BULLETIN OF THE POLISH ACADEMY OF SCIENCES TECHNICAL SCIENCES, Vol. 72(5), 2024, Article number: e149169 DOI: 10.24425/bpasts.2024.149169

SPECIAL SECTION

# Reconfigurable smart metamaterial for energy transfer control in alternating magnetic fields

Adam STECKIEWICZ®\* and Aneta STYPUŁKOWSKA

Bialystok University of Technology, Faculty of Electrical Engineering, Wiejska 45D Str., 15-351 Białystok, Poland

Abstract. The paper presents a metamaterial with reconfigurable effective properties, intended to operate in alternating magnetic fields. The structure of a resonator, based on a series connection of a planar inductor and a lumped capacitor, is expanded using an additional capacitor with a MOSFET transistor. Due to the presence of the controllable active element, it is possible to dynamically change the phase of the current flowing through a meta-cell and shift a frequency response within an assumed range. Since the transistor is driven by the unipolar square wave with a changeable duty cycle and time delay, two closed-loop controllers were utilized to achieve a smart material, able to automatically attain and maintain the imposed resonant frequency. As a result, the complex effective magnetic permeability of the metamaterial can be smoothly changed, during its operation, via an electrical signal, i.e. by adjusting the parameters of a control signal of the active element. The design of the meta-cell, as well as the measuring, data operation and control part, are presented in detail. An illustrative system is examined in terms of achieving the user-defined resonance point of the metamaterial. Transient responses with estimated settling times and steady-state errors and the effective permeability characteristics for the exemplary cases are shown. The meta-cell is also tested experimentally to validate the theoretically determined effective properties.

Keywords: metamaterial; magnetic field; PID controller; smart material; resonance.

### 1. INTRODUCTION

Metamaterials are composites that have extraordinary properties, resulting from an artificially designed structure [1]. By modifying the geometry of a metamaterial unit, it becomes possible to change its electromagnetic (EM) characteristics and acquire permeability and permittivity lower than those of the vacuum [2]. The first considerations of metamaterials were made in [3] by V.G. Veselago, where a lens with negative refractive index had the ability to exceed the diffraction limit. Metaelements are then predisposed to be the building "blocks" of superlenses [4, 5], magnetic shields [6, 7], concentrators [8, 9], or even artificial wormholes [10].

Although metastructures were mostly considered in high-frequency (microwave, optical) EM fields, in which cloaks [11], left-handed materials [12] and antennas [13] were constructed, metamaterials were also developed for magnetic fields in the kHz/MHz regime [14, 15]. The attention was put on cloaking [16,17] and the metamaterial enhanced wireless power transfer (WPT) [18] – resonators (i.e. planar coils with capacitors) were effectively redirecting the power to a receiver upon a major misalignment [19, 20], utilized to shield the leakage flux [21] and used as a transmitting path to transfer the energy wirelessly [22, 23]. Yet, in most cases these structures possessed a single resonant frequency, which limited their practical applications and which may cause problems arising from the variability of parameters (during production and over time) of many

Manuscript submitted 2024-02-12, revised 2024-04-02, initially accepted for publication 2024-04-26, published in September 2024.

meta-cells. For this reason, the methods of overcoming such disadvantages were proposed.

In magnetic metamaterials, the resonant frequency of resonators may be tuned mechanically with the trimmer capacitors [24, 25]; nevertheless, user involvement would be needed. Another approach [26] utilized a switching device, toggled between several capacitors; however, only some discrete capacitances were possible there. In WPT, the tuning was performed using three coupled open bifilar coils, which achieved selfresonant frequency due to the change in the distance between them [27]; this approach can be applied to metamaterials if micromechanical devices (MEMS) are used to shift the position of an inductor. The techniques of reshaping meta-structure, to alter its properties by MEMS, were tested in a GHz [28] and THz [29] regime for relatively small metamaterial units; unfortunately, greater coils, suitable for the kHz and MHz range, would require adequately larger MEMS or some linear actuators to move the resonator.

It is worth noting that the reconfigurable metamaterial should have a high resolution of changeable properties to be able to precisely tune itself to a frequency of an external field. If metacells had a wide spectrum of resonant points, instead of a single resonance, then one metamaterial may become more versatile, and hence used for operating in frequency-varying fields. It may also be expected that the characteristics of such elements will be modified without the need to utilize moving devices or the constant commitment of a user. The desire to meet these practical requirements was the main motivation behind the following research. This led to a design of a meta-cell with an ability to adjust its properties within a wider range of resonant frequencies and with the high resolution, obtainable by an electrical switch-

<sup>\*</sup>e-mail: a.steckiewicz@pb.edu.pl

ing of a semiconducting element. The forecasted applications of the zero-permeability material with adjustable resonance can be proposed as: the magnetic shielding of the areas exposed to the strong EM fields (also the metamaterial, with its sparse inner structure, will weigh less than a solid shield), the reduction of a leakage flux from the WPT system (by putting metamaterial slabs around the edges of transmitting and/or receiving coils), reflecting the magnetic field of the transmitting coil (using a meta-structure behind it) and as the band-stop filter blocking selected frequencies of EM fields.

In this article such novel, reconfigurable metamaterial was presented. The resonant point was tuned due to the usage of an additional capacitor and transistor, driven by a PID controller. As a consequence, each smart meta-cell would be able to autonomously tune to the given resonance frequency. To discuss the introduced system, the paper was organized as follows: in Section 2, the structure of the metamaterial and the control algorithm were presented; in Section 3, the mathematical and simulation models of a meta-cell were contained; in Section 4 the time-domain analysis was performed, with the theoretically and experimentally estimated effective properties; in Section 5, the main conclusions were provided.

### 2. MATERIALS

### 2.1. Structure of the metamaterial

The considered material is intended to operate in magnetic fields with the bandwidth from 10 kHz to 10 MHz. The main attention is caused by a possibility of having the real part of an effective permeability less than permeability of the air. The most interesting is a 'zero-permeability metamaterial', since it can shield magnetic fields efficiently [30–32]. To achieve this, the resonance point ( $f_0$ ) of the material must coincide with the frequency of an incident EM wave. This may be important e.g. in WPT systems where, according to SAE [33], the frequencies of alternating magnetic fields are near 100 kHz, and distribution of the EM field is crucial from a power transfer and EM compatibility point of view.

To directly interfere with the EM wave propagating in a certain direction (as shown in Fig. 1), meta-cells should be orientated perpendicularly to the magnetic field components ( $H_x$  and  $H_y$ ). Hence, a material consisting of magnetically coupled elements, forming a two-dimensional grid of layers (parallel to the Poynting vector  $\mathbf{S}$ ), is needed. The meta-cell in layers, which are separated by a distance h, is the resonator consisting of a multi-turn planar inductor and a lumped capacitor (Fig. 2). The planar coil has a size  $d_w$  and n turns of  $d_c$ -wide conducting paths with a  $d_s$  gap between. The structure is supplemented by a transistor, a second capacitor, and probes to measure the voltage and current of the coil.

