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A new multi-attribute group decision-making method based on probabilistic multi-valued linguistic spherical fuzzy sets for the site selection of charging piles

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Motivated by the concepts of low carbon and environmental protection, electric vehicles have received much attention and become more and more popular all around the world. The expanding demand for electric vehicles has driven the rapid development of the charging pile industry. One of the prominent issues in charging pile industry is to determine their sites, which is a complex decision-making problem. As a matter of factor, the process of charging piles sites selection can be regarded as multi-attribute group decision-making (MAGDM), which is the main topic of this paper. The recently proposed linguistic spherical fuzzy sets (LSFSs) composed of the linguistic membership degree, linguistic abstinence degree and linguistic non-membership degree are powerful tools to express the evaluation information of decision makers (DMs). Based on the concept of LSFSs, we introduce probabilistic multi-valued linguistic spherical fuzzy sets (PMVLSFSs), which can describe DMs' fuzzy evaluation information in a more refined and accurate way. The operation rules of PMVLSFSs are also developed in this article. To effectively aggregate PMVLSFSs, the probabilistic multi-valued linguistic spherical fuzzy power generalized Maclaurin symmetric mean operator and the probabilistic multi-valued linguistic spherical fuzzy power weighted generalized Maclaurin symmetric mean are put forward. Based on the above aggregation operators, a new method for MAGDM problem with PMVLSFSs is established. Further, a practical case of suitable site selection of charging pile is used to verify the practicability of this method. Lastly, comparative anal-

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ysis with other methods is performed to illustrate the advantages and stability of proposed method.

Key words: site selection of charging piles, probabilistic multi-valued linguistic spherical fuzzy sets; generalized Maclaurin symmetric mean; power generalized Maclaurin symmetric mean

1. Introduction

With the enhancement of peoples' awareness of environmental protection, the drawbacks of air pollution caused by fuel vehicles have been widely concerned. Therefore, under the dual pressure of environmental pollution and energy crisis, consumers' demand and acceptance of electric vehicles has entered a hot stage. Given the needs of electric vehicles, electric vehicle charging piles are also developing rapidly and the site selection of charging piles has become a research hotspot [1, 2]. Multi-attribute group decision-making (MAGDM), a method of determining the best schemes, in which multiple attributes pertinent to alternatives are considered by decision makers (DMs), which has been widely used in solving practical problems [3–9]. Therefore, MAGDM provides a new solution for site selection of charging piles. Then, more and more people can be invited to take part in the process of site selection of charging piles. The evaluation information provided by DMs with different social backgrounds, knowledge structure and personal expertise is too vague and complex, so that the methods cannot directly capture the uncertainty of their evaluations. To make it easier under DMs' complex thinking, opinions, and evaluation information, the concept of fuzzy sets (FSs) [10] and intuitionistic fuzzy sets (IFSs) [11] were firstly proposed. As an extension of FSs which are characterized by a membership degree (MD), IFSs with both MD and non-membership degree (NMD) can better describe DMs' evaluation information in MAGDM process. Nevertheless, the application scope of IFS is greatly limited because of its constraint that the sum of the MD and NMD should less than or equal to one. To overcome this shortage, Yager [12] proposed the Pythagorean fuzzy sets (PFSs) which require that the sum of squares of MD and NMD is no more than one. However, in the IFSs and PFSs, only the satisfaction degree and the dissatisfaction degree are considered, which is inadequate, because abstinence degree exists naturally in human nature. Later, Cuong and Kreinovich [13] innovatively added abstinence degree to IFSs and proposed the picture fuzzy sets which contain MD, abstinence degree (AD) and NMD. Recently, considering the similar shortcoming between the picture fuzzy sets and IFs, i.e., the limited use scope, and inspired by the PFSs, the spherical fuzzy sets (SFSs) [14] are proposed as an extension of picture fuzzy sets, which require the sum of squares of MD, AD and NMD is no more than one. Obviously, compared with other early concepts, the structure of SFSs is more consistent with actual evaluations of DMs, and has a wider application potential through a larger range of information expression. As a result, SFSs has become one of the most attractive research fields in MAGDM [15–18].

