors.

The typical example of the work, where transition curves are regarded only as the mathematical object is the work [1]. Mathematical properties of the curves spanned between two straight lines or two circles are in detail presented there. To define these curves, the authors used Bezier curves having the continuity of type G^2 in each point of the curve. This in practice means the continuity of function of the curvature of the curve in the function of current length l, and this satisfies no abrupt changes of a centrifugal force. As the TC, they proposed a spiral, the curve not possessing: discontinuities, inflexion points and extrema of curvature. The another example of the works belonging to this group let also be e.g. [2, 20].

Tari & Baykal in [35] assumed that continuity of function describing a lateral change of acceleration (LCA) of vehicle negotiating the TC in the function of the curve current length l is the most important condition. According to them, this criterion should be absolutely satisfied. The mentioned authors tried to find only the curves, which satisfied this criterion, and the example of a such curve is given by them in the mentioned work. Also Hasslinger, in [10], described the curve possessing LCA continuity, and, as a result – good dynamical properties. To this group also belong works: [7, 17, 18, 26], and [32].

The example of the work belonging to the third group is the work [29]. Mentioned authors applied an advanced model of a railway vehicle and the corresponding simulation software to assess dynamical properties of the railway transition curves. They compared six different shapes of the curves used in practice among themselves. Six criteria were applied to perform it. These criteria were: vehicle body lateral and vertical acceleration, wheel/rail lateral and vertical forces, derailment coefficient, and reduction rate of wheel-load. As the results the authors proposed a new (non-classical) approach, in which the function of curvature should not be the scaled function of superelevation. To this group also belong works like e.g. [3,9,16,21,23–25,27,29,30,32,34], where their authors examine an influence of a given shape of TC on a vehicle dynamics, using a simple or advanced vehicle model.

Kufver, in [22], optimized the length of transition curves taking passenger comfort and track-rail vehicle dynamical interactions into account for only one type of the curve. In his work, he used the European standard published by CEN [6] relating to the passenger ride comfort. In this standard, the formula for the percentage of passengers P_{CT} (both seating and standing) with discomfort feelings was describing, and this formula was used to find the optimum shapes of TCs. Because of the fact that the P_{CT} values in the function of a length curve had a form very close to the second-degree parabola, it was possible to find such a value of l, for which P_{CT} had the minimum value.

A general conclusion from the literature analysis is the following: there are no works, where the full dynamics of the vehicle-track system and optimization methods are considered jointly. The majority of authors use the classical approach to track-vehicle interactions while TC shape' optimization. A simple vehicle model and a demand that physical quantities, which describe the action of vehicle on passengers such as the maximum unbalanced lateral acceleration a and its change should not be exceeded are of interest to them. So the work aimed at approach in which in the TC shape' optimization the advanced rail vehicle model of the vehicle-track system and mentioned mathematically understood optimization methods are used is still a certain novelty.

3. General aim of the work

The general aim of this work is the optimization and assessment of railway polynomial transition curves of 9^{th} and 11^{th} degrees through the use of 3 non-traditional (non-classical) optimization criteria. The 2-axle rail freight vehicle for dynamical simulation and mathematically understood optimization method are used. The high degrees of a polynomial – 9^{th} and 11^{th} – are chosen due to their greater flexibility in transition curve shape modelling. Mentioned non-traditional assessment criteria (quality functions QFs) concern both the lateral (3.1) and vertical (3.3) dynamics of rail vehicle, and creepages in wheel-rail contact (3.2). These three quality functions are as follows:

$$QF_1 = L_C^{-1} \int_0^{L_C} |\ddot{y}_b| \,\mathrm{d}l \tag{3.1}$$

$$\begin{aligned} QF_2 = \max\big(& \left|F_{1lp}\upsilon_{1lp} + F_{2lp}\upsilon_{2lp}\right| + \left|F_{1rp}\upsilon_{1rp} + F_{2rp}\upsilon_{2rp}\right| \\ & + \left|F_{1lk}\upsilon_{1lk} + F_{2lk}\upsilon_{2lk}\right| + \left|F_{1rk}\upsilon_{1rk} + F_{2rk}\upsilon_{2rk}\right|\big) \end{aligned} \tag{3.2}$$

$$QF_3 = L_C^{-1} \int_0^{L_C} |\ddot{y}_b| \,\mathrm{d}l \tag{3.3}$$

where: $\ddot{y}$, $\ddot{\phi}$ – lateral and angular acceleration of vehicle body (around x-axis), F, υ – tangential forces and creepages in wheel-rail contact.

Indices l, r, p, k and b refer to left hand-side, right hand-side, front (leading) wheelset, rear (trailing) wheelset, and vehicle body, respectively.

Quality functions (3.1–3.3) wer chosen not accidentally. Zboinski and Woznica in [40] showed, that adoption of such criterion, improved not only the value of QF, but also the lateral dynamics of vehicle. The wear criteria, which do not possess such properties, are not useful, because the vehicle dynamics improvement is not guaranteed.