Due to the presence of the active element and measuring probes, it becomes possible to create a reconfigurable and self-stabilizing (smart) material. The transistor can be controlled by a pulse width modulated (PWM) signal from an external device, e.g. a microcontroller. Meanwhile, by using an analog-digital converter, the signals from probes will help estimate the state of

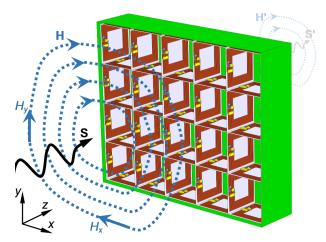
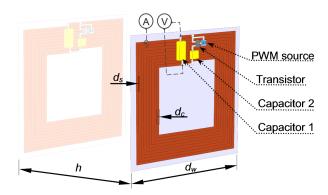


Fig. 1. Exemplary operation of a 2D zero-permeability metamaterial: a magnetic field  $\mathbf{H}$  is passing through several layers of resonators interacting with  $H_x$  and  $H_y$  components; therefore, the magnetic field behind the metamaterial ( $\mathbf{H}'$ ) will be attenuated



**Fig. 2.** Reconfigurable meta-cell with three assemblies arranged in parallel: inductor, capacitor and series connection of a second capacitor and transistor, with the voltage (V) and current (A) probes

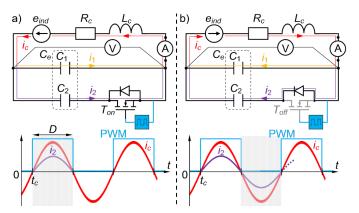
the meta-cell, since a closed-loop PID controller is meant to be used to maintain the desired resonance frequency.

### 2.2. Operation of the reconfigurable meta-cell

Currently developed magnetic metamaterials possess one resonant frequency, since the simplest resonant structure is used – a series circuit with constant RLC parameters. Yet, if some switchable/replaceable capacitors or a variable capacitors were incorporated, then the resonance point could be changed. Unfortunately, mechanical elements (switches, knobs, etc.) have to be manually activated, while different capacitors ensure only several discrete values of the resonant frequency.

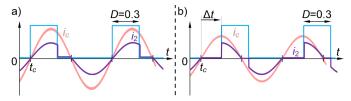
One of the key features expected in material engineering is to modify properties of the material continuously, precisely, using electrical signals without employing moving parts or manual actions. By adding active branch and probes (current and voltage) to the meta-cell (Fig. 3), the presented reconfigurable composite can ensure these requirements.

The idea of operation is as follows: when the inductor current is positive, the transistor is activated by PWM signal (Fig. 3a), thus current flows through  $C_1$  and  $C_2$ . As a result capacitors



**Fig. 3.** Equivalent circuits of a meta-cell at two stages of operation, arising from a signal with duty cycle D = 0.5: (a) the AC current flowing in the "positive" direction; (b) the current in the "opposite" direction

became connected in parallel, giving equivalent  $C_e = C_1 + C_2$ ; when current changes direction (Fig. 3b) and transistor will be properly deactivated, the current will still flow through  $C_1$  and  $C_2$  (utilizing a reverse diode), again resulting in  $C_e = C_1 + C_2$ . Therefore, the flow of the AC current in both directions is satisfied. Yet, if the transistor is not activated at any time, only  $C_1$  will conduct the AC current. It seems that merely two resonant frequencies can arise from these boundary cases, i.e. 50% PWM duty cycle (giving  $C_1 + C_2$ ) or 0% duty cycle (only  $C_1$  in use). Between these extremes intermediate states are achievable (Fig. 4), by changing duty cycle,  $D \in \langle 0; 0.5 \rangle$ , and causing equivalent capacitance,  $C_e \in \langle C_1; C_1 + C_2 \rangle$ . Therefore, one transistor and two capacitors are sufficient to provide any  $f_0$  from the selected range, determined by the chosen  $C_1$ ,  $C_2$ .



**Fig. 4.** Exemplary waveforms of  $i_2$  during: (a) duty cycle D = 0.3; (b) desynchronization of PWM pulse and current of a coil  $(i_c)$  – pulse starts at the time  $t_c + \Delta t$ , causing strong distortion of  $i_2$  waveform

It is crucial to synchronize PWM pulse with the induced current  $i_c$ , since the phase of the external magnetic field (inducing electromotive force  $e_{\rm ind}$ ) varies through time. In other words, PWM pulse has to start at the time  $t_c$  – exactly at the point when  $i_c$  changes to "positive". The time compensation ( $\Delta t$ ) is easy to calculate for a known phase ( $\varphi_{ic}$ ) of  $i_c$ , that is:

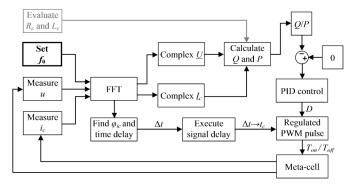
$$\Delta t = f_0^{-1} \left( 1 - \frac{\varphi_{ic}}{360^{\circ}} \right). \tag{1}$$

While if desynchronization between PWM and  $i_c$  appears,  $i_2$  would be distorted and its RMS value would decrease (Fig. 4b), even at an unchanged duty cycle. Therefore, a current probe and a phase estimation algorithm are essential.

The main objective to satisfy is to get a zero-permeability material, interacting with a magnetic field at specific resonant frequency, defined by the user. While the disadvantage of the presented meta-structure is a nonlinear dependence  $C_e = f(D)$ , it may be preferable to adjust D via the PID controller, where the operator enters the desired  $f_0$  and D is set automatically (Fig. 5). To gain this ability, a resonance point must be estimated, to provide feedback to the controller and the Fast Fourier Transform (FFT) ought to be performed. Since if resistance  $(R_c)$  and inductance  $(L_c)$  of a coil were found, and current  $(i_c)$  and voltage (u) were measured, the ratio of a reactive (Q) and active power (P) is:

$$\frac{Q}{P} = \frac{j2\pi f_0 L_c \left| \underline{I}_c \right|^2 + \operatorname{Im} \left[ \underline{U} \underline{I}_c^* \right]}{R_c \left| \underline{I}_c \right|^2 + \operatorname{Re} \left[ \underline{U} \underline{I}_c^* \right]}, \tag{2}$$

where:  $\underline{U}$  – complex value of the measured u;  $\underline{I}_c$  – complex value of the measured  $i_c$ . Simultaneously, Q/P is an output of the closed-loop controller and, in an ideal case, Q/P should be zero. Consequently, at desired  $f_0$ , the control system has to maintain such a duty cycle D where the Q/P ratio is closest to 0.



