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Research paper

Assessment of the impact of the number of girders on the dynamic behaviour of Geiger dome

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Abstract: In this paper, the dynamic behaviour of the tensegrity domes is explored. The consideration includes all cable structures called Geiger domes, i.e., two cases of configurations (with a closed and open upper section) and two variants of the nature of a dome (regular and modified) are taken into account. Particularly, the impact of the number of girders on the natural frequencies is analysed. A geometrically quasi-linear model is used, implemented in an original program written in the Mathematica environment. The results confirm that the number of girders affects the number of infinitesimal mechanisms. However, the dynamic behaviour does not depend on the number of mechanisms. The most important is the nature of a dome and the type of load-bearing girder. Especially, the behaviour of Geiger domes with a closed upper section is specific. In this case, not only the frequencies corresponding to the infinitesimal mechanisms depend on the prestress. There are additional frequencies that depend on prestress. The number of them, and the sensitivity on the initial prestress changes, depends on the number of girders. Generally, for the same number of girders, the natural frequencies of regular domes are higher than for the modified ones.

Keywords: Geiger dome, infinitesimal mechanism, self-stress state, natural frequency

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1. Introduction

The Geiger dome is a ‘cable dome’ constructed with tensed cables and compressed struts. It is the first dome classified as a tensegrity structure, patented by David Geiger in 1988 [1]. This structure is represented by the number of flat load-bearing girders connected with cables. The elements are in a self-equilibrium system of internal forces (self-stress state), which stabilize existing infinitesimal mechanisms. Additionally, the adjustment of prestressing forces allows controlling the static and dynamic properties of the structure. Due to low material demand, reduced wind lift, and resistance to uneven load settling, this type of structure is the best solution for long-span roofs and covers. These lightweight construction systems were employed on the roof of the Olympic Gymnastics Hall in Seoul [2].

After the appearance of the Geiger dome, the ideas of modifying this cable structure have been presented. The leading research concerned the layout of a geometrical mesh of the structure and form-finding new systems. In [3], modifications of the layout by exchanging the ridge cables of the conventional cable dome for hinged ridge beams were proposed. The experimental study and comparative analysis with the original Geiger dome were studied. As part of experimental studies, the construction methods [4] and the construction shape-forming process [5] were also provided. In [6], authors proposed new types of Geiger patent-based domes. In turn, in [7] a new form-finding method for designing irregular and asymmetric cable-strut structures was proposed. To change the shape of the structure (generating new topologies) optimization algorithms were used too [8–10]. The new topology aims to achieve the desired performance criteria, such as e.g., the level of stiffness. In the existing Geiger domes, it is also possible to control the stiffness, for example in [11], to monitor the possible stiffness degeneration the dynamic testing was used.

Searching for stable configurations for Geiger domes is the most important part of each research. The appropriate state of self-stress provides the stability to the elements [12–14] and assures proper response of the structure on load. Due to a non-conventional shape, the investigation of the response of structure not only on simple load conditions, like a self-weight [15] but also more complex ones, for example, non-uniform snow load [16], is extremely important. Generally, the static analysis of the Geiger dome includes the influence of the level of self-stress state on the behaviour of structures. It is a parametrical approach. The case of dynamic analysis is the same, but in the literature known to authors, this area is still understudied. Some papers include dynamic analysis of the Geiger dome [17–20], however, no such an assessment that includes complete parametrical analysis (qualitative and quantitative assessment).

In the paper, the parametrical dynamic analysis of Geiger domes is conducted. Two cases of configurations (with a closed and open upper section) are considered. Additionally, two variants of nature of a dome (regular and modified) are taken into account. This consideration includes all cable structures called Geiger domes. It is well known from the literature [20–22], the number of natural frequencies, which depend on the prestressing, is equal to the number of infinitesimal mechanisms. In the absence of compression, these frequencies are zero, and the corresponding modes of vibration implement the mechanisms. After introducing an initial prestress, the frequencies increase in proportion to the square root of this state. The remaining frequencies are practically insensitive to changes in the level of initial prestress. In the case of Geiger domes, there are a few questions. Firstly, is it possible to control the occurrence of mechanisms by changing the number of girders? Secondly, the behaviour of which type of geometry (regular and modified) is easier to control? Thirdly, is the behaviour the same for domes with the same number of mechanisms? Finally, is the number of natural frequencies depending on the prestressing equal to the number of infinitesimal mechanisms?

To answer these questions, the broadly understood parametric analysis is carried out. The influence of the initial prestress on the dynamic behaviour of Geiger domes is analysed. The complete qualitative and quantitative assessment is performed. Additionally, the formulas on self-equilibrium forces are derived.

