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Research paper

Application of exponential smoothing method to forecasting daily water consumption in rural areas

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Abstract: The size and distribution of water demand within a given structural unit is the basis for the proper operation and planning of the expansion and modernization of the water supply system's elements. In rural areas, particularly in municipalities adjacent to urban-industrial agglomerations, a change in the use of tap water has been increasingly observed. The water consumption for animal breeding or agricultural use, typical of these areas, has been decreasing and even disappearing. Water has been increasingly used for domestic purposes in single- and multi-family housing as well as for other purposes such as watering lawns and filling residential swimming pools. Taking this into account, this paper presents observations regarding daily water consumption in a municipality adjacent to Wrocław together with an analysis of the possibility of using the exponential smoothing method for the short-term forecasting of daily water consumption. The analyses presented in this paper were carried out using STATISTICA 13 software.

Keywords: forecasting, exponential smoothing, pipe network, water consumption

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1. Introduction

In order to optimize and control water distribution in rural areas it has become necessary to determine the actual water distributions and to forecast water demand on this basis. It is crucial to know the pattern of water consumption not only for forecasting but also for proper hydraulic maintenance [1]. It has been increasingly observed that the usage of water mains in rural areas, particularly in municipalities adjoining large urban-industrial agglomerations, changes. Water consumption for agricultural and animal rearing purposes, typical of rural municipalities, has been significantly decreasing and even disappearing. On the other hand, water has been increasingly used for domestic purposes in single- and multi-family dwellings as well as for watering greenery (lawns) and filling residential swimming pools. As the population in these areas grows, so does the demand for water mains. As a result and due to climate changes (increasingly longer rainless periods with intensified insolation and high air temperature) water deficits increasingly often occur in spring-summer periods [2, 3] and smart buildings are necessary to optimize water consumption [4].

Because of the higher water demand in the municipalities adjacent to large agglomerations, which entails increased water production and using proper materials for water pipes which reliability is relatively high [5], water companies look for new solutions concerning, among other things, the optimal control of such processes as: the tapping and distribution of available water resources, proper maintenance of technical condition [6] as well as the distribution and treatment of water [7]. Water sources in Poland and worldwide are limited and water recovery will be needed in future, so sewage inflow should also be considered as the important factor influencing the environment and our future life [8]. In order to ensure the efficient control of these processes a calibrated hydraulic model of the water distribution network and a predictive model of water consumption are required [9].

Water consumption and the demand for it by consumers, as well as sewage runoff, have a complex deterministic-random character. Thus they are subject to certain regularities onto which random properties are superimposed. Since predictive models are increasingly often used in operational practice in water distribution system operation simulation algorithms to determine an optimal real time control (RTC) strategy, they must enable one to quickly make forecasts on the basis of a possibly small number of easily available predictors. Furthermore, the forecasts should include information about the changing number citizens and climate warming [10] as well as about changes of demand pattern caused by holidays [11].

Stochastic models, introduced in an algorithmized form into the control process management structure, are used for the current and short-term forecasting of water consumption. Most often auto regressive integrated moving average (ARIMA) models and time series exponential smoothing methods are used for this purpose [12]. ARIMA class models map the static and dynamic properties of stationary series and certain classes of nonstationary series, interpreted as the result of white noise passing through a finite-dimensional discrete linear filter. They are characterized by different properties, but use a uniform notation and identical methods of estimating parameters for different model types and subclasses. Forecasting methods based on exponential smoothing algorithms are easily applicable in practice and do not require the analysed time series to be assumed stationary. Artificial neural networks can be an alternative to the above methods. Owing to their several features they can be a useful tool for analysing and forecasting time series of short-duration water consumption. The use of artificial neural networks is attractive mainly due to their ability to approximate any nonlinearities and fine-tune the adopted structure on the basis of experimental data or other training images. Moreover, neural model building consists in exploring the available data sets, which provides a basis for the fully automatic estimation of the model describing the observed regularities and correlations. Hence the use of neural models does not require the knowledge of the form of the function describing an existing regularity. Consequently, neural models can be applied in all the cases where the precise laws describing the shaping of the investigated relationships are not known [13, 14].