The rule was to minimize the values of quality functions (3.1–3.3), and find the best (optimum) shapes of railway transition curves for a given criterion. In the work the authors also adopted wide range of circular arc radii values: R = 600 m, 900 m, 1200 m, 2000 m, 3000 m and 4000 m. The idea formulated in such a way reveals wide range of research to be performed. It corresponded to train speed range 20.20–36.17 m/s (see Table 1). The usefulness of combined simulation and optimization methods in TCs formation was checked positively e.g. in [39], so it is sensible to make the efforts to achieve the aim assumed. The increased number of criteria and curve radii were adopted in order to be sure that the final conclusions drawn are correct and objective and can be the source of knowledge for engineer-practitioner.

4. Transition curves adopted

The authors of the current work in research performed, as the type of transition curve, assumed, as previously, a polynomial of n^{th} degree. The assumption $x = l$ (x co-ordinate = length) was made, by them and consequently applied in the further part of the work. It leads to some inaccuracies, although they are not large. The shape of the transition curve is specified in the Cartesian coordinate system. The equation of the curve was as follows:

$$y = \frac{1}{R}\left(\frac{A_n l^n}{l_0^{n-2}} + \frac{A_{n-1} l^{n-1}}{l_0^{n-3}} + \frac{A_{n-2} l^{n-2}}{l_0^{n-4}} + \frac{A_{n-3} l^{n-3}}{l_0^{n-5}} + \ldots + \frac{A_4 l^4}{l_0^2} + \frac{A_3 l^3}{l_0^1}\right) \tag{4.1}$$

Each symbol in Equation (4.1) has the following meaning: y – curve lateral coordinate (offset) [m], R – curve radius [m], A – polynomial coefficient, l_0 – total curve length [m], l – curve current length.

In many earlier works, e.g. in [42], the formulas for the curvature, superelevation ramp slope were presented.

It should be mentioned that here the problem of finding a local minimum is encountered. The optimization procedure used in the research is able to find a global minimum, if it is in a certain neighbourhood of the initial point. Since the possibility of finding the global minimum depends on the appropriately selected starting point, the 3rd degree parabola was additionally used to solve the problem. This curve was used but only in 2 cases:

1) when the initial transition curve applied appeared to be a local minimum and optimization procedure indicated this curve as an optimum solution,
2) for transition curves of which the lengths were not greater than 100 m, provided that the optimum transition curve found had the curvature different than linear.

Each railway transition curve has a nominal (minimum) length l_0. This length can be calculated according to the methods used in engineering practice [8]. The minimum curve length arises from 2 kinematic conditions, having a connection with the fact that limits of two quantities during negotiating a curve should not be exceeded. The first quantity is the velocity of the unbalanced lateral acceleration change, whereas the second one is the velocity of the wheel vertical rise along the superelevation ramp. Calculating the lengths of transition curves in a such way, two values of the lengths were always obtained, of which only the greater one was considered.

5. Rail vehicle model and software used

As already mentioned, a model of 2-axle rail freight car of average values of parameters was adopted in the current studies. It was described many times in earlier works, e.g. in [42]. Due to its relatively simple structure, it expedites computation times, which is preferred in a basic research. This also facilitates interpretation of the results obtained. The structure of the vehicle model is also presented in [42]. The vehicle model is supplemented with discrete models of vertically and laterally flexible track. The mentioned models are also presented in [42]. In the model linearity of the vehicle suspension elements (stiffness and damping elements) was assumed. The same relates to the track models. All key parameters of the whole vehicle-track system adopted in the work are shown in [42].

The algorithm of the optimization procedure applied in the optimization of shape of transition curves is shown in the mentioned work [42]. Within this algorithm two iteration loops are present. The first one is the loop of numerical integration. The loop stops current simulation when the model reaches the assumed distance L_C, being the overall length of the route. The second loop is the optimization process loop. The loop stops, when the number of iterations reaches earlier assumed limit value $i_{\lim}$. This $i_{\lim}$ value means that such number of the simulations of vehicle passage has to be carried out in order to stop the optimization process. In the optimizations made to date $i_{\lim} = 200$ was applied as a typical value. If the optimum solution was found earlier, i.e. for number of steps $i < i_{\lim}$, then the optimization process was stopped automatically and the results obtained were recorded. If no optimum solution was found for $i_{\lim} = 200$, this value had to be increased manually, and the optimization process had to be repeated.

The optimization problem of finding a local minimum of functions (3.1–3.3) defined in the current work is generally typical formulation of static constrained optimization. It is solved with the use of a library procedure that utilizes a moving penalty function algorithm combined with Powell's method of conjugate directions. The objective function QF must be a real-valued function of a assumed number of real-valued inputs. It is worth of noting that the function needs not be differentiable. Due to this fact the algorithm belongs to direct optimization methods. Mentioned method minimizes a quadratic function of n variables.

