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Three faces of balancing: the development of automatic balancing devices for shafts in motion

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In this paper, the authors present a novel construction of an automatic balancing device applicable to balancing shafts working in a heavily polluted environment. The novelty of the presented system lies in the fact that its utilization requires no changes to be made in the already existing shafts. Also, the system is capable of working during the operation of the balanced shaft, so there is no need to stop it. The propulsion system is based on eddy current braking, therefore no wires need to be used in the device. During the development process of the system, three iterations of the device were created. Each iteration is presented, described, and discussed. The advantages and drawbacks of each version are pointed out and explained thoroughly. The correctness of the design was verified by the created devices that were assembled and fixed on shafts to prove the design assumptions.

1. Introduction

The problem of balancing of rotary objects has been investigated over the years, as it is inseparable from such a type of motion. In the case of an axle, shaft, wheel, etc., being put into rotary motion around its axis of rotation, it is necessary to ensure that the actual axis of rotation overlaps the theoretical one. In case their positions differ, the rotary objects lose their equilibrium and start

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to generate vibrations. These in turn are being transferred onto the bearings, the housings and the overall structure, which results in shortening the lifespan of the named elements and affects the fluence of the whole system operation. Therefore, a proper balancing of the system is crucial.

When the rotary unit is a shaft, e.g., an axle of a power fan, it is highly possible that it operates in a polluted environment, such as one present in power plants, cement works, foundries, etc. If so, it is only a matter of time before the unit starts getting covered with the dust and pollution present in the air. Over time the pollution creates a coating on the unit whose unevenly distributed mass generates additional forces in the system, that cause unbalance. It must be pointed out that such a dust covering process is quite slow and the forces it generates are negligibly low in many cases. A serious problem arises in the situation in which a big part of the dust cover rips off suddenly, which causes that a significant force appears in a glimpse.

The classical solution to the problem is to stop the rotary unit completely and perform a balancing procedure, which can be either a one-plane or a two-plane procedure – depending on the balancing and control forces applied in the process [1–3]. The process consumes a significant time, as the dynamic balancing process can be carried out using one of three methods [4–6]:

- amplitude method/trial method,
- phase method/circular dash method,
- amplitude/phase method.

The amplitude/trial method consists in determining the magnitude and the angle of unbalance by measuring the amplitude of vibration of the balancer bearings at various positions of the test mass on the rotating assembly to be balanced. The method essentially consists of two steps. In the first step, the location of the correction mass is determined by measuring the amplitude of vibration of the bearings several times at one rotational speed - corresponding to the resonance speed - and constant test mass of a varying position on the rotor. In the second step, the value of the correction mass is determined by measuring the amplitude of vibration of the bearings at one rotational speed for a constant mounting angle of the test mass, but with a variable value of this mass. The trial method is used for balancing small rotors when no suitable balancing apparatus is available.

In the second method (the phase method), one assumes that, at a constant rotational speed of the balanced rotor, the value of the phase angle between the force induced by the unbalance and the largest displacement of that rotor transverse to its axis is constant and does not depend on the magnitude of unbalance. Both methods involve a simultaneous measurement of the change in the phase angle and the value of the vibration amplitudes of the balancer bearing. Their main advantage is that they allow determining the unbalance in one plane of correction with only two actuations.

The third of the amplitude/phase methods have found wide application in the development of the electronic balancing apparatus. Balancing in a single plane of

correction can be performed with only two actuations, while for dual-plane balancing only three actuations of the rotating assembly are required. The process of balancing is carried out in the own bearings of the shaft being balanced. These methods are currently the best solutions for balancing rotating assemblies. They significantly reduce the time required for balancing. In addition, simultaneous observation of the amplitudes and phases of vibration of the bearings gives a complete picture of the unbalance condition of the rotor.

Typically, stopping the unit for the balancing process involves a minimum of a two-days standstill of the block. This is due to the need to cool down the fan and to carry out a minimum of 3 start-ups for correct balancing. After each start-up, it is necessary to wait for the fan motor to cool down. Many times, 3 start-ups are not enough for balancing the fan in such a way that the vibration velocity on the bearings is below the limit value.