It is noted that all the above fuzzy set concepts serve for quantitative assessments. In reality, qualitative assessments should also be taken into consideration. For example, when a customer expresses his/her evaluation of the overall impression of iPhone 14, linguistic variables (LVs) such as ‘good’, ‘medium’, ‘perfect’ are more often adopted. Therefore, it is of great significance to extend the fuzzy set theory to the LVs environment [19]. Based on LVs, linguistic intuitionistic fuzzy numbers (LIFNs) [20], linguistic Pythagorean fuzzy numbers (LPFNs) [21] and linguistic q-rung orthopair fuzzy numbers (Lq-ROFNs) [22] were proposed and used to solve MAGDM problems. Thereafter, linguistic spherical fuzzy sets (LSFSs) which combine the strengths of LVs and SPFSs, were proposed by Jin et al. [23]. Compared with LIFNs, LPFNs and Lq-ROFNs, LSFSs have a wider range of linguistic evaluation information because they include linguistic membership degree, linguistic abstinence degree and linguistic non-membership degree. Then, Liu et al. [24] extended the Multi-Attributive Border Approximation Area Comparison method to linguistic spherical fuzzy numbers to construct a public evaluation system for shared bicycles. Similarly, Ashraf et al. [25] investigated the spherical linguistic fuzzy Choquet integral weighted averaging operator and applied it in solving decision-making problems. Mathew et al. [26] presented a spherical fuzzy linguistic term scale and showed that the spherical fuzzy AHP-TOPSIS is an effective manner to handle uncertainty in decision-making. For more articles related to LSFSs, the readers can refer [27–29].

In LSFSs, DMs are allowed to use a single linguistic value when expressing their satisfaction degree, abstinence degree and dissatisfaction degree. This characteristic implies that LSFs cannot cope with common situations where DMs hesitate among multiple values. To overcome this drawback, we introduce a new concept of multi-valued linguistic spherical fuzzy sets (MVLSFSs). In MVLSFSs, the MD, AD and NMD can be represented by several LVs, making them an effective tool to describe the hesitation of DMs. Moreover, it is noted that in recent research, the probabilistic linguistic term sets (PLTSs) proposed by Pang et al. [30] have become a hot spot [31–34]. The PLTs allow DMs to have a personal preference for LVs, that is, to give an importance degree for each LV. In a word, the PLTSs provide a flexible manner to express the linguistic fuzzy information. Absorbing the advantages of PLTSs, we further improve MVLSFSs to the probabilistic multi-valued linguistic spherical fuzzy sets (PMVLSFSs) by considering probabilistic information of all LVs. In other words, PMVLSFSs can capture the hesitant information in expert’s evaluation and reduce the reduce information loss. For instance, when evaluating performance of a new smart-phone, the expert gives an evaluation expressed as $d = ((s_3, s_4)|(0.3, 0.7),\ (s_3)|(1),\ (s_0)|(1))$ with the linguistic term set (LTS) being $S = \{s_0 = \textit{extermly bad}, s_1 = \textit{bad}, s_2 = \textit{medium}, s_3 = \textit{good}, s_4 = \textit{perfect}\}$. It can be seen from the evaluation results that the expert is hesitant between s_3 and s_4 when giving the linguistic membership degree and its important degree is 0.3 and 0.7, respectively. In addition, the expert is certain when providing the linguistic abstinence degree and linguistic non-membership degree. Therefore, experts are less constrained in the evaluation process and can express their true feelings more accurately by using PMVLSFSs. Meanwhile, to process fuzzy information, some effective information aggregation operators (AOs) have attracted great attention of researchers [35–39]. The Maclaurin symmetric mean (MSM) [40] operator is well-known for its characteristic of considering the interrelationship among attributes. Later, the generalized Maclaurin symmetric mean (GMSM) operator [41] was proposed, which is more powerful than MSM because it can not only capture the interrelationship, but also reflect the importance of the influence among related attributes by adjust its parameters. In addition, the GMSM operator can be simplified to weight average (WA) operator, Bonferroni mean (BM) operator and MSM operator. The feasibility and superiority of GMSM have been widely proven [42, 43]. The power average (PA) operator [44] is also widely recognized by scholars because it can effectively deal with the unduly high and low values provided by DMs. Based on the flexibility of GMSM operator and the practicability of PA operator, we develop the novel powerful tools to aggregate the information of PMVLSFSs, i.e., the probabilistic multi-valued linguistic spherical fuzzy power generalized Maclaurin symmetric mean (PMVLSFPGMSM) operator and the probabilistic multi-valued linguistic spherical fuzzy power weight generalized Maclaurin symmetric mean (PMVLSFPWGMSM) operator. Then, the PMVLSFPWGMSM operator is used to establish a method to solve MAGDM problems under PMVLSFSs environment.

