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Vibration attenuation and frequency shifting of beam structures using single and multiple dynamic absorbers in the parallel

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The finite element method (FEM) using Ansys program (APDL) was used in this study to evaluate the idea of tuned vibration absorbers applied to a beam construction for the undamped system. The ideal location for the Dynamic Vibration Absorbers (DVAs) and their numbers to be installed on the fixed-fixed beam in order to lessen beam vibration was also investigated. The DVA was coupled to the fixed-fixed beam vibration node for three vibration modes. The natural frequency and frequency response of the beam were calculated in this study using modal and harmonic analysis, respectively. The vibrational characteristics of the F-F beam with and without DVAs were presented. The simulation results demonstrated that the vibration amplitude decreases in the presence of the DVAs and its reduction depends on the locations of the DVAs and its number. In addition, the attached DVAs affect the structural beam vibration. Depending on the modes of vibration, the vibrational peak is the optimal place to attach DVA.

1. Introduction

Forced vibration and free vibration are the two different types of vibration. When no external forces are applied and the system oscillates as a result of internal forces, this is known as free vibration. At one or more basic frequencies, the vibrating system is free, and these frequencies are affected by the mass, stiffness,

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boundary condition, and mass distributions [1]. Vibration reduction of the oscillated structures is one of the major issues in the design process. One of the promised solutions for vibration reductions is the use of dynamic vibration absorbers. When forces are applied to the structure, the dynamic vibration absorber reacts by supplying a force in the opposite direction, so limiting the movement of the beam. It eliminates the steady-state motion of the linked point when appropriately tuned and attached to a vibrating body that responds to harmonic stimulation. The DVA is a device that consists of an auxiliary block spring system that absorbs the vibration of the system with which it is associated [2, 3].

A dynamic vibration absorber, commonly referred to as "a tuned mass absorber", can be connected to a primary dynamic structure to attenuate vibration or sound emission. A mass-spring damper system with a single degree of freedom (SDOF) is a typical passive vibration damper. The DVA can be used to lessen the force or undesired vibration that the resonant mode in the fundamental structure causes. The primary vibration of structures may be decreased by successfully transferring vibration energy to the DVA when DVA was set up to manage vibration at the required frequency. In civil engineering, DVAs are widely used to increase the durability of thin, tall structures [4–6]. According to Shi et al. (2014), suspension factors such as suspension frequency, damping ratio, mounting position, and mass were examined for their effects on the vibration of the car body based on the beam modal analysis and DVA parameter optimization [7]. In all four different experimental settings, according to Zainulabidin and Jaini (2012), the vibration amplitude is reduced, and the DVAs mounted on the beam efficiently absorb the vibration [8]. The study is then validated by Noor Aslamiah (2016) utilizing the (FEA) methodology and ANSYS APDL software. DVA effectively lowers the vibration's amplitude for particular vibration modes [9]. Salleh and Zaman (2016) studied a fixed-fixed plate connected to a vibration absorber using finite elements and utilized a single, lightweight DVA to absorb the vibration of a plate [10]. Ong and Zainulabidin (2019) used FEA with ANSYS APDL software to demonstrate that the DVA can reduce vibration amplitude at the applied natural frequency, regardless of the number of DVAs used [11]. By comparing the amplitude before and after fixing a DVA, Zainulabidin & Jaini (2013) were able to determine that the DVA was successful in absorbing the beam vibration, which in turn lowered the vibration amplitude of the beam structure [12]. Rozlan et al. (2017) showed that combining two to ten (multiple) absorbers reduced vibration more effectively than using only one [13]. In high-speed automobiles, the mode of vertical vibrations of the car body was affected by the equipment of a DVA, according to the research by Sunil K. Sharma and many others published in 2022. The influence of a DVA approach is used to establish the ideal suspension frequency for different types of equipment [14].

Structural dynamics is typically considered in civil, mechanical, and aerospace engineering to be a field concerned with the analysis and characterization of structures' vibratory response. A theory for substructurally synthesizing dissipation under free-wave motion conditions, i.e., waves not constrained to a particular driving frequency, is presented and tested. This concept has ramifications for applications that require a high rigidity and high damping combination [15]. Bloch's theorem was used, along with a sub-structuring technique in which the resonating branch was modelled independently and its effective dynamic stiffness was condensed [16]. Acoustic metamaterial beams contain an isotropic beam with built-in spring-mass vibration absorbers. The structure may absorb waves in one direction or two directions, depending on how the units were linked to the beam [17]. The metamaterial sandwich panels were constructed using a host sandwich panel and periodically connected resonant devices. Panels with and without damping were explored. A significant decrease of vibration and sound was achieved also from periodic design for the panels with varied parameter settings; on the other hand, the reduction features were modified [18].

The vibration control performance of the MDOF DVA equipped with nonlinear HSLDS mount on the car body was analysed using dynamic simulation of a nonlinear model in the time domain. The results reveal that the MDOF DVA can effectively absorb vibration of the car body in many degrees of freedom and improve the vehicle's operating ride quality [19]. The beam tip responses were compared to those simulated by ADAMS software to validate the dependability of the DVA-beam element in handling different types of boundary conditions, and good agreements was observed [20].

This work uses the spring block system, a fundamental vibration-absorbing mechanism, to avoid the vibration of a fixed-fixed beam. Using APDL, finite element analysis is used to perform the investigation. Furthermore, this works aims at providing a comprehensive and innovative approach to the utilization of single and multiple dynamic vibration absorbers (DVA) for controlling not just a single mode shape, but three modes simultaneously. This distinguishes our work from previous studies, such as the one conducted by Jacquot [Ref], which solely focused on a single mode shape and lacked the consideration of multiple locations in the DVA design. In addition, the current work presents a comparative analysis that underscores the fundamental differences between our approach and the existing literature. The example presented by Jacquot solely examined the DVA design with respect to a single location, whereas our work accounts for multiple DVAs and multiple locations, enabling the simultaneous control of three modes. This expanded scope adds substantial value to the field and paves the way for more effective vibration control strategies. Finally, the present work proposes a novel mathematical expression for obtaining the optimal design of the DVA, taking into consideration essential factors such as frequency shift, reduction in beam amplitude, and achieving a trade-off between them. To the best of the author's knowledge, this proposed methodology stands as a unique contribution, not previously explored by other researchers. The results can be used as a guide for the placement of DVA because the objective of this research is to identify the ideal position for DVA installation in order to obtain the highest level of vibration attenuation.

2. Methodology

The beam is fixed at both ends and has a total length of 0.84 m, and is partitioned into 43 nodes (N), each of which is 0.02 m apart. At node 4 of the beam, a harmonic force, F_o, of 28.84 N was applied. The beam density, elasticity modulus, and mass moment are 69 GPa, 2800 kg/m^3 and $1.67 \cdot 10^{-11}$ kg m^2, respectively. The stiffness (K) of the DVA had been chosen so that the values correspond to the first three natural frequencies of the beam, and the dynamic vibration absorber has a mass of 0.1 kg. This study has an initial set of testing scenarios that looked into how tuning influences DVA performance at the vibration modes. There were two cases of testing conditions, the first test was to study the effect of tuning a single DVA at different nodes of vibration modes and the second test was used to study the effect of multiple DVAs placed at deferent nodes for each mode of beam vibrations. The two tests are presented in Tables 1 and 2.

Table 1. Conditions for the case one

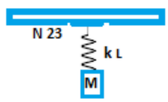
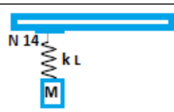
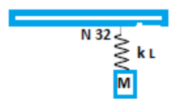
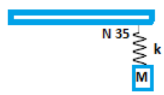
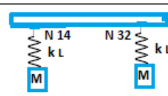
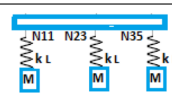
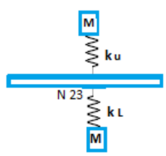
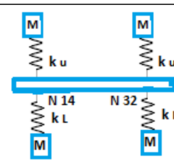
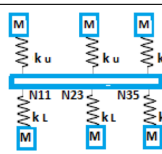
Condition	Configuration	Condition	Configuration
A_1			
B_1		B_2	
C_1		C_3	

Table 2. Conditions for the case two

Condition	Configuration	Condition	Configuration
D		E	
G		H	
M			

2.1. Finite Element Method (FEM)

The FEM is one numerical technique for solving differential or integral problems. The finite element method is used to greatly simplify complex and difficult situations. A geometry is broken down into finite, simple-shaped components for easier and faster solutions [21].