2. State of the art

In the classical approach to the balancing process, the unbalanced unit is out of order for several hours, or even several days in serious cases. The idea to make the balancing process automated has been appearing for almost 80 years. The first information on automatic balancing devices can be found in [7]. All the proposed solutions to the problem are based on three possibilities [8]:

- Adding a material to the unit. One well-known solution for automatic balancing based on the addition of material is the P. Loetzner's device, which was filed with the patent office in 2003 [9]. In this method, a welding head is assumed to move along the balanced shaft. When an imbalance is detected, the welding head adds some material with welding. This method assumes that there are at least two planes in which the surfacing can be performed, so that two-plane balancing of the rotating assembly is possible. Other examples to be mentioned can be found in [10–12].
- Removal of material. A method opposite to the previous one. A good example of it was the invention of J. Perdriat [13]. This is an automatic balancing device that involves attaching an empty or liquid-filled tube to the rotating shaft. If a loss of balance is detected, the tube can be filled with fluid at a specified location and to a specified level, thus ensuring that the shaft is rebalanced. Other examples were described in [14, 15].
- Performing a relative motion of masses fixed to the unit. An example of such a method can be found in the invention of P.C. Stein [16]. In this device, a shell in the form of a ring is mounted to the shaft to be balanced. A pair of balancing elements is then mounted inside the shell on opposite sides of the shaft so that they can move around its outer surface. The movement of the balancing elements is blocked by an automatic clutch, which blocks the movement when the shaft speed is below a certain value, but when this

value is exceeded; the elements can freely move. This idea is also presented in [17, 18].

Most of the solutions that can be found in literature are only theoretical concepts which have never been put in use, or were suited for the use in very narrow applications, i.e., Hard Disk drives [19] or grinding wheels [20]. As for the balancing devices that can be mounted on big diameter shafts, only one solution is currently present in the market [21], the principle of which is presented in Fig. 1.

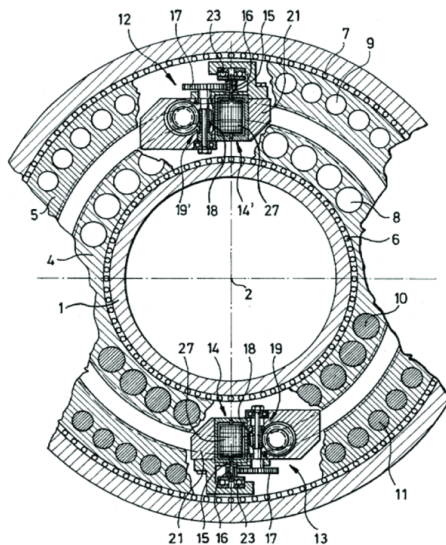


Fig. 1. Cross-section of the existing balancing device solution [21]

It is based on the idea that the balanced shaft is equipped with two balancing devices – each of them mounted next to the bearing's housings. Each device consists of two balancing rings that can independently move relative to each other by making a rotation around the balanced shaft. When the rings are placed in positions opposite to each other, they generate no additional force in the system. Yet, in the case a force of unbalance appears, the balancing rings perform a relative motion to generate an additional force in the system to counteract the force of unbalance and to bring the system back to equilibrium. The rings are put in motion with the use of DC motors located inside the construction (element 27 in Fig. 1). To make it possible to successfully implement the described balancing device into a system, it is necessary to prepare the shaft properly. One needs to have enough space available next to bearing housings to make room for the device. Because of that, there is no the possibility to install the device in many systems present in the industry, which weren't designed to be capable of using the balancing device. Also, in many cases, the whole structure needs to be completely dismantled from the bearings to install the balancing devices, as they are mounted axially.

After taking the named necessities, possibilities, and drawbacks into consideration, it is possible to create an idea of a balancing system free of the above-mentioned disadvantages. According to the assumptions, the novel solution should be created according to the following principles.

- The device volume should be relatively low as it is supposed to be installed next to the bearing housings, where in most industrial cases the available space is a couple of centimeters wide gap in the free shaft's surface. It was then assumed that the thickness of the developed device shouldn't exceed 7 cm.
- The device must be mounted radially, which is necessary to ensure the possibility to install it without the necessity to dismount the shaft from its housings. This approach allows reducing installation time drastically.
- The propulsion system should be created using wireless technology to avoid the necessity to insert any wires to the rotary unit.
- Final assumption is that the system will consist of two balancing units, each of them located next to the shaft bearings to enable two-plane balancing.