**Fig. 5.** Diagram of operation: one-time evaluation of  $R_c$  and  $L_c$  is needed before initialization of the system; next, the desired  $f_0$  is set/changed by the user; then, measurements and calculations are made to produce PWM with proper delay and duty cycle D

# 3. METHODS

# 3.1. Properties of metamaterial units

To represent a metamaterial from the material science point of view, the properties of a solid structure should be determined. Considering applications in magnetic and EM fields, such a composite formed of many identical elements is mainly characterized by relative effective permeability [34]:

$$\underline{\mu}_{\text{eff}} = 1 - \frac{F}{1 - f_0^2 / f^2 + j \left( f_0 / fq \right)},$$
(3)

where q is the quality factor and F may be calculated as [14]:

$$F = \frac{\mu_0 \left[ 0.5 d_w^3 - 4 \left( 0.5 d_w - n d_t \right)^3 - n d_t^3 \right]^2}{9 d_t^2 L_c h d_w^2},$$
 (4)

for  $d_t = d_c + d_s$ . It should be noted that the inductance of a coil  $(L_c)$  in a metamaterial grid can be expressed as a combination

of [14]: self-inductance  $(L_s)$  and the sum of mutual inductances (M) between an arbitrary meta-cell and all the others  $(L_c = L_s + M)$ . Mutual inductances can be found using proper equations provided in [14], while the approximated formula for  $L_s$  is [35]:

$$L_{s} = \frac{c_{1}\mu_{0}n^{2}\left[d_{w} - nd_{c} - (n-1)d_{s}\right]\left(\ln\frac{c_{2}}{v} + c_{3}v + c_{4}v^{2}\right)}{2}\,, \ \ (5)$$

where v is the fill factor:

$$v = \frac{nd_c + (n-1)d_s}{d_w - nd_c - (n-1)d_s}$$
 (6)

and  $c_1$ – $c_4$  are shape coefficients, e.g.  $c_1$  = 1.26,  $c_2$  = 2.08,  $c_3$  = 0.14,  $c_4$  = 0.115 for a square planar coil. In addition, the resistance of the inductor can be found as [14]:

$$R_c = 4n\sigma^{-1}a^{-1}[d_w - nd_c - (n-1)d_s],$$
 (7)

where  $\sigma$  is the electrical conductivity and a is a cross-section of winding (e.g.  $a = d_c l$ , where l is the thickness of windings).

Finally, capacitances may be estimated. The designer has to specify the boundaries ( $f_{\min}$  and  $f_{\max}$ ) of a bandwidth wherein the resonant frequency can be changed. Then, the capacitances can be calculated using the formulae below:

$$C_1 = \left(4\pi^2 f_{\text{max}}^2 L_c\right)^{-1},\tag{8}$$

$$C_2 = \left(4\pi^2 f_{\min}^2 L_c\right)^{-1} - C_1. \tag{9}$$

For precise calculations  $L_c$  must be known; however, for quick design purposes, it is acceptable to assume  $L_c \approx L_s$ .

Furthermore, in (2) the real and imaginary parts of the apparent power of equivalent capacity  $(C_e)$  were taken, since in practical conditions the resistances of capacitors occur. While they have no influence on  $f_{\min}$ ,  $f_{\max}$ , or  $f_0$  the real and reactive power must be considered to control the Q/P value. For the simulation, these resistances can be estimated as:

$$R_x = DF (2\pi f_0 C_x)^{-1}, (10)$$

where:  $C_x$  – the capacitance of  $C_1$  or  $C_2$ ; $R_x$  – the equivalent series resistance (ESR) of  $C_x$ ; DF – the dissipation factor.

### 3.2. Simulation model

A metamaterial was composed of many identical resonators responsible for the periodic distribution of an effective permeability in a 3D space. Thus, due to this periodicity (identity), the analysis could be reduced to a single meta-cell, representing an arbitrary element of the metamaterial grid.

Calculations were performed in *Matlab/Simulink*, where the model and signal operations were established (Fig. 6). The simulation model was divided into appropriate sections: 1) Reconfigurable resonator with probes that route measured values to 2) the Processing unit, where electrical quantities were evaluated and used in 3) the PID controller, which produced the duty cycle to drive 4) the PWM control signal, with the time delay

varying with a phase  $(\varphi_{ic})$  of the current  $(i_c)$ . The computations always began at the time  $t_0=0$  and the initial duty cycle D=0.25. After 0.5 ms (the time needed to wait out the transient state after the circuit started) the switch had been activated and the duty cycle was further transferred from the discrete PID. The resonant frequency  $f_0$ , requested by a user, had to be fed into two blocks: as the switching frequency of the PWM generator and the fundamental frequency of the FFT block, since only sinusoidal components with the set  $f_0$  were extracted from the measured voltage and current to be used in  $\varphi_{ic}$  and Q/P calculations.

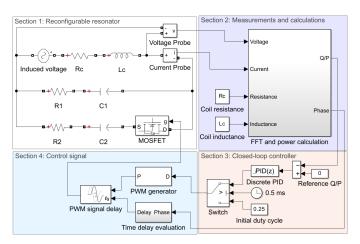


Fig. 6. Simulation model

While the planar inductor and capacitors were modeled as the real components, the MOSFET transistor and two probes were ideal elements in order to distinguish, in this preliminary research, the negative impact of passive components on the behavior of the metamaterial unit.

# 3.3. Experimental stand

The possibility of shifting/changing the resonance point as well as obtaining certain characteristics of the metamaterial were tested based on experimental results. At the beginning, this was done by means of comparing an active metamaterial, namely the simulation model, with its passive equivalent having identical lumped parameters. This may help to estimate a response of the meta-cell when additional capacitors are attached, and examine whether the theoretical characteristics of the active elements are consistent with those of passive RLC elements (Fig. 7). Consequently, the main interest was put on the value of resonant frequency of the meta-cell and frequency dependence of the effective permeability.

The measurement method for the properties mentioned above can be found in [36]. The identification of  $f_0$  and q was based on an admittance characteristic  $(Y = |\underline{Y}|)$  of a metamaterial unit. Resonant frequency occurs at the point where  $Y(f_0) = Y_{\max}$ , while the quality factor,  $q = f_0/(f_h - f_l)$ , was estimated by finding a low  $(f_l)$  and high  $(f_h)$  limit of the Y-parameter bandwidth, so that  $Y(f_l) = Y(f_h) \approx 0.707Y_{\max}$ .

The meta-cell should be placed in an environment filled with air and surrounded by the perfect electric (PEC) and/or magnetic

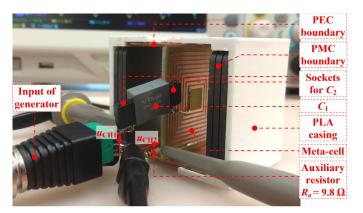


Fig. 7. Experimental setup – the sample was inside PLA casing with PEC and PMC boundaries; voltages on two channels of a scope were measured, giving  $e_{\text{ind}} = u_{\text{CH1}} - u_{\text{CH2}}$  and  $i_c = u_{\text{CH2}}/R_a$ 

(PMC) conductor boundaries (Fig. 7), mimicking the presence of other meta-cells. To imitate the perpendicular position of an element inside the infinitely expansive material, the PEC and PMC were required. The PEC boundaries (top and bottom) were made of 1 mm thick copper plates, while PMC (front and back) were imitated using three 2 mm thick ferrite plates MP1496-000 [37]. All plates were size  $38 \times 38$  mm and secured in a casing, 3D printed of a non-magnetic polylactide (PLA).