Based on the above analysis, the main innovations of this study are divided into four aspects: (1) The concept of PMVLSFSs are proposed, which have the strong ability to describe the personal preference and hesitancy information. The operation rules of PMVLSFSs have also been introduced; (2) The PMVLSFPGMSM and PMVLSFPWGMSM operators based on PA operator and GMSM operator are provided to aggregate the information donated by PMVLSFSs; (3) Based on these operators, a new method for solving MAGDM problems is presented; (4) An illustrative example about an evaluation method for site selection of charging pile is solved by using the novel method. The experimental results and comparison implications are showed to justify the applicability and effectiveness of the proposed method.

The structure of this paper is as follows. Section 2 recalls the notions of LSFSs and basic AOs. Section 3 analyzes the drawback of LSFSs and proposes the PMVLSFSs. The operation rules and comparison method of PMVLSFSs are also given in Section 3. Section 4 derives some AOs for PMVLSFSs and studies their properties. Section 5 describes a new MAGDM method under PMVLSFSs in detail. Section 6 illustrates an example of how to determine the site of charging piles and puts forward the feasibility and superiority of our proposed method through an application of practical case. Section 7 summarizes this article and future research directions.

2. Basic concepts

In this section, we briefly review basic concepts that will be used in the following sections.

2.1. Linguistic spherical fuzzy sets

Definition 1. *[23] Let X be a universe of discourse and $\widetilde{S} = \left\{0 \leqslant \beta \leqslant l \middle| s_\beta\right\}$ be a continuous linguistic set, a linguistic spherical fuzzy set A defined on X is expressed as*

$$A = \left\{(x, s_a(x), s_b(x), s_c(x)) \middle| x \in X\right\}, \tag{1}$$

where $s_a(x)$, $s_b(x)$, $s_c(x) \in S_{[0,l]}$ and s_a, s_b and s_c denote the linguistic membership degree, linguistic abstinence degree and linguistic non-membership degree, respectively. For any $x \in X$, the condition $a^2 + b^2 + c^2 \leqslant l^2$ holds. In addition, we call the ordered pair $\alpha = (s_a, s_b, s_c)$ linguistic spherical fuzzy numbers (LSFNs). The linguistic indeterminacy degree of α is expressed as $\pi = s_{(l^q-(a^2+b^2+c^2))^{1/q}}$.

The basic operations of LSFNs are defined as follows.

Definition 2. *[23] Let $\alpha_1 = \left(s_{a_1}, s_{b_1}, s_{c_1}\right)$ and $\alpha_2 = \left(s_{a_2}, s_{b_2}, s_{c_2}\right)$ and $\alpha = (s_a, s_b, s_c)$ be any three LSFNs, and λ be a positive real number, then*

(1) $\alpha_1 \oplus \alpha_2 = \left(s_{(a_1^2+a_2^2-a_1^2a_2^2/l^2)^{1/2}},\ s_{\frac{b_1b_2}{l}},\ s_{\frac{c_1c_2}{l}}\right)$;

(2) $\alpha_1 \otimes \alpha_2 = \left(s_{\frac{a_1a_2}{l}},\ s_{(b_1^2+b_2^2-b_1^2b_2^2/l^2)^{1/2}},\ s_{(c_1^2+c_2^2-c_1^2c_2^2/l^2)^{1/2}}\right)$;

(3) $\lambda\alpha = \left(s_{(l^2-l^2(1-a^2/l^2)^\lambda)^{1/2}},\ s_{l\left(\frac{b}{l}\right)^\lambda},\ s_{l\left(\frac{c}{l}\right)^\lambda}\right)$;

(4) $\alpha^\lambda = \left(s_{l\left(\frac{a}{l}\right)^\lambda},\ s_{\left(l^2-l^2(1-b^2/l^2)^\lambda\right)^{1/2}},\ s_{\left(l^2-l^2(1-c^2/l^2)^\lambda\right)^{1/2}}\right)$.

The authors also proposed a method to rank any two LSFNs.