The FEA for structural components greatly shortens the design cycle and raises the quality of the finished product. For instance, linear FEA for acoustic analysis is used in the automotive industry to design engines with acceptable temperatures and pressures, improve comfort, analyse vibrations, increase rigidity of the structure, and increase stress life of suspension components, among many other tasks [22]. Beam189 is the name given to the finite element model of the beam structure, which was partitioned into 21 elements. Because it is a fixed ends beam, displacement and rotation of its end are restricted. The DVA spring element is constructed utilising the COMBINI14 element type and longitudinal displacement is the only degree of freedom that is considered. The dynamic vibration absorber system was created by combining MASS21 and COMBIN14 for the lumped mass and spring, respectively.

Modal and harmonic analyses were carried out using the mechanical ANSYS-APDL-19.2. Modal analysis was used to recognise the essential frequency and mode shapes of the beam. The harmonic analysis was used to study the dynamic response of the beam with and without DVA to calculate a linear steady-state response to harmonically varying loads over time. By computing the structure's response at various frequency, an amplitude versus frequency graph is usually constructed.

3. Results and discussion

3.1. Modal analysis of F-F beam

Modal analysis is carried out to determine natural frequencies and the corresponding mode shapes. The vibration will occur at its natural frequency for the 1st, 2nd and 3rd modes presented in Fig. 1 that vibrate at 14.463 Hz, 39.867 Hz, and 78.159 Hz, respectively. To analyse the impact of absorber masses on the natural frequencies of the beam, the natural frequency of the beam was constructed with or without absorber masses. Additionally, Tables 3 and 4 display the natural frequency

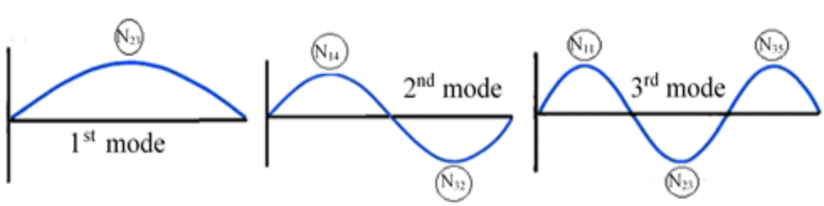


Fig. 1. The first three mode shapes

(ω_n) of a beam connected to a DVA. The symbols M, K_u, K_L and X/L represent the mass of the DVA, stiffness of the DVA located above the beam, stiffness of the DVA located under the beam, and location of the DVA on the beam, respectively.

Table 3. The natural frequency of the fixed-fixed beam linked to one DVA only (one point of DVA application)

Condition	Stiffness values (N/m)	Natural frequencies		
		ω_{n1}	ω_{n2}	ω_{n3}
A_1		X/L = 0.5 (N_{23})		
A_{11}	$K_L = K_1$	7.2807	27.650	39.867
A_{12}	$K_L = K_2$	7.9775	39.867	52.510
A_{13}	$K_L = K_3$	8.0620	39.867	60.033
B_1		X/L = 0.28 (N_{14})		
B_{11}	$K_L = K_1$	8.7367	20.496	45.387
B_{12}	$K_L = K_2$	9.8745	26.982	64.994
B_{13}	$K_L = K_3$	10.006	28.097	69.798
B_2		X/L = 0.71 (N_{32})		
B_{21}	$K_L = K_1$	8.7367	20.496	45.387
B_{22}	$K_L = K_2$	9.8745	26.982	64.994
B_{23}	$K_L = K_3$	10.006	28.097	69.798
C_1		X/L = 0.21 (N_{11})		
C_{11}	$K_L = K_1$	7.5463	17.216	43.376
C_{12}	$K_L = K_2$	9.5880	22.393	54.306
C_{13}	$K_L = K_3$	9.8623	23.846	58.955
C_2		X/L = 0.5 (N_{23})		
C_{21}	$K_L = K_1$	7.2807	27.650	39.867
C_{22}	$K_L = K_2$	7.9775	39.867	52.510
C_{23}	$K_L = K_3$	8.0620	39.867	60.033
C_3		X/L = 0.78 (N_{35})		
C_{31}	$K_L = K_1$	10.052	18.184	43.565
C_{32}	$K_L = K_2$	11.599	25.054	55.766
C_{33}	$K_L = K_3$	11.752	26.576	60.491

Table 4. The natural frequency of the F-F beam linked to multiple DVAs (two or more DVAs attached to the beam)

Condition	Stiffness (N/m)	Natural frequency		
		ω_{n1}	ω_{n2}	ω_{n3}
D		X/L = 0.28 and 0.71 (N_{14} and N_{32})		
D_1	$K_L = K_1$	7.6319	11.547	26.275
D_2	$K_L = K_2$	8.4188	16.595	49.396
D_3	$K_L = K_3$	8.5135	17.565	57.056

Table 4 [cont.]

Condition	Stiffness (N/m)	Natural frequency		
		ω_{n1}	ω_{n2}	ω_{n3}
E		X/L = 0.21, 0.5 and 0.78 (N_{11}, N_{23} and N_{35})		
E_1	$K_L = K_1$	6.6313	12.030	13.198
E_2	$K_L = K_2$	7.1739	18.131	24.557
E_3	$K_L = K_3$	7.2398	19.378	28.489
G		X/L = 0.5 (N_{23})		
G_1	$K_u = K_2$, $K_L = K_1$	5.9496	18.723	39.867
G_2	$K_u = K_3$, $K_L = K_1$	5.9669	19.308	39.867
G_3	$K_u = K_3$, $K_L = K_2$	6.1806	39.867	47.423
H		X/L = 0.28 and 0.71 (N_{14} and N_{32})		
H_1	$K_u = K_2$, $K_L = K_1$	6.3164	10.674	18.644
H_2	$K_u = K_3$, $K_L = K_1$	6.3362	10.724	19.190
H_3	$K_u = K_3$, $K_L = K_2$	6.5908	12.950	46.451
M		X/L = 0.21, 0.5 and 0.78 (N_{11}, N_{23} and N_{35})		
M_1	$K_u = K_2$, $K_L = K_1$	5.2994	11.347	12.965
M_2	$K_u = K_3$, $K_L = K_1$	5.3126	11.393	12.987
M_3	$K_u = K_3$, $K_L = K_2$	5.4643	14.381	20.355

3.2. The fixed-fixed beam harmonic analysis without DVA

The frequency response of the fixed-fixed beam was studied through harmonic analysis. Fig. 2 depicts the frequency response of the fixed-fixed beam when no DVA is attached.

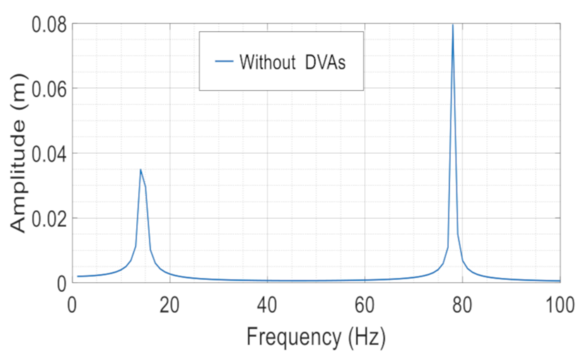


Fig. 2. Dynamic response without DVA

The three peaks indicate the amplitude of vibration that befalls at 14 Hz, 40 Hz, and 70 Hz for the first, second, and third mode of vibration, respectively. The amplitude of the first mode of vibration is $3.49 \cdot 10^{-2}$ m, the second mode is

$6.87 \cdot 10^{-4}$ m, and the third mode is $7.95 \cdot 10^{-2}$ m. Since the amplitude value in the second mode is very low and close to zero, we will impose the reduction and comparing with the first mode. After utilising a DVA to compute the amplitude changes for each condition, the amplitude changes in these three frequencies are monitored and analysed.

3.3. Harmonic analysis of fixed-fixed beam with DVA

The (DVA) is a device made up of an auxiliary mass-spring system that tends to dampen the vibration of the structure to which it is attached, as shown in Fig. 3. Many researchers have examined the use of a DVA in linear systems [23–25]. Two case studies were considered in the present work, attaching single DVA and multiple DVA, respectively, and they are:

Case 1: Investigate the influence of the performance of a **single** DVA.
Fifteen experiments are presented in Table 3 for this case. Only one DVA is attached at a specific point defined by the symbol N (as shown in Fig. 1). The values of the DVA parameters are also varied for each experiment.