Taking the above into consideration, this paper presents findings concerning the daily water consumption in one of the municipalities adjacent to Wrocł aw and an analysis of the possibility of using the exponential smoothing method for the short-term forecasting of daily water consumption.

2. Materials and methods

2.1. Characterization of area under study

The rural municipality of Czernica was selected for the analysis of daily water consumption in the years 2015–2018. This municipality is situated in the east part of the Lower Silesian Province and in the north-west part of the Wrocław County (Fig. 1). The Czernica Municipality partially enters into the functional zone of the City of Wrocław, constituting a part of the Wrocław agglomeration.

In 2018 the municipality had 14 400 inhabitants. Within this area there are 13 village council administrative units [15, 16].

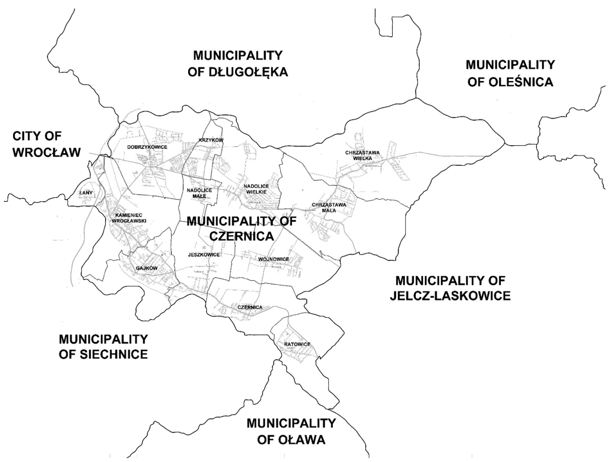

Fig. 1. Location of Czernica Municipality in Wrocław agglomeration

The close proximity to Wrocław has resulted, especially in recent years, in the rapid development of single-family housing and in a change in the villages' character – from agricultural (there are average and poor arable lands within the municipality area) to housing estate residential, entailing a relatively large annual population growth in the municipality (Fig. 2).

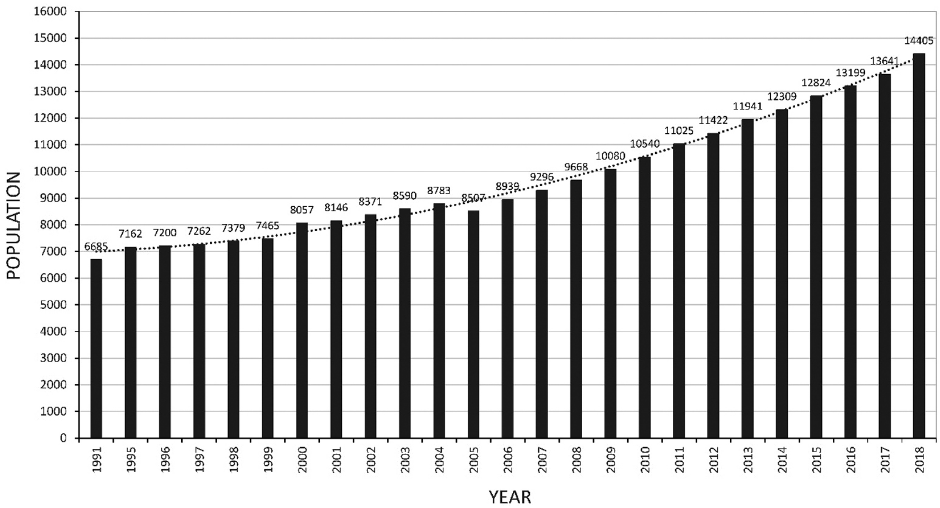

Fig. 2. Population in Czernica Municipality in 1991–2018

As the population has increased, so has the water demand. The average daily water demand in the considered area has increased from 2088.1 m^3/d in 2015 to 2555.3 m^3/d in 2018 (an increase of 467.2 m^3/d – 22.4%). Not only the population growth, but also the development of service centres, shops and educational facilities and the continuous development of the industrial zone in Dobrzykowice have contributed to the increase in water consumption. The water consumption from the water-pipe network in the Czernica Municipality in the years 2015–2018 is illustrated in Fig. 3. Investigations concerned 4 years of exploitation of water pipe network. Historical data of water demand in the past are not available.