Definition 3. *[23] Let* $\alpha = (s_a, s_b, s_c)$ *be a LSFN, then the score function* $S(\alpha)$ *is defined as*

$$S(\alpha) = s_{\left(\frac{l^2+a^2-c^2}{3}\right)^{1/2}}, \tag{2}$$

and the accuracy function $H(\alpha)$ *is expressed as*

$$H(\alpha) = s_{\left(\frac{a^2+b^2+c^2}{3}\right)^{1/2}}, \tag{3}$$

for any two LSFNs α_1 *and* α_2.

(1) If $S(\alpha_1) > S(\alpha_2)$, *then* $\alpha_1 > \alpha_2$;

(2) If $S(\alpha_1) > S(\alpha_2)$, *then*

If $H(\alpha_1) > H(\alpha_2)$, *then* $\alpha_1 > \alpha_2$;

If $H(\alpha_1) = H(\alpha_2)$, *then* $\alpha_1 = \alpha_2$.

2.2. The power average operator

Definition 4. *[44] Let* a_i $(i = 1, 2, \ldots, n))$ *be a set of crisp numbers, then the power average (PA) operator is expressed as*

$$PA(a_1, a_2, \ldots, a_n) = \frac{\sum_{i=1}^{n} (1 + T(a_i))\, a_i}{\sum_{i=1}^{n} (1 + T(a_i))}, \tag{4}$$

where $T(a_i) = \sum_{i=1, i\neq j}^{n} Sup\left(a_i, a_j\right)$ *and* $Sup\left(a_i, a_j\right)$ *denotes the support degree forfrom, satisfying the following properties:*

1) $0 \leqslant Sup\left(a_i, a_j\right) \leqslant 1$;

2) $Sup\left(a_i, a_j\right) = Sup\left(a_j, a_i\right)$;

3) $Sup(a, b) \leqslant Sup(c, d)$, *if* $|a - b| \geqslant |c - d|$.

2.3. The generalized Maclaurin symmetric mean

Definition 5. *[41] Let* a_i $(i = 1, 2, \ldots, n)$ *be a collection of crisp numbers, then the GMSM operator is defined as*

$$GMSM^{(k,\lambda_1,\lambda_2,\ldots,\lambda_k)}(a_1, a_2, \ldots, a_n) = \left(\frac{\sum_{1\leqslant i_1<\ldots<i_k\leqslant n} \left(\prod_{j=1}^{k} a_{i_j}^{\lambda_j} \right)}{C_n^k} \right)^{\frac{1}{\lambda_1+\lambda_2+\ldots+\lambda_k}}, \tag{5}$$

where $\lambda_1, \lambda_2, \ldots, \lambda_k \geqslant 0$, $\lambda_1 + \lambda_2 + \ldots + \lambda_k > 0$, $k = 1, 2, \ldots, n$ *is an integer.*

In addition, we introduce some special cases of GMSM.

Case 1. If $k = 1$, $\lambda_1 = 1$, then the GMSM can be simplified to average operator.

$$GMSM^{(1)}(a_1, a_2, \ldots, a_n) = \left(\frac{\bigoplus_{1 \leqslant i_1 < \ldots < i_k \leqslant n} \bigotimes_{j=1}^{k}}{C_n^k} \left(a_{i_j}\right)^{\lambda_j} \right)^{\frac{1}{\lambda_1+\lambda_2+\ldots+\lambda_k}} = \left(\frac{\bigoplus_{1 \leqslant i_1 \leqslant n} \bigotimes_{j=1}^{1}}{C_n^1} \left(a_{i_j}\right)^{\lambda_j} \right)^{\frac{1}{\lambda_1}} = \frac{1}{n}\sum_{i=1}^{n}(a_i). \tag{6}$$

Case 2. If $k = 2$, then the GMSM can be simplified to BM operator.