Case 2: Investigate the influence of the performance of **multiple** DVAs.
Fifteen experiments are presented in Table 4 for this case. Multiple DVAs are attached at specific points defined by the symbol N (as shown in Fig. 1). Values of the DVA parameters are also varied for each experiment. The symbol K_l and K_u means that the DVA is attached to lower and upper sides of the beam, respectively.

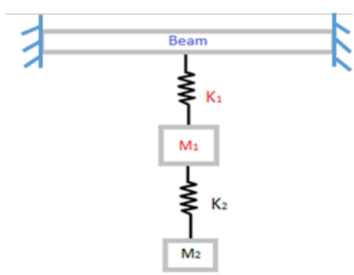


Fig. 3. Schematic representation of a beam structure attached with two DVA

3.4. The effect of each DVA in the 1st mode of vibration

According to Table 3, fifteen various types of conditions were applied, with the first DVA of 14.463 Hz, the second DVA of 39.867 Hz, and the third DVA of 78.159 Hz, in their natural frequency, respectively. These DVAs were placed on the peak and anti-peak of the 1st three modes of beam vibration.

3.4.1. Result of the condition (A_{11}, A_{12}, and A_{13})

According to condition A_{11}, the amplitude reduced in the first, second, and third modes, as shown by the frequency response of DVA in Fig. 4. The system's new responses were similar to those of the original system. The inherent frequency of the system is significantly influenced by the mass of the DVA. The vibration amplitude of the initial mode increased and the new response frequency moved in the conditions A_{12} and A_{13}, proving that the DVA is ineffective.

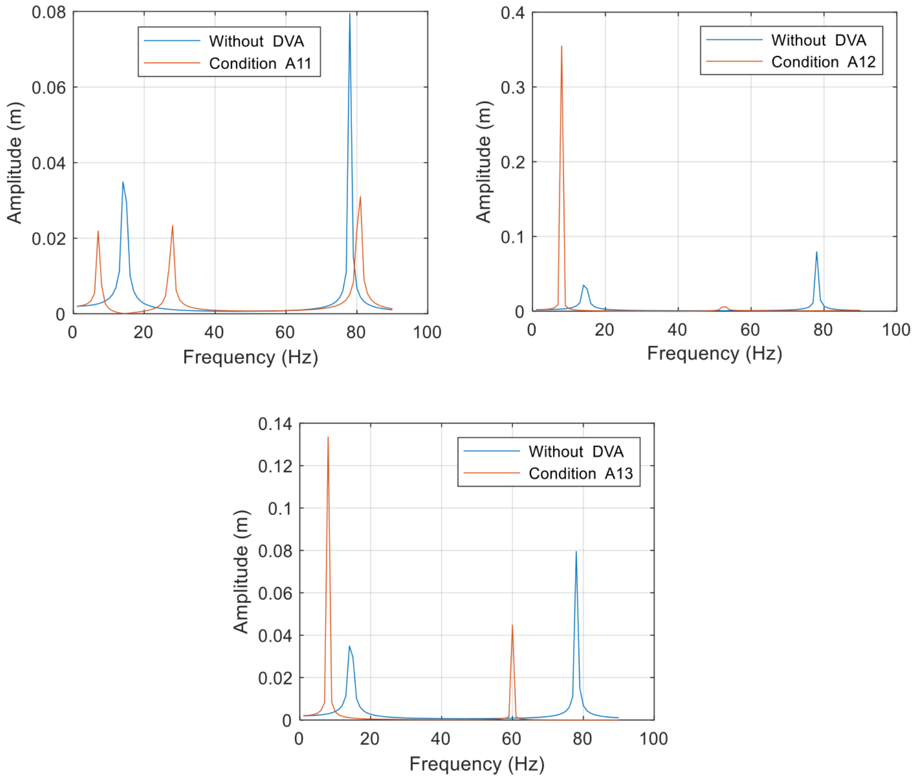


Fig. 4. Comparison of amplitudes between without and with DVA for conditions A_{11}, A_{12} and A_{13}

3.4.2. Result of the condition (B_{11}, B_{12}, and B_{13})

At condition B_{11}, the amplitude decreased in the first, second, and third modes, as shown in Fig. 5. The new response frequencies were similar to those of the original systems. By altering the mass of DVA, it was possible to drastically alter the reaction frequency of the system. The response frequencies changed at conditions B_{12} and B_{13}, and the vibration amplitudes of its first and second modes increased, showing that the absorber did not absorb any vibrations.

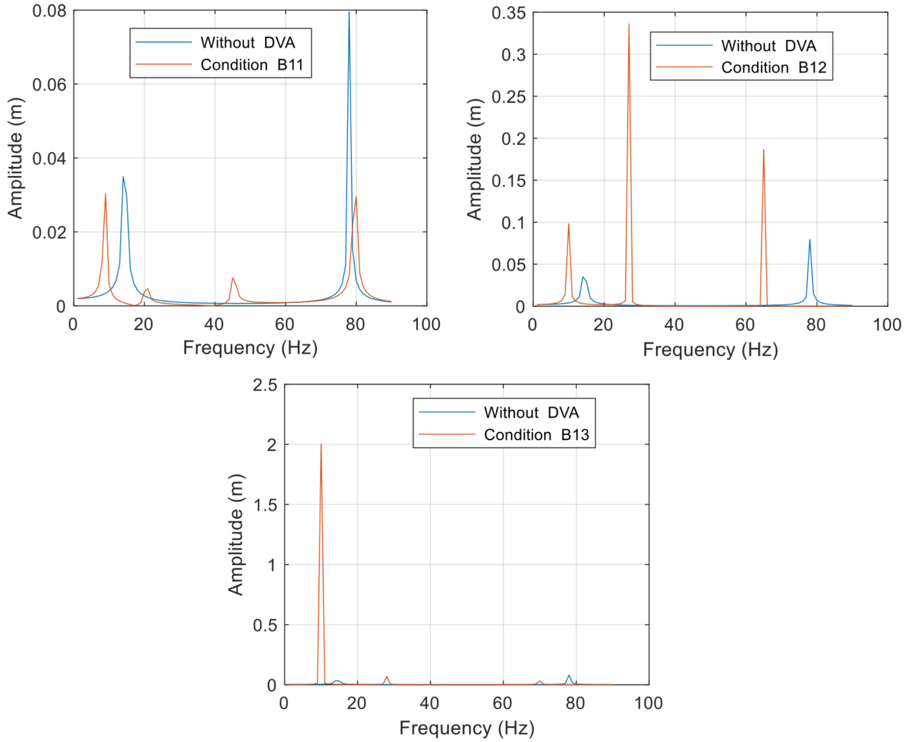
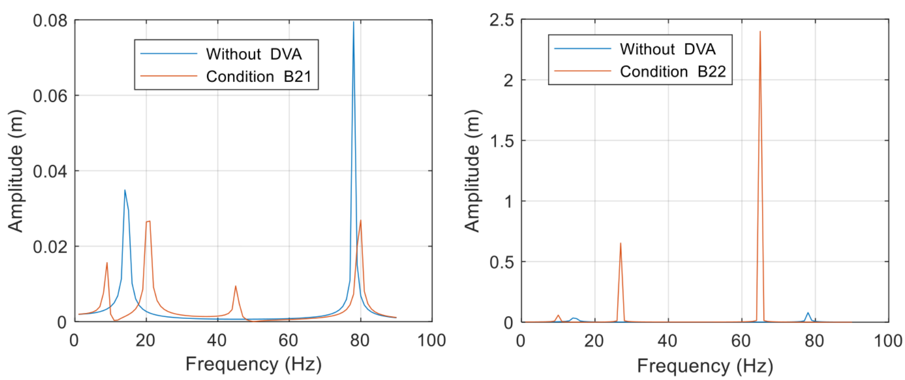


Fig. 5. Comparison of amplitudes between without and with DVA for conditions B_{11}, B_{12} and B_{13}

3.4.3. Result of the condition (B_{21}, B_{22}, and B_{23})

The amplitude reduced in the first and third modes, as can be seen in the condition B_{21} in Fig. 6. The new response frequencies of the system shifted toward those of the original systems. The conditions B_{22} and B_{23} showed a change in the response frequencies and an increase in amplitude of the first and second modes, respectively, which showed that the absorber was not efficient in reducing vibrations.