2. Methods of analysis

The tensegrity cable dome is a n-element space truss ($e = 1, 2, \ldots, n$) with m-degrees of freedom $\mathbf{q}\left(\in \mathbb{R}^{m\times 1}\right) = [q_1\ q_2\ \ldots\ q_m]^T$, consisting of flat load-bearing girders connected by cables. The structure is described by the elasticity matrix $\mathbf{E}\,(\in \mathbb{R}^{n\times n})$, compatibility matrix $\mathbf{B}\,(\in \mathbb{R}^{n\times m})$, and consequent matrix of masses $\mathbf{M}\,(\in \mathbb{R}^{m\times m})$. The explicit matrices forms can be found for example in [20, 23]. This type of dome is characterized by two immanent tensegrity features, i.e., the self-stress state and infinitesimal mechanism. The first feature allows us to define the matrix of initial prestress $\mathbf{K}_G(\mathbf{S})\,(\in \mathbb{R}^{m\times m})$, whereas the second makes it possible to control the dynamic behaviour. Both features depend only on the compatibility matrix [20, 24–26]. Zero eigenvalues of the matrix $\mathbf{B}\,\mathbf{B}^T\,(\in \mathbb{R}^{n\times n})$ are responsible for existing the self-stress states, whereas zero eigenvalues of the matrix $\mathbf{B}^T\mathbf{B}\,(\in \mathbb{R}^{m\times m})$ – for existing the infinitesimal mechanisms. The self-stress state is considered as an eigenvector $\mathbf{y}_S$ related to the zero eigenvalue of the matrix $\mathbf{B}\,\mathbf{B}^T$. The self-equilibrium systems of longitudinal forces $\mathbf{S}$ depend on the eigenvector $\mathbf{y}_S$ and initial prestress level S ($\mathbf{S} = \mathbf{y}_S S$). The dynamic analysis of the Geiger dome is a parametrical approach. It leads to the determination of the impact of the initial prestress level S on the frequency of vibrations. The natural vibration equation associated with the small motions of a tensegrity structure is given in terms of the well-known generalized eigenvalue equation:

$$\left[\mathbf{B}^T\mathbf{E}\,\mathbf{B} + \mathbf{K}_G(\mathbf{S}) - (2\pi f)^2\mathbf{M}\right]\widetilde{\mathbf{q}} = \mathbf{0} \tag{2.1}$$

where: f – the frequency of natural vibrations, $\widetilde{\mathbf{q}}$ – the amplitude vector.

The modal analysis (2.1) leads to a determination of the natural frequencies of vibrations f_i. The omission of the influence of prestress ($\mathbf{K}_G(\mathbf{S}) = \mathbf{0}$) in equation (2.1) leads to the zero frequency of natural vibrations. These zero values correspond to the forms of vibrations that implement the mechanisms.

3. Results

In the paper, the dynamic parametric analysis of four cases of Geiger domes is performed. The two variants of the geometry of the load-bearing girders – type A [27] (Fig. 1a) and type B [13] (Fig. 1b) – are considered. For both types the regular (RG – Fig. 2a, b) and modified (MG – Fig. 2c, d) structures are used. The modification of the Geiger patent (RG) is to add additional cables connecting the top nodes. Additionally, a different number of load-bearing girders (6, 8, 10, and 12) are considered. The diameter of the domes is equal to 12 m and the height – to 3.25 m. It is assumed that the cables in tensegrity domes are made of steel S460N. The cables type A with Young modulus 210 GPa [28] are used. The struts are made of hot-finished circular hollow sections (steel S355J2) with the Young

