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Research paper

Effect of the hole size and the open area value on the effective Young's modulus of perforated sheets with a right pattern of cylindrical holes

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Abstract: Perforated sheets are materials which – maintaining good mechanical properties – are characterized by reduced mass in comparison to full sheets. Their elastic properties are important features that are considered in the context of these materials' design applications. Compared to full sheets, they are characterized by reduced mass while simultaneously preserving good strength properties. This article presents an experimental and numerical analysis of the effect of key parameters of the hole mesh (open area, hole diameter and orientation relative to the direction of greatest hole concentration) in association with the type of material and sheet thickness t on the value of the effective Young's modulus of perforated sheet. A significant influence of open area (the share of holes in the sheet, as a percentage) and orientation angle was determined. On the basis of experimental results and computer simulations, a mathematical dependency allowing for calculation of this parameter's value was proposed. The average deviation of calculated values from experimental is less than 4%.

Keywords: computer simulation, effective Young's modulus, FEM, mathematical formula, perforated sheet

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1. Introduction

Perforated sheets have broad applications, from mining and construction, through electronics, all the way to art [1–3]. Compared to full sheets, they are characterized by reduced mass while simultaneously preserving good strength properties. This means that it is necessary to precisely determine the functional properties of structural materials of this type, with special consideration of elastic properties. Determination of the value of Young's modulus and Poisson's ratio is possible both experimentally and numerically.

The properties of perforated sheets largely depend on the perforation parameters. The presence of holes causes strong anisotropy of mechanical properties, above all. These properties depend on the type of perforation, open area value, hole shape and profile, as well as the methods of making holes [1,2,4,5]. This means that a correct experimental and mathematical description of the behavior of materials of this type under various loads is more complex than in the case of full sheets.

Metalworking of perforated sheets, particularly their deformation limit, lies within the area of scientists' interests [1, 6–10]. Issues of determining dependencies of their elastic properties on perforation parameters are also studied. In the work of Muzykiewicz et al. [2], perforated sheet with a straight configuration of cylindrical holes, made of deep drawing steel, was subjected to experimental analysis. Both the influence of perforation itself and the orientation of load with respect to the direction of greatest hole concentration were analyzed. High anisotropy of properties, related to the presence of holes in the sheet, and a drop in average values of the tested parameters were determined. A method of determining the weight loss factor of perforated sheet was proposed for the case of the same material and the same sheet thickness. Article [4] analyzes the influence of a sheet's relative density on its stiffness. It was determined that as the ratio of the effective density of perforated sheet to the density of full sheet rises, its stiffness increases curvilinearly, particularly within the range from 0.6 to 1 of the relative density value. A linear dependency was accepted in the remaining range (below 0.6). In article [11], Malkin presented a theoretical model allowing for determination of effective elastic properties, derived on the basis of the deformation energy of the elementary area between holes. Works [12, 13] examine thin perforated sheets with a triangular configuration of holes and small bar width. Among other things, a formula is presented, by means of which it was possible to determine values of effective Young's moduli for constant values of other parameters. Articles [14, 15] investigate the elastic properties of perforated sheets of varying thickness and varying Poisson ratios of the base material. Studies were carried out through bending and tensile tests. Differences were observed between values obtained based on Malkin's and Horvay's models and values obtained experimentally. A classification of perforated sheets according to the ratio of the material's thickness g to hole diameter d was proposed. It was determined that in the case of g/d lower than 2, there is a significant difference in the value of the effective Young's modulus and Poisson ratio determined in bending and tensile tests. For g/d values greater than 2, the studied elastic properties, determined on the basis of both tests, were similar. Work [16] conducted numerical and experimental analysis of the influence of the hole pitch to diameter ratio on the elastic properties of perforated sheets. A similar analysis was carried out in articles [17, 18]. In these works, a decrease in the value of effective elastic properties was determined, particularly in the case of small distances between holes.

In the present article, the influence of selected factors on the value of the effective Young's modulus (E') was analyzed experimentally and numerically. An attempt was made to determine a mathematical dependency based on experimental and simulation results, allowing for determination of the E' value depending on the most important parameters: open area P and angle of sample orientation γ (direction of sheet loading relative to direction of greatest hole concentration) of a perforated sheet with a right pattern of cylindrical holes.

2. Research methodology

In contrast to the modulus of elasticity of a full sheet, the effective Young's modulus of a perforated sheet is not a material constant. It depends on the perforation parameters and properties of the parent sheet, such as: type of perforation, percentage share of holes (open area), hole shape, orientation (angle of measurement) relative to the direction of greatest hole concentration, as well as the type of material, anisotropy of sheet properties, its thickness and temperature.

