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Research paper

Preliminary optimization technique in the design of steel girders according to Eurocode 3

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Abstract: The problem of optimal design of a steel plated girder according to the Eurocode 3 is considered. Code regulations admit the Finite Element Analysis (FEA) in designing plated structures with variable cross-sections. A technique of determining an approximate solution to the optimization problem is presented. It is determined a solution of a control theory optimization task, in which Eurocode requirements regarding the Ultimate Limit State (bearing capacity, local and global stability) as well as Serviceability Limit State (flexural rigidity) are used as appropriate inequality constraints. Static analysis is performed within the framework of linear elasticity and Bernoulli-Euler beam theory making an account for second-order effects due to prescribed imperfections. Obtained solutions, after regularization, may be used for direct verification with the use of FEA or as the first guess for iterative topology optimization algorithms. Code requirements governing the determination of optimal shape are visualized in the constraint activity diagram, which is a proposed tool for analysis of optimization process.

Keywords: Eurocode, imperfections, Pontryagin's Maximum Principle, second order effects, steel structure, structural optimization

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1. Introduction

General aim of any design process in civil engineering may always be considered as an optimization task regarding chosen objective and constraints – most commonly, it is minimizing the total cost of construction while meeting all building code requirements. In practice, it is usually done by designers with the use of a conceptually simple approach of "trial and error". Automatization of such a process enables analysis of huge number of possibly optimal solutions. Yet, still an uncertainty is preserved weather the determined solution is truly optimal or not.

As opposed to the heuristic methods described above, optimization problems stated within specific mathematical framework – e.g. Euler – Lagrange equations, Bellman equation, Pontryagin's Maximum Principle – provide a solution about which it is possible to say that it fulfils optimality conditions. Mentioned formalisms provide only necessary and not sufficient condition for optimality, however, under certain convexity conditions imposed on objective and constraints functions the Pontryagin's Principle may provide also a sufficient condition [1, 2].

Both approaches, the (meta)heuristic as well as "analytical" ones, are successfully applied in the problem of structural optimization. The problem of optimal design of steel structures has been extensively investigated with the use of multiple different methods. A comprehensive study of the problem of optimization of steel structures – involving not only material consumption but also cost of fabrication, construction of joints etc. – may be found in [3]. Moving asymptotes algorithm was used in [4] in topology optimization of plane frame structures. The problem of relatively higher cost of construction works regarding structural joint motivated the introduction of a joint penalty, which results in lowering the number of joints in optimized layouts. Evolutionary algorithms were used in [5] in the task of topology optimization of perforated I-section steel beams. Genetic algorithms (GA) are also commonly used in structural optimization: both topology and element size optimization of tall buildings was performed in [6], optimization of a moment-resisting steel frame reducing the joint manufacturing costs according to a number of trade-off curves was performed in [7]. Combining fuzzy logic with GA enables reduction of computational time [8]. In [9] truss-shaped steel frames' layouts were optimized with respect to minimization of either compliance or maximal stress, making an account for both global and local stability requirements – gradient-based methods are used for topology optimization. Generalized Reduced Gradient method was used in [10] in the task of optimization of cross-section of Cold-Formed Steel channel sections being the primary load-carrying sections of a portal frame. Also Particle Swarm Optimization (PSO) algorithms emerged to be efficient the design of CFS sections [11]. Comparing PSO with Non-dominated Sorting GA suggested that PSO may emerge more efficient in optimizing 2D and 3D moment resisting steel frames [12]. PSO algorithm emerged also to be faster in optimization of steel frame than differential evolution algorithm, even despite lower convergence rate [13]. Combining PSO with appropriate cellular automata was used in layout optimization of steel trusses [14]. A very comprehensive survey on the application of metaheuristic population-based optimization algorithms may be found in [15]. The method used in current research is derived from the control theory problems in which there is only a single independent variable – time. Proposed approach is based on observation that plane bar structures may be described with a single spatial variable. The dependent state variables describe both axial and transverse displacements of each cross-section as well as corresponding internal forces according to the Bernoulli-Euler beam theory. It was shown that the formalism of Pontryagin's Maximum Principle emerges to be a useful tool in optimization of bar structures. It enables finding optimal dimensions of an assumed bar section [16–19] as well as the layout of the whole structure (arch curvature control) [20]. Control theory approach is also capable of making an account for both multiple construction and operation phases [21, 22] as well as the global stability [23]. Recently it was also shown that the method of partial discretization (one-dimensional discretization, commonly referred to as "method of lines") of a two-dimensional problem of bending of a thin elastic problem enable application of Pontryagin's Maximum principle in order to find optimal plane distribution of thickness of such plate [24].

If obtained solution is to have any practical meaning and value, it must be erectable and so it must satisfy all code requirements. Some examples of optimization procedures making an account for standard design rules may be found in [25, 26] in which gradient-based methods were used to solve optimization task.

The aim of this paper is to contribute to the methods of solving of an optimization task making an account for requirements imposed by chosen structural design standards (Eurocodes). Appendices B and C of EN 1993-1-5 [27] explicitly mention only the Finite Element Analysis as an admissible method for the design of plated structural elements of variable cross-sections. The technique proposed in this article may be considered an add-on for hybrid algorithms, which performs preliminary optimization in determining the first approximation of optimal solution.

One specific aspect of this aim is the EN 1993-1-1 [28] requirements regarding verification of local and global stability. A common approach is to use the buckling length coefficients – for more complex structural systems their values cannot be easily determined, so they are often overestimated for safety. An optimal approach regarding this issue is to perform the second-order analysis with initial imperfections – then the bearing capacity of the cross-section is no longer reduced, however, refined static analysis must be performed.