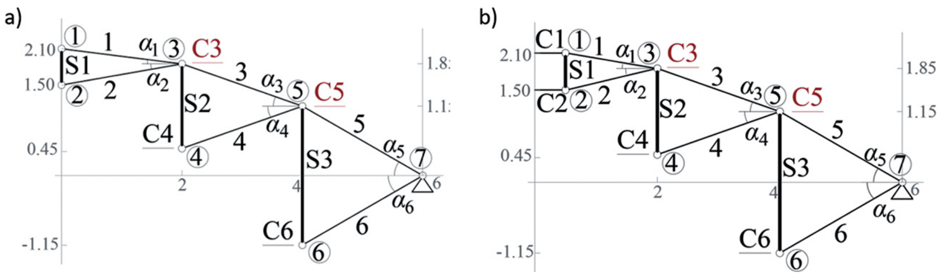

Fig. 1. Load-bearing girders of Geiger dome: a) type A, b) type B

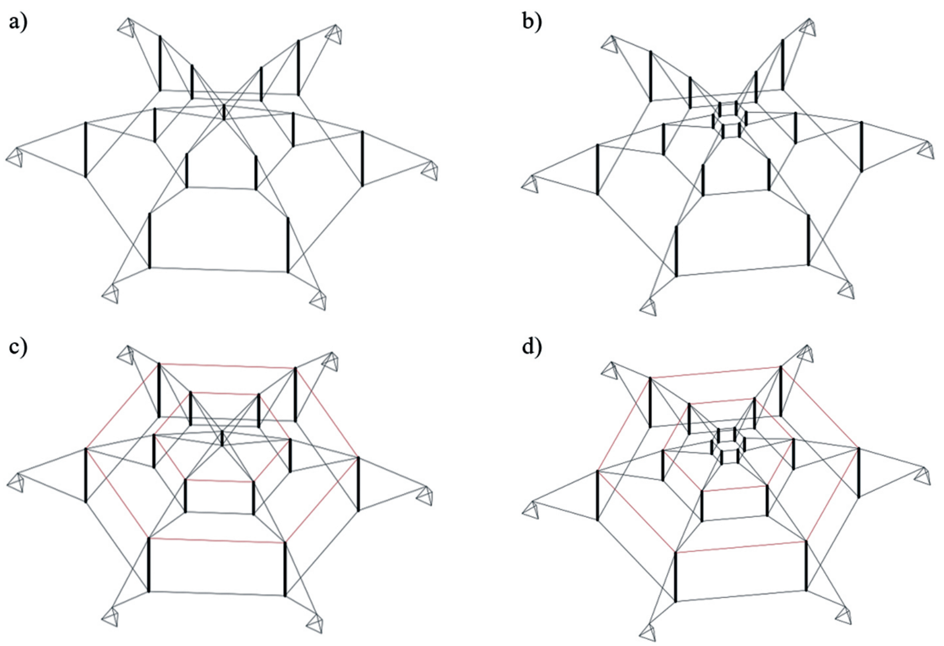

Fig. 2. Geiger domes: a) RG 6A, b) RG 6B, c) MG 6A, d) MG 6B

modulus 210 GPa. The density of steel is equal $\rho = 7860$ kg/m^3. The cables with the diameter $\phi = 20$ mm and load-bearing capacity $N_{Rd} = 110.2$ kN are taken into account. For struts, there are rods with a diameter $\phi = 76.1$ mm and thickness $t = 2.9$ mm with lengths 0.6 m, 1.4 m, and 2.3 m and load-bearing capacity $N_{Rd} = 224.3$ kN, 170.5 kN, and 107.1 kN respectively. The domes are supported in all external nodes. The calculations were made with the use of the quasi-linear model implemented in a proprietary program written in the Mathematica environment.

3.1. Qualitative assessment

This qualitative assessment is required to determine the immanent features like infinitesimal mechanisms and self-equilibrium systems of longitudinal forces (self-stress states) which stabilize mechanisms. The summarized results are contained in Table 1.

Table 1. Results of the qualitative analysis of Geiger domes

No. of the load-bearing girders	No. of nodes	No of d.o.f	No. of elements (n)	No. of struts (ns)	No. of mechanisms (nm)	No. of self-stress states
Type A						
6	32	78	61 (73)	13	18 (8)	1 (3)
8	42	102	81 (97)	17	22 (8)	1 (3)
10	52	126	101 (121)	21	26 (8)	1 (3)
12	62	150	121 (145)	25	30 (8)	1 (3)
Type B						
6	42	108	78 (90)	18	31 (21)	1 (3)
8	56	144	104 (120)	24	41 (27)	1 (3)
10	70	180	130 (150)	30	51 (33)	1 (3)
12	84	216	156 (180)	36	61 (39)	1 (3)

(.) – the results for the modified domes (MG).

Generally, the modification of the regular dome leads to the reduction of the number of infinitesimal mechanisms, but at the same time, the number of self-stress states increases. In the case of modified domes (MG) of type A, independently on the type and on the number of load-bearing girders, eight mechanisms were identified, whereas for type B the number of mechanisms (nm) depends on the number of struts (ns):

$$nm = ns + 3 \tag{3.1}$$

In the case of the regular dome (RG), the number of mechanisms depends on the type of dome:

$$\text{type A}: \quad nm = ns + 5; \qquad \text{type B}: \quad nm = 0.5(n - ns) + 1 \tag{3.2}$$

In turn, the number of self-stress states does not depend on the number of load-bearing girders. In the case of regular domes (RG), only one self-stress state was identified (Table 2), while in the case of modified domes (MG) – three ones. Since neither of the three states correctly identifies the type of elements, a superimposed self-stress state is necessary (Table 3). The values on the self-stress forces $\mathbf{y}_S$ are normalized in such a way that the minimum compressed force in struts is equal to –1 (Note! In Tables 2 and 3 the values of self-stress state for domes with 6, 8, 10, and 12 load-bearing girders are presented).

Table 2. Values of self-stress state $\mathbf{y}_S$ of the regular Geiger dome (RG)

Type A						Type B					
el.	$\mathbf{y}_S$	el.	$\mathbf{y}_S$	el.	$\mathbf{y}_S$	el.	$\mathbf{y}_S$	el.	$\mathbf{y}_S$	el.	$\mathbf{y}_S$
S1	–0.3804 (6) –0.5072 (8) –0.6341 (10) –0.7609 (12)	**1**	0.5112			**S1**	–0.0845	**1**	0.5142	**C1**	0.5072 0.6627 0.8207 0.9799
S2	–0.3043	**2**	0.3678			**S2**	–0.3043	**2**	0.3721	**C2**	0.3623 0.4734 0.5862 0.6999
S3	–1.0000	**3** **4**	0.9213	**C4**	0.8696 1.1361 1.4070 1.6799	**S3**	–1.0000	**3** **4**	0.9213	**C4**	0.8696 1.3614 1.4070 1.6799
		5 **6**	2.0061	**C6**	1.7391 2.2723 2.8140 3.3597			**5** **6**	2.0061	**C6**	1.7391 2.2723 2.8140 3.3597