The sample was made on an FR-4 laminate with a  $l=105~\mu m$  thick copper layer. Firstly, the resistance ( $R_c$ ) and inductance ( $L_c$ ) were measured by LCR bridge Hameg 8118. Then, capacitor  $C_1=46.4~\rm nF$  was inserted between the coil and the Rigol DG4062 generator ( $R_{\rm in}=50~\Omega$ ) – which produced sinusoidal voltage to imitate the electromotive force. Thirdly, the Rigol DS2072 oscilloscope measured the voltage and current of the meta-cell at frequencies from 250 to 600 kHz. Fourthly, admittance was calculated and then capacitor  $C_2=100~\rm nF$  was added in parallel to  $C_1$  and again the Y-parameter was found. To validate another intermediate resonant frequency,  $C_2$  had been removed and  $C_2'=33~\rm nF$  was attached. At the end, upon the identification of  $f_0$  and q, permeability was evaluated using (3) and (4), and compared with those derived from the simulation model.

The structure of the meta-cell is volumetrically extensive, causing material to be "spacious"; hence miniaturization must be considered. Crucial is the size  $(d_w)$  of an inductor: by lowering it several times, the width can drop below 1 cm, but this decreases the F and  $L_s$ , which then make  $\underline{\mu}_{\text{eff}}$  deteriorate. To counteract this, one may consider a paramagnetic material, in the form of solid plates, placed on the top and bottom of the inductor. This will increase the relative permeability and partially compensate for the decrease in F and  $L_s$ . In a smaller element the wounds will be thinner so, to maintain a low  $R_c$ , the height (t) of the wounds must also be elevated.

### 4. RESULTS AND DISCUSSION

The main analysis focused on the operation of the reconfigurable resonator and on the examination of quality factors (settling time  $t_s$  and steady-state error  $\Delta y$ ) arising from control of the

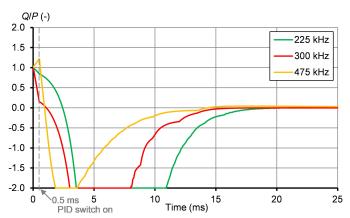
resonance point. Since the reference value was Q/P = 0, well-known  $\pm 5\%$  tolerance bands of the final value could not been used; hence, the absolute value  $Q/P = \pm 0.05$  was utilized as the criterion for reaching the steady state.

At the same time, the waveforms of the Q/P ratio, duty cycle D and current of the coil  $i_c$  (and its total harmonic distortion, THD) were presented. Transient responses were also shown for different phases of an induced voltage, to demonstrate the efficacy of the time delay compensation. All the results in Section 4.1 and Section 4.2 were obtained from the time-domain simulation model, discussed in Section 3.2. But subsequently, in the last section the effective relative permeability was calculated for three cases, compliant with those studied during the experiments, where the equivalent passive RLC resonator was tested and effective permeabilities were found.

The illustrative resonator was analyzed with the parameters as follows: square planar coil (e.g. Fig. 2) of  $d_w = 36$  mm in size, having n = 9 turns made of copper ( $\sigma = 5.6 \cdot 10^7$  S/m); each turn  $d_c = 1.1$  mm wide with  $d_s = 0.5$  mm separation and l = 105 µm thickness; and distance  $h = d_w$  between the layers. The lower and upper limit of the resonant frequencies was chosen as  $f_{\min} = 200$  kHz and  $f_{\max} = 500$  kHz, giving  $C_1 = 50.35$  nF and  $C_2 = 264.31$  nF. The equivalent series resistances of  $C_1$  and  $C_2$  were found based on (10) for DF = 0.0021. Meanwhile, the lumped parameters of the inductor,  $R_c = 0.12 \Omega$  and  $L_c = 2.01$  µH, were estimated using (5)–(7) and the formula for mutual inductance from [14]. The gains of the PID controller were following:  $K_P = 0.01$ ,  $K_I = 10$ ,  $K_D = 3 \cdot 10^{-4}$ ; whereas the sampling frequency was set to 25 MHz, which is achievable with modern processing units and data acquisition cards.

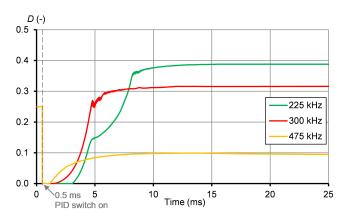
# 4.1. Transient responses

The results for exemplary desired resonant frequencies (225, 300 and 475 kHz) were shown in Fig. 8 and Fig. 9. The step responses started at 0.5 ms and, for convenience, the visualization of Q/P has been limited from -2 to 2.



**Fig. 8.** Exemplary transient responses (Q/P) for three resonant frequencies:  $f_0 = 225$  kHz,  $f_0 = 300$  kHz and  $f_0 = 475$  kHz

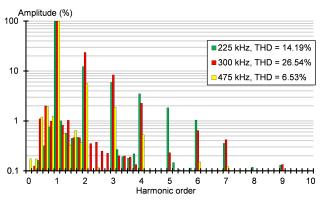
It can be seen that, despite the initial value D = 0.25, the controller increased the duty cycle from 0 to some steady-state value. Meanwhile, Q/P initially decreased below -2 (for several mil-



**Fig. 9.** Exemplary waveforms of duty cycles (*D*) for three resonant frequencies:  $f_0 = 225 \text{ kHz}$ ,  $f_0 = 300 \text{ kHz}$  and  $f_0 = 475 \text{ kHz}$ 

liseconds), but then successfully increased from negative values to the reference Q/P=0. During the transient state, oscillations (between 4 and 9 ms) were found on waveforms of  $f_0=225~\rm kHz$  and  $f_0=300~\rm kHz$ , nevertheless without an impact on the operation of the system, since stabilization on the reference value was always attained. Hence, tuning of the meta-cell was possible within the designed limits – whether near the boundaries ( $f_{min}$ ,  $f_{max}$ ) or for intermediate resonant frequencies.

The FFT analysis (Fig. 10) has shown the distortion of a current ( $i_c$ ). Considered examples presented not only raised THD values, but also more detail disorders. The undesired contents appeared mostly from the 2nd to 7th harmonics, with a descending trend of amplitudes decreasing from several percent of the fundamental frequency amplitude to its tenths. Moreover, by looking at the 2nd and 3rd harmonic, one may find that lower values of their amplitudes remain in correlation with also lower THD. Therefore, to reduce current distortion, an elimination of these harmonics would be crucial. Further analysis showed negligible amplitudes of higher harmonics. In contrast, sub- and interharmonics (up to the 4th harmonic) were present with relatively lower, but still noticeable, values (from 0.1% to 2.1%).