1.1. Assumptions on performed analysis

The problem considered in this paper is minimizing the total structural volume of material used in design of structure of assumed class. The plane deformation of one-dimensional structure is analysed with an account for second order effects within the framework of linear elasticity. Plastic deformation is disregarded in the design process. It is assumed that due to the constructional solution, the girder is not susceptible to out of plane buckling and lateral torsional buckling. Structural connections of members and their parts are not the subject of this analysis.

2. General description of the structure

The structure under consideration is a symmetric three-span I-section steel beam (Fig. 1). All span lengths are assumed to be equal $L = 6$ m.

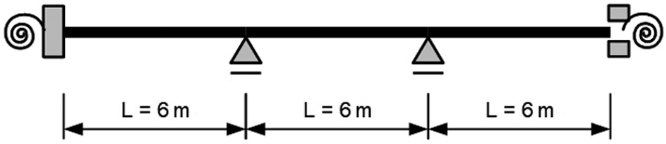

Fig. 1. Layout of designed steel beam

Global stability analysis of a plane or spatial structure would require non-local problem formulation which is beyond the scope of applicability of presented approach – for this reason only girder is analysed and its interaction with posts is modelled with the use of flexible supports the stiffness of which is equal to:

$$k_{\phi} = \frac{4EI_{\text{col}}}{H} \tag{2.1}$$

where height of the column $H = 3$ m, and flexural rigidity EI_{col} is assumed to be the same as the rigidity of the joist connected with it:

$$EI_{\text{col,left}} = EI_{\text{beam}}(x = 0)$$
$$EI_{\text{col,right}} = EI_{\text{beam}}(x = 3L)$$

The beam is made from steel S235 (according to EN 10025-2) of Young modulus $E = 210$ GPa, Kirchhoff modulus $G = 81$ GPa, Poisson ratio $\nu = 0.3$ and characteristic yield strength $f_y = 235$ MPa. Material density is assumed to be $\rho_s = 7850$ kg/m^3.

3. Actions on the structure

Following loads are assumed to be applied to the structure: Structure's and finishing elements' (trapezoidal sheet metal coating on cold-bent steel section purlins) dead load (DL), snow load (SN) and several levels of axial compressive force (N). Snow load is determined according to EN 1991-1-3 (with Polish national annex) [29], for building located in Krakow, Poland. The optimization is performed for of uniform axial load.

The assumed magnitudes of axial force are assumed to be in relation with the critical buckling force, which is estimated for the beam with rigid supports and constant cross-section as a minimum root of the following secular equation:

$$\begin{aligned}(19\kappa 4\kappa^3)\sin^3\kappa + \left[\left(6 - 16\kappa^2\right)\cos(\kappa) + 10\kappa^2 - 6\right]\sin^2\kappa \\ + \left(20\kappa\cos(\kappa) + 3\kappa^3 - 20\kappa\right)\sin(\kappa) + 6\kappa^2\cos(\kappa) - 6\kappa^2 = 0\end{aligned} \tag{3.1}$$

Fundamental value of the critical force is equal to $N_{\text{crit}} = 3.8567EI/L^2$. Variation of flexural rigidity EI in obtained solutions determine the range of predicted value of critical force as (10 kN;189 kN).

4. Load combinations

The limit states are considered: ULS STR and SLS according to EN 1990 with Polish national annex. As to ULS STR state an alternative of combinations (6.10a/b) of EN 1990 [30] is considered. As to SLS state a characteristic combination (6.14b) of EN 1990 is considered. Preliminary analysis indicate that it is the combinations given by (6.10b) that is less favourable. Following combinations are analysed:

– Design combinations (DC) (6.10b): $\xi\gamma_G \text{DL} + \gamma_Q \text{SN} + \gamma_Q\psi_0 N$
– Characteristic combination (CC) (6.14b): $\text{DL} + \text{SN} + \psi_0 N$

where: $\xi = 0.85$, $\gamma_G = 1.35$, $\gamma_Q = 1.50$, $\psi_1 = 0.20$, $\psi_0 = 0.6$. There are six decisive live load combinations (Fig. 2).