3. Propulsion system

To fulfill the assumption regarding the propulsion system, it was decided to create a system using the eddy currents phenomenon. The initial idea was that outside the balancing device there will be located a ring of coils, which can be referred to as a stator. By wounding coils alternately and powering them, an electromagnetic field is generated. Now, if one inserts a moving conductor in this field, an eddy current will be generated. Therefore, if this conductor is an input to the balancing system, it will be possible to use the eddy currents as propulsion to the system. This idea is presented in Fig. 2.

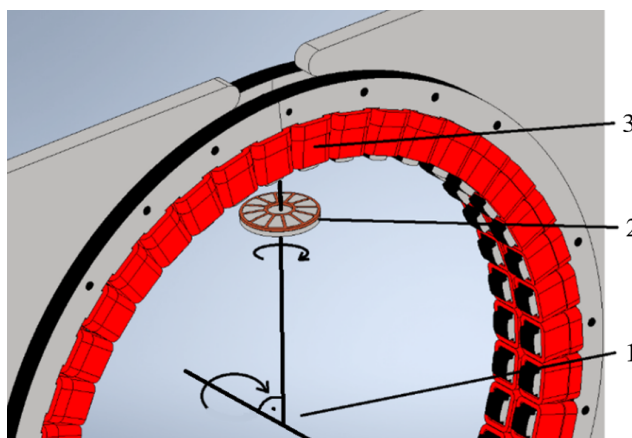


Fig. 2. The idea of the developed propulsion system

A shaft to be balanced is pointed out by (1). A balancing device is mounted on it and fixed, therefore they move with the same angular velocity. The input to the device (2) is a propulsion plate made with a conductive material. It moves along the overall system, yet its axis of rotation is set perpendicular to the shaft's axis of rotation. As the shaft rotates, the propulsion plate containing current-carrying elements moves in a constant magnetic field generated by stators (3). This movement then causes that, according to Faraday's law, there appears an electromotive force of induction, followed by the formation of eddy currents, which, according to Lenz's law, counteract this motion, thus generating braking forces. Moving each of the two stators away from the axis of the plate causes that these forces move away with respect to the plate axis, that is, they induce a moment of force that causes the plate to rotate with respect to its axis. The rotary motion of the plate is then translated into the motion of the balancing mass, which is a part of the device. The masses need to be set in motion relative to that of the balanced shaft. It is also necessary to use two such balancing masses in each device to use their relative motion to generate a balancing force vector. An algorithm of balancing was created – the idea behind it is as follows: one introduces a known unbalance into the system twice (it is obtained by overdriving the balancing rings by a known preset angle). The first test is carried out for the device near one bearing housing, while the second test is carried out by overdriving the device at the second bearing housing after the balancing rings at the first bearing have returned to the equilibrium position. The algorithm, having the information about the introduced unbalance and the change in the amplitude and phase of vibrations at the bearings, determines the system's matrix. Based on the determined matrix, the algorithm calculates the desired position of the disks to balance the force generating the unbalance. A detailed description of the process will be the subject of forthcoming publications.

4. First iteration of the balancing device

It must be noted that the first iteration of the balancing device was created only to explore the subject and the possibility of using the designed gear types as the drive transfer method. The designed device is presented in Fig. 3.

The device consists of three main modules pointed out with numbers. The first module is the input plate (1) that rotates with the overall system in the magnetic field. In the process, an eddy current is generated and the plate starts to rotate. The rotation of the plate is transferred to the rotation of the outer ring (2). Mass distribution in this ring is uneven as its purpose is to generate the balancing force. The outer balancing ring is set in rotation with respect to the fixed inner ring (3). Its purpose is to be installed permanently on the balanced shaft and to serve as a base for the whole system. It always rotates with the same angular velocity as the balanced shaft. As it can be seen, the outer ring is split into two uneven parts. The inner ring is one solid part, yet at this point, it is not a problem, as this iteration is not intended to be practically realized and mounted on any shaft.

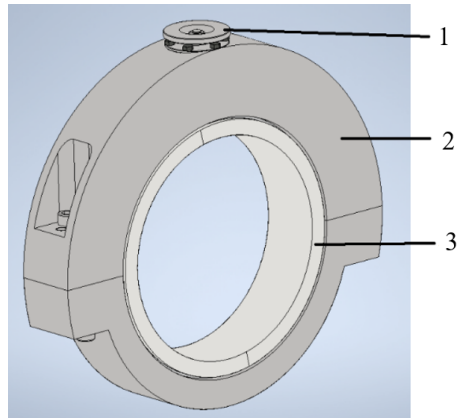


Fig. 3. First design iteration of the device

It was necessary to fully control the rotation of the input plate to ensure that it rotates only when needed. To do so, a self-locking mechanism was used to transfer the drive from the input plate to the outer ring. The designed mechanism is presented in Fig. 4.