Table 3. Values of self-stress state $\mathbf{y}_S$ of the modified Geiger dome (MG)

Type A						Type B					
el.	$\mathbf{y}_S$	el.	$\mathbf{y}_S$	el.	$\mathbf{y}_S$	el.	$\mathbf{y}_S$	el.	$\mathbf{y}_S$	el.	$\mathbf{y}_S$
S1	–0.2277 (6) –0.3036 (8) –0.3795 (10) –0.4554 (12)	**1**	0.3060			**S1**	–0.0506	**1**	0.3076	**C1**	0.3034 0.3964 0.4909 0.5862
S2	–0.2646	**2**	0.2201			**S2**	–0.2646	**2**	0.2225	**C2**	0.2167 0.2830 0.3505 0.4185

Continued on next page

Table 3 – *Continued from previous page*

Type A						Type B					
el.	$\mathbf{y}_S$	el.	$\mathbf{y}_S$	el.	$\mathbf{y}_S$	el.	$\mathbf{y}_S$	el.	$\mathbf{y}_S$	el.	$\mathbf{y}_S$
S3	–1.0000	**3** **4**	0.8010	**C3**	0.2356 0.3078 0.3812 0.4551	**S3**	–1.0000	**3** **4**	0.8010	**C3**	0.2356 0.3078 0.3812 0.4551
		5 **6**	2.0061	**C4**	0.7560 0.9877 1.2233 1.4606			**5** **6**	2.0061	**C4**	0.7560 0.9877 1.2233 1.4606
				C5	0.2270 0.2968 0.3676 0.4389					**C5**	0.2270 0.2968 0.3676 0.4389
				C6	1.7391 2.2720 2.8139 3.3597					**C6**	1.7391 2.2720 2.8139 3.3597

Due to the regular Geiger dome is consisting of flat girders (Fig. 3), the formulas on self-equilibrium forces are possible to derive. These formulas (Table 4) depend on type of

Table 4. Formulas on self-equilibrium forces for the regular Geiger dome

<table>
<tr><td>type A</td><td>type B</td></tr>
<tr><td colspan="2">$N_1 = \text{constant}$
$N_i = N_{i-1}\dfrac{\sin(\alpha_{i-1})}{\sin(\alpha_i)}; \quad i = 2,4,6,8,\ldots$
$N_j = \dfrac{N_{j-2}\cos\left(\alpha_{j-2}\right) + N_{j-1}\cos\left(\alpha_{j-1}\right)}{\cos\left(\alpha_j\right)}; \quad j = 3,5,7,\ldots$</td></tr>
<tr><td colspan="2">$N_{Ck} = 0.5N_k\dfrac{\cos(\alpha_k)}{\cos(\beta)}; \quad k = 4,6,8,\ldots$</td></tr>
<tr><td></td><td>$N_{C1} = 0.5N_1\dfrac{\cos\left(\alpha_1\right)}{\cos\left(\beta\right)}$
$N_{C2} = 0.5N_2\dfrac{\cos\left(\alpha_2\right)}{\cos\left(\beta\right)}$</td></tr>
<tr><td>$N_{S1} = ngN_2\sin\left(\alpha_2\right)$</td><td>$N_{S1} = N_2\sin\left(\alpha_2\right)$</td></tr>
<tr><td colspan="2">$N_{Sn} = N_{2n}\sin\left(\alpha_{2n}\right); \quad n = 2,3,4\ldots$</td></tr>
</table>

load-bearing girders, on is the angle of inclination of cables of gird – α (Fig. 3) and is the angle between perimeter cables – 2β (Fig. 4).