The water pipes system supplying consumers in the area under study consists of the Czernica Municipality group water main into which water flows from the Nadolice Wielkie Water Treatment Plant and from the Wrocław water grid (Wrocław Municipal Company LLC) via a booster station. The water main is owned by the Czernica Municipality and managed by Czernica Municipal Services Company LLC. All the Municipality's villages (about 99% of the households) are connected to the water supply system.

The considered water pipe network of the group water distribution system is made of mainly PVC pipes and in a smaller proportion of PE pipes, with nominal diameters ranging from 90 to 225 mm. The network's water mains form a circumferential-terminal system (Fig. 4).

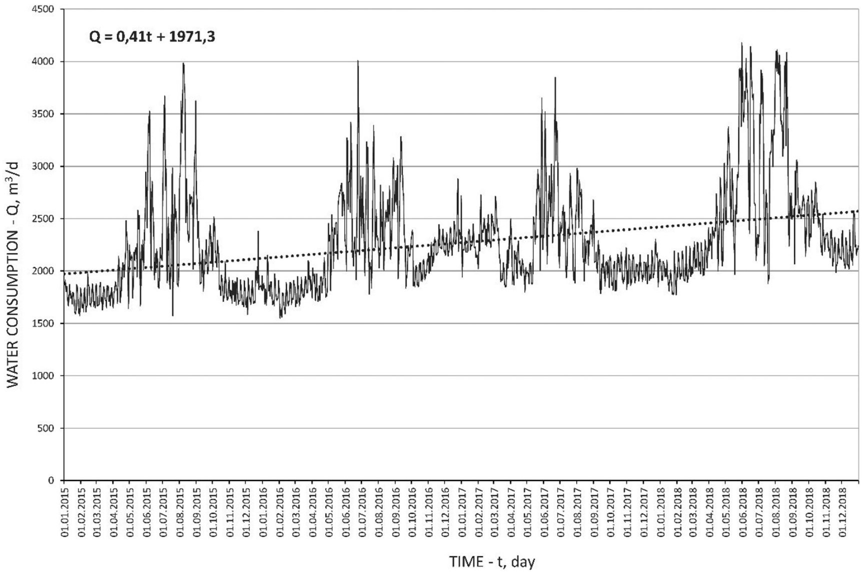

Fig. 3. Water consumption in the Czernica community in 2015–2018

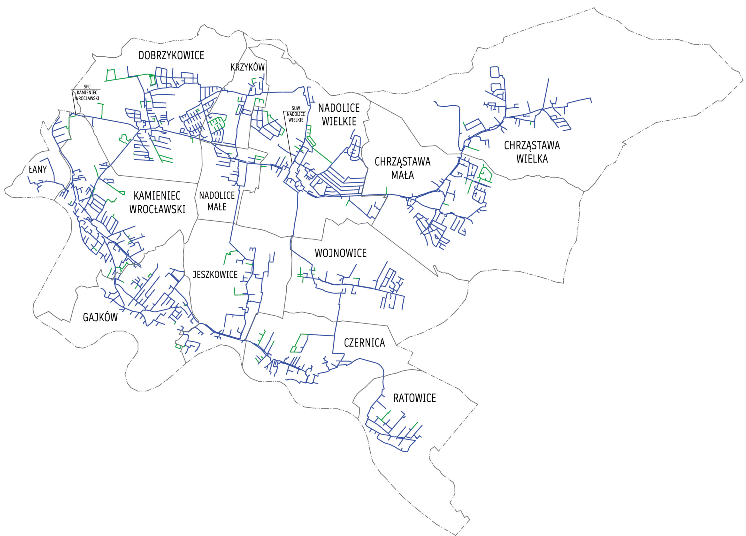

Fig. 4. Water-pipe network structure in the Czernica community

The total length of analysed water-pipe network (water mains and distribution pipes) was equalled to ca. 238 km. The water losses, including network flushing, amount to about 15% of the average daily water demand. This value is satisfactory and correct, and is caused by relatively low failure frequency resulting from good materials used for building and proper maintenance.