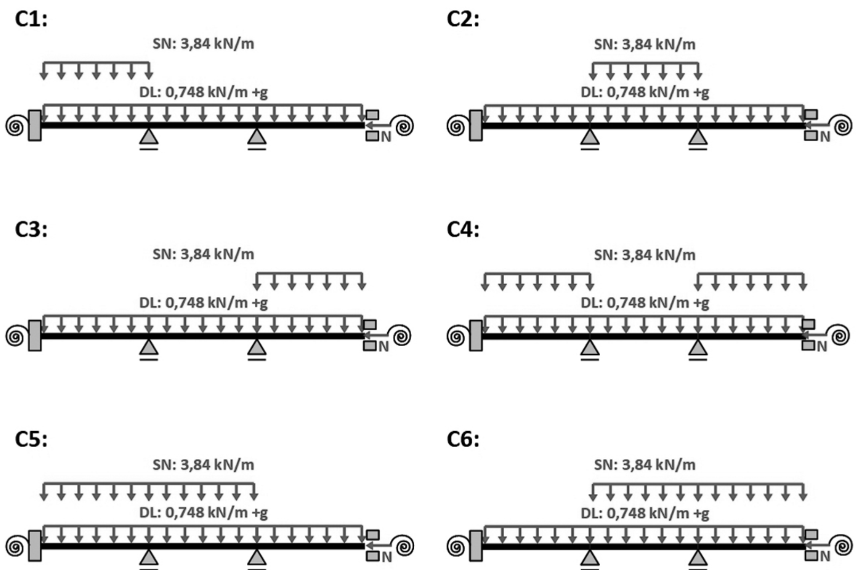

Fig. 2. Considered load combinations

Each design combination is analysed for 5 design values of an axial load $\{0, -25, -50, 75, -100\}$ kN. Combinations C2 and C4 are decisive with respect to maximum deflection in the middle and in the edge span respectively. As a result 8 load combinations are considered 6 design combinations (DC1, DC2, DC3, DC4, DC5, DC6) and 2 characteristic combinations (CC2, CC4).

5. Mathematical formulation of the problem

5.1. State variables

For each j-th load combination ($j = 1, 2, \ldots, 8$) a set of 6 state variables may introduced for a general 2D frame element:

$$
\begin{aligned}
X_1^{[j]}(x) &= u^{[i,j]}(x) \\
X_2^{[j]}(x) &= EA(x)\frac{\mathrm{d}}{\mathrm{d}x}u^{[j]}(x) = N^{[j]}(x) \\
X_3^{[j]}(x) &= w^{[j]}(x) \\
X_4^{[j]}(x) &= \frac{\mathrm{d}}{\mathrm{d}x}w^{[j]}(x) = \phi^{[j]}(x) \\
X_5^{[j]}(x) &= -EI(x)\frac{\mathrm{d}^2}{\mathrm{d}x^2}w^{[j]}(x) = M^{[j]}(x) \\
X_6^{[j]}(x) &= -EI(x)\frac{\mathrm{d}^3}{\mathrm{d}x^3}w^{[j]}(x) = Q^{[j]}(x)
\end{aligned}
\tag{5.1}
$$

where $u^{[j]}$, $w^{[j]}$, $\phi^{[j]}$ are longitudinal and transverse displacement and deflection angle of beam due to j-th load combination respectively and $N^{[j]}$, $M^{[j]}$, $Q^{[j]}$ are axial force, bending moment and shear force due to j-th load combination respectively.

5.2. State equations

State equations are derived for a linear-elastic bar making an account for initial imperfections and second-order effects. Constitutive law between curvature and bending moment is as follows:

$$
\kappa(x) \approx \frac{\mathrm{d}^2 w(x)}{\mathrm{d}x^2} = -\frac{M(x)}{EI(x)}
\tag{5.2}
$$

The bending moment distribution is expressed as follows (see Fig. 3):

$$
\begin{aligned}
M(x) = {} & M_B - Q_B \cdot (L - x) + N_B \cdot (w_B - w(x) - w_0(x)) \\
& - \int_{y=x}^{L} q(y) \cdot (y - x)\,\mathrm{d}y + \int_{y=x}^{L} n(y) \cdot (w(y) + w_0(y) - w(x) - w_0(x))\,\mathrm{d}y
\end{aligned}
\tag{5.3}
$$

where $w_0(x)$ is the initial deflection due to imperfection. Eurocode 3 suggests to consider the imperfection of member's axis as a sum of sway and bow imperfection, the magnitudes of which depend on geometry of structure.

Eq. (5.2) and Eq. (5.3) lead to the integro-differential equation. The formalism of Pontryagin's Maximum Principle requires the problem to be stated as a system of the first

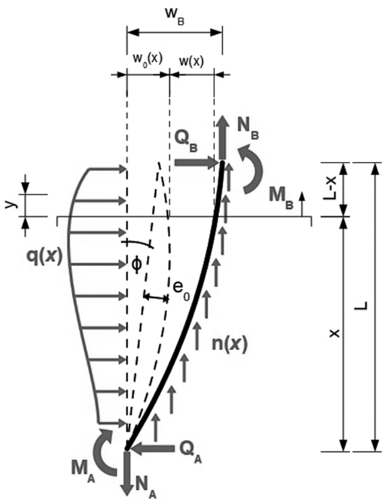

Fig. 3. Deformation and nodal loads for a beam element with global sway imperfection and local bow imperfection involving the second-order effects and load's tracking

order ordinary differential equations. The use of general Leibniz formula

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_{a(x)}^{b(x)} f(x,y)\,\mathrm{d}y = \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x,y)\,\mathrm{d}y + f(x,b(x))\frac{\partial b}{\partial x} - f(x,a(x))\frac{\partial a}{\partial x} \tag{5.4}$$

lead us to the following state equations for a single member:

$$\begin{aligned} \frac{\mathrm{d}^3 w}{\mathrm{d}x^3} &= -\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{M(x)}{EI}\right] \\ &= \frac{1}{EI}\left[-Q_B - \int_{y=x}^{L} q(x)\,\mathrm{d}y + \left(N_B + \int_{y=x}^{L} n(y)\,\mathrm{d}y\right)\cdot\left(\frac{\mathrm{d}w}{\mathrm{d}x} + \frac{\mathrm{d}w_0}{\mathrm{d}x}\right)\right] \end{aligned} \tag{5.5}$$

$$\frac{\mathrm{d}^4 w}{\mathrm{d}x^4} = \frac{1}{EI}\left[q(x) + \left(N_B + \int_{y=x}^{L} n(y)\,\mathrm{d}y\right)\cdot\left(\frac{\mathrm{d}^2 w}{\mathrm{d}x^2} + \frac{\mathrm{d}^2 w_0}{\mathrm{d}x^2}\right) - n(x)\cdot\left(\frac{\mathrm{d}w}{\mathrm{d}x} + \frac{\mathrm{d}w_0}{\mathrm{d}x}\right)\right] \tag{5.6}$$