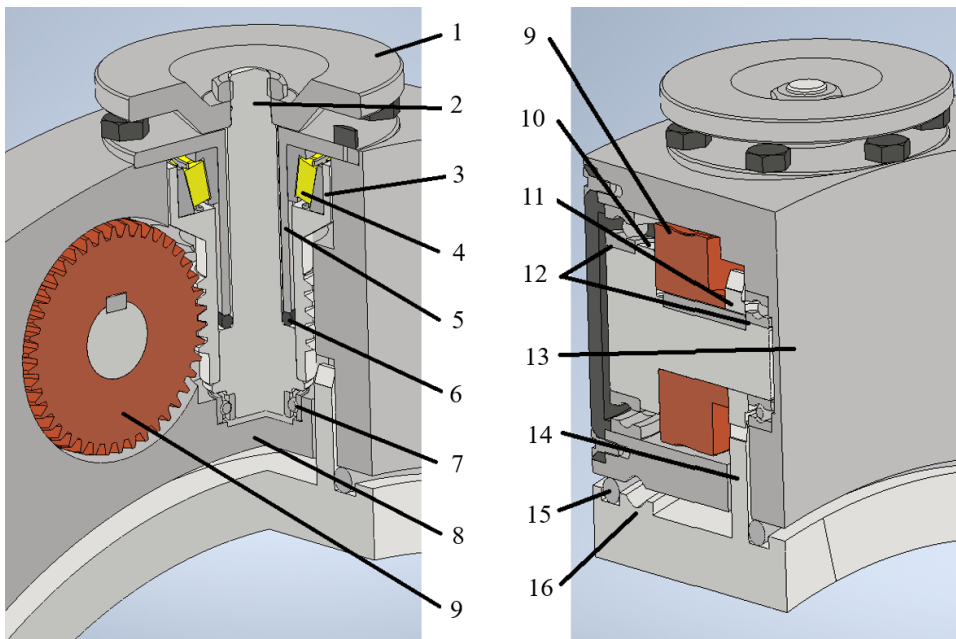


Fig. 4. Cross-section of the drive transfer mechanism

Fig. 4 shows the input to the system. The input plate (1) is mounted on the input shaft (2). This shaft is combined with the sleeve (3) using high friction. This assembly is supported by two bearings, the angular one (4) and the radial (7).

The upper, angular bearing is mounted using a T-shaped sealing sleeve (5) with an additional rubber sealing (6). The lower, radial bearing is mounted in the body of the outer ring (8). In addition, the sleeve (3) is an input to the worm gear – it is meshed with the worm wheel (9).

In the following construction of the transmission, the input is the previously mentioned worm wheel (9) that is mounted on the shaft (10) along with the cogwheel (11). This shaft is mounted using two radial ball bearings (12) in the body of the outer ring (13). The mentioned cogwheel (11) meshes with the ring gear (14) which is a part of the earlier-mentioned inner ring of the device that is fixed on the balanced shaft. Between the inner and outer rings, there are located two rubber sealings (15 and the bearing rail (16) to guide the motion of the rings.

Therefore, the rotation of the input plate (1) enforces rotation of the worm gear (3, 9) which is transferred to the rotation of the cog wheel (11) around the ring gear (14). It results in the rotation of the outer ring in relation to the inner ring.

There are two main forces present in the mechanism: the centrifugal force and the resultant reaction force in the worm gear. The centrifugal force acts on all rotating mechanisms displaced from the shaft axis of rotation. In the case of the worm wheel, cogwheel, and their shaft, the force will be dissipated by the radial bearings (12) supporting the shaft. In the case of the worm, it will be transferred through angular bearing onto the sealing sleeve and dissipated there.

As it was mentioned, the designed device is assumed to work in a heavily polluted environment. Therefore the mechanisms need to be sealed against the dust. There is a rubber sealing between the sealing sleeve and the input shaft to prevent pollution to enter along the input plate and the shaft. The second part of the rubber sealing is located between the inner and outer rings.