2.2. Methods of forecasting daily water distributions

The time series of daily water consumption in the Czernica Municipality in the years 2015–2018 were used to build predictive models. There is lack of daily net water consumption directly in all consumers. It was assumed that in the carried out analysis the amount of water registered by flowmeters in pumping stations is taken into account. It was assumed that water losses in the analysed water systems (below 15% of daily average water demand) did not disrupt the pattern of daily water consumption by consumers. The effectiveness of the exponential smoothing method in forecasting the values of the time series was evaluated.

Exponential smoothing has become very popular as a method of forecasting many types of time series of deterministic-random phenomena. It allows one to determine future values (forecasts) by calculating a moving average of a certain kind, where the current and immediately preceding observations are assigned a greater weight than appropriately older observations with an exponentially declining trend. The method was independently proposed by Brown and Holt in the middle of the last century [17] and later modified by Winters [17]. Brown's model is applicable to time series which do not show any (upward) trend and the fluctuations in their values are the result of only random factors. Holt's model is used to smooth time series showing upward linear trends, in which only random fluctuations occur. The Winters model is applicable to forecasting the values of time series showing both (linear, exponential or damped) trends and seasonal and random fluctuations [17, 18]

Since the time series analysed in this paper shows an upward linear trend (Fig. 3), the effectiveness of forecasting its values was tested (for a linear trend without seasonality) using the exponential smoothing method in accordance with Holt's linear model. Short range forecast was proposed (day by day) which influence on possibility to neglect seasonality. In the model the forecast was determined from equation [19]:

$$\hat{y}_t = F_n + (t - n) \cdot S_n \quad t > n \tag{2.1}$$

where:

$\hat{y}_t$ – a forecast of variable y over time t,
F_n – the smoothed value of the forecasted variable,
S_n – a trend increment estimate,
n – the number of terms in the time series,

and the following model equations were assumed:

$$F_{t-1} = \alpha \cdot y_{t-1} + (1 - \alpha) \cdot (F_{t-2} + S_{t-2}) \tag{2.2}$$

$$S_{t-1} = \beta \cdot (F_{t-1} - F_{t-2}) + (1 - \beta) \cdot S_{t-2} \tag{2.3}$$

where:

F_{t-1} – the smoothed value of the forecasted variable over time $t - 1$,
S_{t-1} – the smoothed value of the trend increment over time $t - 1$,
α, β – smoothing parameters taken from interval $[0, 1]$.

For the calculation of a forecast for the first observation the initial values (trend) were assumed according to the formulas:

$$F_0 = y_1 - \frac{S_0}{2} \tag{2.4}$$

$$S_0 = (y_n - y_1)\,/\,(n-1) \tag{2.5}$$

The accuracy of the forecasts made was determined using the following absolute measures [20, 21] mean error (ME), mean absolute error (MAE), mean squared error (MSE), root mean squared error (RMSE). Also from statistical point of view Theil index is necessary to calculate:

Theil index

$$I^2 = \frac{n \cdot \text{MSE}}{\sum\limits_{t=1}^{n} y_t^2} \tag{2.6}$$

and relative percentage measures

$$V_{\text{MAE\%}} = \frac{\text{MAE}}{\overline{y}} \cdot 100\% \tag{2.7}$$

$$V_{\text{RMSE\%}} = \frac{\text{RMSE}}{\overline{y}} \cdot 100\% \tag{2.8}$$

where:

y_t – variable y over time t,

$\hat{y}_t$ – a forecast of variable y over time t,

$\overline{y}$ – the average of the time series:

$$\overline{y} = \frac{1}{n} \cdot \sum_{t=1}^{n} y_t \tag{2.9}$$

n – the number of terms in the time series.

Statistical analyses of residual series were carried out to test the adequacy of the predictive model. It is assumed that if a model is correct, then as the series size is increased, the terms of the series will become uncorrelated and will be subject to approximately a normal distribution with an average equal to zero, and to finite variation $N(0, \sigma^2)$, i.e. they will approach the white noise series. Thus tests of predictive model adequacy for the observed time series are tests of the residual series concordance with the white noise process. Considering the above, the distribution of residual frequencies and tests of the goodness of fit with the normal distribution were carried out. The analyses presented in this paper were carried out using STATISTICA 13 software.