6. The results of the optimizations

Generally, there are two main objectives of presenting the results of the optimization. First of all, the authors of this work want to show, that the method of transition curves finding implemented by them works correctly. The second objective is to show the usefulness of the results of the optimization obtained (first of all optimum transition curves shapes) in railway engineering practice.

In each optimization process, the results of curves' optimization always consisted of:

- values of optimum polynomial coefficients A'_i clearly defining formulae for both optimum transition curve and the curvature corresponding to it,
- value of QF applied to correspond to optimum polynomial coefficients,
- graphical representation of searching for optimum TC represented by curvatures (superelevation ramps),
- graphical representation of the lateral displacements y_b and accelerations y''_b of vehicle body and wheelsets y_l, y''_l, respectively.
- graphical representation of creepages in wheel-rail contact.

To give the results of the work more practical character, the authors assumed that each curve obtained in the optimization process can have the curvature, which can be qualified to one of 5 groups (types). These 5 mentioned groups are:

1) type 1 – curvature is the curvature of standard transition curves of 9th [29] and 11th degree [41],
2) type 2 – curvature is something between the curvature of standard TC of 9th or 11th degree and 3rd degree parabola,
3) type 3 – linear curvature (like approximately for 3rd degree parabola),
4) type 4 – curvature has a convex character. It has slope subtype (4a) or G_1 continuity subtype (4b) at the beginning of TC and it has slope at the end,
5) type 5 – curvature (superelevation ramp) has a concave character.

The same distinction relates also to superelevation ramps.

In Tables 1 and 2 the authors presented the results of the optimization – the types of transition curves obtained in the their research. The values of parameters assumed in the optimization are: curve radius R, lateral unbalanced acceleration limit in a circular arc a, vehicle velocity v, cant d, degree of polynomial n, and transition curve length l_0.

The vehicle velocities v in the majority cases were very close to 30 m/s, whereas the cant d ranged between 20 mm and 150 mm. It is obvoius that resulted in different resultant values of $a_{\lim}$ in circular arc CC. A much smaller cant should not be used, because it increased vehicle velocity and this velocity in a such case could approach a critical velocity of vehicle.

Going to the analysis of the types of transition curves presented in Tables 1 and 2, we see that in general optimization procedure has found the different optimum shapes of transition curves. Among all cases analysed, the largest number of transition curves found had the type no. 3 of the curvature. It is the most visible for transition curves with the smaller lengths l_0 (not greater than 100 m), for all values of a circular arc, and all three quality functions assumed.

Table 1. The results of the optimizations – the types of transition curves – 600 m, 900 m, 1200 m

R [m]	a_{lim} [m/s^2]	v [m/s]	d [mm]	n	l_0 [mm]	QF_1	QF_2	QF_3
600	–0.3	20.20	150	9	118.35	2	1	2
				11	133.10	2	1	3
600	0	24.26	150	9	142.15	2	1	2
				11	159.86	2	1	2
600	0.3	27.71	150	9	162.36	2	1	3
				11	182.58	5	1	2
600	0.6	30.79	150	9	180.46	1	1	2
				11	202.94	1	1	1
900	–0.3	24.74	150	9	144.96	3	1	3
				11	163.01	1	1	3
900	0	29.70	150	9	174.02	5	1	4a
				11	195.70	5	1	3
900	0.3	30.28	110	9	129.93	4a	1	3
				11	146.31	3	1	3
900	0.6	30.37	65	9	77.11	4b	3	3
				11	86.71	4b	3	3
1200	–0.3	28.57	150	9	167.40	2	1	2
				11	188.25	3	1	3
1200	0	24.26	75	9	71.07	3	3	3
				11	79.93	3	3	3
1200	0.3	30.14	70	9	82.41	3	3	3
				11	92.68	4a	2	3
1200	0.6	36.17	75	9	105.98	4b	3	3
				11	119.18	4b	4a	3

In mentioned cases transition curve with any other type of curvature (due to non-linear curvature) is found by the program as the curve, which gives larger values of quality functions adopted. Approximatevely linear curvature of 3$^{\text{rd}}$ degree parabola, due to its linear nature, has no the inflexion point in its middle part. Therefore, the existence of a such point has greater weight (importance) on vehicle dynamics, than the existence of G^0 continuity (geometric continuity) in terminal points of a curvature.

