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An Unexpected Result on Modelling the Behavior of A/D Converters and the Signals They Produce

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***Abstract*—In this paper, we show that the signal sampling operation considered as a non-ideal one, which incorporates finite time switching and operation of signal blurring, does not lead, as the researchers would expect, to Dirac impulses for the case of their ideal behavior.**

***Keywords*—modelling of signal sampling operation**

I. Introduction

THERE exists in the literature a variety of models of the sampling operation of analog signals as well as of the associated models of the behavior of analog-digital (A/D) converters that carry out this operation in a real world. Further, notice that these two terms are used in papers and textbooks interchangeably (that is as synonyms of one another). Moreover, among the models mentioned above are those that describe the sampling operation (or equivalently the behavior of an A/D converter) in an idealized way and also those that take into account non-idealities of the signal sampling process, to a greater or lesser extent. Let us now take a closer look at some of them.

Probably, the most prominent among them is the one that assumes that the sampling process is to generate a weighted Dirac comb (weighted with an analog signal samples). This idealized model is commonly used in the literature; see, for example, such excellent and prominent textbooks as [1]–[11]. It is used in the context of calculations of the spectrum of a sampled signal. That is of the spectrum of a signal appearing at the output of an A/D converter.

An example sampling of an analog signal modeled with the use of the above model is presented in Fig. 1. Precisely, it is the visualization shown in Fig. 1c) and denoted as the signal $x_D(t)$, where t means a continuous time variable. The analog signal that was sampled here is presented by curve $x(t)$ of Fig. 1a). Furthermore, $t \in \mathrm{R}$ with R standing for the set of reals and the lower index D at $x_D(t)$ standing for the name of Dirac. Moreover, $T \in \mathrm{R}$ in Fig. 1 means the period of a uniform signal sampling illustrated there.

Fig. 1 (precisely Fig. 1b)) illustrates also the second idealized model that occurs in the literature (see, for example, [4], [5]). It consists simply of ideal samples of an analog signal (that is of ones obtained in an ideal manner), and these samples are just visualized as shown in Fig. 1b). They form the so-called discrete time signal $x_K(k)$, where $k \in \mathrm{Z}$ stands for the so-called discrete time variable, but the lower index K for the name of Kronecker. Further, Z above means the set of integers.

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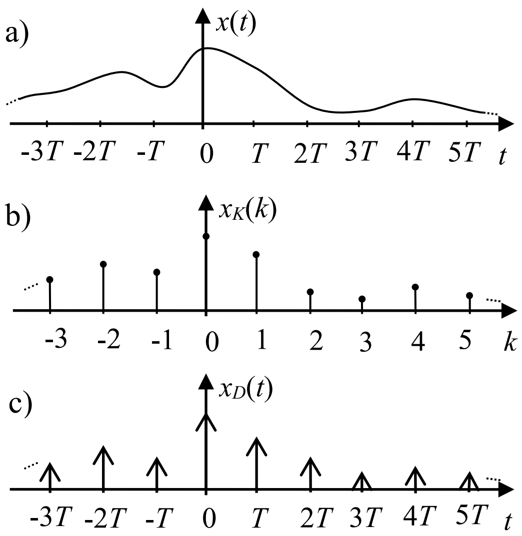

Fig. 1. Visualization of two types of modelling of the signal that appears at the output of an A/D converter – ideal case: a) example analog signal before performing sampling operation (that will be performed in an ideal manner); b) the set of samples of the ideally sampled signal of Fig.1a) viewed as the so-called discrete time signal; c) the set of samples mentioned above viewed as a signal of a continuous time in form of a series of weighted Dirac deltas [12]. Remark: this figure is based on ones, which were used by this author in his other papers, for example, in [13].

As we know, the Fourier analysis is inextricably linked with the signals of a continuous time (that is with those functions in which the argument $t \in \mathrm{R}$ has a physical interpretation of time). And, all the problems occurring in this analysis with the models visualized by $x_K(k)$ and $x_D(t)$ in Fig. 1 lie in that they are not functions of the continuous variable $t \in \mathrm{R}$. The object $x_K(k)$ is a nice visualization of the samples of the signal $x(t)$ often used in the literature (see, for example, [4], [5]), however, it is not a function of *t*. But, this object is of course a function of the variable $k \in \mathrm{Z}$. On the other hand, the object $x_D(t)$ works admittedly with the independent variable $t \in \mathrm{R}$, but it is not obviously a function; as it is a sequence of the weighted Dirac deltas (which are not functions [12]).