**Fig. 10.** Harmonics of the steady-state current  $(i_c)$  for three resonant frequencies:  $f_0 = 225$  kHz,  $f_0 = 300$  kHz and  $f_0 = 475$  kHz

The distortion of  $i_c$  was studied for all possible resonant frequencies, along with the steady-state duty cycle (Fig. 11). THD was rapidly rising right after increasing  $f_0$  over the lower

limit  $f_{\rm min}$ , reaching a peak at THD = 26.54% ( $f_0$  = 300 kHz). Then distortion decreased until THD  $\approx 0$  ( $f_0$  = 500 kHz). The current of an inductor is consequently distorted for intermediate  $f_0$ , with the lowest values for the resonant frequencies near the limits of the range  $f_{\rm min}-f_{\rm max}$ .

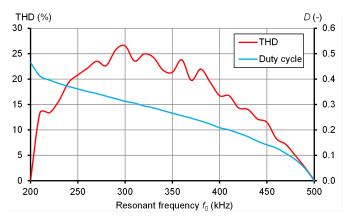


Fig. 11. Total harmonic distortion (THD) and steady-state duty cycle (D) for different specified resonant frequencies  $(f_0)$  of a meta-cell

Steady-state values of a duty cycle (Fig. 11) proved that a resonance point of a meta-cell can be reconfigured by finding unambiguous D for any resonant frequency between  $f_{\min}$  and  $f_{\max}$ . Additionally, the confirmed non-linear relation  $D = f(f_0)$  may lead to potential problems with precise control of  $f_0$ , using open-loop control, where prior evaluation of the  $D = f(f_0)$  characteristic would also be necessary. Hence, it seems essential to utilize the closed-loop controller.

For all resonant frequencies, the PID controller was able to effectively set the duty cycle, which was interpreted as keeping the steady-state error  $(\Delta y)$  of the Q/P ratio within the  $\pm 0.05$  span (Fig. 12). Generally speaking, the error was unpredictable, both its values and sign, with relation to D or  $f_0$ . Similar conclusions came from an analysis of the settling time  $(t_s)$ . The only exceptions were the  $f_0$  set near the  $f_{\min}$  limit, where  $t_s$  were approximately twice longer. For the other resonant frequencies  $t_s$  comprised within 11.50–17.22 ms. Thus, these values were comparable as well acceptable from the dynamics point of view

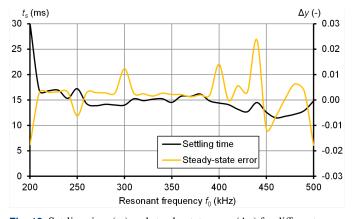


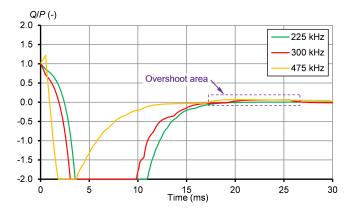
Fig. 12. Settling time  $(t_s)$  and steady-state error  $(\Delta y)$  for different specified resonant frequencies  $(f_0)$  of a meta-cell

 despite the relatively fast reaction of the system, it would be possible to change the resonant frequency at least several dozen times per second.

### 4.2. Impact of the magnetic field phase

The phase of external magnetic field may vary, causing directly related changes of a phase  $(\varphi_e)$  of an induced voltage  $(e_{\text{ind}})$ , resulting in a time shift of the current of a coil  $(i_c)$ . At the end of Section 2.2 this current and its phase  $(\varphi_{ic})$  were proposed to be identified, to perform additional control in the system – the ongoing synchronization of the PWM pulse with  $i_c$ . In this section, the impact of the phase  $(\varphi_e)$  of the external excitation has been studied to show adequacy of the utilized time delay of the PWM signal.

Illustrative responses of the reconfigurable system for  $\varphi_e = 90^\circ$  (Fig. 13) indicated similar waveforms as for  $\varphi_e = 0^\circ$  (Fig. 8) and an ability to stabilize Q/P closely at a reference value. Yet, such as for other tested phases, the relatively low overshoot appeared after 15 ms, but the PID controller was still able to accomplish the anticipated steady state. The drawback, however, was the nearly twice longer settling time, giving the steady state over approximately 30 ms.

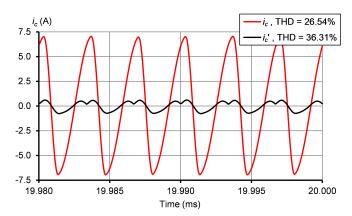


**Fig. 13.** Exemplary transient responses (Q/P) for three resonant frequencies  $(f_0 = 225 \text{ kHz}, f_0 = 300 \text{ kHz}, f_0 = 475 \text{ kHz})$  at  $\varphi_e = 90^\circ$ 

Here, a comparison between the system with and without phase synchronization can be also made. While having an inappropriate initialization of a PWM pulse it is possible to achieve  $Q/P \approx 0$  at different D and  $t_s$ , the unquestionable fact is the impact on the current of the metamaterial unit. In Fig. 14 one may observe that both higher THD and lower admittance appeared, comparing the system without synchronization  $(i'_c)$  to the PWM and current being synchronized with each other  $(i_c)$ .

During the use of the PWM time delay, the phase of the magnetic field had a minor influence on THD and D. The values of parameters remained nearly unchanged throughout an entire range of  $\varphi_e$  (Fig. 15). As a result, the amplitude and phase of the EM field interacting with the reconfigurable unit will not affect its performance, making the material versatile.

On the other hand, settling times were affected by the phase (Fig. 16), where recurring increase (10 ms or more) of  $t_s$  was present for some  $\varphi_e$  (e.g. 30°, 90°, 150°). For the remaining



**Fig. 14.** Waveforms of the current when the phase synchronization of PWM pulse is used  $(i_c)$  and without any synchronization  $(i'_c)$ , for an exemplary case of  $f_0 = 300$  kHz and  $\varphi_e = 90^\circ$ 

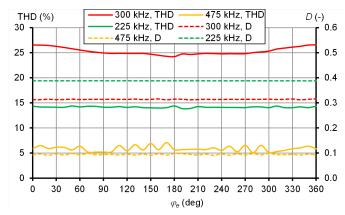


Fig. 15. Total harmonic distortion (THD) and steady-state duty cycle (D) for different phases ( $\varphi_e$ ) of the induced voltage/magnetic field

phases  $t_s$  was comparable as for  $\varphi_e = 0^\circ$  (Fig. 12). The longer  $t_s$  can be explained by the appearance of a slight overshoot (Fig. 13), resulting from the need of extra time for the PWM delay controller to synchronize pulses with the current. However, it did not have a significant effect on the steady-state error, since  $\Delta y$  remained within the  $\pm 0.05$  band. Moreover, by looking at Fig. 15 and Fig. 16, it can be concluded that PID must not

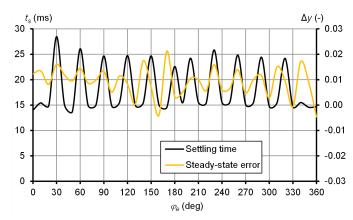


Fig. 16. Settling time  $(t_s)$  and steady-state error  $(\Delta y)$  for different phases  $(\varphi_e)$  of the induced voltage/magnetic field for  $f_0 = 300$  kHz

retune the duty cycle while phase changes (D would be roughly constant at some  $f_0$ ), but only time delay control has to perform the time shift of PWM pulses.