Looking at Tables 1 and 2, the further certain observations can be made. For one quality functions assumed – QF_2 – we see a tendency (trend) to variation of type of TCs in

Table 2. The results of the optimizations – the types of transition curves – 2000 m, 3000 m, 4000 m

R [m]	a_{lim} [m/s^2]	v [m/s]	d [mm]	n	l_0 [mm]	QF_1	QF_2	QF_3
2000	–0.3	30.05	115	9	134.99	1	1	3
				11	151.80	3	3	5
2000	0	24.26	45	9	42.64	3	3	3
				11	47.95	3	3	3
2000	0.15	30.87	50	9	60.29	3	3	3
				11	67.80	3	3	3
2000	0.3	34.47	45	9	60.60	3	3	4b
				11	68.15	3	3	3
3000	–0.6	29.10	135	9	153.45	2	1	2
				11	172.57	3	1	3
3000	–0.3	29.40	90	9	103.35	4b	4b	3
				11	116.23	3	3	3
3000	0	31.30	50	9	61.13	3	3	3
				11	68.74	3	3	3
3000	0.15	30.66	25	9	29.94	3	3	3
				11	33.67	3	3	3
4000	–0.6	29.45	125	9	143.79	2	2	2
				11	161.71	2	1	3
4000	–0.3	29.85	80	9	93.28	3	2	3
				11	104.90	3	2	3
4000	0	30.24	35	9	41.34	3	3	3
				11	46.49	3	3	3
4000	0.15	33.50	20	9	26.17	3	3	3
				11	29.43	3	3	3

accordance with the rule $3 \rightarrow 2 \rightarrow 1$ versus increasing transition curve length. For quality function QF_1 in some measure it is also valid. We see also that the increase of the transition curve length causes existence of TC of types 1 and 2, which curvatures have the inflexion point in their middle part. In case of type 1 the existence of G^1 continuity in terminal points of a curve is also a fact.

It is also worth noting that 2 quality functions (QF_1 and QF_3) concern vehicle dynamics – first one – lateral dynamics, and the third one – vertical dynamics. Here, the dynamical characteristics used in quality functions construction have their largest values with in transition curve zone. The last quality function (QF_2) concerns creepages (and consequently a wear) in wheel-rail contact. The creepages have their largest possible values just in the circular arc.

Despite the differences, which can be observed between quantities used in three quality functions, in all cases analyzed, there is a mentioned certain general tendency $3 \rightarrow 2 \rightarrow 1$. There are still however relatively many cases (types of curvatures – no. 4 and 5), which do not confirm this fact.

Figs. 1–3 present types of curvatures (and also superelevation ramps funtions) in the function of the total length of the transition curves for three quality functions used – QF_1, QF_2, and QF_3.

The results presented in three mentioned figures show certain general tendency (4), $3 \rightarrow 2 \rightarrow 1$ in changing the types of curvatures in function of the length of the transition curves. It is not however confirmed in all cases analysed. The mentioned results also confirm that in general the lateral dynamics of rail vehicles is not coupled with their vertical dynamics. In railway engineering practice, it is not possible to determine or predict the courses of vertical displacements and accelerations based on the lateral displacements and accelerations. There are only some similarities between them – they have the largest values in transition curves and they approach to a fixed value in circular arc. Therefore, the results – the optimum types of railway TCs – obtained through the optimization processes for both lateral (QF_1) and vertical dynamics (QF_3) of the vehicle in general were different.

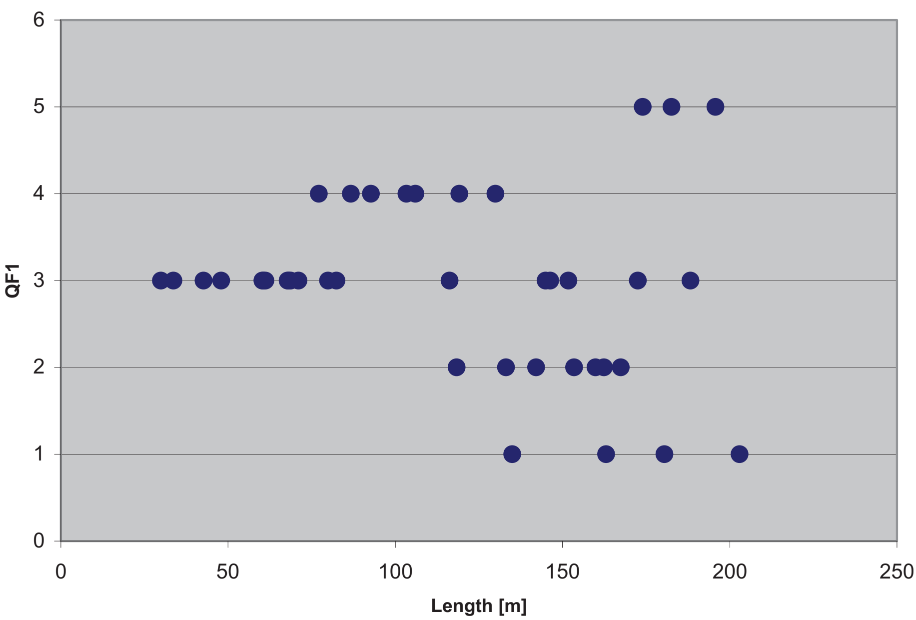

Fig. 1. Types of curvatures (superelevation ramps) in the function of the length of the transition curves for QF_1