Some researchers, who are aware of the above problems, propose to use, instead, descriptions of the sampling process that take into account its non-ideal characteristics. Why? Because then, as well known, the generated waveforms have the form of functions of a continuous time variable *t*, so their Fourier analysis is unproblematic. And, these signals have well-defined spectra.

Such an approach, as sketched above and with an extensive justification, was presented in [14]. It was also argued there that the waveforms generated in a non-ideal sampling process are good representatives of the (ideal) samples (being numbers), appearing at the output of an A/D converter. And, implicitly, the spectrum of the sequence of these signals can be related with the spectrum of the sequence of signal samples (in a sense of their spectral representatives, whatever that would mean).

In [14], two kinds of non-idealities in a non-ideal model of the sampling process are considered. Namely, the following ones:

a) A non-perfect switching that is modeled by a switch which does not sample an analog signal immediately but needs a finite time to perform this operation; let us denote it by $\tau \in \mathrm{R}$; $0 < \tau \leq T$. Obviously, because of this fact each sample is not a number but a relevant fragment of the sampled signal of a finite duration equal to τ.
b) A signal blurring effect, i.e. distortion of the portion of the sampled signal having a finite duration equal to τ (mentioned in point a) above) due to the presence of signal transients and due to non-idealities of the realizing electronics, which, taken together, is in [14] referred to as a sample pulse blurring.

The analysis presented in [14] is not however convincing, particularly at the points where it attempts to show that the model presented therein leads, in the limit, to receiving the results which are predicted by the ideal model. For instance, the formulas derived in [14] do not explain why the aliasing phenomenon in the spectrum of the sampled signal disappears in the limit when the parameter τ goes to zero. Furthermore, the blurring of the sample pulse completely disappears in the limit (this pulse then becomes a Dirac impulse (Dirac delta)). That is in the case when the impulse response of an equivalent device used to interpret the impulse blurring effect becomes a Dirac delta.

It is shown in this short paper that consideration of the two non-idealities mentioned above together (not separately) does not lead to the occurrence of the weighted Dirac impulses at the sampling instants. Then, we get the weighted Kronecker time functions (impulses) [13]. And, certainly, this result will be astonishing for the researchers [1] –[11], [14] defending the model illustrated in Fig. 1c) (where the so-called comb of such weighted Dirac deltas, being the sample pulses, is shown). They will look at this obviously unexpected outcome in disbelief.

However, before deriving the aforementioned result, we will dwell a little longer on consideration of some aspects of the sampled signal blurring effect itself. We do this in the next section; the outcomes achieved will turn out to be also interesting.

II. Modelling of the Signal Blurring Effect

The author of this paper has gained some experience in analyzing and modelling of the blurring effects of model-based characteristics of sensors as well as of signals acquired in measurements (with the use of these sensors) [15]. This expertise will be helpful here to see in the right light the model of the non-ideal sampling operation sketched briefly above and an ideal one which is reflected by it.

Note that any blurred single impulse, whose origin lies in a Dirac impulse, can be well approximated (see, for example, [16-18]) by a shifted Gaussian function $h_G(t-\mu)$ having the following form:

$$h_G(t-\mu) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2\right], \tag{1}$$

whereby, in this paper, the independent variable in the function (1) stands for the time variable t. Furthermore, μ and σ are its shaping parameters: the first one means the time shift of $h_G(t)$ on the timeline, but the second determines its width. When $\sigma \to 0$, then this function behaves as a shifted Dirac delta $\delta(t-\mu)$, as argued, for example, in [16-18]. Or, in other words, the latter can be expressed in the following way: the function $h_G(t-\mu)$ becomes for $\sigma = 0$ a Dirac delta appearing at the point $t = \mu$. Note also further that it ceases then to be a function because the Dirac delta is not [12].