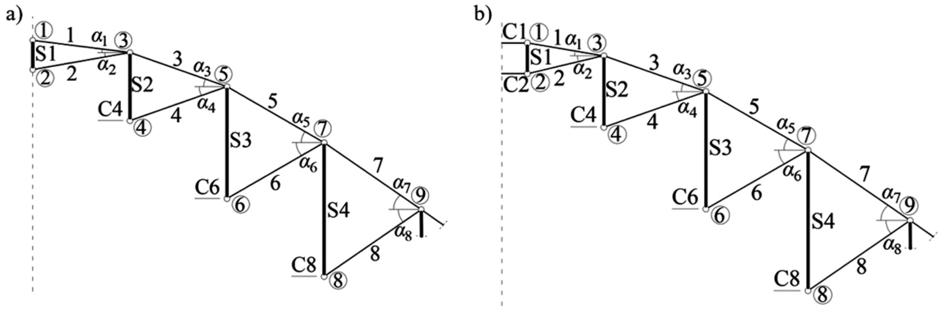

Fig. 3. Load-bearing girders of the Geiger dome: a) type A, b) type B

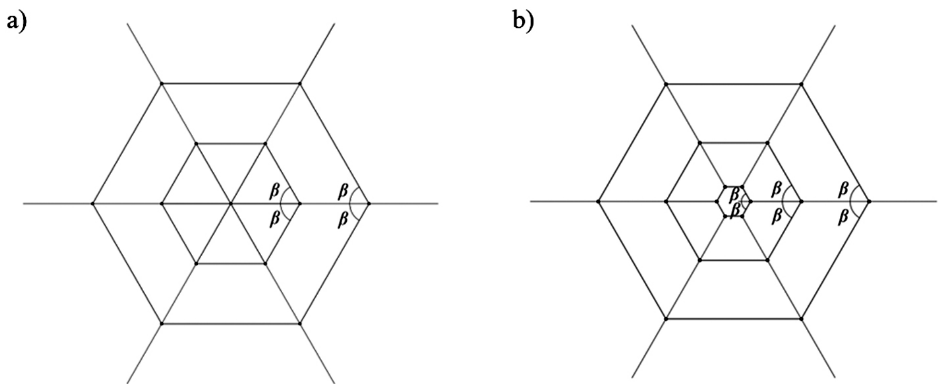

Fig. 4. View from the top of the regular Geiger domes: a) RG 6A, b) RG 6B

3.2. Quantitative assessment

The quantitative assessment leads to determining the influence of the initial prestress level $S(\mathbf{S} = \mathbf{y}_S S)$ on the natural frequency f_i. The maximum prestress levels are assumed as $S_{\max} = 50$ kN – the maximum effort of the structures is equal to 0.9. In Fig. 5, the first and last frequencies corresponding to the infinitesimal mechanisms are showed. In the case, if $S = 0$ these frequencies are zero and after introducing an initial prestress, the frequencies increase. As we can see, the higher frequencies are more sensitive to the change in prestressing. Additionally, the dynamic behaviour of domes heavily depends on the type of load-bearing girder. In the case of a regular dome of type A (Fig. 5a), the range of changes in frequencies is practically insensitive to changes in the number of girders – the last frequency, which corresponds to a mechanism for RG 6A ($f_{18} = 16.4$ Hz) is almost the same as the last for RG 12A ($f_{30} = 16.7$ Hz). However, in the case of the modified dome (Fig. 5b), the results are not entirely convergent, the value of the eighth frequency f_8 for $S_{\max}$

for the MG 6A dome is equal 12.3 Hz, whereas for MG 12A – (13.7 Hz). It is completely different in the case of a dome of type B. The influence of prestress forces depends on the number of girders both for regular (Fig. 5c) and modified (Fig. 5d) domes. The first natural frequency for all domes is almost the same ($f_1(S_{max})$ = 6.1 Hz÷6.4 Hz), but the last frequency, which corresponds to the mechanism, is different. In the case of regular domes for S_{max} this frequency is f_{18}(RG 6B) = 38.5 Hz and f_{61}(RG 12B) = 74.7 Hz, whereas for modified – f_{21}(MG 6B) = 29.8 Hz and f_{39}(MG 12B) = 57.8 Hz.