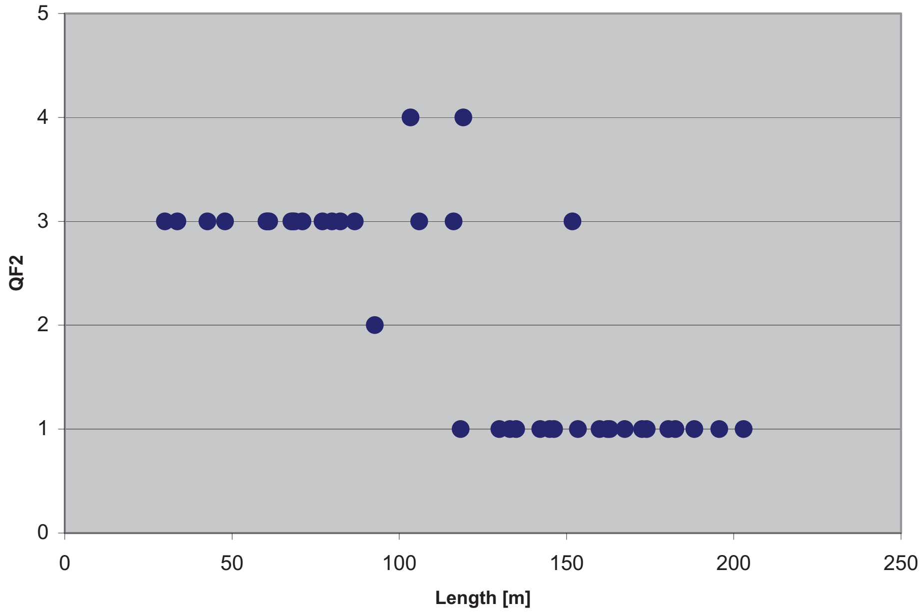

Fig. 2. Types of curvatures (superelevation ramps) in the function of the length of the transition curves for QF_2

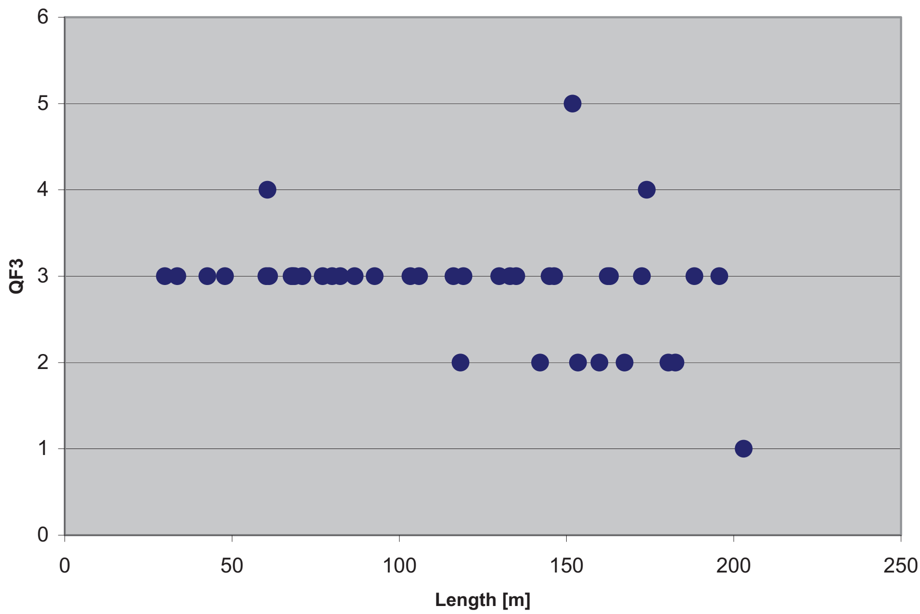

Fig. 3. Types of curvatures (superelevation ramps) in the function of the length of the transition curves for QF_3

7. Conclusions

In the current work the authors presented the effective method of the optimization of railway polynomial transition curves. They used 2-axle rail vehicle model and mathematically understood optimization method. Taking all the results obtained into account, it can be observed that in the computational and numerical sense the optimization tool used was in 100% reliable and it fulfilled the authors' expectations.

The local minima problem, despite it was present in the work done with the authors' software was not troublesome in the research. Rational adoption of starting points allowed to find the acceptable optimum solutions without performing extra optimization processes very often. We one should also note that optimization processes performed did not need extremely long calculation times. Moreover, it is also possible to use other optimization methods for the approach proposed if one thinks it could make the analysis faster or more accurate.

In the work the authors showed that the use of different criteria for the assessment of the shape of the railway polynomial TCs (dynamical and wear criteria), and different circular arc radii, generally resulted in different shapes of the curves. The adoption of more quality functions and different starting points in the shape optimization process enabled to look wider than before (e.g. the work [41]) at the transition curves of higher – 9th and 11th – degrees in the context of their dynamical properties.