For the purposes of this research, one type of perforation was adopted – a right pattern of cylindrical holes of varying diameter, uniformly distributed throughout the sheet. Analysis of the influence of open area P value (Eq. (2.1) [2]), sample orientation relative to the direction of greatest hole concentration φ and thickness t on the elastic properties of perforated sheet made of different materials was conducted. A mathematical dependency was proposed, allowing for determination of the value of the effective Young's modulus for an open area within the range from 0 to 0.785 (the open area value at which the distance between hole centers is equal to the hole diameter and bar width is equal to 0) and for different sheet orientations. For this purpose, experimental and numerical tests were carried out on sheets with varying hole diameters (2 and 10 mm – experiment; 2, 5 and 10 mm – simulation), open area (from 0.0035 to 0.747) and thickness values (from 0.25 to 8 mm). As a rule, an identical open area value was accepted, regardless of hole diameter. The distance between hole centers s (so-called pitch) was the variable.

$$P = 0.785 \cdot \frac{d^2}{s^2} \tag{2.1}$$

Experimental tests were conducted similarly as described in work [2]. For this purpose, samples were cut at three angles (0°, 45°, 90°) with respect to the direction of greatest hole concentration from perforated sheets made of EN AW-1050A aluminum alloy in H14 state (yield point YS = 99 MPa, tensile strength UTS = 107 MPa, elongation A = 12%) with a thickness of 1 mm. The 0° direction corresponded to the sheet rolling direction. Every sample was subjected to ten cyclic deformations (tension – unloading) within the elastic range, determined based on a static tensile test. For each variant of orientation, five samples were used. Measurement of deformation was conducted by means of an external biaxial Instron extensometer with a measuring resolution of ±0.25% (Fig. 1). In effect, a series of force-elongation charts were obtained. Force values were divided by the total cross-section of the sample subjected to tension. Based on the $\sigma - \varepsilon$ charts obtained in this way, values of the effective Young's modulus were determined for the given direction of measurement using linear regression (Fig. 2). The average value of the effective Young's modulus for the sheet was determined from the dependency given by Eq. (2.2).

$$E'_{av} = \frac{E'_0 + 2E'_{45} + E'_{90}}{4} \tag{2.2}$$

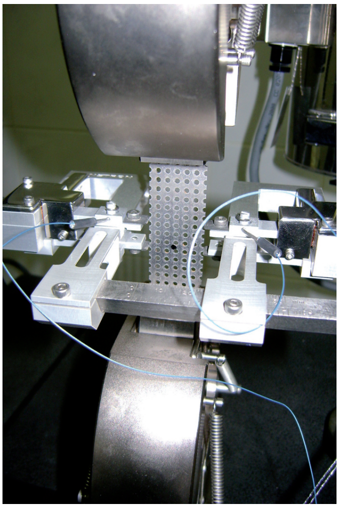

Fig. 1. An example of a perforated sheet sample during stretching with an extensometer

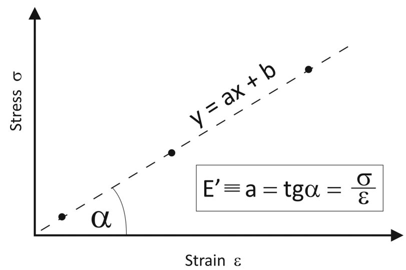

Fig. 2. Scheme of Young's modulus determination

Perforated sheets with significantly different values of the base material's Young's modulus (Table 1) were subjected to numerical analysis. Simulation of uniaxial tension was carried out in SolidWorks commercial software. Geometric models of perforated

Table 1. Basic properties of base materials used in numerical analysis

Material	Young's modulus, GPa	Poisson's ratio	Yield point, MPa	Ultimate tensile strength, MPa	Elongation after fracture, %
1050A-H14	69	0.33	103	110	10
M1E-R240	120	0.34	200	250	15
S355JR	210	0.28	275	450	12

sheets, for the given direction, were made with an identical number of holes, regardless of open area and hole diameter. The model's fastening and the method of its loading are presented in Fig. 3. A solid mesh composed of three-dimensional solid tetrahedral elements and the curvature-based mesher was used. This allowed the mesh to be densified in the area of the holes. The maximum size of the mesh element was based on the smallest model dimension and was equal to one-third of its value. The smallest value of an element was defined as half the maximum value of the element. Forces with values of 10, 50 and 100 N were applied. The maximum value of the tensile force was selected at this level (100 N) to prevent local plastic deformation in the perforated sheets. Due to the elastic nature of deformations, a linear-elastic model of the base material, described by Eq. (2.3), was applied in simulations. In the case of perforated sheets, the total deformation is the sum of material deformation and hole deformation, where the latter has the dominant effect on deformation of sheet [1,2]. Therefore, an isotropic model of the base material was used in the work. Numerical analysis was conducted for the directions of 0° and 45° with respect to the direction of greatest hole concentration (Fig. 4).

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{zx}\end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu & \nu & 0 & 0 & 0\\ \nu & 1-\nu & \nu & 0 & 0 & 0\\ \nu & \nu & 1-\nu & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{zx}\end{Bmatrix} \tag{2.3}$$

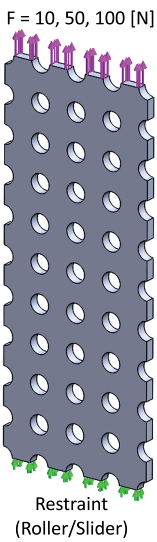

Fig. 3. Method of fastening and loading of the virtual model

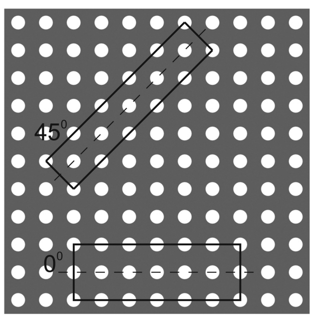

Fig. 4. Arrangement of virtual models on the sheet's plane

3. Results and discussion

3.1. Experiment

Table 2 presents values of the effective Young's modulus as well as the perforation parameters of the sheets applied in the experiment. Distributions of effective Young's modulus on the sheet's plane are given in Fig. 5. In general, higher values of the effective Young's modulus were obtained for directions conforming to the directions of greatest hole concentration (0°, 90°).