Now, let us recall the following convolution integral convention [12], [19]:

$$x_o(t) = h(t) = \int_{-\infty}^{\infty} h(\varphi)\delta(t-\varphi)\,d\varphi \tag{2}$$

that is used in circuits and systems theory to identify the so-called impulse response $h(t)$ of a (non-pathological) linear system (device) by applying the Dirac delta to its input. (Note that this is really only a notational convention because the integral in (2) does not exist at all, even in the Lebesgue's sense [12], [20], [21]. Moreover, the Dirac delta is not an usual physical (deterministic) signal, demonstrating only one value at each time instant [9].) Obviously, when the input signal, say $x_i(t)$, in (2) is such that $x_i(t) \neq \delta(t)$, then in general (2) exists in meaning of a standard convolution integral. And finally, for the sake of description completeness, $x_o(t)$ in (2) stands for the system's output signal. (When $x_i(t) = \delta(t)$, then $x_o(t) = h(t)$.) Furthermore, note also that the convention expressed by (2), when applied to our function $h(t) = h_G(t-\mu)$, gives $x_o(t) = h(t-\mu)$.

The latter result suggests that the blurring effect can be interpreted as processing of the Dirac delta $\delta(t)$ through a virtual device having the impulse response $h_G(t-\mu)$. And, for an illustrative purpose, it is of course useful. However, see that concluding from this that a "true" convolution integral of the form (2) with any input physical signal $x_i(t-\varphi)$ instead of $\delta(t-\varphi)$ holds, would lead to a pure illusion. Why? Simply because the electronics implementing the signal sampling operation (or any of its components) does not exhibit such an impulse response. Or, in other words, it is forbidden to replace the Dirac delta in (2) through a signal of another form in the description of the blurring effect that is presented in this section.

Let us now turn our attention to some peculiarities of idealizing the above description (that is ubiquitous in the literature; example of this is the presentation in [14]). To this end, we recall that the convention used in (2) applies also [12], [19] in the case, when $h(t)=\delta(t)$. That is we can then write

$$x_o(t)=\delta(t)=\int_{-\infty}^{\infty}\delta(\varphi)\delta(t-\varphi)\,d\varphi \,. \tag{3}$$

Furthermore, the convention (3) applied to the shifted Dirac delta $h(t)=\delta(t-\mu)$ gives the shifted $x_o(t)=\delta(t-\mu)$, accordingly. Next, see that with the impulse responses $h_{kT}(t)=x(kT)\delta(t-kT)$, $k\in \mathrm{Z}$, associated with the subsequent points of sampling on the timeline, kT, we get the sampled signal description illustrated in Fig. 1c); that is in a form of a sequence of the weighted Dirac deltas.

In what follows, we consider the sampling operation at each sampling instant as related with its own impulse response. Whereby each of them is characterized by an own multiplying constant equal to $x(kT)$ and an own time shift $\mu=kT$, $k\in \mathrm{Z}$.

Let us now critically evaluate the results, we have just obtained by idealization of the function (1) used to describe the signal blurring effect. That is by assuming $\sigma\to 0$ in (1), what leads to $\delta(t-\mu)$ [16-18]. As a result, we got impulses in the form of the weighted Dirac deltas at the sampling instants that, obviously, do not represent any usual physical (deterministic) signals [9].) Even worse, the blurring effect has completely disappeared in this representation. So, one may simply ask what it was all for. After all, one could assume a priori that the signal sampling means simply generation of the weighted Dirac deltas – as was done, for example, in [4].

Further, observe occurrence of a certain interesting effect of idealization of the blurring impulse responses given by (1) and multiplied by the values of the signal samples $x(kT)$, $k\in \mathrm{Z}$. Namely, they could be then interpreted as the impulse responses of resistances [22], [23]. Why? Because they have then the following form: $h_{kT}(t)=x(kT)\delta(t-kT)$, which is identical, in form, with the formula: a value of the resistance multiplied by the Dirac delta [22]. But beware, they cannot be regarded as "true" ones because, as already said, they describe only virtual devices (similarly as $x(kT)h_G(t-kT)$ considered just before by interpreting (2)) used exclusively for illustrative purposes.

Finally, let us summarize our considerations in this section with the following illustration. Take into account our "virtual resistors" mentioned above and apply (2) with $\delta(t)$, that is correctly chosen in it. This gives

$$\int_{-\infty}^{\infty} x(kT)\delta(\varphi-kT)\delta(t-\varphi)\,d\varphi = x(kT)\delta(t-kT), \tag{4}$$

what represents the impulse at the instant kT on the timeline according to the sampling model visualized in Fig. 1c). Further, the sequence of such impulses describes the weighted periodization [24] of the Dirac delta. And, this is a correct result within the model of signal sampling in force [1-11], [24].