# 4.3. Theoretical and experimental frequency response

The final investigation should concern properties of the material. Reconfigurable metamaterial units act as meta-cells having quasi-equivalent capacitance  $C_e$  and resistance  $R_e$ . A resonance was achieved by switching the capacitor  $C_2$ , but also causing the current distortion. This raises a question of preserving identical effective permeability characteristics as those arising from a passive RLC. Thus, in this section the Y-parameter was obtained for the two meta-cells: one measured for the passive element (presented in Fig. 7) and one simulated for the active element (using the model in Fig. 6) with the lumped parameters  $(R_c, L_c, C_1, C_2)$  of the first one. Then, the effective permeability of the simulated meta-cell could be compared with  $\mu_{\rm eff}$  of the passive metamaterial unit having a lumped  $C = C_e$ , previously presented in Section 3.3.

At the beginning, resistance and inductance of the meta-element were measured, giving  $R_c = 259.95 \pm 8.09 \text{ m}\Omega$  and  $L_c = 2.20 \pm 0.12 \,\mu\text{H}$ . These values were inserted into the model, along with  $C_1 = 46.06$  nF and  $C_2 = 241.79$  nF (with DF = 0.21%) to acquire resonant frequencies 500–200 kHz. Then, simulations were conducted for  $f_0 = \{500 \, \text{kHz}; 388.61 \, \text{kHz}; 289.34 \, \text{kHz}\}$  corresponding to  $C_e = \{C_1; C_1 + 100 \, \text{nF}; C_1 + 33 \, \text{nF}\}$ . As a result, the Y-parameter (Fig. 17) for these three cases was extracted from the simulated smart meta-cell and from the measurements of the passive resonator.

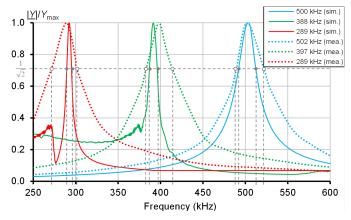


Fig. 17. Admittances of the reconfigurable (sim.) and passive (mea.) meta-cell, with indicated (grey dashed lines) bandwidth

It is clearly seen that for  $f_0 = 500$  kHz, simulated and measured characteristics had similar smooth shape, such as in a series RLC circuit, with some acceptable inconsistencies in lower and upper frequencies. Yet, the situation changed visibly at lower  $f_0$ . In the passive meta-cell, the quality factor drops due to wider bandwidth, but in the reconfigurable system it behaved quite the opposite – the frequency response was narrowing while  $f_0$  was decreasing. Furthermore, "ripples" have appeared at the left arms of the simulated characteristics, probably due to transistor operation. This was a crucial difference between metamaterial

with an active control and the classic one. To conclude, the reconfigurable unit will not have properties identical to the passive element; hence, it cannot be modelled or adequately replaced with the equivalent, lumped RLC meta-cell.

The main parameters were gathered in Table 1. Relative differences between maximum admittances ( $\Delta Y_{\rm max}$ ) as well as resonant ( $\Delta f_0$ ), lower ( $\Delta f_l$ ) and higher ( $\Delta f_h$ ) frequencies were below 28% and indicated sufficient agreement of simulation and experiment. Nevertheless, even the small changes in  $f_l$  or  $f_h$  values lead to much more significant divergencies in the quality factor ( $\Delta q$ ), reaching nearly 79%. These results can be explained based on two factors: a) the above-mentioned specific behavior of the active meta-cell; b) the impact of equivalent series resistance of capacitors ( $R_e$ ). This parameter was additionally evaluated, showing that practical implementation of the metamaterial may face the issue of too high capacitor resistance. A large polypropylene capacitive element will be required in order to minimize ESR ( $R_e$ ) and thus increase  $Y_{\rm max}$  and q.

**Table 1**Results of simulation and measurements

Type	Simulated			Measured		
$Y_{\text{max}}$ (S)	3.56	3.31	3.20	2.57	2.62	2.60
$\Delta Y_{ m max}$	_	_	_	-27.84%	-21.04%	-18.83%
$f_0$ (kHz)	500.00	388.61	289.34	502.00	397.00	289.00
$\Delta f_0$	-	-	_	0.40%	2.16%	-0.12%
$f_l$ (kHz)	487.90	383.60	286.40	489.00	383.00	271.00
$\Delta f_l$	-	-	_	0.23%	-0.16%	-5.38%
$f_h$ (kHz)	508.70	393.00	292.80	522.00	414.00	301.00
$\Delta f_h$	_	_	_	2.61%	5.34%	2.80%
q (-)	24.04	41.34	45.21	15.21	12.81	9.63
$\Delta q$	_	_	_	-36.72%	-69.02%	-78.69%
$R_e \text{ (m}\Omega)$	20.6	41.9	52.8	128.8	122.3	125.3

Effective permeabilities were shown in Fig. 18–20. Discrepancies between the simulated and measured characteristics were directly combined with different quality factors. The lower q re-

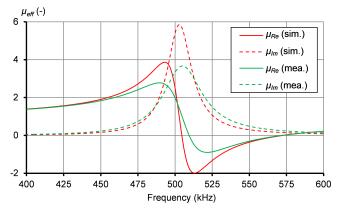


Fig. 18. Real and imaginary parts of effective permeability from simulation (500 kHz, sim.) and measurement (502 kHz, mea.)

sulted in an attenuation of real and imaginary parts, leading to values of relative permeability closer to 1. In each case zero-permeability was reached; however, the frequency at which it occurred was shifted towards higher f. By analyzing (3), it could be found that q was responsible for these effects. Moreover, for the lower  $f_0$  the extreme values of  $\mu_{\rm Re}$  and  $\mu_{\rm Im}$  should decrease. Meanwhile, for a simulated meta-cell, the quality factor increased for the lower resonant frequencies, giving  $\mu_{\rm eff}$  with higher extremes, e.g.  $\min(\mu_{\rm Re}) = -1.98$  at  $f_0 = 500$  kHz, while  $\min(\mu_{\rm Re}) = -4.51$  at  $f_0 = 289.34$  kHz. This opens up the possibility of preserving roughly similar  $\mu_{\rm eff}$ , despite the imposed resonant frequencies, when the meta-cell is tuned using the proposed approach.

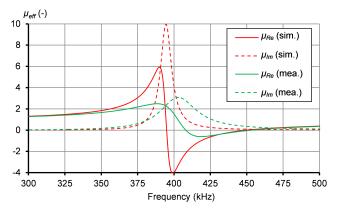


Fig. 19. Real and imaginary parts of effective permeability from simulation (388.61 kHz, sim.) and measurement (397 kHz, mea.)

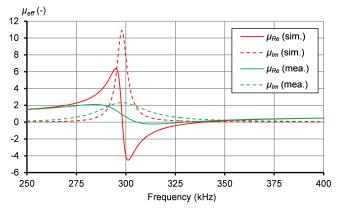


Fig. 20. Real and imaginary parts of effective permeability from simulation (289.34 kHz, sim.) and measurement (289 kHz, mea.)