Table 2. Values of effective Young's modulus for the tested perforated sheets (standard deviation values are given in parentheses)

d mm	s mm	Percentage open area %	E'_0 GPa	E'_{45} GPa	E'_{90} GPa	E'_{av} GPa	$\Delta E'$ GPa
10	14	40.051	26.6 (0.28)	13.0 (0.30)	27.5 (0.27)	20.0	14.1
2	3	34.889	29.8 (0.27)	18.2 (0.29)	30.3 (0.28)	24.1	11.9
10	15		29.8 (0.25)	17.7 (0.35)	30.0 (0.27)	23.8	12.2
2	3	19.625	41.3 (0.26)	34.2 (0.34)	41.0 (0.29)	37.7	7.0
10	15		41.1 (0.27)	35.2 (0.27)	42.6 (0.36)	38.5	6.7
2	3	3.140	64.4 (0.31)	62.9 (0.32)	64.2 (0.28)	63.6	1.4
10	50		63.6 (0.25)	61.3 (0.27)	65.0 (0.34)	62.8	3.0
2	30	0.349	68.1 (0.20)	68.5 (0.24)	68.1 (0.22)	68.3	-0.4

For the defined percentage share of open area, average values of the modulus of elasticity for the analyzed hole diameters coincided, regardless of the open area value (Table 2). The difference did not exceed 2.5%.

The values of the index of anisotropy of distribution of effective Young's modulus E' on the plane of the perforated sheet ($\Delta E'$), given in Table 2, were determined according to the dependency given by Eq. (3.1). As the share of open area rose, so too did the value of the $\Delta E'$ index (Fig. 6). The non-uniformity of distribution of effective Young's modulus grew. This means that the dominant factor influencing the values of effective Young's modulus

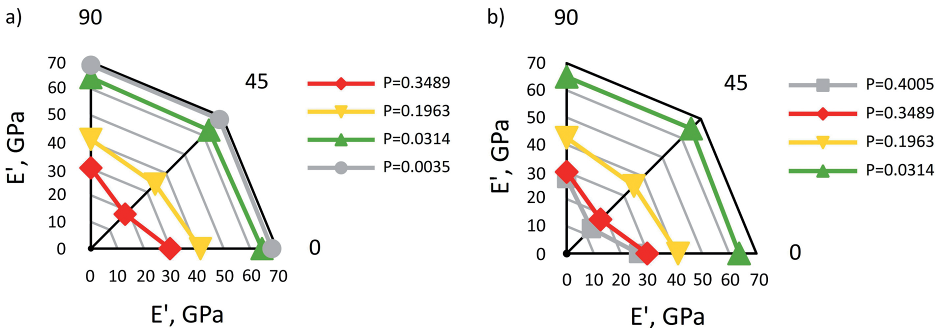

Fig. 5. Distributions of effective Young's modulus of aluminum sheets perforated with holes of diameter 2 mm (a) and 10 mm (b)

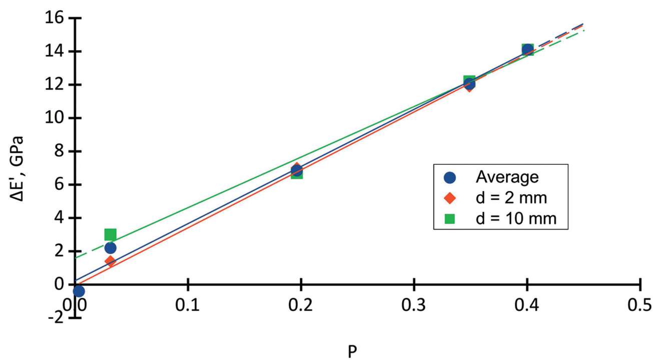

Fig. 6. Change of effective Young's modulus distribution anisotropy index $\Delta E'$ as a function of open area

of the perforated sheet and the nature of its distribution on the sheet's plane is the ratio of hole area to sheet area, particularly at higher open area values. For a small open area, the $\Delta E'$ index was similar to the value determined for full EN AW-1050A sheet, which amounted to –0.6.

$$\Delta E' = \frac{E'_0 + E'_{90} - 2E'_{45}}{2} \tag{3.1}$$

Similarly as in articles [2, 4, 5, 12, 16–18], in the case of the studied sheets, the increase in the percentage share of open area caused a drop in effective Young's modulus values, regardless of sample orientation and hole diameter. This change was curvilinear in nature, both in the case of average values and in individual directions (Fig. 7a. As the share of open area grows (with the increase of open area value), the scatter between values of the effective Young's modulus E' also grew, determined for the principal directions (0° and 90°) vs 45° (Fig. 7b).