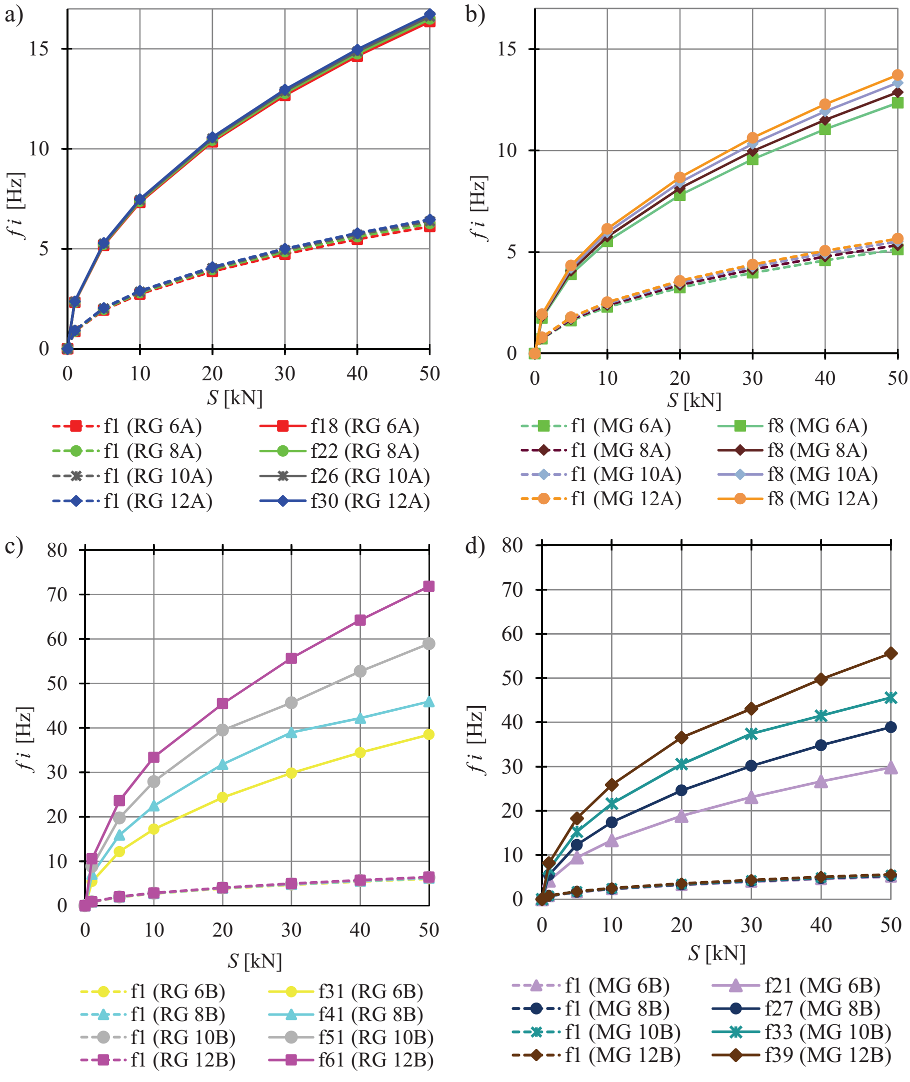

Fig. 5. Influence of the initial prestress S on the natural frequency: a) RG iA, b) MG iA, c) RG iB, d) MG iB

The number of natural frequencies, which in the case of $S = 0$ are zero, is equal to the number of infinitesimal mechanisms and the forms of vibration realize these mechanisms. For example, in Fig. 6 the forms of vibration realize mechanisms for RG 6A dome is showed. It is interesting, there are eight different forms of vibration but six different frequencies ($f_2 = f_3$ and $f_5 = f_6$). In the case of other domes, it is the same.

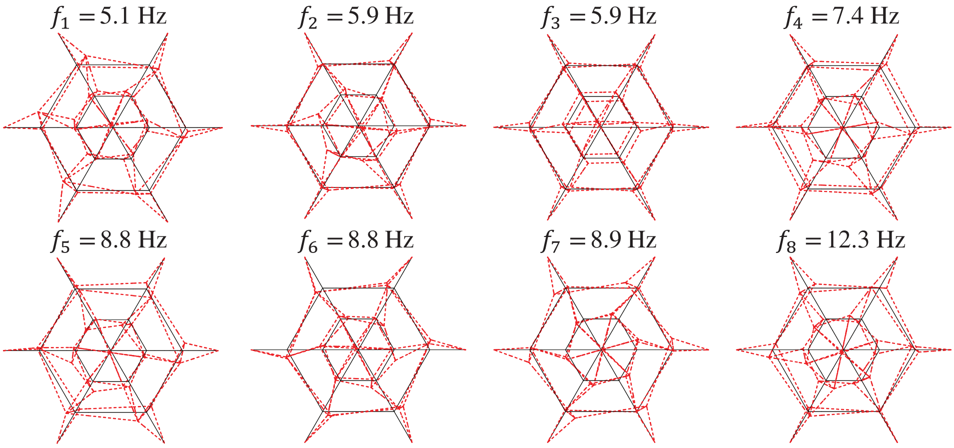

Fig. 6. Forms of vibration for the MG 6A dome (values of frequencies for $S_{\max}$)

As we can see, the dynamic behaviour of Geiger domes depends both on the case of dome (regular, modified) and on the type of load-bearing girder (type A, type B). Generally, for the same number of girders the natural frequencies of regular domes are higher than in the case of modified ones. Additionally, the Geiger domes of type A are the specific structures. It is well known that the number of natural frequencies, depending on the prestressing, is equal to the number of infinitesimal mechanisms (f_{nm}), but in the case of Geiger domes of type A it is different. In this case the number of dependent frequencies f_{total} is greater and depends on the number of girders (ng):

$$f_{\text{total}} = f_{nm} + f_{\text{add}}; \quad f_{\text{add}} = (ng - 3) \tag{3.3}$$

In Fig. 7 the last frequency corresponding to the infinitesimal mechanism (f_{nm}), the next additional depended of prestress (f_{add}) and the first independent of prestress (f_{const}) ones are showed. The results for domes built with 6 (Figs. 7a, b) and 12 (Figs. 7c, d) girders are presented. In the absence of prestress ($S = 0$) the frequency f_{nm} is equal zero, and after introducing prestress S the values f_{nm} increase in nonlinear way. Whereas, the behaviour of additional frequency depended of prestress (f_{add}) is different. In the absence of prestress f_{add} is not zero and dependence on the prestress is almost linear. Additionally, the number of frequencies f_{add}, and the sensitivity on the initial prestress changes, depends on the number of girders. More sensitive to the changes are higher frequencies. In turn, the value of first frequency independent of prestress (f_{const}) for all Geiger domes are at the same level $f_{\text{const}} = 40.7$ Hz÷44.5 Hz (Table 5).