Assuming that distributed normal load is constant along member's axis allow us to write:

$$\frac{\mathrm{d}^3 w}{\mathrm{d}x^3} = \frac{1}{EI}\left[-Q_B - \int_{y=x}^{L} q(x)\,\mathrm{d}y + [N_B + n\cdot(L-x)]\cdot\left(\frac{\mathrm{d}w}{\mathrm{d}x} + \frac{\mathrm{d}w_0}{\mathrm{d}x}\right)\right] \tag{5.7}$$

$$\frac{\mathrm{d}^4 w}{\mathrm{d}x^4} = \frac{1}{EI}\left[q(x) + [N_B + n\cdot(L-x)]\cdot\left(\frac{\mathrm{d}^2 w}{\mathrm{d}x^2} + \frac{\mathrm{d}^2 w_0}{\mathrm{d}x^2}\right) - n\cdot\left(\frac{\mathrm{d}w}{\mathrm{d}x} + \frac{\mathrm{d}w_0}{\mathrm{d}x}\right)\right] \tag{5.8}$$

The above fourth order differential equation may be rewritten as a system of the first order equations of the following form:

$$\frac{\mathrm{d}w}{\mathrm{d}x} = \varphi, \qquad \frac{\mathrm{d}\varphi}{\mathrm{d}x} = -\frac{1}{EI}\left[m + M_B + N_B \cdot w_B\right]$$

$$\frac{\mathrm{d}m}{\mathrm{d}x} = t + Q_B + \left[\left[N_B + n(L-x)\right]\frac{\mathrm{d}w_\phi}{\mathrm{d}x}\right]$$

$$\frac{\mathrm{d}t}{\mathrm{d}x} = -\left[q + \left[N_B + n(L-x)\right]\left(-\frac{1}{EI}\left[m + M_B + N_B w_B\right] + \frac{\mathrm{d}^2 w_e}{\mathrm{d}x^2}\right) - n\left(\varphi + \frac{\mathrm{d}w_e}{\mathrm{d}x}\right)\right]$$

Derivatives of the initial deflection according to EN 1993-1-1 are as follows:

$$\frac{\mathrm{d}w_e}{\mathrm{d}x} = \pm\frac{4e}{L^2}(2x - L), \qquad \frac{\mathrm{d}^2 w_e}{\mathrm{d}x^2} = \pm\frac{8e}{L^2}, \qquad \frac{\mathrm{d}w_\phi}{\mathrm{d}x} = \phi$$

In the present analysis no account for sway imperfection w_ϕ as well as axial distributed load n is made and end-node deflection w_B is assumed 0 for each span. For each j-th load combination ($j = 1, 2, \ldots, 8$) a following system of governing equations must be thus satisfied:

$$(5.9) \qquad \begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}x}X_1^{[j]}(x) = \dfrac{1}{EA(x)}\left[X_2^{[j]}(x)\right] \\ \dfrac{\mathrm{d}}{\mathrm{d}x}X_2^{[j]}(x) = 0 \\ \dfrac{\mathrm{d}}{\mathrm{d}x}X_3^{[j]}(x) = X_4^{[i,j]}(x) \\ \dfrac{\mathrm{d}}{\mathrm{d}x}X_4^{[j]}(x) = -\dfrac{1}{EI(x)}\left[X_5^{[i,j]}(x)\right] \\ \dfrac{\mathrm{d}}{\mathrm{d}x}X_5^{[j]}(x) = X_6^{[i,j]}(x) \\ \dfrac{\mathrm{d}}{\mathrm{d}x}X_6^{[j]}(x) = -\left[q(x) + N_B \cdot \left(-\dfrac{1}{EI(x)}X_5^{[i,j]}(x) + \dfrac{\mathrm{d}^2 w_e}{\mathrm{d}x^2}\right)\right] \end{cases} \qquad j = 1, \ldots, 8$$

Axial displacements and forces are determined uniquely by assumed axial load and thus they are excluded from the problem formulation. Total number of state variables is 32 (Table 1).

Table 1. Layout of designed steel beam

	Load combination							
	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$	$j = 6$	$j = 7$	$j = 8$
$w^{[j]}$	X_1	X_5	X_9	X_{13}	X_{17}	X_{21}	X_{25}	X_{29}
$\phi^{[j]}$	X_2	X_6	X_{10}	X_{14}	X_{18}	X_{22}	X_{26}	X_{30}
$M^{[j]}$	X_3	X_7	X_{11}	X_{15}	X_{19}	X_{23}	X_{27}	X_{31}
$Q^{[j]}$	X_4	X_8	X_{12}	X_{16}	X_{20}	X_{24}	X_{28}	X_{32}

5.3. Control variables

We introduce following control variables (Fig. 4): U_1 – width of flanges, U_2 – height of web, U_3 – thickness of flanges, U_4 – thickness of web.