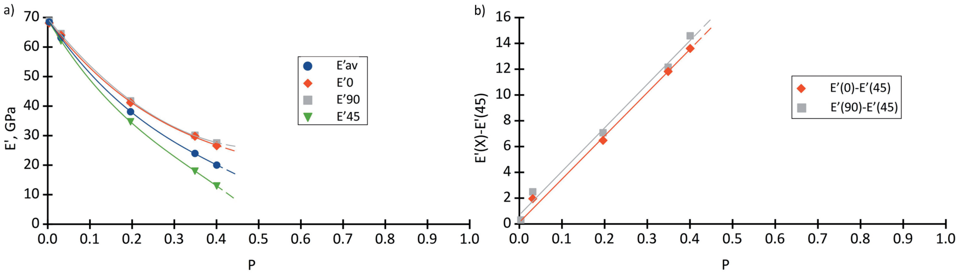

Fig. 7. Change of effective Young's modulus value (a) and difference $E'_{0(90)} - E'_{45}$ (b) as a function of open area

3.2. Simulation

Numerical simulations of tension of perforated sheet were carried out for samples with different orientations relative to the direction of greatest hole concentration, within the range of open area values from 0 to 0.785. The hole diameters were 2, 5 and 10 mm (Table 3). Similarly as in the experiment, a slight influence of hole diameter on the value of the effective Young's modulus was determined for the same open area value. As the share of holes in the sheet grew, the value of the effective Young's modulus decreased, regardless of sample orientation (Fig. 8a). At the same time, within the open area range up to approx. 0.47, an increase in the difference between E' values, determined for the tested directions (0°, 45°), was also noted. These results coincide with the data obtained in the experiment (Table 2 and 3). For open area values above 0.47, difference $E'_0 - E'_{45}$ decreased, tending towards zero (Fig. 8b).

Table 3. Average values of the effective Young's modulus of perforated sheet, corresponding to the open area values applied in the experiment

d mm	s mm	Percentage share of holes %	E'_0 GPa	E'_{45} GPa	E'_{av} GPa	$\Delta E'$ GPa
2	3	34.889	30.7	18.9	24.8	11.8
5	7.5		30.6	18.0	24.3	12.6
10	15		30.6	18.0	24.3	12.6
2	4	19.625	42.8	35.3	39.1	7.5
5	10		42.7	34.9	38.8	7.8
10	20		42.6	34.9	38.8	7.7
2	10	3.140	63.3	62.9	63.1	0.4
5	25		63.0	62.6	62.8	0.4
10	50		63.2	62.8	63.0	0.4

In the case of the conducted computer simulation, the value of the effective Young's modulus distribution anisotropy index $\Delta E'$ was equivalent to the difference $E'_0 - E'_{45}$ (Fig. 8b). Throughout the entire range of the analyzed open area, the change of this index value was curvilinear in nature. Similarly as in the experiment, the scatter of E'_0 and E'_{45} values increased as the share of holes in the perforated sheet grew, within the perforation value range up to approx. 0.47. However, it was determined that, after reaching the maximum (for $P \cong 0.47$), this difference decreased, tending towards zero.

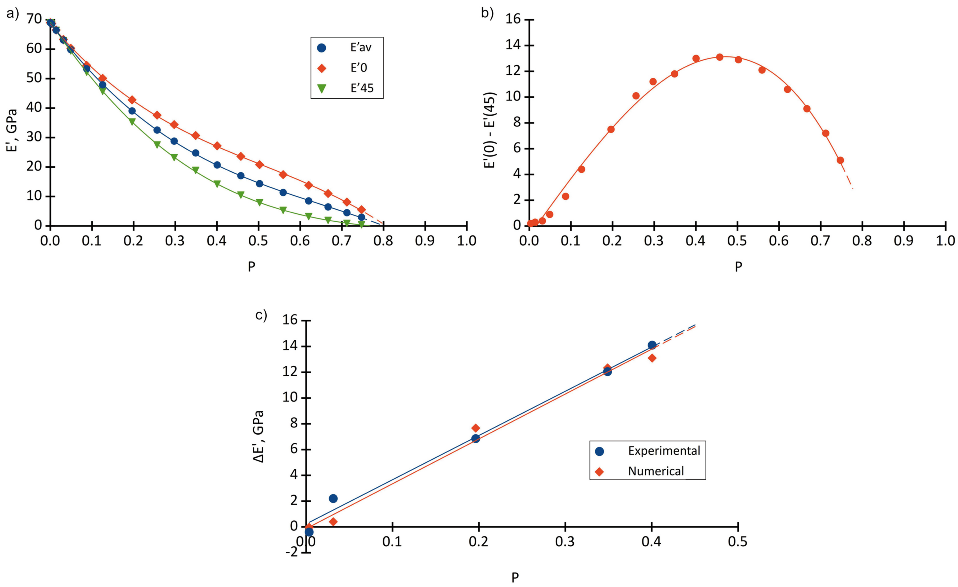

Fig. 8. Influence of the share of holes on the value of the effective Young's modulus (a), difference $E'_0 - E'_{45}$ (b) and change of the value of the $\Delta E'$ index (for P up to approx. 0.4) (c) as a function of open area

Within the P range up to 0.40051 (range of open area values applied in the experiment), it is possible to plot the regression line for the calculated values of the $\Delta E'$ index, similarly as in the case of experimentally obtained values (Fig. 8c). The computer simulation made it possible to investigate those open area values that were difficult to achieve under industrial conditions.