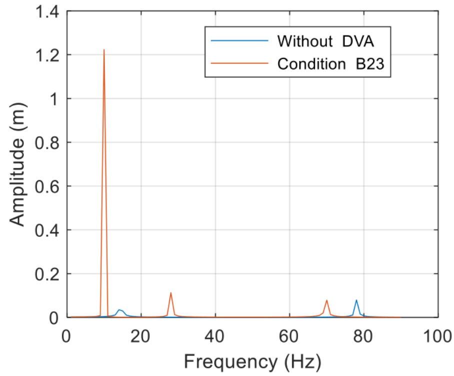


Fig. 6. Comparison of amplitudes between without and with DVA for conditions B_{21}, B_{22} and B_{23}

3.4.4. Result of the condition (C_{11}, C_{12}, and C_{13})

The condition C_{12} in Fig. 7 shows that the amplitude of all vibrations decreased. The new response frequencies of the system approached those of the initial systems. However, not all the types of vibration exhibit a reduction in amplitude, as shown by the condition C_{13} (the DVA is not functional). According to condition C_{11}, the

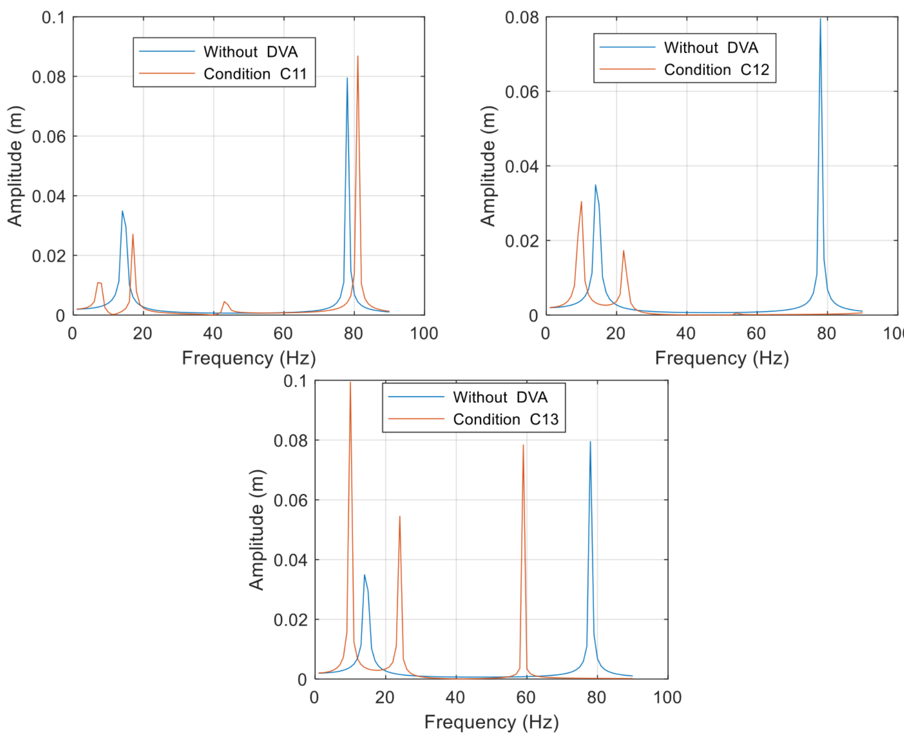


Fig. 7. Comparison of amplitudes between without and with DVA for conditions C_{11}, C_{12} and C_{13}

amplitude of vibration for the first mode decreased by 22.23%, while increased by 9.98% for the third mode. The frequencies shifted, implying that the tuned absorber only vibrates with the beam at two or three new peaks.

3.4.5. Result of the condition (C_{31}, C_{32}, and C_{33})

As seen in Fig. 8, condition C_{33} caused the vibration amplitude for the first mode to increase by 10.31% while decreased by 70% for the third mode. The frequencies subsequently reverted to their initial settings. The DVA was able to absorb vibration for the first and third modes, but not for the second mode for the condition C_{32}. Since the tuned absorber vibrated together with the beams at three additional frequencies under condition C_{31}, the amplitude of vibration in all modes increased.

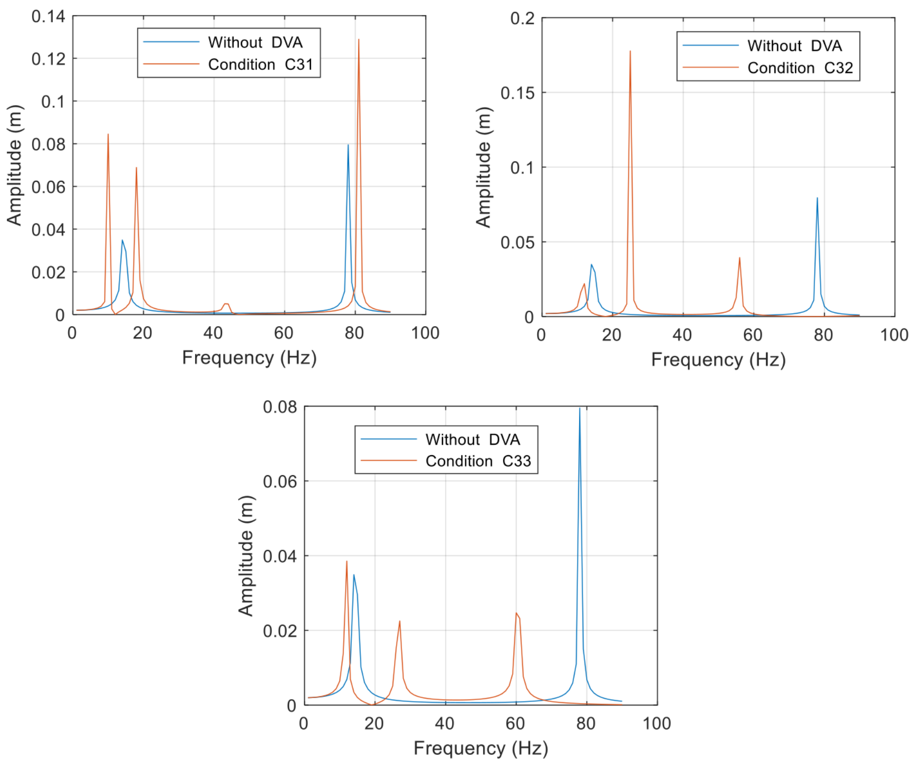


Fig. 8. Comparison of amplitudes between without and with DVA for conditions C_{31}, C_{32} and C_{33}

Table 5 shows that all vibration modes could be effectively absorbed by DVA placements, with the exception of the case when they were situated at the node of the second vibration mode, which increased the vibration amplitude. In comparison to the third vibration mode, the amplitude of the second mode increased as a result of the second DVA's inertial impact at the vibration node, which led to a smaller