The conclusions coming from the creation of the first iteration of the design are that it is possible to create a self-locking mechanism to transfer the drive originating from eddy currents, yet some changes need to be done. Additional tests were performed on rubber sealings of the proposed shape, which showed that the friction torque between them and other mechanism elements was so high that device was not able to move at all. Therefore, some other type or shape of sealing needed to be implemented. Also, a proper technology to create a split construction of the inner ring should be used. And finally, the shape of the outer ring must be redesigned to create the possibility of two outer rings be present on one inner ring at the same time.

5. Second iteration

The idea is that the novel balancing system consists of two units – each of them mounted next to bearing housings. Such a solution provides the possibility of performing a two-plane balancing. In the cases where it isn't possible to install two devices or there is no necessity to do so, only one unit can be used to perform one-plane balancing. Regardless of which version is to be used, each unit consists

of three elements: an inner ring and two outer balancing rings. The idea is that the inner ring is installed on the balanced shaft radially next to the bearing housing. It is possible due to the fact that it is split into two halves and its width is lower than approximately 7 cm, which makes it possible to mount it on almost any shaft present in the industry without the need to dismount the unit. The first iteration of the device was never actually constructed. The approach to the second iteration was different, both the technology and the construction were thoroughly redesigned to create a model ready for manufacturing.

The design of the overall device is presented in Fig. 5.

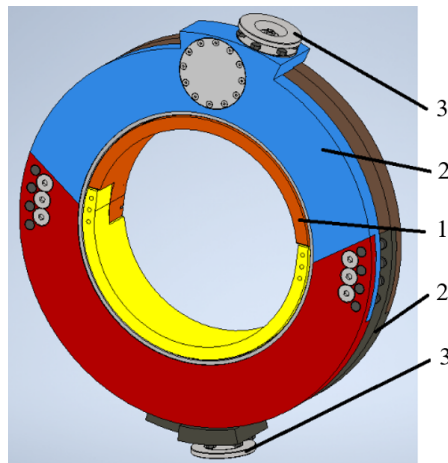


Fig. 5. The overlook of the second version of the device

As it was assumed, the device consists of the inner ring (1) that is fixed to the balanced shaft and is split into two halves, which are connected using fitted screws. There are two outer rings (2) that can move independently. Each input ring has its input plate (3). The outer rings are also split into halves and connected with screws and pins.

The design of the transmission was slightly changed and is presented in Fig. 6.

Similarly as in the previous version, the input plate (1) is mounted on the input shaft (2) which is also an input to the worm gear. In this version, the lip sealing (3) is applied, whose base is mounted in the input plate and the seals are in contact with the sealing plate (4). The input shaft is mounted using two ball bearings, the axial (5) and the radial one (8). The bearings are installed in the body of the outer ring (6). The axial bearing is fastened using a circlip which is not shown in the model diagram to increase the clarity of the picture. The worm meshes with the worm wheel (7).

The main changes in this module consist in reducing the number of elements to a minimum and simplifying the technological process. Also, the sealing has been changed from an O-ring to a lip sealing. The lips of the sealing touch the bottom

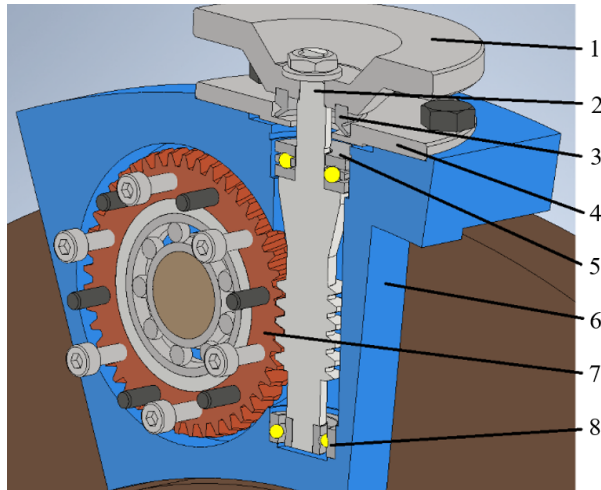


Fig. 6. Cross-section of the drive transfer mechanism

part of the sealing plate when the shaft is not in motion. As it starts rotating, the lips are bent by the centrifugal force and start touching the sides of the sealing plate, ensuring the sealing.