Figure 9 presents an example of change in value of the effective Young's modulus value as a function of the sample's angle of orientation with respect to the direction of greatest hole concentration. Regardless of the open area value, its value changes periodically as a function of angle with a constant period. It accepts the highest values for angle $\varphi = 0 + \frac{\pi}{2} n$, and the lowest for $\varphi = \frac{\pi}{4} + \frac{\pi}{2} n$ $(n = 0, 1, 2, 3)$. From this, it is possible to describe changes of effective Young's modulus values depending on the angle of orientation on the plane of the sheet using the trigonometric cosine function with the basic period $\frac{\pi}{2}$.

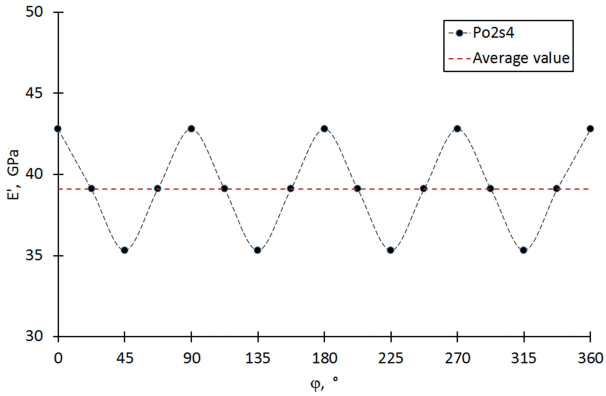

Fig. 9. Periodic change in effective Young's modulus value as a function of angle of sample orientation for Po2s4 aluminum perforated sheet (right perforation, hole diameter 2 mm, pitch 4 mm)

As part of numerical analyses, the influence of the type of perforated sheet material and its thickness on effective Young's modulus values was examined. The greater the value of the modulus of elasticity of the base material, the higher the E' value of the perforated sheet for any open area (Fig. 10a). As the open area value increased, the difference between calculated Young's modulus values decreased. This confirms the dominant influence of the presence of holes, particularly at high open area values, on the value of effective Young's modulus. At the same time, it was determined by dividing the E' value by the Young's modulus value of the base material E, that values of the effective modulus of elasticity for the analyzed materials (Al1050A, M1E, S355JR) coincide throughout the entire range of possible open area values (Fig. 10b).

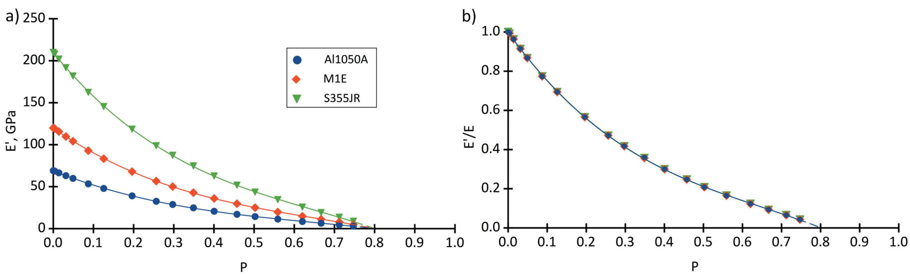

Fig. 10. Influence of material type on effective Young's modulus value of the perforated sheet: (a) absolute E' values, (b) relative E'/E values

Numerical simulations demonstrated linear growth of the effective Young's modulus value along with sheet thickness for the set open area value (Fig. 11). Within the analyzed range of material thickness (from 0.25 to 8 mm), the difference between E'/E values amounted from approx. 0.3% (P = 0.0035) to approx. 6.5% (P = 0.712). However, in design practice, the open area of perforated sheets with a straight configuration of cylindrical holes usually does not exceed the value of 0.5. For this open area value, the difference in the relative value of the effective Young's modulus was at the level of 4.5%. In comparison to the influence of the sample's open area and orientation, this influence is negligible enough to have been omitted in the further part of this article.

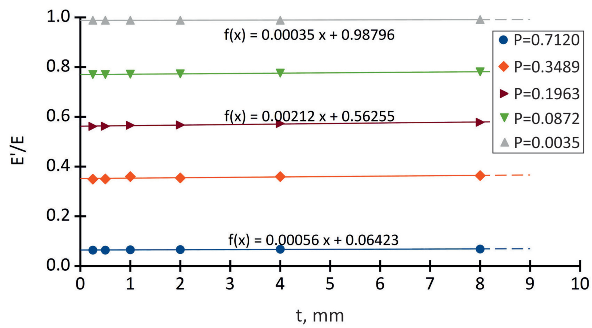

Fig. 11. Influence of material thickness on relative value of effective Young's modulus, for example open area values

3.3. Mathematical formula for determining E'/E

Based on conducted analyses, it was determined that the effective Young's modulus E' of perforated sheets with a straight configuration of cylindrical holes mainly depends on three parameters:

- Young's modulus of sheet material E,
- open area P,
- orientation of direction of measurement with respect to direction of greatest hole concentration φ.