Table 5. Amplitude percentage changes (APC %) for case-1

Cond. N_{23}	1st mode Ampl. (m)	APC %	Shift (Hz)	2nd mode Ampl. (m)	APC %	Shift (Hz)	3rd mode Ampl. (m)	APC %	Shift (Hz)
Without DVA	$3.49 \cdot 10^{-2}$	Ref.		$6.87 \cdot 10^{-4}$	Ref.		$7.95 \cdot 10^{-2}$	Ref.	
A_1									
A_{11}	$2.19 \cdot 10^{-2}$	−37.24	−7	$2.33 \cdot 10^{-2}$	−33.23	−12	$3.10 \cdot 10^{-2}$	−61	+3
A_{12}	$3.54 \cdot 10^{-1}$	+914	−6	$3.02 \cdot 10^{-6}$	−99.55	0	$6.02 \cdot 10^{-3}$	−92.42	−25
A_{13}	$1.33 \cdot 10^{-1}$	+281	−6	$2.03 \cdot 10^{-4}$	−70.45	0	$4.48 \cdot 10^{-2}$	−43.64	−18
B_1									
B_{11}	$3.03 \cdot 10^{-2}$	−13.18	−5	$7.59 \cdot 10^{-3}$	−78.2	+5	$2.95 \cdot 10^{-2}$	−62.89	+2
B_{12}	$9.81 \cdot 10^{-2}$	+181	−4	$3.36 \cdot 10^{-1}$	+862	−13	$1.86 \cdot 10^{-1}$	+133	−13
B_{13}	2.003	+5639	−4	$6.89 \cdot 10^{-2}$	+97	−12	$3.22 \cdot 10^{-2}$	−59.49	−8
B_2									
B_{21}	$2.64 \cdot 10^{-2}$	−24.35	+6	$9.49 \cdot 10^{-3}$	−72.8	+5	$2.69 \cdot 10^{-2}$	−66.16	+2
B_{22}	$5.82 \cdot 10^{-2}$	+66.76	−4	$6.52 \cdot 10^{-2}$	+79	−13	2.39	+2906	−13
B_{23}	1.22	+3395	−4	$1.12 \cdot 10^{-1}$	+220	−12	$7.87 \cdot 10^{-2}$	−1.006	−8
C_1									
C_{11}	$2.71 \cdot 10^{-2}$	−22.34	+3	$4.53 \cdot 10^{-3}$	−87.02	+3	$8.68 \cdot 10^{-2}$	+9.18	+3
C_{12}	$3.04 \cdot 10^{-2}$	−12.89	−4	$1.73 \cdot 10^{-2}$	−50.42	−18	$2.31 \cdot 10^{-4}$	−99.70	0
C_{13}	$9.94 \cdot 10^{-1}$	+2748	−4	$5.45 \cdot 10^{-2}$	+56.16	−16	$7.83 \cdot 10^{-2}$	−1.50	−19
C_3									
C_{31}	$8.45 \cdot 10^{-2}$	+142	−4	$5.05 \cdot 10^{-3}$	−85.53	+3	$1.28 \cdot 10^{-1}$	+61	+3
C_{32}	$2.20 \cdot 10^{-2}$	−36.96	−2	$1.77 \cdot 10^{-1}$	+407	−15	$3.94 \cdot 10^{-2}$	−50.44	−22
C_{33}	$3.85 \cdot 10^{-2}$	+10.31	−2	$2.25 \cdot 10^{-2}$	−35.53	−13	$2.47 \cdot 10^{-2}$	−68.93	−18

percentage of the drop caused by the original reduction in amplitude. The goal of each DVA was to reduce vibration at a given location, however, when a DVA was made to focus on a single vibration mode, it might also be able to lower vibration for other modes because the DVA effect changed the initial vibration of the beam. However, the natural frequency would be preserved if the block was positioned at the node of the designated vibration mode. Furthermore, it was believed that the DVA would work better if it was placed close to both the vibratory peak and the stimulation source. This may be seen by contrasting the DVA mode's measurements at the peak of the first and third mode, respectively. The most successful example in Table 5 is the Case-1, where DVA decreased vibration in each of the three vibration modes based on the amplitude percentage change (APC) of the specific mode compared to the one without DVA. The negative sign of the percentage change means a reduction in the amplitude, while the positive sign means an increasing in the amplitude (which is undesirable). In addition, the system frequency varies, increases or decreases, based on the DVA design parameters and its location. This causes a shift to the left or to the right of the frequency at the peaks, as shown in Table 5. The negative sign means that the system frequency decreased due to addition of the DVA, while the positive sign means that system frequency increased. Frequency shift and its importance will be discussed in Section 5.

3.5. Effect of DVA placement of the beam at case-2

A number of simulations were run according to Table 4 to analyse the vibration characteristics of a fixed-fixed beam after the installation of several DVAs at particular locations.

3.5.1. Result for condition (D1, D2 and D3)

The first DVA is installed at N_{14} from the fixed end of the beam, and the second DVA is installed at N_{32}. Fig. 9 illustrates how the conditions D_2 and D_1 reduced the amplitude and brought the frequencies closer to those of the initial system. Although the amplitude was decreased in condition D_3, the DVA did not function for the third mode. The condition that best absorbed vibrations was found to be D_2.

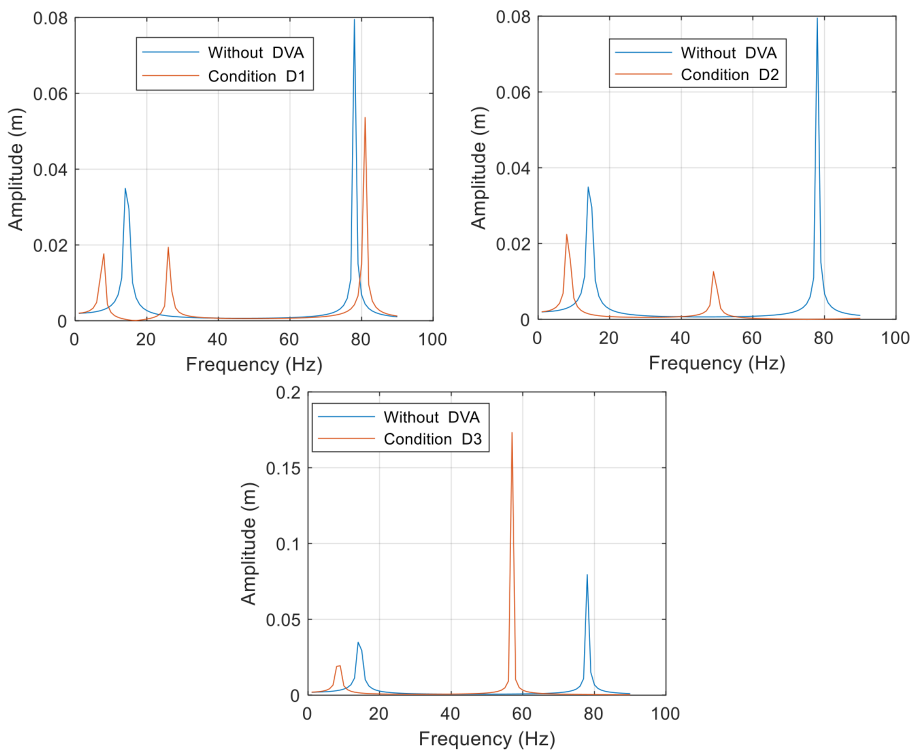


Fig. 9. Comparison of amplitudes between without and with DVA for conditions D_1, D_2 and D_3

3.5.2. Result for condition (E_1, E_2 and E_3)

As seen in Fig. 10, the vibration amplitude was successfully decreased in the E_1 condition when the absorber was tuned to the first and second mode of the beam. The amplitude of vibration was also successfully decreased in situations E_2 and E_3, where the absorber was in the third mode of the beam. The system response frequency was similar to that of the original system for all situations. The absorber in the first mode did not, however, reduce the amplitude in the E_2 condition.

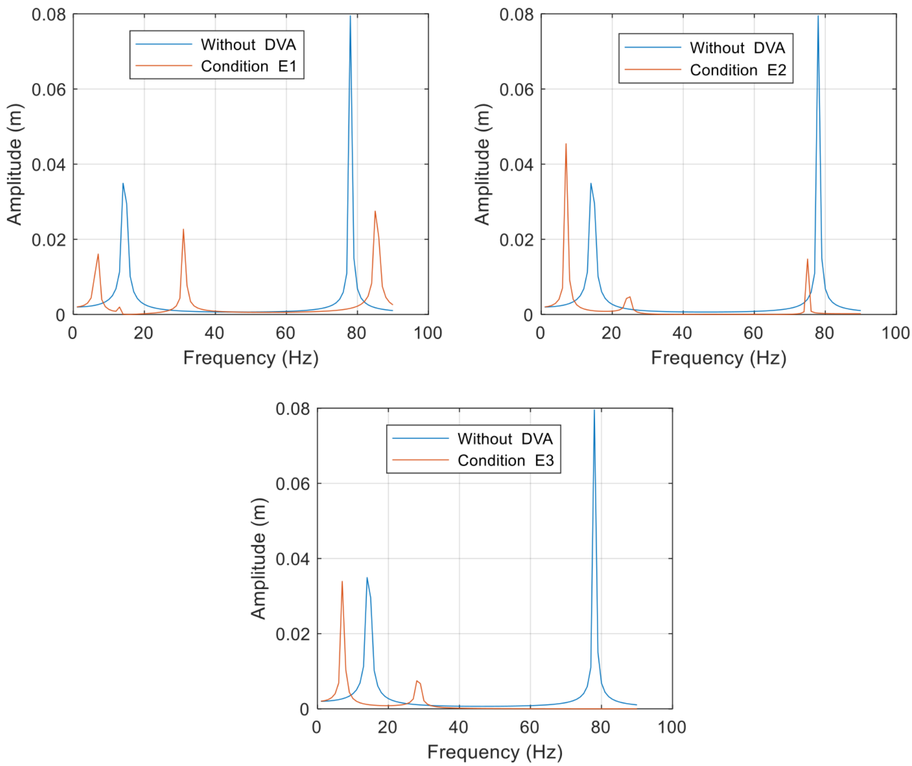


Fig. 10. Comparison of amplitudes between without and with DVA for conditions E_1, E_2 and E_3

3.5.3. Result for condition (G_1, G_2 and G_3)

A single DVA was used in this work and was mounted in the middle of the beam at N_{23}. As seen in Fig. 11, condition G_3 successfully lowered the amplitude for the third mode while leaving the second amplitude unchanged. Furthermore, the new response frequency was similar to that of the initial system. The third mode of vibration was successfully suppressed under all conditions. The DVA did not function for the first mode under the circumstances G_1 and G_2, as the first mode did not exhibit a decrease in beam vibration amplitude.