As for the dissipation of forces, the centrifugal force acting on the input shaft and the plate is dissipated by the axial bearing (5), while the residual force of the worm gear is mainly dissipated by the radial bearing (8). Angular bearings present in the first iteration were changed into these for practical reasons – it was assumed that it might be necessary to assemble and disassemble the mechanism several times in the initial phase, which might result in damaging the bearings. Therefore, typical ball bearings were selected in this case.

A different view of the transmission is presented in Fig. 7. It also presents many significant changes in the mechanism in comparison to the previous design iteration.

The input to the further part is the worm wheel (7). It is fixed to the cog wheel (8). This module is mounted using two radial bearings (9) on the shaft (10) which is an element of the cover plate and is fixed in the body of the outer ring with 12 Allen screws, as it is the component that must ensure highly reliable alignment because bearings and the gears are mounted on it, so it is better to use more screws in case any of them loosens and breaks the alignment of the transmission. The bearings (9) are distanced using a circlip. The cogwheel (8) meshes with the ring gear (11) which is an element of the inner ring. The inner ring is equipped with two ring gears – each one used to drive the other outer ring. Also, there are two bearing rails (12) to guide the outer rings. There are also two types of sealing: two lip sealings (13) and one hollow O-ring sealing (14).

The working principle of the mechanism remained unchanged compared to the first version. The main difference lies in the shape of the outer ring bodies to

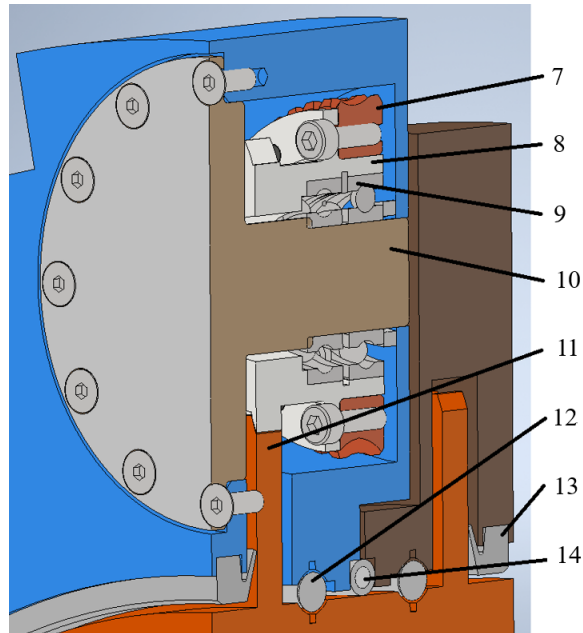


Fig. 7. The further part of the transmission mechanism

guarantee their coexistence on the inner ring, as it can be seen in Fig. 7. Also, the rubber sealings were changed – in the case of the outer parts of the bodies of outer rings, they were changed to lip sealings to ensure possibly low friction torque. The same concerns the sealing of the inner part of the body. As the pollution can get inside the mechanism through the gap between the outer rings, a hollow O-ring type is used there, with the assumption that it touches only the edges of the outer ring bodies, therefore the generated friction torque remains minimal.

The second version of the device was created and installed on a test stand, which can be seen in Fig. 8.

It was used to perform initial tests of the device. The correctness of the installation procedure was tested, the correctness of the propulsion system using eddy currents was also checked, and initial drive characteristics were determined. The outcomes of these tests can be found in [22].

Yet, there were additional results of the experiments that mainly pointed out the necessity of changing construction of some parts of the device. The detected problems were related to the problematic installation of the device and the high possibility of damaging its elements in some specific situations. It was found during an unplanned crash test event, when accidentally the balancing elements of outer rings moved too close to each other and the resulting balancing force became so large that the resulting vibrations of the shaft lead to a crash of the input plate and stator elements. Therefore, it was decided to design and create the third version.