This dependency was formulated mathematically as given by Eq. (3.2).

$$E' = f(E, P, \varphi) \tag{3.2}$$

Relative effective Young's modulus E'/E is defined by a more general formula, independent of the type of sheet material (Eq. (3.3)).

$$\frac{E'}{E} = f(P, \varphi) \tag{3.3}$$

The dependency of this quantity on sample orientation φ is given by Eq. (3.4). As stated in the previous section, the effective Young's modulus value for the given open area value changes periodically as a function of sample orientation. The quadrupled value of angle γ arises directly from the value of the period for the analyzed data (Fig. 9), which is four times smaller than the base period of the standard cosine function. Factor A was defined as the function's amplitude, while B – the graph's offset from the abscissa.

$$\frac{E'}{E} = A \cdot \cos(4\varphi) + B \tag{3.4}$$

The first part of Eq. (3.4) – $A \cdot \cos(4\varphi)$ – mainly corresponds to the influence of sample orientation with respect to the direction of greatest hole concentration, while factor A depends on the open area of the perforated sheet and does not exceed the value of 0.1 (Fig. 12). It was described by a third-order polynomial function, given by Eq. (3.5).

$$A(P) = a_A P^3 + b_A P^2 + c_A P + d_A \tag{3.5}$$

Coefficient values were determined on the basis of the equation of the polynomial regression function and amount to: $a_A = -0.6757$, $b_A = 0.1912$, $c_A = 0.2753$, $d_A = -0.0038$.

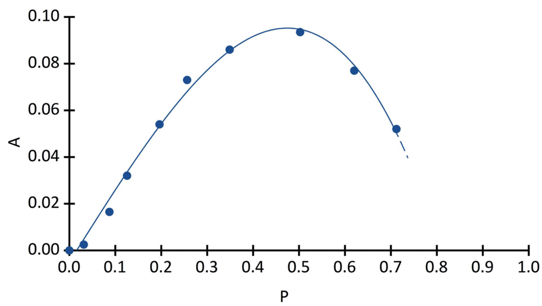

Fig. 12. Change of A factor value as a function of open area

Factor B in Eq. (3.4) is equivalent to the mean value of the relative effective Young's modulus E'/E for the given open area. It mainly depends on the relative share of holes in the sheet's area and can be described with the help of Eq. (3.6).

$$B(P) = \frac{E'_{av}}{E}(P) = a_B P^3 + b_B P^2 + c_B P + d_B \tag{3.6}$$

Coefficient values amount to, respectively: $a_B = -1.8413$, $b_B = 3.4467$, $c_B = -2.8393$, $d_B = 0.9979$.

Finally, the dependency defining the effective Young's modulus of perforated sheets with a straight configuration of cylindrical holes can be presented in the form of the following Eq. (3.7), for determination of the relative value, or Eq. (3.8) for calculation of values for a specific sheet material.

$$\frac{E'}{E}(P,\varphi) = A(P) \cdot \cos(4\varphi) + \frac{E'_{av}}{E}(P) \tag{3.7}$$

$$E'(E,P,\varphi) = \left[A(P) \cdot \cos(4\varphi) + \frac{E'_{av}}{E}(P)\right] \cdot E \tag{3.8}$$

In these equations:

$$A(P) = -0.6757P^3 + 0.1912P^2 + 0.2753P - 0.0038$$

$$\frac{E'_{av}}{E}(P) = -1.8413P^3 + 3.4467P^2 - 2.8393P + 0.9979$$

where: E – Young's modulus of full sheet from which perforated sheet was made.

The presented mathematical formula by Eq. (3.7) was evaluated. For this purpose, values obtained by using this formula were compared with data obtained experimentally. An insignificant deviation of the values determined on the basis of the proposed mathematical dependency with respect to experimental results was observed, in the case of both average values of the relative effective Young's modulus (Fig. 13a) and the distribution of its values on the plane of the sheet (Fig. 13b). The average deviation of calculated values from experimental values does not exceed 4%.

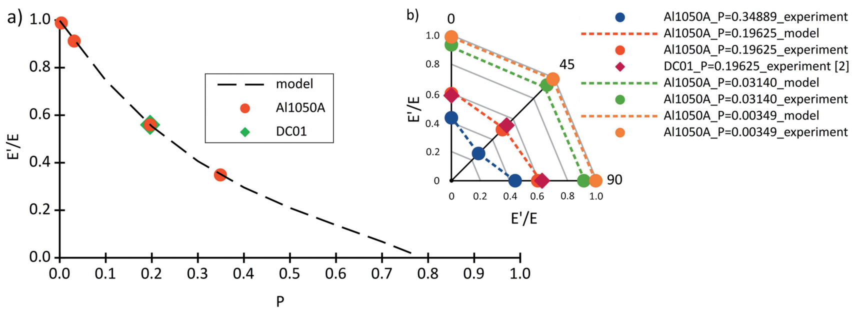

Fig. 13. Comparison of E'/E values obtained on the basis of Eq. (3.7) and from experiment (A11050A, DC01 [2])

4. Summary

This article proposed a mathematical formula, based on which it is possible to determine the value of the effective Young's modulus (relative and absolute) of perforated sheet as a function of open area and sample orientation (direction of sheet loading) with respect to the direction of greatest hole concentration. Good fit of the values determined based on the model with experimental results was determined. The average deviation of calculated values from experimental data did not exceed 4%.