3. Selected results of analysis

The practical effectiveness of forecasting daily water consumption in the selected municipal water pipe system by means of the exponential smoothing of time series in accordance with Holt's linear model is illustrated by the analytical results presented in Table 1 and in Fig. 5 and in Fig. 6.

Table 1. Measures of accuracy of forecasts made for time span 2015–2018

Exponential smoothing (Holt's linear model): F_0 = 1866, S_0 = 0.2566, α = 0.977, β = 0.000		
Measure of accuracy	Unit	Value
Mean error, ME	m^3/d	–0.00063
Mean absolute error, MAE	m^3/d	151.8
Mean squared error, MSE	m^3/d	54870.8
Root mean squared error, RMSE	m^3/d	234.2
Theil index, I^2	–	0.01016
Relative absolute percentage error, $V_{MAE\%}$	%	6.69
Relative means squared percentage error, $V_{RMSE\%}$	%	10.32

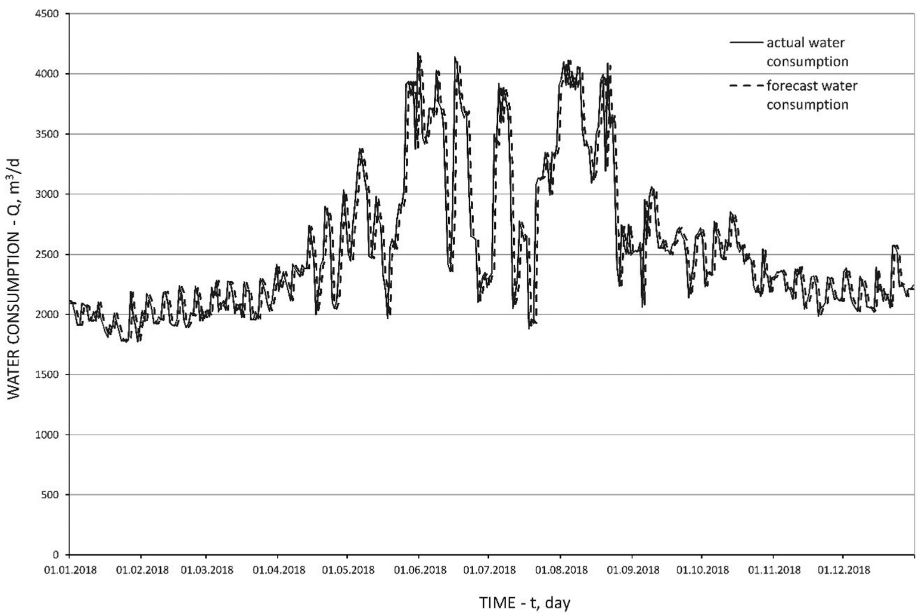

Fig. 5. Actual and forecasted daily water consumption in 2018 in Czernica Municipality

The analyses showed that in the prediction period the absolute value of actual daily water consumption in the considered water pipes system on average deviated from the forecast by 151.8 m^3/d while the root mean squared error amounted to 2342 m^3/d.

The relative forecast errors are at a satisfactory level – the relative absolute percentage error amounts to 6.7% while the relative mean squared percentage error is equal to 10.3%.

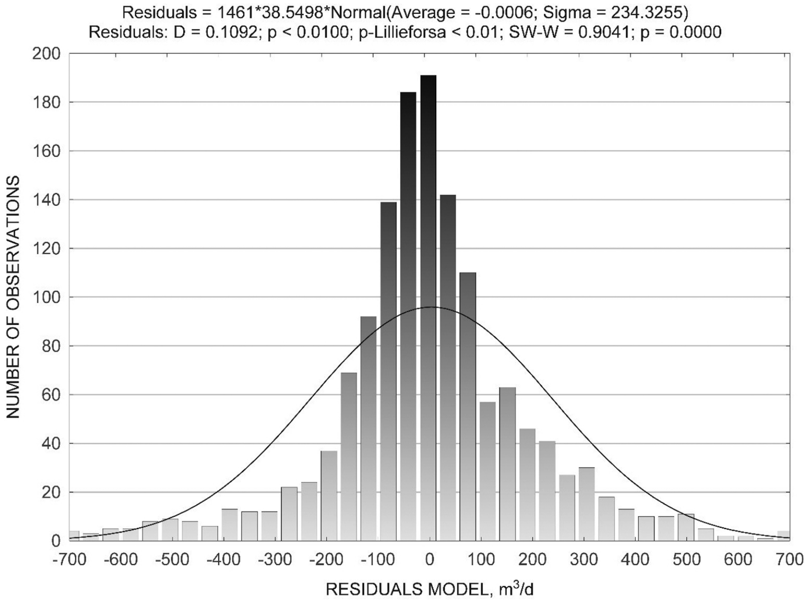

Fig. 6. Residual frequency distribution of the adopted model for generating forecasts

Water demand modelling with such low errors could be considered as proper tool for water companies.