$$GMSM^{(2)}(a_1, a_2, \ldots, a_n) = \left(\frac{\bigoplus_{1 \leqslant i_1 < \ldots < i_k \leqslant n} \bigotimes_{j=1}^{k}}{C_n^k} \left(a_{i_j}\right)^{\lambda_j} \right)^{\frac{1}{\lambda_1+\lambda_2+\ldots+\lambda_k}} = \left(\frac{\bigoplus_{1 \leqslant i_1 < \ldots < i_2 \leqslant n} \bigotimes_{j=1}^{2}}{C_n^2} \left(a_{i_j}\right)^{\lambda_j} \right)^{\frac{1}{\lambda_1+\lambda_2}}$$
$$= \left(\frac{\bigoplus_{1 \leqslant i < j \leqslant n}}{n(n-1)} \left((a_i)^{\lambda_1} \times (a_j)^{\lambda_2}\right) \right)^{\frac{1}{\lambda_1+\lambda_2}} = \left(\frac{\bigoplus_{1 \leqslant i,j \leqslant n; i \neq j}}{n(n-1)} \left((a_i)^{\lambda_1} \times (a_j)^{\lambda_2}\right) \right)^{\frac{1}{\lambda_1+\lambda_2}} = BM^{(\lambda_1,\lambda_2)}(a_1, a_2, \ldots, a_n). \tag{7}$$

Case 3. If $\lambda_1 = \lambda_2 = \ldots = \lambda_k = 1$, then the GMSM mean can be simplified to MSM operator.

$$GMSM^{(k)}(a_1, a_2, \ldots, a_n) = \left(\frac{\bigoplus_{1 \leqslant i_1 < \ldots < i_k \leqslant n} \bigotimes_{j=1}^{k}}{C_n^k} \left(a_{i_j}\right)^{\lambda_j} \right)^{\frac{1}{k}} = MSM^{(k)}\left(a_1, a_2, \ldots, a_n\right). \tag{8}$$

3. Probabilistic multi-valued linguistic spherical fuzzy sets

In this section, we analyze the shortcoming of the LSFSs and propose a novel concept of probabilistic multi-valued linguistic spherical fuzzy sets (PMVLSFSs). In addition, the definition, and the basic operation rules of PMVLSFSs are also introduced.

3.1. Motivations of proposing PMVLSFSs

In LSFSs [23] MD, AD and NMD are expressed by linguistic terms. In real life, linguistic terms are like natural languages so that LSFSs have a wider range of information expression capabilities. Although LSFSs can process the language evaluation information, its application is also limited. It is because a single value of linguistic membership degree, linguistic abstinence degree and linguistic non-membership degree fail to express the experts' hesitation and uncertainty. Actually, DMs always hesitant among a collection of possible linguistic terms when determined the MD, AD and NMD. To expand the scope of linguistic information expression, we propose the concepts of MVLSFSs, which have a strong ability to capture the complex personal preferences of decision-making experts in the evaluation process. To intuitively describe the motivation of the proposal, we provide the following example.

Example 1. Suppose there is an expert who was invited to evaluate an electric vehicle from three aspects: price, comfort, and brand. Let S be a pre-defined LTS, and $S = \{s_0 = \textit{extermly bad}, s_1 = \textit{bad}, s_2 = \textit{medium}, s_3 = \textit{good}, s_4 = \textit{perfect}\}$. Then, the evaluation information provided by the decision-making expert is listed in Table 1.

Table 1: DM's evaluation information in Example 1

	linguistic membership degree	linguistic abstinence degree	linguistic non-membership degree
Price	$\{s_4, s_5\}$	$\{s_1, s_2\}$	$\{s_0, s_1, s_2\}$
Comfort	$\{s_1, s_2, s_3\}$	$\{s_0, s_1\}$	$\{s_3, s_4\}$
Brand	$\{s_2, s_3\}$	$\{s_3, s_4\}$	$\{s_5, s_6\}$

From Table 1, we find that the expert is hesitant between s_4 and s_5 when providing the linguistic membership degree regarding the attributes of price. In addition, the expert is also hesitant between s_1 and s_2 when giving the linguistic abstinence degree and hesitant among s_0, s_1 and s_2 when expressing linguistic non-membership degree. It is worth noting that the overall evaluation results fail to be expressed by using LSFSs. Specifically, the evaluation opinions cannot be expressed using single element. In real life, due to the existence of various factors, such as personal character or experience, people often hesitate when making decisions. Therefore, it is meaningful to propose a powerful tool which can express a wider range of the linguistic evaluation information. Based on it, we propose the concept of MVLSFSs.

3.2. The definition of PMVLSFSs

In this section, we first propose the concept of MVLSFSs. Afterwards, we further extend MVLSFSs to PMVLSFSs by taking the probabilistic information into consideration.