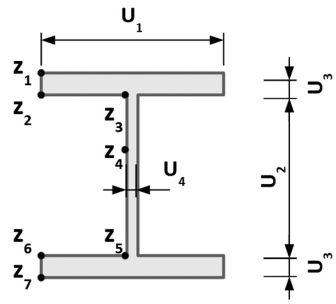

Fig. 4. Dimensions of I-section as control variables

5.4. Cross-sectional forces

Transverse shear force Q and bending moment M – in i-th section of the beam in the j-th load combination may be expressed as follows:

$$Q^{[j]}(x) = X_{4(j-1)+4}(x)$$
$$M^{[j]}(x) = X_{4(j-1)+3}(x) \tag{5.10}$$

Geometrical characteristics of the I-profile cross-sections are as follows:

$$A(x) = 2U_1(x)U_3(x) + U_2(x)U_4(x) \tag{5.11}$$

$$I(x) = \frac{U_4(x)\,[U_2(x)]^3}{12} + 2\cdot\frac{U_1(x)\,[U_3(x)]^3}{12} + 2U_1(x)U_3(x)\left(\frac{U_2+U_3}{2}\right)^2 \tag{5.12}$$

5.5. Boundary and compatibility conditions

Boundary conditions for each j-th load combination ($j = 1, \ldots, 8$) are:

$$\begin{cases} w^{[j]}(0) = 0 \\ \phi^{[j]}(0) = -\dfrac{1}{k_\phi}M^{[j]}(0) \end{cases} \tag{5.13}$$

$$\begin{cases} w^{[j]}(3L) = 0 \\ \phi^{[j]}(3L) = \dfrac{1}{k_\phi}M^{[j]}(3L) \end{cases} \tag{5.14}$$

Displacement and force compatibility conditions between spans for each i-th load combination ($i = 1, \ldots, 8$) are:

$$\begin{cases} w^{[i]}\left(L^-\right) = 0 & \text{zero deflection at support} \\ w^{[i]}\left(L^+\right) = 0 & \text{zero deflection at support} \\ \phi^{[i]}\left(L^-\right) = \phi^{[j]}\left(L^+\right) & \text{angle of deflection continuity} \\ M^{[i]}\left(L^-\right) = M^{[j]}\left(L^+\right) & \text{bending moment continuity} \end{cases} \tag{5.15}$$

$$\begin{cases} w^{[i]}\left(2L^-\right) = 0 & \text{zero deflection at support} \\ w^{[i]}\left(2L^+\right) = 0 & \text{zero deflection at support} \\ \phi^{[i]}\left(2L^-\right) = \phi^{[j]}\left(2L^+\right) & \text{angle of deflection continuity} \\ M^{[i]}\left(2L^-\right) = M^{[j]}\left(2L^+\right) & \text{bending moment continuity} \end{cases} \tag{5.16}$$

We have 32 boundary conditions and 64 compatibility conditions in total.

6. ULS code requirements in form of inequality constraints

An account for design requirements of EN 1993 is made in the formalism of Pontryagin's Maximum Principle by introduction of proper set of inequality constraints.

6.1. Class of the cross-section

Class 3 cross-section is assumed, as no account for plastic deformation is made in the mathematical formulation of the problem. It must be noted, however, that the use of class 1 or class 4 sections may potentially provide a more favourable solution to the problem. In the considered load combinations no tensile axial force occurs so it may be assumed that ratio of edge stresses in web $\psi > -1$. Under this assumption following inequality constraints must be satisfied in order to consider the cross-section as being class 3:

$$\frac{U_2}{U_4} \le \frac{42\varepsilon}{0.67 + 0.33\psi} \tag{6.1}$$

$$\frac{U_1 - U_4}{U_3} \le 28\varepsilon \tag{6.2}$$

where $\psi = \sigma_{\max}/\sigma_{\min}$, $\varepsilon = \sqrt{235\ \text{MPa}/f_y}$ and $\sigma_{\max}$, $\sigma_{\min}$ are the signed values of normal stress at web edges – maximal and minimal, respectively $(\sigma_{\max} \ge \sigma_{\min})$.

6.2. Shear lag effect

Eurocode 3 part 1–5 [27] allow for neglecting the shear lag effect if

$$\frac{U_1 - U_4}{2} < \frac{L_e}{50} \tag{6.3}$$

where L_e is the effective span length: $L_e = 0.85L$ for edge span, $L_e = 0.7L$ for intermediate span and $L_e = 0.5L$ for supported cross-section. It is assumed that large rigidity of cross-section fixed to the column in corner joint prevents the occurrence of the shear lag effect.

6.3. Web instability due to shear stresses

Stability of a not-ribbed web against shear stresses is provided if only:

$$\frac{U_2}{U_4} < 72\varepsilon\frac{1}{\eta} \tag{6.4}$$

where η is a coefficient depending on grade of steel. For chosen steel S235 $\eta = 1.2$.

6.4. Bending with shearing and axial force – point strength verification

Limit state condition for class 3 cross-sections under combined load is given by equations:

$$\left(\frac{\sigma_x}{f_y/\gamma_{M0}}\right)^2 + 3\left(\frac{\tau_{xz}}{f_y/\gamma_{M0}}\right)^2 \leq 1,0 \tag{6.5}$$

$$\sigma_x(x,z) = \frac{N_{Ed}(x)}{A} + \frac{M_{Ed}(x)}{I}z \tag{6.6}$$

$$\tau_{xz}(x,z) = \frac{S(z)V_{Ed}(x)}{b(z)I} \tag{6.7}$$

Design values of cross-sectional forces $N_{Ed}M_{Ed}V_{Ed}$ are in general functions determined by state variables according to Eq. (5.1). Partial safety factor for material $\gamma_{M0} = 1.0$. Instead of line search along z-axis for maximum equivalent stress, the limit state condition will be checked only at 7 characteristic points denoted in the Fig. 4 as z_k $(k = 1, 2, \ldots, 7)$.