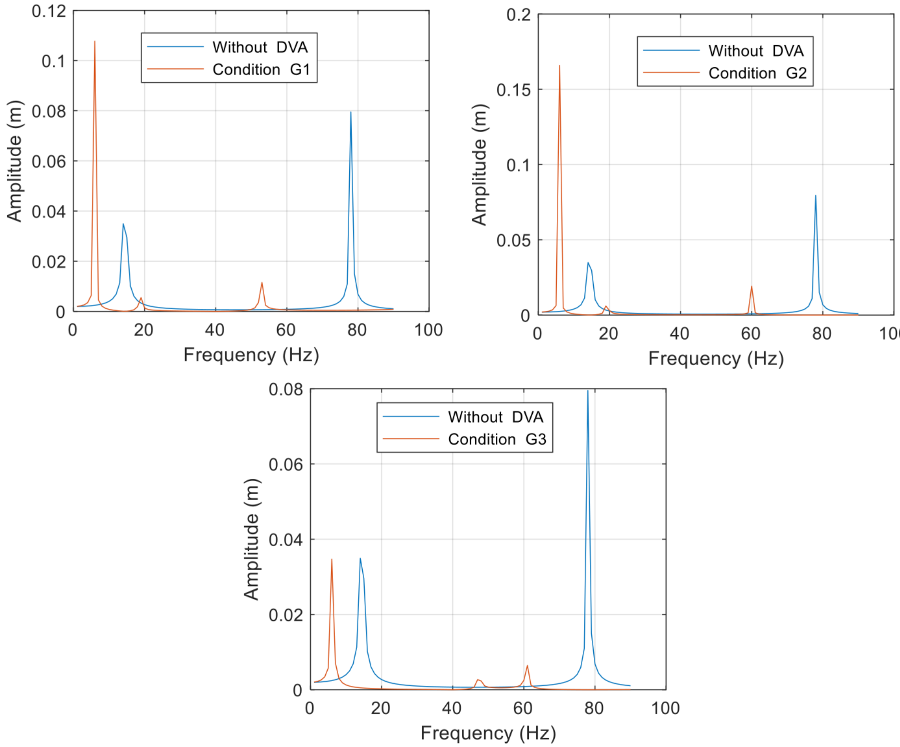
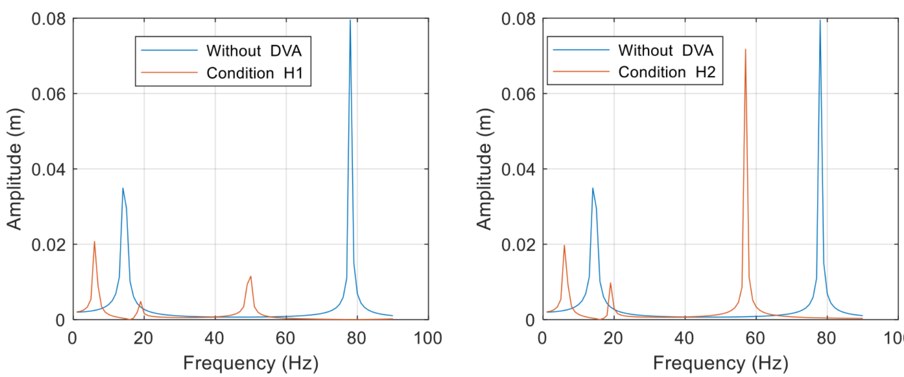


Fig. 11. Comparison of amplitudes between without and with DVA for conditions G_1, G_2 and G_3

3.5.4. Result for condition (H_1, H_2 and H_3)

Two DVAs are connected in parallel at N_{14} and N_{32} from the fixed beam in the vibration mode shape. All three kinds of vibration are successfully minimized, as seen in Fig. 12. The H_1, H_2, and H_3 conditions resulted in a decrease in vibration amplitude.



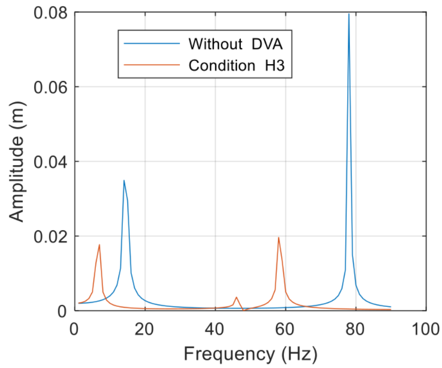


Fig. 12. Comparison of amplitudes between without and with DVA for conditions H_1, H_2 and H_3

3.5.5. Result for condition (M_1, M_2 and M_3)

At positions N_{11}, N_{23}, and N_{35} from the fixed beam, three DVA are parallelly attached to the beam. All three peaks of vibration are effectively decreased for all conditions, as seen in Fig. 13. The new response frequencies are comparable to those of the original system. The DVA mass affects the system's fundamental frequency, demonstrating that the DVA tuned to the beam is successful in reducing vibration amplitude.

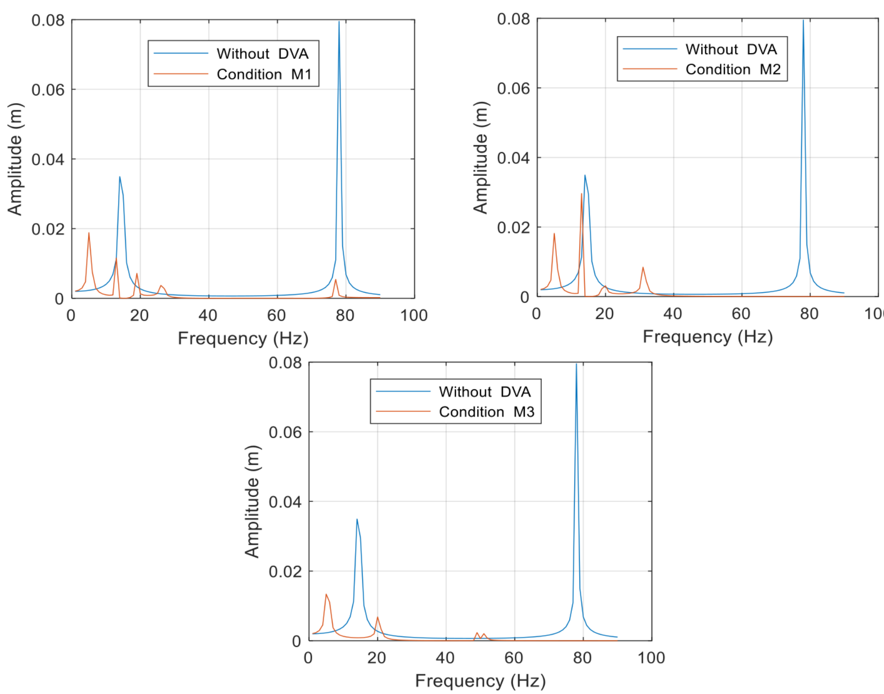


Fig. 13. Comparison of amplitudes between the cases without and with DVA for conditions M_1, M_2, and M_3

A summary of the results of the amplitude reduction is shown in Table 6, where in some situations it was possible to absorb the second vibration mode without changing the capacity. The first and third vibration modes also experienced notable drops in vibration amplitude. The optimum vibration reduction state was represented by the M_3 ($K_u = K_3$, $K_L = K_2$), where the amplitude reduction of the vibration mode was greatest at the first vibratory peak. In E_2, G_1, and G_2, the amplitude of the first mode reduced, and in condition D_3, the third mode reduced. In such cases, the addition of DVA did not help reducing the amplitude of other vibration mode forms but rather boosted the vibration of the first or third vibration mode. Conditions D_3, E_2, G_1, and G_2, though, may demonstrate that the attached DVA decreased the vibration of the beam.