Definition 6. *Let X be a universe of discourse and* $\widetilde{S} = \left\{0 \leqslant \beta \leqslant l \middle| s_\beta\right\}$ *be a continuous linguistic set, a multi-valued linguistic spherical fuzzy set M defined on X is expressed as*

$$M = \left\{(x, g_M(x), t_M(x), h_M(x)) \middle| x \in X\right\}, \tag{9}$$

where $g_M(x) t_M(x) h_M(x) \subseteq S$ *are three sets of linguistic terms, denoting the possible linguistic membership degree, linguistic abstinence degree and linguistic non-membership degree of the element* $x \in X$ *to the set* M*. Additionally, we call the ordered pair* $\beta = (g, t, h)$ *a multi-valued linguistic spherical fuzzy element (MVLSFE), satisfying* $\mu^2 + v^2 + \eta^2 \leqslant l^2$*, where* $s_\mu \in g$*,* $s_v \in t$ *and* $s_\eta \in h$*.*

By taking the probability of each element into account, we propose the PMVLSFSs.

Definition 7. *Let X be a fixed set and* $\widetilde{S} = \left\{s_\alpha \middle| 0 \leqslant \alpha \leqslant l\right\}$ *be continuous linguistic term set with odd cardinality. A probabilistic multi-valued linguistic spherical fuzzy sets (PMVLSFSs) P is expressed as*

$$P = \left\{\left\langle x, g_P(x)\middle| p(x), t_P(x)\middle| q(x), h_P(x)\middle| m(x)\right\rangle\middle| x \in X\right\}. \tag{10}$$

The component $g_P(x)|p(x)$, $t_P(x)|q(x)$ and $h_P(x)|m(x)$ are three sets of some possible values, denoting the possible linguistic membership degree, linguistic abstinence degree and linguistic non-membership degree of the element $x \in X$ to the set P, respectively, where $g_M(x) t_M(x) h_M(x) \subseteq S$. $p(x)$, $q(x)$ and $m(x)$ are the probabilistic information of $g_P(x)$, $t_P(x)$ and $h_P(x)$, respectively. The ordered pair $d = (g|p, t|q, h|m)$ is called a probabilistic MVLSFE (PMVLSFE), such that $\mu^2 + v^2 + \eta^2 \leqslant l^2$, $0 \leqslant p_i \leqslant 1$, $0 \leqslant q_j \leqslant 1$, $0 \leqslant m_y \leqslant 1$, $\sum_{i=1}^{\#g} p_i = 1$, $\sum_{j=1}^{\#t} q_j = 1$ and $\sum_{y=1}^{\#h} m_y = 1$, where $s_\mu \in g$, $s_v \in t$, $s_\eta \in h$, and $\#g$, $\#t$ and $\#h$

denote the numbers of values in g, t, h, respectively. For example, an evaluation information of alternative is represented by a PMVLSFE. Assume the PMVLSFE is $d = \big((s_3, s_4)\big|(0.4, 0.6), (s_2, s_4)\big|(0.4, 0.6), (s_5, s_6, s_3)\big|(0.4, 0.2, 0.4)\big)$, we can obtain $\#g = 2$, $\#t = 2$ and $\#h = 3$ respectively.

3.3. Basic operations of PMVLSFSs

Definition 8. *Let $d = (g|p, t|q, h|m)$, $d_1 = (g_1|p_1, t_1|q_1, h_1|m_1)$ and $d_2 = (g_2|p_2, t_2|q_2, h_2|m_2)$ be any three PMVLSFEs, and λ be a positive real number, then*

(1) $d_1 \oplus' d_2 = \bigcup_{\mu_1\in g_1,\mu_2\in g_2,v_1\in t_1,v_2\in t_2,\eta_1\in h_1,\eta_2\in h_2}$

$$\left\{\left\{s_{\left(\mu_1^2+\mu_2^2-u_1^2u_2^2/l^2\right)^{1/2}}|p_{\mu_1}p_{\mu_2}\right\}, \left\{s_{\frac{v_1v_2}{l}}|q_{v_1}q_{v_2}\right\}, \left\{s_{\frac{\eta_1\eta_2}{l}}|m_{\eta_1}m_{\eta_2}\right\}\right\};$$