The problem was the negative impact of coil and capacitors resistances, which lowered q and shifted the frequency of zero-permeability. Realizations of such structure will require thicker planar inductors (200  $\mu$ m copper at least) and low ESR capacitors. Otherwise, relative permeability at low resonant frequencies will not reach the near-zero region and the performance of metamaterial as the magnetic shield will be restricted. Therefore, the main constraints of the discussed material mostly came from resistivity of the components. Effective permeability should possess as extreme as possible maximum and minimum

values. They are directly related with the current of a coil; yet  $i_c$  is limited by resistances of the inductor  $(R_c)$ , capacitors (ESR) and even a transistor  $(R_{DS(\text{on})})$ . This is why the real metamaterial will not be able to ensure e.g.  $\max(\mu_{Re})$  several dozen times higher than permeability of the air. Another limitation is the physical size of a resonator, which should be more compact in order to shrink the thickness of the material. The reduction of the meta-cell below a few millimeters will not always be feasible, due to usage of a polypropylene capacitor (with a low DF, but greater size than a ceramic one) and a requirement of a high value of the F factor, responsible for shaping  $\mu_{eff}$  characteristics.

Also, the reconfigurable meta-cell with MOSFET and PID controller possesses a different frequency response than a passive resonator; thus, the introduced system should be designed/analyzed with a time-domain simulator, rather than using a frequency-domain RLC circuit.

### 5. CONCLUSIONS

The metamaterial with changeable resonant frequency was presented. Reconfigurability was achieved through the usage of an additional capacitor, transistor and regulated PWM pulse. The "smart" ability came from a PID controller, producing PWM with a variable duty cycle and time delay, as well as probes measuring voltage and current of a coil, helping to obtain self-control of the resonance point.

The simulation model confirmed that nearly zero Q/P ratio of the specified resonant frequency can be gained and successfully maintained by the PID controller. However, resonance was possible only within the frequency range determined by  $C_1$  and  $C_2$  capacitors. Since these capacitances can be chosen during the design stage, it is possible to acquire the desirable spectrum of adjusted  $f_0$ . As a result, the introduced algorithms and the structure of meta-cells enable automatic electric-pulse-driven reconfiguration of properties.

The settling time was in the order of several milliseconds (average  $t_s = 15.02$  ms) and steady-state errors were below 0.025 (average  $|\Delta y| = 0.0055$ ). Despite the usage of the effective time delay compensation of PWM, the phase of external magnetic field has an impact on both indicators. The resulting current waveforms were sinusoidal, but at the same time distorted (average THD = 16.5%) – yet, due to reduced values, the abovementioned results may still be acceptable.

In the end, effective permeability was calculated for three cases. A comparison between simulated and measured values showed that characteristics which allowed obtaining zero and negative permeability as well as resonant frequencies were comparable and shifted after changing the capacitance of  $C_2$ . The resistance of meta-cell will not affect a control algorithm (Q/P) mostly relay on inductance of a coil, and measured voltage and current), whereas it will negatively decrease the effective permeability values. Thus, low-resistance inductors and capacitors have to be used to build metamaterial units.

The future works will focus on improvement of the introduced design. One of the promising suggestions can be the usage of two transistors in a common source configuration, which may

help to reduce THD, due to control of the current flow in a negative direction. Moreover, miniaturization should be tested; hence, planar inductors with a paramagnetic core and thicker wounds will be made. Finally, the next experimental verification is planned to compare characteristics of a theoretical active metamaterial with its physical realization. The focus would also be put onto the ability to achieve a high quality factor by e.g. minimization of the equivalent resistances of resonators in a practically operating material.

### **ACKNOWLEDGEMENTS**

This work was supported by the Ministry of Science and Higher Education in Poland at the Bialystok University of Technology under research subsidy No. WZ/WE-IA/7/2023.

### **REFERENCES**

- [1] R. Ghatak and A. Gorai, "Metamaterials: Engineered Materials and its Applications in High Frequency Electronics," *Encyclopedia of Materials: Electronics*, vol. 3, pp. 419–440, 2023, doi: 10.1016/B978-0-12-819728-8.00022-X.
- [2] J.J. Yang, Y. Francescato, S.A. Maier, F. Mao, and M. Huang, "Mu and epsilon near zero metamaterials for perfect coherence and new antenna designs," *Opt. Express*, vol. 22, no 8, pp. 9107– 9114, 2014, doi: 10.1364/OE.22.009107.
- [3] V.G. Veselago, "Electrodynamics of substances with simultaneously negative values of  $\varepsilon$  and  $\mu$ ," Sov. Phys. Usp., vol. 10, pp. 509–514, 1968, doi: 10.1070/PU1968v010n04ABEH003699.
- [4] Z. Manzoor, "Aperiodic hyperbolic metamaterial superlens with random distribution," *Optik*, vol. 242, p. 167290, 2021, doi: 10.1016/j.ijleo.2021.167290.
- [5] S. Haxha, F. AbdelMalek, F. Ouerghi, M.D.B. Charlton, A. Aggoun, and X. Fang, "Metamaterial Superlenses Operating at Visible Wavelength for Imaging Applications," *Sci. Rep.*, vol. 8, p. 16119, 2018, doi: 10.1070/PU1968v010n04ABEH003699.
- [6] W. Jiang et al., "Room-temperature broadband quasistatic magnetic cloak," NPG Asia Mater., vol. 9, p. e341, 2017, doi: 10.1038/am.2016.197.
- [7] M. Boyvat and C. Hafner, "Molding the flow of magnetic field with metamaterials: magnetic field shielding," *PIER*, vol. 126, pp. 303–316, 2012, doi: 10.2528/PIER12022010.
- [8] C. Navau, J. Prat-Camps, and A. Sanchez, "Magnetic Energy Harvesting and Concentration at a Distance by Transformation Optics," *Phys. Rev. Lett.*, vol. 109, p. 263903, 2012, doi: 10.1103/PhysRevLett.109.263903.
- [9] F. Sun and S. He, "Static magnetic field concentration and enhancement using magnetic materials with positive permeability," PIER, vol. 142, p. 579, 2014, doi: 10.2528/PIER13082102.
- [10] J. Prat-Camps, C. Navau, and A. Sanchez, "A magnetic wormhole," *Sci. Rep.*, vol. 5, p. 12488, 2015, doi: 10.1038/srep12488.
- [11] C. Qian *et al.*, "Deep-learning-enabled self-adaptive microwave cloak without human intervention," *Nat. Photonics*, vol. 14, pp. 383–390, 2020, doi: 10.1038/s41566-020-0604-2.
- [12] T. Ramachandran, M.R.I. Faruque, and M.T. Islam, "A dual band left-handed metamaterial-enabled design for satellite applications," *Results Phys.*, vol. 16, p. 102942, 2020, doi: 10.1016/j.rinp.2020.102942.