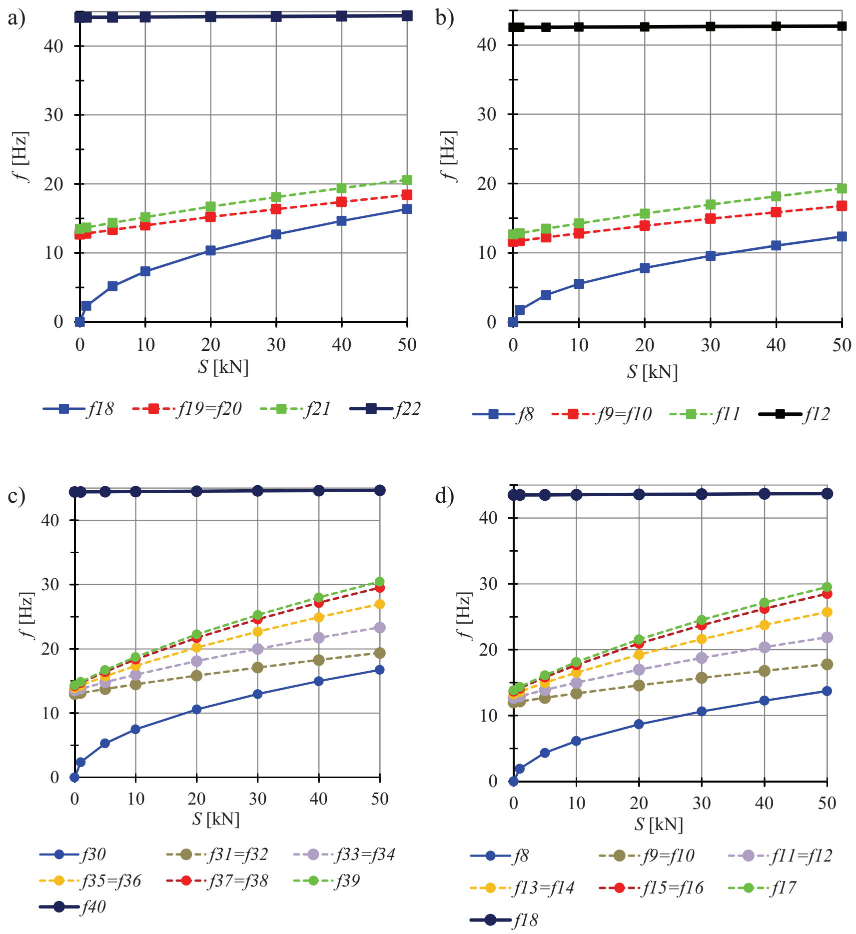

Fig. 7. Influence of the initial prestress S on the natural frequency: a) RG-6A, b) MG-6A, c) RG-12A, d) MG-12A

Table 5. Values of the first independent on prestress natural frequency (f_{const}) of Geiger domes

No. of the load-bearing girders	Regular dome			Modified dome		
	i	f_i ($S = 0$) [Hz]	f_i ($S = 50$) [Hz]	i	f_i ($S = 0$) [Hz]	f_i ($S = 50$) [Hz]
Type A						
6	(22)	44.16	44.39	(12)	42.56	42.74
8	(28)	44.41	44.66	(14)	43.12	43.32
10	(34)	44.45	44.71	(16)	43.38	43.59
12	(40)	44.41	44.67	(18)	43.50	43.71

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Table 5 – *Continued from previous page*

No. of the load-bearing girders	Regular dome			Modified dome		
	i	f_i $(S = 0)$ [Hz]	f_i $(S = 50)$ [Hz]	i	f_i $(S = 0)$ [Hz]	f_i $(S = 50)$ [Hz]
Type B						
6	(32)	41.87	42.08	(22)	40.73	40.90
8	(42)	42.00	42.24	(28)	41.22	41.40
10	(52)	41.86	42.10	(34)	41.34	41.54
12	(62)	41.58	41.83	(40)	41.26	41.47

4. Conclusions

In this paper, the dynamic behaviour of the Geiger dome is explored. Particularly, the impact of the number of girders on the natural frequencies is analysed. Two types of load-bearing girders, i.e., with a closed (type A) and open (type B) upper section, are considered. Additionally, two variants of geometry (regular and modified) are taken into account. The considerations contained in this paper answer four questions, i.e., is it possible to control the occurrence of mechanisms by changing the number of girders? The behaviour of which type of geometry (regular and modified) is easier to control? Is the behaviour the same for domes with the same number of mechanisms? Is the number of natural frequencies depending on the prestressing equal to the number of infinitesimal mechanisms?