Now, let us consider for a moment the notation (2) again to check what would happen there for signals other than the Dirac impulse $\delta(t)$. That is with the following assumption: $x_i(t)=x(t)\neq\delta(t)$ in place of $\delta(t)$. Substituting this in (2) together with the impulse response of any of our "virtual resistors" would give

$$\int_{-\infty}^{\infty} x(kT)\delta(\varphi-kT)x(t-\varphi)\,d\varphi = x(kT)x(t-kT), \tag{5}$$

what would lead to the weighted periodization of the signal to be sampled. (By the way, note that by normalizing the signal $x(t)$ in (5) for the individual time instants $kT, k\in \mathrm{Z}$, with respect to the successive weights $x(kT), k\in \mathrm{Z}$, we would get a perfect periodization of the signal to be sampled (which follows from the summation of impulses given by (5)).)

Evidently, the result (5) is not fully correct (in the strict sense) within the model of signal sampling being in force in the literature. From the comparison of (4) with (5), it follows however that the closer the form of the normalized signals $x(t-kT)/x(kT)$ to that of the Dirac impulse, the better the approximation of (4) with (5). On the other hand, this approximation is getting worse, as this difference is getting bigger. Consequently, this does not allow us to identify the signal $x(t)$ with an input signal applied to a A/D converter and to treat (5) as a convolution-integral-like representation of the sampling operation performed by the A/D converter at the time instant kT.

It is worth noting however that such a representation as the aforementioned one can be derived from (4) – by cleverly transforming this relation. To this end, let us rewrite the right-hand side of it as

$$\int_{-\infty}^{\infty} x(\varphi)\delta(\varphi-kT)\delta(t-\varphi)\,d\varphi \,. \tag{6}$$

Note that to obtain (6) from (4) a standard sifting property of the Dirac delta [2], [9], [10], [12] has been applied. That is here $x(kT)\delta(\varphi-kT)=x(\varphi)\delta(\varphi-kT)$.

In the next step, let us introduce a new auxiliary variable $\gamma = t - \varphi$ in (6). This gives

$$\begin{aligned} &-\int_{\infty}^{-\infty} x(t-\gamma)\delta(t-\gamma-kT)\delta(\gamma)d\gamma = \\ &= \int_{-\infty}^{\infty} \{\delta(t-\gamma-kT)\delta(\gamma)\} x(t-\gamma)d\gamma = \\ &= \int_{-\infty}^{\infty} \delta(\gamma)\{\delta(t-\gamma-kT)x(t-\gamma)\}d\gamma \quad . \end{aligned} \tag{7}$$

Observing now the second and the third line in (7), we see that really the convolution-integral-like representation of the sampling operation performed by a A/D converter (on its input signal $x(t)$) at each of the sampling instants kT, $k \in \mathrm{Z}$, really exists. And, it can be interpreted in two ways. First, see that the middle expression in (7) shows that the impulse response of the sampling operation associated with the time instant $kT, k \in \mathrm{Z}$, is equal to $\delta(t-\gamma-kT)\delta(\gamma)$; this impulse response belongs to the category of time-dependent ones [25]. [26]. Then, respectively, the impulse response of the sampling operation associated with all the sampling instants kT will be the sum of them over all the k's; denoting it by $h_{1\Sigma}(t)$, we will have $h_{1\Sigma}(t,\gamma) = \sum_{k=-\infty}^{\infty} \delta(t-\gamma-kT)\delta(\gamma)$.

By the way, we draw here the reader's attention to the occurrence of products of the Dirac deltas in the above expression, which some researchers [27], referring to L. Schwartz [28], consider unacceptable (incorrect). That this is not the case is shown, among others, in [29]–[33].

The interpretation of that convolution shown in the last expression on the right-hand side of (8) is left to the reader.

III. Non-Perfect Switching and Signal Blurring Effect Considered Together

In [14], two kinds of non-idealities of the sampling process, which are listed in Introduction, are considered separately (it is not known why). Here, let us consider both of them together, as forming jointly a signal sampling operation. So, with that in mind, we can write

$$x_{okT}(t) = \int_{-\infty}^{\infty} h_B(\lambda)\left(H_{S,kT}x\right)(t-\lambda)d\lambda \tag{8}$$

where $x_{okT}(t)$ means the part of the A/D converter output signal associated with the sampling instant kT, i.e. the impulse appearing as a result of sampling at this instant. The symbol $h_B(t)$ in (8) means the impulse response of a linear filter responsible for the signal blurring effect, but $H_{S,kT}(\cdot)$ is the operator describing behavior of the non-perfect switching (it works on the signal $x(t)$). Here, we assume their descriptions as follows:

$$h_B(t) = \begin{cases} 1/\tau & \text{if } 0 \le t \le \tau \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

and

$$\left(H_{S,kT}x\right)(t) = \begin{cases} x(t) & \text{if } kT \le t \le kT+\tau \\ 0 & \text{otherwise} \end{cases} \quad , \tag{10}$$

where the second lower subscript kT at $H_{S,kT}(\cdot)$ is used to note the association of this windowing operator with the sampling instant kT ; moreover, its duration is τ .