(2) $d_1 \otimes d_2 = \bigcup_{\mu_1\in g_1,\mu_2\in g_2,v_1\in t_1,v_2\in t_2,\eta_1\in h_1,\eta_2\in h_2}$

$$\left\{\left\{s_{\frac{\mu_1\mu_2}{l}}|p_{\mu_1}p_{\mu_2}\right\}, \left\{s_{\left(v_1^2+v_2^2-v_1^2v_2^2/l^2\right)^{1/2}}|q_{v_1}q_{v_2}\right\}, \left\{s_{\left(\eta_1^2+\eta_2^2-\eta_1^2\eta_2^2/l^2\right)^{1/2}}|m_{\eta_1}m_{\eta_2}\right\}\right\};$$

(3) $\lambda d = \bigcup_{\mu\in g,v\in t,\eta\in h}\left\{\left\{s_{\left(l^2-l^2(1-\mu^2/l^2)^\lambda\right)^{1/2}}|p_\mu\right\}, \left\{s_{l\left(\frac{v}{l}\right)^\lambda}|q_v\right\}, \left\{s_{l\left(\frac{\eta}{l}\right)^\lambda}|m_\eta\right\}\right\};$

(4) $d^\lambda = \bigcup_{\mu\in g,v\in t,\eta\in h}$

$$\left\{\left\{s_{l\left(\frac{\mu}{l}\right)^\lambda}|p_\mu\right\}, \left\{s_{\left(l^2-l^2\left(1-v^2/l^2\right)^\lambda\right)^{1/2}}|q_v\right\}, \left\{s_{\left(l^2-l^2\left(1-\eta^2/l^2\right)^\lambda\right)^{1/2}}|m_\eta\right\}\right\}.$$

Theorem 1. *Let $d = \big(g\big|p, t\big|q, h\big|m\big)$, $d_1 = \big(g_1\big|p_1, t_1\big|q_1, h_1\big|m_1\big)$ and $d_2 = \big(g_2\big|p_2, t_2\big|q_2, h_2\big|m_2\big)$ be any three PMVLSFEs, then,*

(1) $d_1 \oplus' d_2 = d_2 \oplus' d_1$;
(2) $d_1 \otimes d_2 = d_2 \otimes d_1$;
(3) $\lambda(d_1 \oplus' d_2) = \lambda d_2 \oplus' \lambda d_1$;
(4) $\lambda_1 d_1 \oplus' \lambda_2 d_1 = (\lambda_1 + \lambda_2)d_1$;
(5) $d_1^\lambda \otimes d_2^\lambda = (d_1 \otimes d_2)^\lambda$;
(6) $d_1^{\lambda_1} \otimes d_1^{\lambda_2} = d_1^{\lambda_1+\lambda_2}$.

Proof. It is easy to proof the Theorem 1, we omitted the specific proof here.

3.4. Comparison method of PMVLSFSs

Definition 9. *Let $d = \big(g\big|p, t\big|q, h\big|m\big)$ be a PMVLSFE, then the score function of d is defined as*

$$\mathrm{S}(d) = s_{\sqrt{\left(l^2+\sum_{i=1,u\in g}^{\#g} u_i^2 p_i-\sum_{i=1,\eta\in h}^{\#h} \eta_i^2 m_i\right)/2}} \tag{11}$$

and the accuracy score function is defined as

$$H(d) = s_{\sqrt{\left(\sum\limits_{i=1,u\in g}^{\#g} u_i^2 p_i + \sum\limits_{i=1,v\in t}^{\#t} v_i^2 q_i + \sum\limits_{i=1,\eta\in h}^{\#h} \eta_i^2 m_i\right)}}, \tag{12}$$

where the #g, #t and #h are the numbers of the values in g, t and h, respectively.

Let $d_1 = (g_1|p_1, t_1|q_1, h_1|m_1)$ and $d_2 = (g_2|p_2, t_2|q_2, h_2|m_2)$ be any two PMVLSFEs,

1. If $S(d_1) > S(d_2)$, then d_1 is better than d_2
2. If $S(d_1) = S(d_2)$, then
   If $H(d_1) > H(d_2)$, then d_1 is better than d_2
   If $H(d_1) = H(d_2)$, then d_1 is equal to d_2.