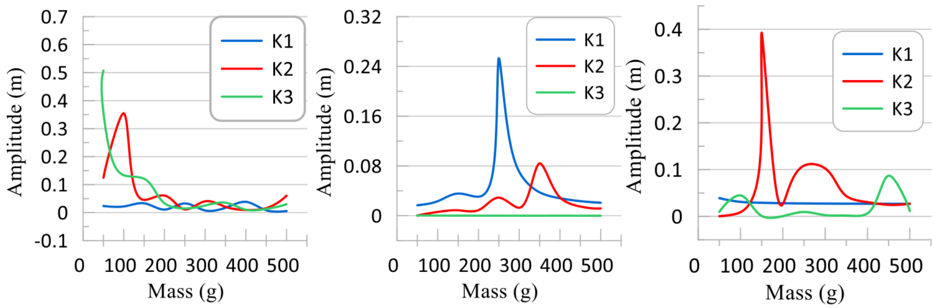
Table 6. Amplitude percentage change (APC%) for case-2

Cond. N_{23}	1^{st} mode			2^{nd} mode			3^{rd} mode		
	Ampl. (m)	APC. %	Shift (Hz)	Ampl. (m)	APC. %	Shift (Hz)	Ampl. (m)	APC. %	Shift (Hz)
Without DVA	$3.49 \cdot 10^{-2}$	Ref.		$6.87 \cdot 10^{-4}$	Ref.		$7.95 \cdot 10^{-2}$	Ref.	
D_1	$1.76 \cdot 10^{-2}$	−49.57	−6	$1.94 \cdot 10^{-2}$	−44.41	−14	$5.36 \cdot 10^{-2}$	−32.74	+3
D_2	$2.24 \cdot 10^{-2}$	−35.81	−6	$7.21 \cdot 10^{-4}$	−97.93	0	$1.26 \cdot 10^{-2}$	−84.15	−29
D_3	$1.94 \cdot 10^{-2}$	−44.41	−5	$7.09 \cdot 10^{-4}$	−97.97	0	$1.73 \cdot 10^{-1}$	+117.6	−21
E_1	$1.61 \cdot 10^{-2}$	−53.86	−7	$2.27 \cdot 10^{-2}$	−34.95	−19	$2.75 \cdot 10^{-2}$	−65.40	+7
E_2	$4.54 \cdot 10^{-2}$	+30.08	−7	$4.70 \cdot 10^{-3}$	−86.53	−15	$1.47 \cdot 10^{-2}$	−81.50	−3
E_3	$3.39 \cdot 10^{-2}$	−2.85	−7	$7.45 \cdot 10^{-3}$	−78.65	−12	$1.47 \cdot 10^{-7}$	−99.99	0
G_1	$1.07 \cdot 10^{-1}$	+206.5	−8	$5.51 \cdot 10^{-3}$	−85.24	−21	$1.15 \cdot 10^{-2}$	−85.53	−25
G_2	$1.65 \cdot 10^{-1}$	+372.7	−8	$6.05 \cdot 10^{-3}$	−82.66	−21	$1.92 \cdot 10^{-2}$	−75.84	−18
G_3	$3.47 \cdot 10^{-2}$	−0.57	−8	$2.68 \cdot 10^{-3}$	-92.32	+7	$6.44 \cdot 10^{-3}$	−91.89	−17
H_1	$2.07 \cdot 10^{-2}$	−40.68	−8	$4.80 \cdot 10^{-3}$	−86.24	−21	$1.15 \cdot 10^{-2}$	−85.53	−28
H_2	$1.97 \cdot 10^{-2}$	−43.55	−8	$9.76 \cdot 10^{-3}$	−72.03	−21	$7.17 \cdot 10^{-2}$	−9.81	−21
H_3	$1.77 \cdot 10^{-2}$	−49.28	−7	$3.63 \cdot 10^{-3}$	−89.59	+6	$1.96 \cdot 10^{-2}$	−75.34	−20
M_1	$1.88 \cdot 10^{-2}$	−46.13	−9	$3.69 \cdot 10^{-3}$	−89.42	−14	$5.42 \cdot 10^{-3}$	−93.18	−1
M_2	$2.96 \cdot 10^{-2}$	−15.18	−1	$3.10 \cdot 10^{-3}$	−91.11	−20	$8.45 \cdot 10^{-3}$	−89.37	−37
M_3	$1.33 \cdot 10^{-2}$	−61.89	−9	$6.81 \cdot 10^{-3}$	−80.48	−20	$2.32 \cdot 10^{-3}$	−97.08	−29

4. Parametric study

4.1. Effect of mass on a harmonic response for different stiffness values

The harmonic response of the F-F beam for each of the three distinct mode forms, with variable DVA mass values, is shown in Fig. 14 a, b, and c, respectively, that show the responses for the first, second, and third modes. Both the mass and stiffness levels have an impact on the dynamic response, and the response varies depending on the appropriate mode. For the first mode, the maximum amplitude at N_{23} (X/L = 0.5) (K3) can be significantly changed even by small changes in the mass values at lower levels of applied stiffness. Minor variations have been found for the second and third modes, on the other hand. The resonance-induced alterations in the response are particularly pronounced in the second and third modes (K1 and K2, respectively).

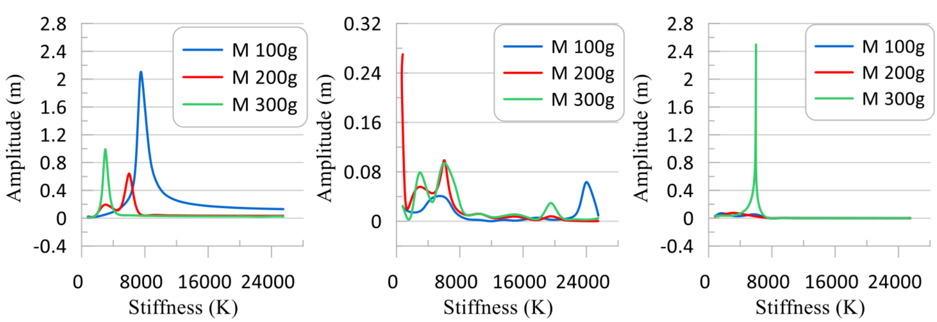


(a) (b) (c)

Fig. 14. The influence of mass on harmonic responsiveness for three mode shapes (a, b, and c)

4.2. Effect of stiffness on a harmonic response for different mass values

In Fig. 15, the harmonic response of the F-F beam is presented for the three different mode shapes at various DVA stiffness values. The responses for the first, second, and third modes are illustrated in Figs. 15a, b, and c, respectively. The values of both springs and stiffness are crucial in determining the dynamic response, and the response varies depending on the corresponding mode. For instance, in the 1st mode with a smaller mass value of 100 g, there is a significant variation in the maximum amplitude node 23 (X/L = 0.5) with changes in stiffness values. Conversely, minor variations are observed for the 2nd and 3rd modes. Meanwhile,



(a) (b) (c)

Fig. 15. The influence of stiffness on harmonic responsiveness for three mode shapes (a, b, and c)

the most substantial change in the response is observed at 200 g and 300 g for the second and third modes, respectively, due to the resonance of the three modes that can be achieved at these specific mass values in this test.