- [13] V.P. Sarin, R.K. Raj, P.S. Sreekala, and K. Vasudevan, "A Meta-material Inspired Low-Scattering Electric Quadrupole Antenna," Wireless Pers. Commun., vol. 132, pp. 131–145, 2023, doi: 10.1007/s11277-023-10595-x.
- [14] A. Steckiewicz, "Homogenization of the vertically stacked medium frequency magnetic metamaterials with multi-turn resonators," *Sci. Rep.*, vol. 12, p. 20333, 2022, doi: 10.1038/s41598-022-24809-y.
- [15] W. Lee and Y.-K. Yoon, "Wireless Power Transfer Systems Using Metamaterials: A Review," *IEEE Access*, vol. 8, pp. 147930– 147947, 2020, doi: 10.1109/ACCESS.2020.3015176.
- [16] J. Zhu et al., "Three-dimensional magnetic cloak working from dc to 250 kHz," Nat. Commun., vol. 6, p. 8931, 2015, doi: 10.1038/ncomms9931.
- [17] A. Steckiewicz, "High-frequency cylindrical magnetic cloaks with thin layer structure," *J. Magn. Magn. Mater.*, vol. 534, p. 168039, 2021, doi: 10.1016/j.jmmm.2021.168039.
- [18] Z. Zhang, B. Zhang, B. Deng, X. Wei, and J. Wang, "Opportunities and challenges of metamaterial-based wireless power transfer for electric vehicles," *Wirel. Power Transf.*, vol. 5, no. 1, pp. 9–19, 2018, doi: 10.1017/wpt.2017.12.
- [19] S. Yoon, T. Lim, and Y. Lee, "Multifunctional coil technique for alignment-agnostic and Rx coil size-insensitive efficiency enhancement for wireless power transfer applications," *Sci. Rep.*, vol. 13, p. 22838, 2023, doi: 10.1038/s41598-023-50094-4.
- [20] X. Huang, C. Zhang, L. Cong, R. Cai, F. Yang, and C. Lu, "Development and prospects of metamaterial in wireless power transfer," *IET Power Electron.*, vol. 14, pp. 2423–2440, 2021, doi: /10.1049/pel2.12189.
- [21] C. Rong, L. Yan, L. Li, Y. Li, and M. Liu, "A Review of Metamaterials in Wireless Power Transfer," *Materials*, vol. 16, no. 17, p. 6008, 2023, doi: 10.3390/ma16176008.
- [22] J. Alberto, U. Reggiani, L. Sandrolini, and H. Albuquerque, "Accurate Calculation of the Power Transfer and Efficiency in Resonator Arrays for Inductive Power Transfer," *PIER B*, vol. 83, pp. 61–76, 2019, doi: 10.2528/PIERB18120406.
- [23] J. Alberto, U. Reggiani, L. Sandrolini, and H. Albuquerque, "Fast Calculation and Analysis of the Equivalent Impedance of a Wireless Power Transfer System Using an Array of Magnetically Coupled Resonators," *PIER B*, vol. 80, pp. 101–112, 2018, doi: 10.2528/PIERB18011704.
- [24] D. Shan, H. Wang, K. Cao, and J. Zhang, "Wireless power transfer system with enhanced efficiency by using frequency reconfigurable metamaterial," *Sci. Rep.*, vol. 12, 331, pp. 1–11, 2022, doi: 10.1038/s41598-021-03570-8.
- [25] W. Lee and Y.-K. Yoon, "Tunable metamaterial slab for efficiency improvement in misaligned wireless power transfer," *IEEE Microwave Wirel. Compon. Lett.*, vol. 30, no. 9, pp. 912–915, 2020, doi: 10.1109/LMWC.2020.3015680.
- [26] Z. Dai, Z. Fang, H. Huang, Y. He, and J. Wang, "Selective omnidirectional magnetic resonant coupling wireless power transfer with multiple-receiver system," *IEEE Access*, vol. 6, pp. 19287– 19294, 2018, doi: 10.1109/ACCESS.2018.2809797.
- [27] C.M. de Miranda and S.F. Pichorim, "A Three-Coil Wireless Power Transfer System using Self-Resonant Open-Bifilar Coils," *Int. J. Electron. Commun.*, vol. 154, p. 154300, 2022, doi: 10.1016/j.aeue.2022.154300.



- [28] J.P. Turpin, J.A. Bossard, K.L. Morgan, D.H. Werner, and P.L. Werner, "Reconfigurable and Tunable Metamaterials: A Review of the Theory and Applications," *Int. J. Antennas Propag.*, vol. 2014, p. 429837, 2014, doi: 10.1155/2014/429837.
- [29] C. Xu, Z. Ren, J. Wei and C. Lee, "Reconfigurable terahertz metamaterials: From fundamental principles to advanced 6G applications," *iScience*, vol. 25, 2022, doi: 10.1016/j.isci.2022.103799.
- [30] J. Besnoff, M. Chabalko, and D.S. Ricketts, "A Frequency-Selective Zero-Permeability Metamaterial Shield for Reduction of Near-Field Electromagnetic Energy," *IEEE Antennas Wirel. Propag. Lett.*, vol. 15, no. 1, pp. 654–657, 2015, doi: 10.1109/LAWP.2015.2466172.
- [31] C. Lu *et al.*, "Shielding the magnetic field of wireless power transfer system using zero-permeability metamaterial," in *Proc. 14th IET International Conference on AC and DC Power Transmission (ACDC 2018)*, 2018, pp. 1812–1815, doi: 10.1049/joe.2018.8678.
- [32] C. Lu et al., "Magnetic Shielding of Wireless Power Transfer Using Zero Permeability Metamaterial Slab," in Proc. IEEE 1st China International Youth Conference on Elec-

- *trical Engineering (CIYCEE)*, 2020, pp. 1–5, doi: 10.1109/CIYCEE49808.2020.9332735.
- [33] SAE International Technical Standard, "Wireless Power Transfer for Light-Duty Plug-in/Electric Vehicles and Alignment Methodology," SAE Standard J2954\_202010, revised October 2020, issued May 2016, doi: 10.4271/J2954\_202010.
- [34] H. Chen *et al.*, "A meta-substrate to enhance the bandwidth of metamaterials," *Sci. Rep.*, vol. 4, p. 5264, 2014, doi: 10.1038/srep05264.
- [35] S.S. Mohan, M. del Mar Hershenson, S.P. Boyd, and T.H. Lee, "Simple accurate expressions for planar spiral inductances," *IEEE J. Solid-State Circ.*, vol. 34, no. 10, pp. 1419–1424, 1999, doi: 10.1109/4.792620.
- [36] J. Liu, Z. Gong, S. Yang, H. Sun, and J. Zhou, "Practical model for metamaterials in wireless power transfer systems," *Appl. Sci.*, vol. 10, p. 8506, 2020, doi: 10.3390/app10 238506.
- [37] Laird. "Ferrite Plates for Wireless Charging MP & 33 Series." [Online]. Available: https://www.laird.com/sites/default/files/mcp-ds-plates-0714.pdf [Accessed: 10 Jan. 2024].