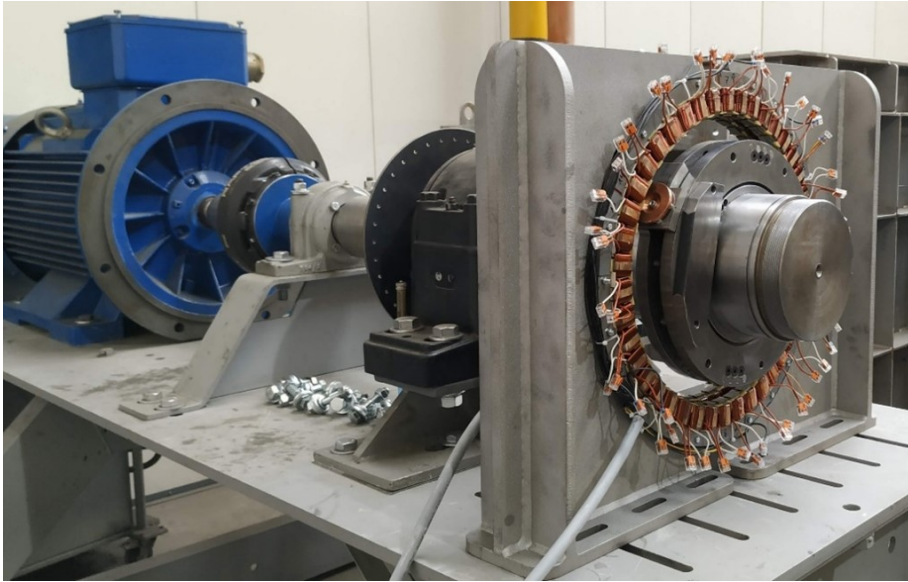


Fig. 8. The test stand with the second version of the device

6. Final iteration

As the second iteration of the device was successfully created and tested, several drawbacks were identified. The named issues concerned design correctness and were applicability-oriented. All of the identified problems were corrected and the improvements were implemented in the final design iteration of the automatic balancing device. This corrected device was built, assembled, and successfully tested.

It was identified that the main problem with the installation of the devices arose because it was split into two halves. It was assumed that splitting it into three elements would make it possible to overcome this problem. Besides, an additional element of the outer ring was implemented to protect the input plate from being damaged. The last change was introduced in the method of sealings.

The third and final version of the balancing device was designed and is presented in Fig. 9.

One might notice several holes present in the body's outer ring. They were added in this iteration to ensure that it would be possible to add or remove masses in the holes present in the device, in order to change the unbalance of the rings to the desired range.

Additionally, several changes were introduced to the construction of the input plate to maximize its magnetic conductivity. The details can be found in [20]. The same applies to the construction details of the stator and the powering system. These details will be a subject of future publications.

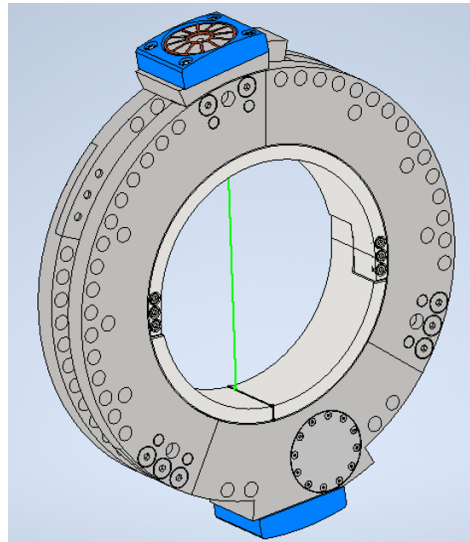


Fig. 9. General outline of the third iteration of the device design

The main changes made in the mechanism lie in the input to the system, as the final version of the input plate was implemented. Also, the bearings were changed one more time to make the worm installation easier and faster. The details of these changes can be seen in Fig. 10.

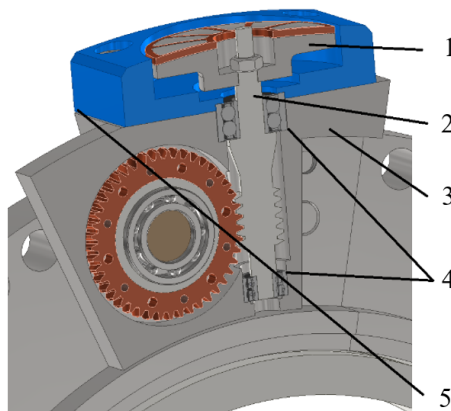


Fig. 10. The details of changes made in the mechanism

The input to the system is a modified version of the propulsion plate (1). It is mounted on the worm (2), which is mounted in the body on the outer ring (3) with two angular covered bearings (4). The upper one is fixed by the cover (5), which is a new element of the construction. In the second iteration, the propulsion

plate was fixed using a single nut with a split lock washer to prevent spontaneous unscrewing. The tests showed that it was not enough in some cases, therefore the shaft was lengthened to use a self-locking nut and a counter nut, if necessary.