7. SLS code requirements in form of inequality constraints

SLS is verified by limiting instant reversible elastic deformation according to National Annex to EC 3 part 1–1. Allowable deflection of beam is equal to:

$$u_{\max} = w^{[j]}(x) \leq \frac{L}{250}, \quad j = 7, 8 \tag{7.1}$$

where index j corresponds with characteristic combinations CC2, CC4. Long-term actions constitute a small fraction of total load no rheological effects need to be accounted for.

8. Optimization task structure

8.1. Objective function

The volume minimization was assumed to be an optimization criterion. The total structural volume of material used may be expressed as follows:

$$V = \int_0^{3L} A(x)\,\mathrm{d}x \tag{8.1}$$

An additional state variable $X_{33}(x) = V(x)$ and state equation was assumed:

$$\frac{\mathrm{d}}{\mathrm{d}x} X_{33}(x) = A(x) \tag{8.2}$$

The optimization problem of the Lagrange functional in the form:

$$J\left(\mathbf{X}(x), \mathbf{U}(x)\right) = \int_0^{3L} A(x)\,\mathrm{d}x \tag{8.3}$$

where $\mathbf{X}(x)$, $\mathbf{U}(x)$ are vectors of state variables and controls respectively, is transformed into a Mayer problem with functional

$$J\left(\mathbf{X}(x), \mathbf{U}(x)\right) = X_{33}\left(\mathbf{X}(3L), \mathbf{U}(3L)\right) \tag{8.4}$$

and initial condition $X_{33}\left(\mathbf{X}(0), \mathbf{U}(0)\right) = 0$.

9. Results

The optimization task described above was solved with the use of Dircol software [31]. Performed calculation were convergent for the design vales of axial load not greater than approximately 100 kN. In the first stage of optimization all 4 controls were considered variable. Plate thickness was assumed to be not less than 3 mm. Range of variation of considered controls in obtained solutions determined an approximate optimal constant value. Then, thickness of flanges U_3 as well as thickness of web U_4 were assumed constant according to results obtained from multiple tries. The result of performed analysis was to assume common dimensions $U_3 = 4.5$ mm and $U_4 = 3$ mm for all considered magnitudes of axial force. This determines $e_0 = L/250$. Web height and flange width changes along beams length are presented in Fig. 5–9 for five chosen values of axial force.

In Table 2 resulting total material volume of the girder corresponding with each value of the compressive force is presented.

Obtained solutions give an insight into the structure of optimal solution, in particular it is possible to observe which one of the prescribed constraints is decisive in determining the

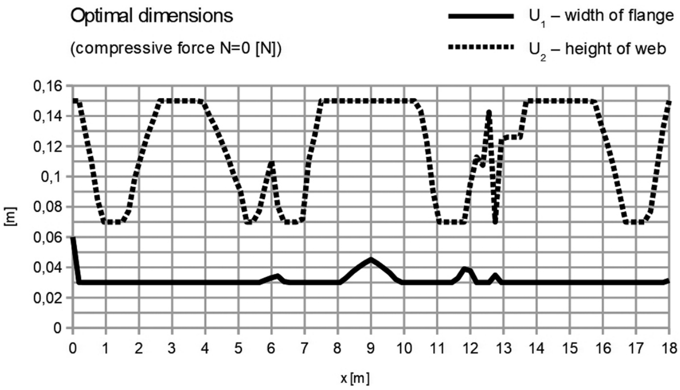

Fig. 5. Flange width and web height distribution in optimized girder – compressive force $N = 0$ N

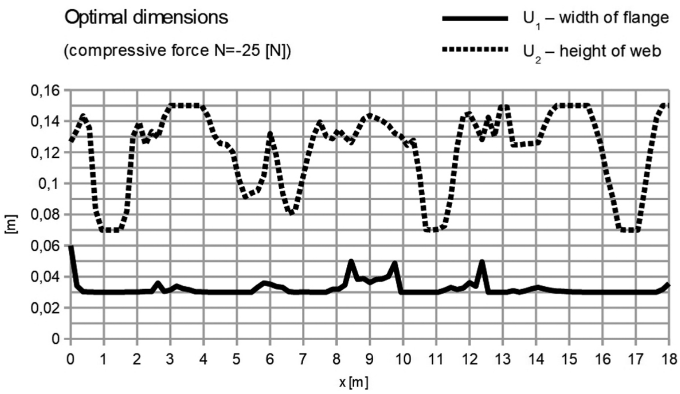

Fig. 6. Flange width and web height distribution in optimized girder – compressive force $N = -25$ N

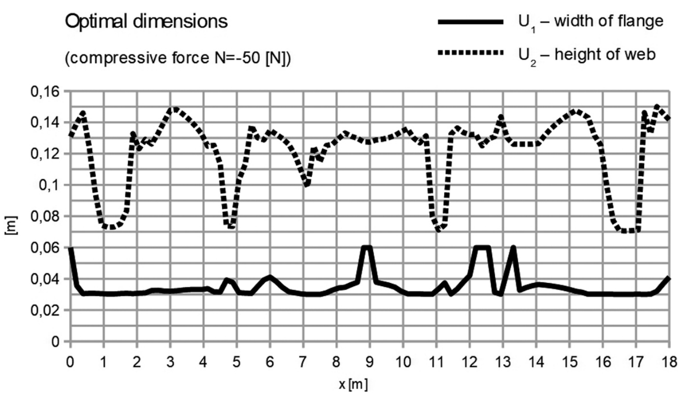

Fig. 7. Flange width and web height distribution in optimized girder – compressive force $N = -50$ N