The work [40] has shown that the railway transition curves of 9th and 11th degrees have the types of curvature 1 and 2 only for the cases with great lengths of the curve. Therefore, the results presented in the mentioned work make only a certain fragment of the results presented in the current work. In general the current work has shown that the curves of 9th and 11th degrees may have different shapes, which ones in particular depend on the length of the curve mainly.

The basic detailed conclusions, which can be drawn from the study are as follows: for the curves of relatively short and the most used lengths ($l_0 < 100$ m) in most cases the optimum transition curve was the curve of the type 3. For the curves of medium lengths ($l_0 > 100$ m and $l_0 < 180$ m) the optimum transition curves usually had the curvature of 1, 2 and 3 types. For the curves of very great lengths ($l_0 > 180$ m) in most cases the best dynamical properties had the curves with the curvature of type 1.

The results obtained in the current article make a novelty taking non-conventional methods of testing the properties of railway polynomial transition curves into account. Proposed curves of 9th and 11th degrees with the maximum number of polynomial terms have better properties than standard curves of mentioned degrees and the traditionally used the 3rd degree parabola. New shapes of the curves can be regarded as those ones with dynamical properties not found in the world literature. Proposed curves are not included nowadays in any design document, like standard EN 13803 and standard PKP PLK ST T1 06.

Due to the fact that the dynamics of the vehicle is not the only criterion for the assessment the TC shape in further studies, the authors think about multi-criteria optimization. The first criterion could generally be dynamics of the vehicle and the second – wear in the wheel-rail contact. In this context, the careful considerations should be performed, how to form the quality function and what weights – assigned to the each criteria of assessment – should be assumed.

References

[1] A. Ahmad, J. Ali, "G^3 transition curve between two straight lines", in *Proceedings 5th CGIV'08*. IEEE Computer Society, 2008, pp. 154–159.

[2] A. Ahmad, R. Gobithasan, J. Ali, "G^2 transition curve using quadratic Bezier curve", in *Proceedings of the Computer Graphics, Imaging and Visualisation Conference*. IEEE Computer Society, 2007, pp. 223–228.

[3] Z. Barna, L. Kisgyorgy, "Analysis of hyperbolic transition curve geometry", *Periodica Polytechnica Civil Engineering*, 2015, vol. 59, no. 2, pp. 173–178.

[4] J.A. Bonneson, "A kinematic approach to horizontal curve transition design", *Transportation Research Record*, 2000, vol. 1737, pp. 1–8.

[5] G. Bosurgi, A. D'Andrea, "A polynomial parametric curve (PPP) for design of horizontal geometry of highways", *Computer-Aided Civil and Infrastructure Engineering*, 2012, vol. 27, no. 4, pp. 304-312; DOI: 10.1111/j.1467-8667.2011.00750.x.

[6] BS EN 12299:2009 Railway applications. Ride comfort for passengers. Measurement and evaluation.

[7] N. Eliou, G. Kaliabetsos, "A new, simple and accurate transition curve type, for use in road and railway alignment design", *European Transport Research Review*, 2014, vol. 6, no. 2, pp. 171–179; DOI: 10.1007/s12544-013-0119-8.

[8] C. Esveld, *Modern Railway Track*. MRT-Productions, 2001.

[9] S. Fischer, "Comparison of railway track transition curves types", *Pollack Periodica*, *An International Journal for Engineering and Infrastructure Science*, 2009, vol. 4, no. 3, pp. 99–110; DOI: 10.1556/pollack.4.2009.3.9.

[10] H. Hasslinger, *Measurement proof for the superiority of a new track alignment design element, the so-called "Viennese Curve"*. ZEVRail, 2005.

[11] E. Jacobs, "Die sinusoide als neuzeitliches trassierungselement", *Vermessung-Ingenieur*, 1987, vol. 87, pp. 3–9.

[12] Q.P. Jiang, "Study of the new type of transition curve of road", *China Journal of Highway and Transportation*", 2002, vol. 15, no. 2.

[13] D. Kahler, "Ein übergangsbogen für den S-Bahnverkehr mit linearer Überhöhungsrampe", *Vermessungstechnik und Raumordnung*, 1990, vol. 52, pp. 10–18.

[14] D. Kahler, "Übergangsbögen zur ausrundung der neigungswechel im schienen-schnellverkehr", *Zeitschrift für Vermessungwesen*, 1990, vol. 115, pp. 154–162.

[15] W. Kik, "Comparison of the behaviour of different wheelset-trackmodels", in *Proceedings of the 12th IAVSD Symposium on the Dynamics of Vehicles on Roads and on Tracks*, *Vehicle System Dynamics*, G. Sauvage, Ed. Amsterdam: Swets & Zeitlinger. 1992, pp. 325–339.

[16] L.T. Klauder, S.M. Chrismer, J. Elkins J, "Improved spiral geometry for high-speed rail and predicted vehicle response", *Rail Track and Structures*, 2003, vol. 6, pp. 15–17.