The calculated value of the Theil index is close to zero (0.01016) which indicated that the forecast is not loaded and the model is flexible. Also the statistical analysis of the residual series indicates that the predictive model is well fitted to the character of the investigated daily water consumption process.

Only the difference between the statistics: MAE and RMSE, which may indicate relatively large errors, raises some doubts as to the model used. Therefore the errors of all the generated forecasts were subjected to analysis and some of the errors were found to reach 25–50%, resulting in the above mentioned difference between the measures. Proposed method of forecasting is one of the easiest. Probably in the future the modelling of water demand should be expanded using other prediction methods as e.g. machine learning techniques which should influence on avoiding high models' error.

4. Conclusions

Thanks to the current computer techniques and the increasingly wider range of available computer software it has become possible to use novel algorithms for forecasting. The rapid development of computerization supplied measuring systems with continuous data transmission (most often to a control room) for the purposes of current control and archiving various parameters in computer databases, which has significantly facilitated forecasting.

As a result, more and more attempts have been made to implement novel methods helping to improve the optimization of such processes as: water distribution, water and sewage treatment, etc. This has had a huge impact on the economics of water utility companies. It should be noted, however, that even extensive archival data and the best forecasting methods may prove insufficient to generate forecasts with a satisfactory level of reliability if the operator lacks sufficient knowledge about the given process.

The one rural municipality was chosen for the analysis of daily water consumption in the time span 2015–2018. During four years of investigations the water demand significantly increased (ca. 22%). The exploitation data of water consumption were used to make the predictive model based on exponential smoothing. The relative forecast errors could be acceptable from engineering point of view – the relative absolute percentage error equals to 6.7% and the relative mean squared percentage error amounts to 10.3%. Water demand predictions with such errors could be considered as promising tool for water companies.

The presented results and analysis of the effectiveness of the current forecasting of daily water consumption indicate that prediction by means of exponential smoothing according to Holt's linear model is of relatively high quality. The errors in forecasted daily water consumption values are at an acceptable level, as confirmed by the calculated measures of the accuracy of the forecasts made. Only in few cases, when the series values would change rapidly, it was observed that the predictive model did not follow the actual values.

The topic indicated in the paper seems to be very important and will be expanded in future authors' investigations. Water demand depends on many random factors and its forecasting is crucial due to proper management in water companies.

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Zastosowanie metody wygładzania wykładniczego do prognozowania dobowego zużycia wody w obszarach wiejskich

Słowa kluczowe: prognozowanie, sieć przewodów, wygładzanie wykładnicze, zużycie wody

Streszczenie:

Wzrost zapotrzebowania na wodę w gminach przyległych do dużych aglomeracji, a co za tym idzie wzrost produkcji wody, zmuszają przedsiębiorstwa wodociągowe do szukania nowych rozwiązań dotyczących między innymi optymalnego sterowania takimi procesami jak: ujmowanie i rozdział dyspozycyjnych zasobów wodnych, dystrybucja oraz oczyszczanie wody i ścieków. Aby zapewnić skuteczne sterowanie tymi procesami wymagany jest między innymi skalibrowany model hydrauliczny sieci dystrybucji i model prognostyczny poboru wody. Do bieżącego i krótkoterminowego prognozowania poboru wody wykorzystywane są modele stochastyczne, wprowadzane w postaci zalgorytmizowanej do struktury zarządzania procesem sterowania. Najczęściej stosowane są scałkowane modele autoregresji i średniej ruchomej ARIMA oraz metody wygładzania wykładniczego