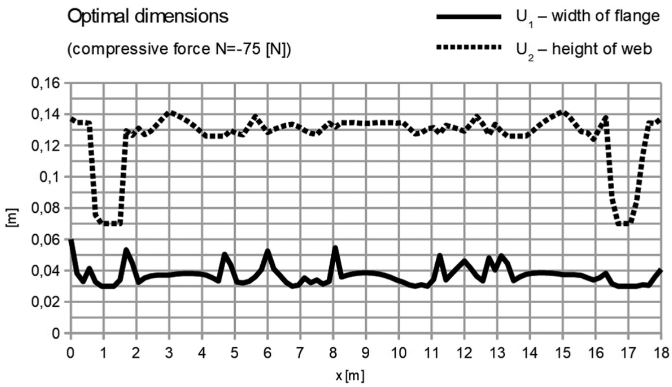

Fig. 8. Flange width and web height distribution in optimized girder – compressive force $N = -75$ N

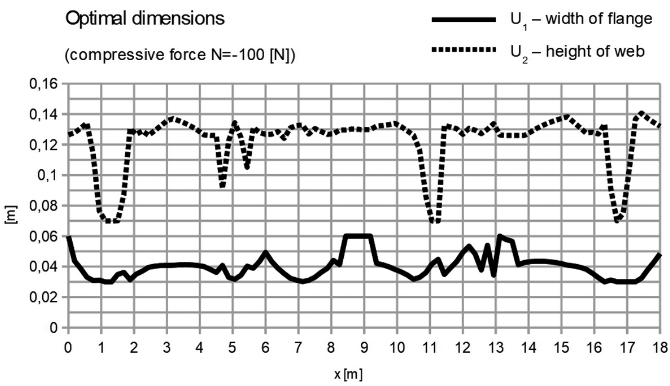

Fig. 9. Flange width and web height distribution in optimized girder – compressive force $N = -100$ N

Table 2. Considered load combinations

N [kN]	V [m^3]
0	0.011468
–25	0.011750
–50	0.0122183
–75	0.012869
–100	0.013130

optimal control. Equality or inequality constraint is termed to be "active", if the equality holds. Such an active constraint is the one which shapes the optimal solution Constraint activity diagram is presented in Fig. 10.

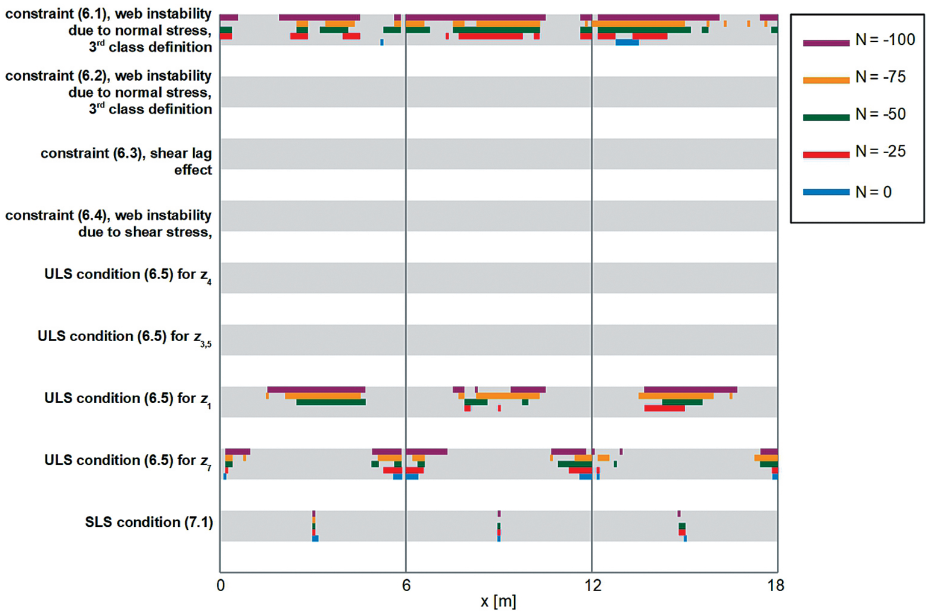

Fig. 10. Constraint activity diagram

Obtained solutions are obviously unconstructible – determined optimal controls need to be regularized. Actual dimensions of optimized girder may be determined as a piece-wise linear upper-bound envelope of controls (see: Fig. 11). It is also important to note, that – unless the girder is a subject of a large-scale repetitive fabrication – high cost of construction of a section of variable dimensions thwarts any possible savings due to smaller consumption of material. In such cases a more efficient solution is to use girders with constant height of web and width of flanges.