4.3. Effect of DVA location on a harmonic response for different mass values

The harmonic response of the F-F beam for three mode forms at various DVA positions is shown in Fig. 16. The response for the first mode with M = 100 g is

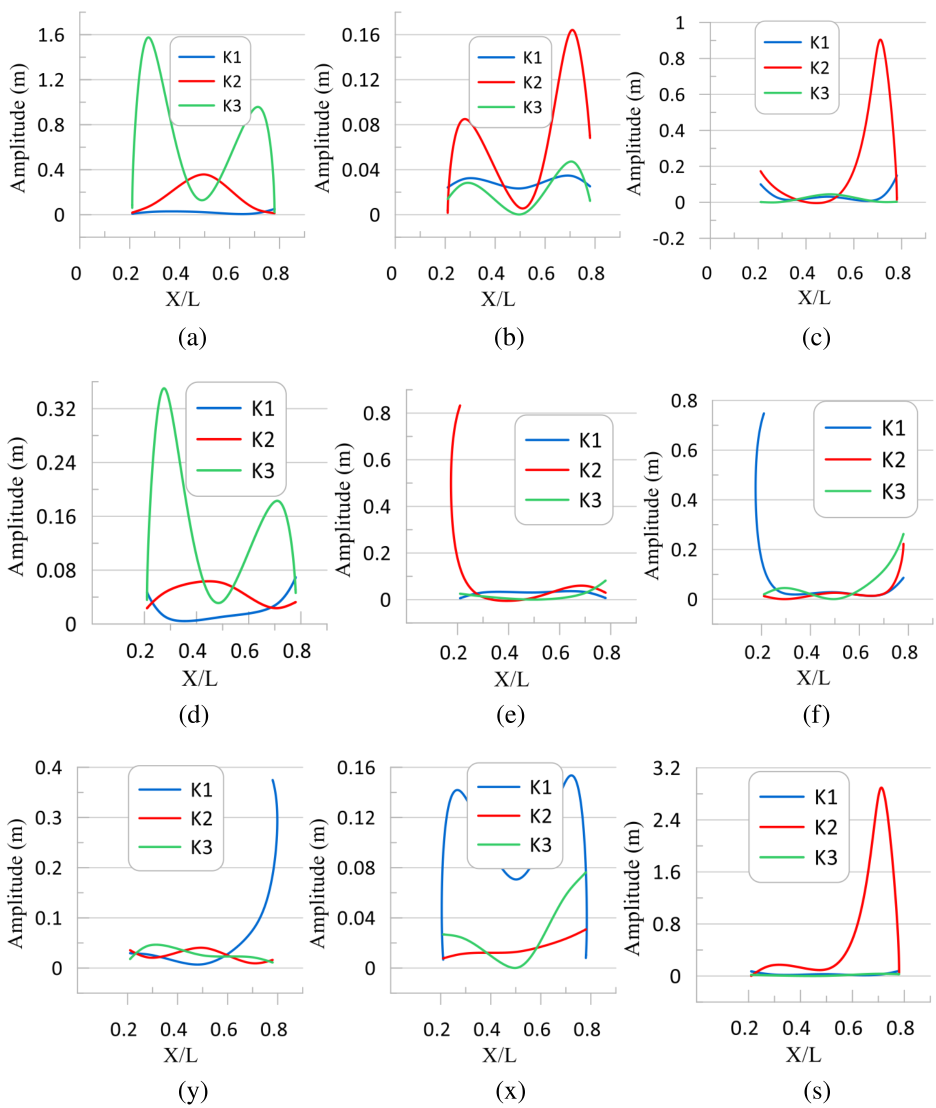


Fig. 16. The effect of DVA locations in harmonic response for three mode shapes; (a, b, and c) for M = 100 g; (d, e and f) for M = 200 g; (y, x, and s) for M = 300 g

shown in Fig. 16 (a, b, and c), (d, e, and f) for M = 200 g, and (y, x, and s) for M = 300 g. The stiffness, mass, and position are some of the variables that affect the dynamic response; the responses change depending on the corresponding mode. For instance, the stiffness (K3–K1) and the lower value of applied mass (100 g–300 g) for the first mode have a considerable impact on where the maximum amplitude node 23 (X/L = 0.5) is located. For the second and third modes, however, small alterations are seen. Due to the resonance of the three modes that can be achieved at these values for the three masses and stiffness in this test, the most notable changes in response for the second and third modes happen at (100 g–300 g with K2–K1) and (100 g–300 g with K2–K1), respectively.

5. Overall amplitude change percentage and frequency shift

As noticed previously, the results showed that adding DVAs at specific locations affects the percentage change in the amplitude and makes a shift in the resulted system frequency. The importance of frequency shift effect is that, in some circumstances, machinery may be built to function within a certain frequency range. The effectiveness and efficiency of the machinery may be impacted if the inclusion of a DVA causes the system frequency to shift outside of the required range. For instance, a change in the system frequency could affect the critical speed of rotors, which would result in excessive vibration, wear, and possibly failure. In addition, in some applications, sensors may be employed to gauge a machine or vibration levels of structures. The sensors may no longer reliably record the vibration amplitude if the inclusion of a DVA shifts the system frequency, resulting in inaccurate results and consequently incorrect maintenance or repair choices.

According on that, it is important that when constructing a DVA, a compromise must be made between minimizing any frequency shift in the system and reducing the vibration amplitude. In some circumstances, lowering the vibration amplitude might be the main objective, and a DVA-induced frequency shift might be acceptable or even desirable. In other circumstances, limiting the frequency shift could be more crucial, especially if the system is sensitive to frequency changes similar to those in our application in this work. Therefore, when building and installing a DVA, it is crucial to carefully evaluate the specific design and operating circumstances of the system and to optimize the design to achieve the necessary amount of vibration reduction while reducing any negative effect. The present work proposes a novel mathematical expression for obtaining the optimal design of the DVA, taking into consideration essential factors such as frequency shift, reduction in beam amplitude, and achieving a trade-off between them. To the best of author's knowledge, the proposed methodology stands as a unique contribution, not previously explored by other researchers.

The authors suggested a simple, but efficient, mathematical expression based on the numerical values of the results to make the trade-off mentioned above. This expression involves the amplitude percentage change and the corresponding frequency shifting ($\sum$ APC% – $\sum$ shift frequency) for the three modes for all presented cases in this work and illustrated in Table 7. The results showed for case-1 that the condition C_{12} is the best selection based on the mathematical expression above, whereas condition B_{13} produced the worst reduction value. In the second scenario, condition M_3 showed the best selection while condition G_2 was the worst.

Table 7. The relationship between the reduction of vibration and shifting frequency

Condition Case-1	$\sum$ APC% – $\sum$ shift freq.	Condition Case-2	$\sum$ APC% – $\sum$ shift freq.
A_{11}	−14.981	D_1	–11.231
A_{12}	+823.519	D_2	–120.401
A_{13}	+275.396	D_3	+81.709
B_{11}	−19.781	E_1	–40.754
B_{12}	+1278.489	E_2	–30.461
B_{13}	+5784.999	E_3	–68.001
B_{21}	−17.821	G_1	+114.219
B_{22}	+3154.249	G_2	+3654.689
B_{23}	+3722.483	G_3	–70.324
C_{11}	+41.939	H_1	–136.961
C_{12}	−52.521	H_2	–42.901
C_{13}	+2896.149	H_3	–102.721
C_{31}	+251.959	M_1	–120.241
C_{32}	+415.086	M_2	-121.171
C_{33}	+5.339	M_3	-164.961

Finally, future directions of the research will focus on improving the design and manufacturing procedures to make them more effective and affordable, creating more sophisticated models to accurately predict their performance, and looking into new applications and industries where they can be used. The study of intelligent materials and sensors in dynamic vibration absorbers is also of interest, since it may help develop systems that are more adaptable and quick-reacting. Further study may be successful in examining the usage of dynamic vibration absorbers in combination with other vibration control strategies, such as active and passive dampers.

6. Conclusion

Finite element analysis was used to explore the effect of tuned vibration absorber location on the beam structure. The results show that the placement of the absorber, mass, and stiffness all have a substantial impact on the amplitude of the beam vibration.

- The use of DVA could represent an effective method for vibration attenuation of the beam structure.

- The mass and stiffness of the DVA used in this work show a significant effect on the harmonic response of the beam. Their effects depend on the mode of vibration required to attenuate and the DVA location.
- Adding a DVA to the main system shifts the frequency of the harmonic response to the left or right of the main system frequency without DVA, depending on the combined effect of DVA mass and spring values.
- Optimal attenuation performance can be attained by choosing optimal values for DVA mass, spring, and DVA location. That means that for every single DVA location there are critical values of DVA mass and stiffness.
- The tuned absorber that exhibits a considerable amplitude at any frequency, but no significant peak amplitude introduced by the beam, vibrates in its mode.

The test findings clearly show a relationship between frequency shift and vibration. Condition C_{12} produced the biggest reduction in vibration amplitude for the first case with an average reduction of 54.33% for all three modes. In contrast, condition M_3 demonstrated the most substantial reduction in vibration amplitude in the second case with an average reduction of 79.81% for all three modes. Vibration amplitude can be reduced even further by adding more DVA units, but only if they are placed properly.

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