[17] A. Kobryn, "New solutions for general transition curves", *Journal of Surveying Engineering*, 2014, vol. 140, no.1, pp. 12–21; DOI: 10.1061/(ASCE)SU.1943-5428.0000113.

[18] W. Koc, "New transition curve adapted to railway operational requirements", *Journal of Surveying Engineering*, 2019, vol. 145, no. 3; DOI: 10.1061/(ASCE)SU.1943-5428.0000284.

[19] W. Koc, "The method of determining horizontal curvature in geometrical layouts of railway track with the of moving chord", *Archives of Civil Engineering*, 2020, vol. 66, no. 4; DOI: 10.24425/ace.2020.135238.

[20] W. Koc, K. Palikowska, "Analiza sposobów modelowania krzywizny – krzywe Bezier a metoda analityczna", *Zeszyty Naukowo-Techniczne SITK in Cracow*. *Conference Proceedings*, 2012, vol. 99, no. 3, pp. 225–237 (in Polish).

[21] W. Koc, K. Palikowska, "Ocena dynamiczna wybranych sposobów łączenia elementów trasy o zróżnicowanej krzywiźnie", *Technika Transporu Szynowego*, 2012, vol. 18, no. 9 (in Polish).

[22] B. Kuvfer, "Optimisation of horizontal alignments for railway – procedure involving evaluation of dynamic vehicle response", Ph.D. Thesis, Royal Institute of Technology, Stockholm, 2000.

[23] X. Li, M. Li, C. Ma, J. Bu, L. Zhu, "Analysis on mechanical performances of high-speed railway transition curves", in *ICCTP 2009: Critical Issues In Transportation Systems Planning, Development, and Management*. ACSE, 2009, pp. 1–8.

[24] X. Li, M. Li, H. Wang, J. Bu, M. Chen, "Simulation on dynamic behaviour of railway transition curves", 2010, in *ICCTP 2010: Integrated Transportation Systems: Green, Intelligent, Reliable*. ASCE, 2010, pp. 3349–3357.

[25] X. Li, M. Li, J. Bu, H. Wang, "Comparative analysis on the linetype mechanical performances of two railway transition curves", *China Railway Science*, 2009, vol. 30, no. 6, pp. 1–6.

[26] X. Li, M. Li, J. Bu, Y. Shang, M. Chen, "A general method for designing railway transition curve algebraic equations", in *ICCTP 2010: Integrated Transportation Systems: Green, Intelligent, Reliable*. ASCE, 2010, pp. 3340–3348.

[27] S.L. Lian, J.H. Liu, X.G. Li, W.X. Liu, "Test verification of rationality of transition curve parameters of dedicated passenger traffic railway lines", *Journal of the China Rail Society*, 2006, vol. 28, no. 6, pp. 88–92.

[28] M. Lindahl, *Track geometry for high-speed railways*. Stockholm: Department of Vehicle Engineering Royal Institute of Technology, 2001.

[29] X.Y. Long, Q.C. Wei, F.Y. Zheng, "Dynamic analysis of railway transition curves", *Proc. IMechE, Part F: Journal Rail and Rapit Transit*, 2010, vol. 224, no. 1, pp. 1–14; DOI: 10.1243/09544097JRRT287.

[30] Y. Michitsuji, Y. Suda, "Improvement of curving performance with assist control on transition curve for single-axle dedicated passenger traffic railway lines", *Journal of the China Railway Society*, 2006, vol. 28, no. 6, pp. 88–92.

[31] W. Oleksiewicz, C. Kraśkiewicz, "Układy geometryczne rozjazdów na kolejach dużych prędkości", in *Teoretyczne podstawy budownictwa. Konstrukcje inżynierskie*, vol. 3, S. Jemioło, M.D. Gajewski, Eds. Oficyna Wydawnicza Politechniki Warszawskiej, 2013 (in Polish).

[32] A. Pirti, M.A. Yucel, T. Ocalan, "Transrapid and the transition curve as sinusoid", *Tehnicki Vjesnik*, 2016, vol. 23, no. 1, pp. 315–320.

[33] J. Pombo, J. Ambrosio, "General spatial curve joint for rail guided vehicles: kinematics and dynamics", *Multibody System Dynamics*, 2003, vol. 9, no. 3, pp. 237–264; DOI: 10.1023/A:1022961825986.

[34] T.-I Shen, C.H. Chang, K.Y. Chang, C.C. Lu, "A numerical study of cubic parabolas on railway transition curves", *Journal of Marine Science and Technology*, 2013, vol. 21, no. 2, pp. 191–197; DOI: 10.6119/JMST-012-0403-1.

[35] E. Tari, O. Baykal, "A new transition curve with enhanced properties", *Canadian Journal of Civil Engineering*, 2005, vol. 32, no. 5, pp. 913–923.

[36] D. Vermeij, "Design of a high speed track", *HERON*, 2000, vol. 45, no. 1, pp. 9–23.