Derivation of the mathematical formula given by Eq. (3.7) was preceded by experimental tests and numerical simulations of the influence of fundamental parameters (open area, hole diameter, loading orientation, type and thickness of material) on the value of the effective Young's modulus of perforated sheet. Based on analyses, a significant influence of the percentage share of holes on the sheet's surface (open area) and angle of measurement on the E'/E value was determined. Sheet thickness and hole diameter itself (for the set open area) affect the effective Young's modulus value to a small extent. The dependency of the mean value of this value on open area is curvilinear in nature, and it was described by means of a third-order polynomial. In the case of the influence of sample orientation with respect to the direction of greatest hole concentration (loading direction of perforated sheet) on the relative value of effective Young's modulus, a periodic change of its value as a function of angle φ was determined. This is combined with the anisotropic distribution of strength properties on the plane of the perforated sheet, accordingly to the type of perforation [1]. This dependency can be presented using the cosine function. Its factors (A and B)

are characterized by variability of value (they depended on the open area of the perforated sheet) and were also described by means of third-order polynomial functions. At the same time, factor B is equivalent to the average value of the relative effective Young's modulus (E'_{av}/E) for the given open area. The type of material influences solely the absolute value of the perforated sheet's effective Young's modulus. This means that, during analysis of the influence of only perforation on the elastic properties of such a material (such a structural material), any base material can be applied. The diameter of a cylindrical hole does not affect the effective Young's modulus value of perforated sheet. Its values, for a given percentage share of holes in the sheet, were similar to one another for different hole diameter values. The ratio of the squares of hole diameter d and pitch s has a greater influence on the value of the effective modulus of elasticity than the specific hole diameter value in the perforated sheet.

References

[1] W. Muzykiewicz, *Deformability of perforated sheets*. Cracow: Wydawnictwo Naukowe "AKAPIT", 2013.

[2] W. Muzykiewicz, M. Wieczorek, M. Mroczkowski, P. Pałka, and Ł. Kuczek, "The modulus of elasticity of steel sheet with right pattern of perforation", *Metal Forming*, vol. 27, no. 4, pp. 283–300, 2016.

[3] P. Stempin and W. Sumelka, "Numerical analysis of road acoustic screen", *Archives of Civil Engineering*, vol. 66, no. 2, pp. 191–210, 2020, doi: 10.24425/ACE.2020.131805.

[4] A. Öchsner, W. Winter, and G. Kuhn, "Elastic-plastic Behaviour of Perforated Aluminium under Tension and Compression", *Technische Mechanik*, vol. 21, no. 2, pp. 101–108, 2001.

[5] H. Khatam, L. Chen, and M.J. Pindera, "Elastic and plastic response of perforated metal sheets with different porosity architectures", *Journal of Engineering Materials and Technology*, vol. 131, no. 3, pp. 1–14, 2009, doi: 10.1115/1.3086405.

[6] M. Vimal, T. Pridhar, P. Viswanathan, and R. Subramanian, "Design and structural analysis of perforated sheet metals", *International Journal of Advanced Scientific and Technical Research*, vol. 4, no. 3, pp. 416–421, 2013.

[7] W. Muzykiewicz, K. Świątkowski, and P. Domagała, "Formability of perforated sheet metal", in P*roceedings of the First Australasian Congress on Applied Mechanics, ACAM-96*, *Melbourne, Australia*, vol. 2. 1996, pp. 907–912.

[8] R. Chiba, H. Takeuchi, and R. Nakamura, "Forming-limit prediction of perforated aluminium sheets with square holes", *The Journal of Strain Analysis for Engineering Design*, vol. 50, no. 6, pp. 391–404, 2015, doi: 10.1177/0309324715594735.

[9] W. Muzykiewicz, "Non-uniformity of plastic deformation of perforated sheet metal", in *Proceedings of the 8th International Conference on Metal Forming - Metal Forming 2000*, *Cracov, Poland*. 2000, pp. 397–404.

[10] G. Venkatachalam, S. Narayanan, and C. Sathiya Narayanan, "Prediction of limiting strains for square pattern – square hole perforated commercial pure aluminium sheets", *Advanced Materials Research*, vol. 548, pp. 382–386, 2012, doi: 10.4028/www.scientific.net/AMR.548.382.

[11] I. Malkin, "Notes on a theoretical basis for design of tube sheets of triangular layout", *Transactions ASME*, vol. 74, pp. 387–396, 1952.

[12] G. Horvay, "Thermal Stresses in Perforated Plates", in *Proceedings of the First U. S. National Congress of Applied Mechanics*, *Illinois, US*. 1951.

[13] G. Horvay, "The plane-stress problem of perforated plates", *Journal of Applied Mechanics*, vol. 19, no. 3, pp. 355–360, 1952, doi: 10.1115/1.4010511.

[14] M.M. Leven, "Preliminary report on deflection of tube sheets", Westinghouse Electric Corp. Research Labs., Pittsburgh, Research Report WAPD-DLE-320, 1959.

[15] R.C. Sampson, "Photoelastic analysis of stresses in perforated material subjected to tension or bending", Westinghouse Electric Corp. Research Labs., Pittsburgh, Research Report WAPD-DLE-319, 1959.