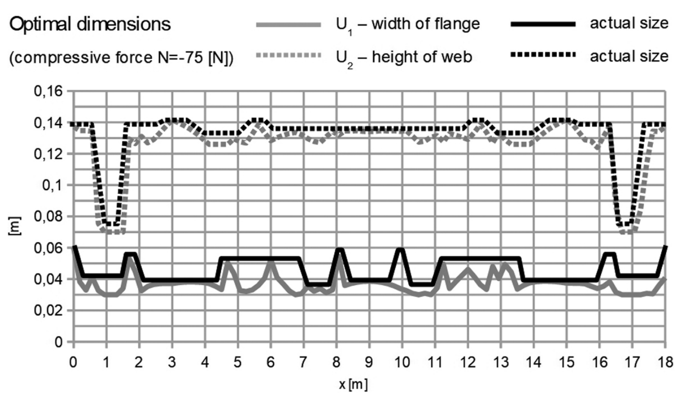

Fig. 11. Symmetric upper-bound envelope determining actual dimensions of optimized girder

As it was emphasized in the introduction, obtained solution cannot be considered an optimal design of steel girder unless the SLS and ULS are verified. According to Appendices B and C of EN 1993-1-5, it may be done with the use of Finite Element Method. Such a verification need to take into account finite-strain deformation, imperfections, global buckling modes as well as assumed connections (welding, bolting). Such a verification is beyond the scope of this research. Determined solution may be also used as a first try for iterative topology optimization algorithms.

10. Summary and conclusions

The article deals with the problem of first-step approximation in the problem of optimization of a steel girder according to the Eurocode regulations. For the sake of simplicity of examples of application of the presented preliminary optimization technique it has been assumed that deformation is plane and interaction with vertical structural elements has been modelled with flexible supports. An optimization task was formulated within the framework of control theory. It was solved with the use of Pontryagin's Maximum Principle in Dircol software. Thicknesses of flange and web, which were assumed to be constant for the whole girder, were first estimated assuming that corresponding control may vary within certain range. Then variable flange width and web's height were determined. Total required volume of material is obviously the smaller, the smaller is the axial compressive force applied to the beam. The formulated optimization task is characterized by following specific features:

– It is an autonomous optimization problem. In such cases resulting Hamiltonian should be piece-wise constant, what was indeed observed up to precision of a numerical solution.
– Boundary conditions for the considered BVP may be classified as the 3rd type (or Robin type) boundary conditions. They depend on controls which are not known in advance.
– Numerical solutions of the considered formulation are very sensitive to initial values in iterative solution procedures – this regards in particular the range of admissible values of controls.

Regarding the obtained solutions, one may conclude that obtained solutions satisfy the optimality condition with precision 10^{-4} and feasibility condition with precision 10^{-5}. Finding optimal solution required approximately 4000–5000 iterations with total computation time approximately 10–40 seconds. The statement of the problem is symmetric, so should be the optimal solution. The symmetry of the solution was not imposed in advance. It can be seen that it is only approximately preserved. Satisfying necessary condition for optimality by non-symmetric solutions may be explained either by finding a saddle point (sufficient condition is violated) or by finding only a local optimum. Regarding the determined optimal shape and constraint activity diagram one may notice that:

– Magnitude of compressive force does not influence the maximum and minimum sizes of height of web and width of flange in considerable way. For larger magnitudes of

axial force, larger dimensions are required in larger parts of the section, compared to the solutions obtained for smaller magnitudes of force.
- Second-order effects may significantly influence the distribution of cross-sectional forces, amplifying their extremal values. Magnitude of this amplification depends on magnitude of axial force and on load combination. Maximum value of compressive force amplified the maximum transverse shear force with 0–10%, compared to beam without axial force. Maximum bending moments were similarly amplified with 0–30%.
- The dominant role in determining the optimal shape of the girder plays the ULS condition regarding the strength against normal stress (top and bottom fibres) as well as the condition regarding web stability due to normal stress.
- The greater is the magnitude of compressive force, the larger is the zone in which the ULS and stability constraints are active.

Obtained solutions, however, do determine the girder's geometry which may be then directly verified with the use of FEA. These solutions may also provide a first guess try in topology optimization tasks, speeding up the process of optimization with the use of more sophisticated methods. Proposed method may be generalized for the case of three-dimensional structures, however, complexity and size of the optimization task becomes significantly larger.

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Metoda wstępnej optymalizacji w projektowaniu dźwigarów stalowych zgodnie z Eurokodem 3

Słowa kluczowe: efekty drugiego rzędu, Eurokod, imperfekcje, konstrukcje stalowe, optymalizacja, zasada maksimum Pontriagina

Streszczenie:

Rozważany jest problem optymalnego projektowania blachownicy stalowej zgodnie z Eurokodem 3. Zapisy normowe dopuszczają stosowanie Metody Elementów Skończonych (MES) w projektowaniu blachownic o zmiennym przekroju poprzecznym. Przedstawiono metodę wyznaczania przybliżonego rozwiązania zagadnienia optymalizacji. Jest ono wyznaczane jako rozwiązanie problemu optymalizacyjnego teorii sterowania, w którym wymagania Eurokodu dotyczące Stanu Granicznego Nośności (nośność, lokalna i globalna stateczność) i Stanu Granicznego Użytkowalności (sztywność giętna) wykorzystane są jako ograniczenia nierównościowe. Analiza statyczna przeprowadzona jest w ramach liniowej teorii sprężystości dla modelu belki Bernoulliego – Eulera z uwzględnieniem efektów drugiego rzędu z uwagi na zadane imperfekcje. Uzyskane rozwiązania, po stosownych modyfikacjach, mogą podlegać weryfikacji z wykorzystaniem MES lub mogą zostać wykorzystane jako pierwsze przybliżenie w iteracyjnych algorytmach optymalizacji topologicznej. Wymagania normowe rządzące wyznaczaniem optymalnego kształtu zostały zwizualizowane na schemacie aktywności ograniczeń, który proponowany jest jako narzędzie analizy procesu optymalizacji.

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