The study confirms that the number of girders affects the number of infinitesimal mechanisms. However, it is not the most important. A much more important effect on the dynamic behaviour has a type of load-bearing girder. The domes of type B are easier to control. In this case, the influence of prestress forces depends on the number of girders both for regular and modified domes. The range of changes in frequencies is much bigger than in the domes of type A. Additionally, the number of natural frequencies depending on the level of prestress corresponds to the infinitesimal mechanisms. In the case of $S = 0$ these frequencies are zero, and after introducing an initial prestress they increase in a nonlinear way. The impact is greater with a lower level of prestress. In turn, in the case of a regular dome of type A, the range of changes of frequencies is practically insensitive to changes in the number of girders, despite the number of mechanisms differs – there are 18 mechanisms for domes built with 6 girds and 30 ones for domes build with 12 girds. However, in the case of the modified dome of type A, the results are not entirely convergent, despite the number of mechanisms does not depend on the number of girds (there are 8 mechanisms). The behaviour of models with the same number of mechanisms is different. In addition, it should be noted, that the Geiger domes of type A are the specific structures. In this case, not only the frequencies corresponding to the infinitesimal mechanisms depend on the prestress. There are additional frequencies that depend on prestress. The number of them, and the sensitivity on the initial prestress changes, depends on the number of girders.

To sum up, the dynamic behaviour of Geiger domes depends both on the nature of a dome (regular, modified) and on the type of load-bearing girder (type A, type B). Generally, for the same number of girders, the natural frequencies of regular domes are higher than for the modified ones.

Additionally, the formulas on self-equilibrium forces for the regular Geiger dome are derived.

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Ocena wpływu liczby dźwigarów na dynamiczne zachowanie kopuły Geigera

Słowa kluczowe: częstotliwość drgań, kopuła Geigera, nieskończenie mały mechanizm, stan samonaprężenia

Streszczenie:

W artykule zbadano zachowanie dynamiczne kopuł Geigera. W sposób szczególny przeanalizowano wpływ liczby dźwigarów nośnych na częstotliwość drgań własnych. Analizie poddano dwa typy dźwigarów nośnych tj. z zamkniętą (typ A) oraz otwartą (typ B) górną częścią dźwigara. Dodatkowo wzięto pod uwagę dwa typy geometrii kopuły (zwykłą i zmodyfikowaną). Przedstawione rozważania odpowiadają na następne pytania tj. czy jest możliwa kontrola liczby mechanizmów poprzez zmianę liczby dźwigarów nośnych? Jaki typ kopuły (zwykła czy zmodyfikowana) jest łatwiejszy do kontroli? Czy zachowanie kopuł z taką samą liczbą mechanizmów infinitezymalnych jest podobne? Czy liczba częstotliwości drgań własnych, zależnych od wstępnego sprężenia, jest równa liczbie nieskończenie małych mechanizmów?

Analiza potwierdziła, że liczba dźwigarów nośnych ma wpływ na liczbę nieskończenie małych mechanizmów. Jednak zachowanie dynamiczne kopuł zależy głównie od geometrii kopuły oraz od typu dźwigara nośnego, a nie od liczby mechanizmów. Łatwiejszymi konstrukcjami do kontroli są

kopuły zwykłe oraz zmodyfikowane typu B. W ich przypadku wpływ wstępnego sprężenia zależy od liczby dźwigarów. Zakres zmian częstotliwości drgań własnych jest znacznie większy niż w przypadku kopuł typu A. Dodatkowo w przypadku kopułtypu B, liczba częstotliwości drgań własnych, zależnych od wstępnego sprężenia, odpowiada liczbie mechanizmów. W przypadku kiedy siła sprężająca S jest równa zero, częstotliwości i te wynoszą zero, a przy zwiększeniu siły sprężającej – rosną nieliniowo. Nieliniowe zachowanie jest bardziej widoczne przy niższym poziome wstępnego sprężenia. Z kolei w przypadku zwykłej kopuły typu A, zakres zmian częstotliwości drgań jest praktycznie niewrażliwy na zmianę liczby dźwigarów, pomimo, że liczba mechanizmów jest różna. Dla kopuły zbudowanej z 6 dźwigarów nośnych zidentyfikowano 6 mechanizmów, a dla kopuły zbudowanej z 12 dźwigarów – 30 mechanizmów. Jednakże, w przypadku zmodyfikowanej kopuły typu A wyniki nie są do końca zbieżne, pomimo że liczba mechanizmów jest niezależna od liczby dźwigarów (zidentyfikowano 8 mechanizmów). Zachowanie kopuł z tą samą liczbą mechanizmów jest różne. Dodatkowo należy zauważyć, że kopuły Geigera typu A są specyficzne. W tym przypadku od wstępnego sprężenia zależą nie tylko te częstotliwości drgań, które odpowiadają nieskończenie małym mechanizmom. Pojawiają się dodatkowe częstotliwości drgań, które również są zależne od wstępnego sprężenia. Liczba takich dodatkowych częstotliwości oraz ich wrażliwość na zmianę poziomu wstępnego sprężenia, jest zależna od liczby dźwigarów nośnych.

Podsumowując, zachowanie dynamiczne kopuł Geigera zależy zarówno od rodzaju geometrii konstrukcji (zwykła lub zmodyfikowana), jak i od typu dźwigara nośnego (typ A lub B). Generalnie, w przypadku tej samej liczby dźwigarów nośnych, częstotliwości drgań własnych są wyższe dla kopuł zwykłych, niż dla zmodyfikowanych.

Dodatkowo w pracy, dla zwykłej kopuły Geigera, zostały wyprowadzone wzory na siły wstępnego sprężenia.

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