As one can see, no rubber sealing is used in the construction. The only gap through which the dust can enter the mechanism is between the input plate and the cover. Yet, the dust is stopped there using the cover of the bearing. No other changes were made in the remaining part of the mechanism. Also, the hollow O-ring sealing visible in Fig. 7 was completely removed and replaced with a high-density grease barrier.

The designed device was later created and assembled on the test stand presented in Fig. 8. The second version of the device installation procedure was as follows. The inner ring halves were placed on the shaft and fixed together with screws and pins. Next, one half of the outer ring was put underneath the shaft, and the balls were put in the bearing rail. To prevent them from falling down, a small amount of grease was used to glue them. Rubber sealings were put after that and such a prepared ring half was pushed towards the inner ring. The combination of the inner ring and the first half of the outer ring with all other elements inside was then fixed together and rotated around the shaft by 180 degrees. Then, the procedure was repeated with the second half of the outer ring, after which the outer ring halves were fixed using screws and pins. After that, the installation of the worm gear was carried out, starting from the worm wheel.

As mentioned, the installation was problematic, even in laboratory conditions, therefore it had to be changed. In the third version of the device, the two bigger outer ring thirds are pushed together towards the shaft and fixed. Next, the sealings are pushed inside, and the combination of outer ring thirds is set axially with the shaft and the inner ring. After that, the balls are inserted until they fill the available space. The rest of the balls are put on top of the inner ring on the remaining rail part, which is uncovered at this point. Finally, the remaining third of the outer ring is installed and fixed to other elements of the outer ring. Everything ends with the installation of the worm gear. The procedure was tested and it was proved that the assumptions were correct.

The installed version of the device has been subjected to a series of tests: determination of drive characteristics (dependence of balancing ring rotation angle and shaft angular velocity for different currents), determination of the dependence of balancing ring rotation angle and stator engagement time, verification of various configurations of switching of the stators and testing the accuracy of the positioning of rings in both directions of motion. All the tests were done in industrial conditions to verify the correctness of the device characteristics and the proposed balancing algorithm. The results will be the subject of forthcoming publications.

It needs to be clarified that the device is made for a shaft with a diameter of 200 mm. In the case of power fan shafts, typically used bearings diameter ranges from 80 to 180 mm, but there is a step on the shaft next to the bearing, so the mounting diameter actually ranges from 100 to 200 mm. For the smaller shafts, it

is possible to use a transition sleeve and it is possible to add or remove masses in the holes present in the device, changing the unbalance of the rings to the desired range.

7. Summary

In this paper we described the necessity to perform balancing of rotary units, applying special focus on ones working in heavily polluted environments, as they are most likely exposed to the danger of becoming rapidly unbalanced. The applicability of an automatic balancing device in such units is obvious. Therefore, we presented an innovative idea of an automatic balancing device that can be easily installed on almost every shaft of the rotary unit, such as a power fan. The innovativeness of the idea lies in the fact that there is no need for additional preparation of the shaft to be balanced, as the required elements (the installation area) are present in most of the rotary units applied in the industrial practice. Also, there is no need to dismount the shaft to be balanced from its bearings. It is only needed to stop the unit for a couple of hours to install the device, its propulsion system, and the sensor system.

In the presented work, three iterations of the device design were presented. The first one was used only to test whether the design assumptions were possible to be met or not. During the design process, it became clear the requirements are realistic, therefore it was possible to create the next, improved design.

The second iteration of the construction was created while taking a great care of the technological and design details. The device was created, installed on a test stand, and tested. The performed experiments showed that the design assumptions were correct and the device could work under the defined work setup and conditions. Yet, some unpredicted construction problems, such as long and troublesome installation procedure or the possibility of damage of the propulsion system elements were identified.

Therefore, the final iteration of the design was proposed. The final version was designed and created anew. It was later installed on the test shaft and tested once again to verify the correctness of its work as well as the correctness of the improvements introduced. All the tests were successful and proved that the device was ready to work under the defined conditions and perform its main task. The created device is capable of performing the automatic balancing process of shafts working in heavily polluted environments without the necessity to stop their work in the process.

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