Note also that we use in this section another form of the impulse response of the "blurring filter". Now, $h_B(t)$ given by (9) instead of $h_G(t)$ exploited in the previous section. We do this to simplify our calculations. (Note that both of these functions describe the Dirac delta equally well in the limiting case, i.e. when $\sigma \to 0$ for $h_G(t)$ and when $\tau \to 0$ for $h_B(t)$ [10], [12].) Also, note that we assume here the same value of the duration of the function $h_B(t)$ and of the windowing operation $H_{S,kT}(\cdot)$; this enables us to simplify calculations.

Observe now that from taking into account in (8) the given form of the functions (9) and (10), it follows that the former, i.e. the convolution given by (8), results in non-zero values only if the following inequalities:

$$kT \le t-\lambda \le kT+\tau \text{ and } 0 \le \lambda \le \tau \quad , \tag{11}$$

are satisfied. Further, it follows from (11) that this holds for the following times:

$$kT + 2\tau \ge t \ge kT \tag{12}$$

and otherwise the signal $x_{okT}(t)$ is equal to zero.

See now that using (9), (10), and (12) in (8) allows us to rewrite the latter as

$$x_{okT}(t) = \begin{cases} \displaystyle\int_0^{t-kT} \frac{x(t-\lambda)}{\tau} d\lambda & \text{when } 0 \le t-kT \le \tau \\ \displaystyle\int_{t-kT-\tau}^{\tau} \frac{x(t-\lambda)}{\tau} d\lambda & \text{when } \tau \le t-kT \le 2\tau \\ 0 & \text{otherwise} \end{cases} \quad . \tag{13}$$

In the next step, let us apply the so-called mean value theorem [9], [12] to (13). As a result, we get

$$x_{okT}(t)=\begin{cases}\dfrac{x(\xi_t)}{\tau}(t-kT) \text{ with } \xi_t\in\langle 0,t-kT\rangle \\ \text{when } 0\le t-kT\le\tau \\ \\ \dfrac{x(\xi_t)}{\tau}(t-kT-\tau) \text{ with } \xi_t\in\langle\tau,t-kT\rangle \\ \text{when } \tau\le t-kT\le 2\tau \\ \\ 0 \text{ otherwise}\end{cases} \quad . \tag{14}$$

A somewhat tedious but essentially simple analysis of (14) leads to the conclusion that in the case of $\tau\to 0$ a very good approximation of this signal is the following triangular one:

$$x_{okT}(t)\cong\begin{cases}x(kT+\tau)(t-kT)/\tau \\ \text{when } kT\le t\le kT+\tau \\ \\ x(kT+\tau)(-t+kT+2\tau)/\tau \\ \text{when } kT+\tau\le t\le kT+2\tau \\ \\ 0 \text{ otherwise}\end{cases} \quad . \tag{15}$$

From (15), it follows that when $\tau\to 0$, then $x_{okT}(t)\to x(kT)\delta_{k,t/T}$, where $\delta_{k,t/T}$ means the so-called shifted Kronecker time function [13], [34], [35]. And, obviously, $x(kT)\delta_{k,t/T}$ (unexpected result) differs from $x(kT)\delta(t-kT)$ (expected result). Thus, a realistic modelling of signal sampling operation does not lead to the appearance of Dirac pulses in the limit case (ideal case), as it would be expected by researchers who believe in the current model.

IV. Conclusion

The analysis presented in this paper shows that the currently used models for modelling the signals produced at outputs of A/D converters lead to their contradictory descriptions (and this, of course, is an unexpected result, not recognized until now by the researchers working in the area of signal processing). The problem here is a correct description of the time waveforms at the outputs of A/D converters for the purposes of spectral analysis of these signals. A consistent and unified model for the above tasks is still waiting to be invented.

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