[37] Y.L. Xu, Z.L, Wang, G.Q. Li, S. Chen, Y.B. Yang, "High-speed running maglev trains interacting with elastic transitional viaducts", *Engineering Structures*, 2019, vol. 183, pp. 562–578; DOI: 10.1016/j.engstruct.2019.01.012.

[38] J.Q. Zhang , Y.H. Huang, F. Li, "Influence of transition curves on dynamics performance of railway vehicle", *Journal of Traffic and Transportation Engineering*, 2010, vol. 10, no. 4, pp. 39–44.

[39] K. Zboinski, "Dynamical investigation of railway vehicles on a curved track", *European Journal of Mechanics A-Solids*, 1998, vol. 17, no. 6, pp. 1001–1020.

[40] K. Zboinski, "Numerical studies on railway vehicle response to transition curves with regard to their different shape", *Archives of Civil Engineering*, 1998, vol. 44, no. 2, pp. 151–181.

[41] K. Zboinski, P. Woznica, "Optimisation of railway polynomial transition curves: a method and results", in *Proceedings of the First International Conference on Railway Technology: Research, Development and Maintenance*, J. Pombo, Ed. Civil-Comp Press, 2012.

[42] K. Zboinski, P. Woznica, "Combined use of dynamical simulation and optimisation to form railway transition curves", *Vehicle System Dynamics*, 2018, vol. 56, no. 9, pp. 1394–1450; DOI: 10.1080/00423114.2017.1421315.

Optymalne kolejowe krzywe przejściowe dla różnych promieni łuku kołowego

Słowa kluczowe: kolejowe krzywe przejściowe, optymalizacja, dynamika kolejowa, symulacja komputerowa

Streszczenie:

Celem pracy była optymalizacja kolejowych wielomianowych krzywych przejściowych 9. i 11. stopnia z wykorzystaniem niestandardowych kryteriów oceny krzywej oraz modelu pojazdu szynowego. Jako wspomniane kryteria oceny autorzy pracy zastosowali tu minimalizację wartości całki ze zmiany przyspieszenia poprzecznego i kątowego nadwozia pojazdu po długości drogi oraz poślizgów w kontakcie koło–szyna.

W pracy jej autorzy użyli jednego modelu pojazdu kolejowego. Był nim model 2-osiowego wagonu towarowego o uśrednionych wartościach parametrów, który jest rozważany w stanie ładownym. Prosta konstrukcja pojazdu skutkuje akceptowalnymi czasami obliczeń, co jest korzystne w dużej liczbie optymalizacji. W modelu tym przyjęto liniowość zawieszenia pojazdu – liniową sztywność i tłumienie elementów zawieszenia pojazdu. To samo zastosowano w modelu toru.

Wykorzystany w pracy model zawiera wszystkie kluczowe elementy modeli dynamicznych pojazdów szynowych, takie jak: kluczowe elementy masowe (zestawy kołowe i nadwozie pojazdu), elementy zawieszenia (elementy sprężyste i tłumiące), koła i geometrię szyn opisaną przez rzeczywisty, nieliniowy kształt ich profili. Poza tym, styczne siły kontaktowe są obliczane przy użyciu uproszczonej nieliniowej teorii kontaktu J.J. Kalkera. Ponadto, model pojazdu jest uzupełniony modelem toru. Może on być traktowany jako zaawansowany model dynamiczny, zwłaszcza gdy porówna się go do punktu materialnego reprezentującego pojazd w tradycyjnych metodach oceny i kształtowania krzywych przejściowych.

W pracy przyjęto następujące wartości promienia łuku kołowego R równe: 600 m, 900 m, 1200 m, 2000 m, 3000 m oraz 4000 m. Dla konkretnych wartości R i przechyłki d, autorzy zawsze obliczali prędkość pojazdu, zgodnie ze wzorami tradycyjnie przyjętymi w praktyce inżynierskiej.

Przyjęto, że każda krzywa otrzymana w pracy ma krzywiznę (oraz rampę przechyłkową), która została zakwalifikowana do jednej z 5 grup. Wspomniane 5 grup (typów) to:

- typ 1 – krzywizna jest w praktyce zbliżona do krzywizny wzorcowej 9. i 11. stopnia,
- typ 2 – krzywizna ma kształt pośredni pomiędzy krzywizną wzorcową 9. i 11. stopnia, a parabolą 3. stopnia, krzywizna ta ma styczność typu G^1 w skrajnych punktach,
- typ 3 – krzywizna quasi-liniowa, bardzo zbliżona do krzywizny paraboli 3. stopnia,
- typ 4 – krzywizna ma wklęsły charakter, jest ostra (4a) lub ma ciągłość typu G^1 (4b) na początku KP i zawsze ostra na końcu KP,
- typ 5 – krzywizna ma wypukły charakter i styczność typu G^0 na początku i końcu krzywej.

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