[16] R. Bailey and R. Hicks, "Behavior of perforated plates under plane stress", *Journal of Mechanical Engineering Science*, vol. 2, no. 2, pp. 143–161, 1960.

[17] D.C. Webb, K. Kormi, and S.T.S. Al-Hassani, "Use of FEM in performance assessment of perforated plates subjected to general loading conditions", *International Journal of Pressure Vessels and Piping*, vol. 64, no. 2, pp. 137–152, 1995, doi: 10.1016/0308-0161(94)00078-W.

[18] M.M. Cepkauskas and J. Yang, "Equivalent properties for perforated plates – an analytical approach", in *18th International Conference on Structural Mechanics in Reactor Technology*, *SMiRT 2005, Beijing, China*. 2005, pp. 1225–1232.

Wpływ wielkości otworu i prześwitu na efektywny moduł Younga blach perforowanych z prostym układem otworów cylindrycznych

Słowa kluczowe: blacha perforowana, efektywny moduł Younga, MES, opis matematyczny, symulacja komputerowa

Streszczenie:

Przedstawiona praca związana jest z numeryczną oraz doświadczalną analizą właściwości sprężystych blachy perforowanej z prostym układem otworów cylindrycznych. Zmiennymi były: rodzaj materiału bazowego (materiały o różnej wartości modułu Younga), grubość materiału, średnica otworu oraz skok (odległość między środkami sąsiednich otworów w kierunku ich najgęstszego upakowania) – przy zachowaniu stałej wartości średnicy otworu oraz wielkość otworu dla ustalonego skoku. W każdym z rozważanych wariantów, kierunki najgęstszego upakowania otworów perforacji były zgodne z kierunkiem walcowania blachy i poprzecznym. Analizę numeryczną przeprowadzono w zakresie wartości prześwitu P od 0 do 0,785. Wartość $P = 0,785$ została określona dla granicznego przypadku skoku, równego średnicy otworu, dla którego krawędzie otworów stykają się ze sobą. W konsekwencji, nie jest możliwe wykonanie blachy o takiej perforacji oraz niemożliwe jest określenie właściwości dla takiego materiału. Materiałami, jakie zastosowano w badaniach numerycznych, były: aluminium 1050A (E = 69 GPa), miedź M1E (E = 120 GPa), stal S355JR (E = 210 GPa). Pozwoliło to na określenie zależności efektywnego modułu Younga E' od rodzaju materiału i modułu sprężystości podłużnej blachy macierzystej (pełnej). Badania doświadczalne przeprowadzono dla czterech wybranych prześwitów, w zależności od średnicy otworu. W przybliżeniu, wynosiły one, odpowiednio: 40,05%, 34,89%, 19,63%, 3,14% dla średnicy otworu równego 10 mm; oraz 34,89%, 19,63%, 3,14%, 0,35% dla d = 2 mm. Materiałem blachy byłstop aluminium EN AW-1050A w stanie H14. W obu przypadkach analiz (numeryczna, doświadczalna) określano wartość efektywnego modułu Younga dla analizowanych zmiennych w funkcji orientacji próbki $\varphi(0, 45, 90^\circ)$ względem kierunku najgęstszego upakowania otworów, połączonego z kierunkiem walcowania ($\varphi = 0^\circ$). Na podstawie badań określono rozkłady efektywnego modułu Younga w płaszczyźnie blachy. Wyznaczono również względne wartości E' (E'/E), które to pozwalają na uniezależnienie wyników od rodzaju zastosowanej blachy (jej właściwości).

Na podstawie uzyskanych wyników stwierdzono istotny wpływ wartości prześwitu oraz orientacji próbki na wartości efektywnego modułu Younga. Efekt ten byłniezależny od grubości blachy, rodzaju materiału i średnicy otworu. Jednocześnie zauważono, że, zarówno średnica otworu jak i grubość mają mały wpływ na wartość E'. Wyniki doświadczalne i numeryczne pokazały, natomiast, że w pewnym zakresie wartości prześwitu rozbieżność wartości efektywnego modułu Younga dla orientacji 0° (90°) i 45° rośnie, a następnie maleje. Maksimum $\Delta E'$ osiągnięto dla wartości P w przybliżeniu równego 0,47 (∼ 47%). Stwierdzono również periodyczność wartości efektywnego modułu

Younga w funkcji orientacji próbki. Wynosiła ona $\pi/2$. W konsekwencji możliwe było opisanie zależności $E = f(\varphi)$ lub $E'/E = f(\varphi)$ za pomocą funkcji trygonometrycznej cosinus. Zdefiniowano poszczególne parametry równania. Stwierdzono, że zarówno parametr A jak i B równania można opisać za pomocą funkcji wielomianowej trzeciego stopnia. Z tym, że A nie przekraczał wartości 0,1, a B był tożsamy ze średnią względną wartością efektywnego modułu Younga dla danego prześwitu. Pozwoliło to na skonstruowanie empirycznej zależności matematycznej, pozwalającej na określenie wartości modułu Younga blachy perforowanej z prostym układem otworów cylindrycznych w zależności od prześwitu, jak i od orientacji względem kierunku najgęstszego upakowania otworów. Różnica pomiędzy wartościami określonymi za pomocą modelu, a wartościami doświadczalnymi, nie przekraczała 4%.

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