szeregów czasowych. Modele klasy ARIMA odwzorowują właściwości statyczne i dynamiczne szeregów stacjonarnych i pewnych klas szeregów niestacjonarnych, interpretowanych jako wynik przejścia białego szumu przez dyskretny filtr liniowy skończenie wymiarowy. Charakteryzują się one różnymi właściwościami przy jednolitym zapisie formalnym oraz identycznych metodach estymacji parametrów dla różnych typów i podklas modeli. Metody prognozowania oparte na algorytmach wygładzania wykładniczego są łatwe do praktycznego zastosowania i nie wymagają założenia o stacjonarności analizowanego szeregu czasowego. W niniejszej pracy przedstawiono obserwacje dotyczące dobowego zużycia wody w jednej z gmin przyległej do Wrocławia wraz z analizą możliwości zastosowania metody wygładzania wykładniczego do krótkoterminowego prognozowania dobowego poboru wody. Analizowany w pracy szereg czasowy wykazuje tendencję wzrostu liniowego, więc sprawdzono skuteczność prognozowania jego wartości przy zastosowaniu metody wygładzania wykładniczego z uwzględnieniem trendu liniowego bez sezonowości – wg modelu liniowego Holta. W celu sprawdzenia adekwatności modelu prognostycznego wykonano analizy statystyczne dotyczące szeregu reszt. Zakłada się, że jeśli model jest prawidłowy, to wraz ze zwiększeniem liczebności szeregu elementy szeregu staną się nieskorelowane i będą w przybliżeniu podlegać rozkładowi normalnemu o średniej równej zeru oraz skończonej wariancji – $N(0, \sigma^2)$, czyli zbliżą się do szeregu białego szumu. Testy związane z adekwatnością modelu prognostycznego do zaobserwowanego szeregu czasowego są zatem testami zgodności szeregu reszt z procesem białego szumu. W tym kontekście, dla przyjętego modelu prognostycznego wykonano rozkład częstości reszt wraz z testami zgodności z rozkładem normalnym. Przedmiotowa sieć wodociągowa grupowego systemu dystrybucji wody zbudowana jest z przewodów o średnicach nominalnych od 90 do 225 mm, głównie z PVC oraz w mniejszej części z PE. Główne przewody sieci tworzą układ obwodowo-końcówkowy. Straty wody wraz z płukaniem sieci kształtują się na poziomie ok. 15% średniego dobowego zapotrzebowania na wodę. Przeprowadzone analizy wykazały, że średnio, w okresie predykcji, rzeczywiste wartości dobowego poboru wody w analizowanym systemie wodociągowym odchylały się co do bezwzględnej wartości, od prognoz, o 151,8 m^3/d, natomiast pierwiastek błędu średniokwadratowego wyniósł 234,2 m^3/d. Względne błędy prognozy kształtują się na zadowalającym poziomie – procentowy względny błąd absolutny wynosi 6,7%, natomiast procentowy względny błąd średniokwadratowy jest równy 10,3%. Obliczona wartość współczynnika Theila jest bliska zeru (0,01016), jak również analiza statystyczna szeregu reszt wskazuje na dobre dopasowanie modelu prognostycznego do charakteru rozpatrywanego procesu dobowego poboru wody. Jedynie różnica pomiędzy statystykami MAE i RMSE może wskazywać na błędy o stosunkowo dużych wartościach budzi pewne wątpliwości co do zastosowanego modelu. Dlatego poddano analizie błędy wszystkich wygenerowanych prognoz, która wykazała, że pojedyncze z nich osiągają wartości od 25% do 50%, co powoduje wspomnianą różnicę między określonymi miernikami. Zaprezentowane rezultaty oraz analiza efektywności bieżącego prognozowania dobowych poborów wody wskazują na relatywnie dobrą jakość predykcji za pomocą metody wygładzania wykładniczego z wykorzystaniem modelu liniowego Holta. Błędy prognozowanych wartości dobowego poboru wody kształtują się na akceptowalnym poziomie co potwierdzają obliczone mierniki dokładności uzyskanych prognoz. Jedynie w nielicznych przypadkach, wówczas gdy wartości szeregu zmieniały się w dynamiczny sposób, zaobserwowano, że model prognostyczny nie nadąża z prognozą za wartościami rzeczywistymi.

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