3.5. The distance of PMVLSFSs

Definition 10. *Let $d_1 = (g_1|p_1, t_1|q_1, h_1|m_1)$ and $d_2 = (g_2|p_2, t_2|q_2, h_2|m_2)$ be any two PMVLSFEs, then the distance between d_1 and d_2 is defined as*

$$\begin{aligned} d(d_1, d_2) = \frac{1}{l^2(\#g + \#t + \#h)} \Bigg(& \sum_{i=1}^{\#g} \left| \left(u_1^{\sigma(i)}\right)^2 p_{u_1^{\sigma(i)}} - \left(u_2^{\sigma(i)}\right)^2 p_{u_2^{\sigma(i)}} \right| \\ & + \sum_{i=1}^{\#t} \left| \left(v_1^{\sigma(i)}\right)^2 q_{v_1^{\sigma(i)}} - \left(v_2^{\sigma(i)}\right)^2 q_{v_2^{\sigma(i)}} \right| \\ & + \sum_{i=1}^{\#h} \left| \left(\eta_1^{\sigma(i)}\right)^2 m_{\eta_1^{\sigma(i)}} - \left(\eta_2^{\sigma(i)}\right)^2 m_{\eta_2^{\sigma(i)}} \right| \Bigg), \end{aligned} \tag{13}$$

where the #g, #t and #h are the numbers of the values in g, t and h, respectively.

Example 2. Assume that $d_1 = ((s_3, s_4, s_2)|(0.3, 0.3, 0.4), (s_2, s_3)|(0.5, 0.5), (s_5, s_6, s_3)|(0.3, 0.2, 0.5))$ and $d_2 = ((s_5, s_4, s_3)|(0.4, 0.4, 0.2), (s_5, s_3)|(0.4, 0.6), (s_2, s_3)|(0.6, 0.4))$ are two PMVLSFEs, then the distance between d_1 and d_2 is:

Step 1. Before calculating the distance, the values of PMVLSFEs should be normalized. Due to space issues, the normalization rules are omitted here. The normalization values of d_1 and d_2 are

$$d_1' = ((s_2, s_3, s_4)|(0.4, 0.3, 0.3),\ (s_2, s_3)|(0.5, 0.5),\ (s_6, s_5, s_3)|(0.2, 0.3, 0.5)),$$

$$d_2' = ((s_3, s_4, s_5)|(0.2, 0.4, 0.4),\ (s_3, s_5)|(0.6, 0.4),\ (s_2, s_3)|(0.6, 0.4)).$$

Step 2. From the Definition 9, it can be found that the number of members of g_1 and g_2, t_1 and t_2, h_1 and h_2 should be the same. Therefore, the shorter PMVLSFE should add values to make the numbers are same. The added value is calculated by formula, and the specific calculation method is omitted due to space issues. The extended d_2 is displayed as follows:

$$d_1' = \left((s_2, s_3, s_4)\middle|(0.4, 0.3, 0.3),\ (s_2, s_3)\middle|(0.5, 0.5),\ (s_6, s_5, s_3)\middle|(0.2, 0.3, 0.5)\right),$$
$$d_2' = \left((s_3, s_4, s_5)\middle|(0.2, 0.4, 0.4),\ (s_3, s_5)\middle|(0.6, 0.4),\ (s_2, s_3, s_3)\middle|(0.6, 0.2, 0.2)\right).$$

Step 3. The distance between d_1 and d_2 can be obtained by Eq. (13)

$$d\left(d_1, d_2\right) = \frac{1}{3^2(3+2+3)} \times (9.1 + 8.9 + 13.2) = 0.4333.$$

In the next article, the PMVLSFEs mentioned are all normalized. Therefore, when calculating the distance between any two PMVLSFEs, step 1 can be simplified.

4. Aggregation operators of PMVLSFSs and their properties

In this section, some aggregation operators of PMVLSFSs are introduced and their properties are also explored.

4.1. The Probabilistic Multi-valued Linguistic Special Fuzzy Power Generalized Maclaurin Symmetric Mean (PMVLSFPGMSM) Operator

Definition 11. *Let* $d_i = \left(g_i\middle|p_{g_i}, t_i\middle|q_{t_i}, h_i\middle|m_{h_i}\right)$ $(i = 1, 2, \ldots, n)$ *be a collection of PMVLSFEs, then the probabilistic multi-valued linguistic special fuzzy power generalized Maclaurin symmetric mean (PMVLSFPGMSM) operator is defined as*

$$PMVLSFPGMSM^{(k)}\left(d_1, d_2, \ldots, d_n\right) = \left(\frac{\bigoplus_{1\leqslant i_1<\ldots<i_k\leqslant n} \bigotimes_{j=1}^{k} \left(\frac{n\left(1+T\left(d_{i_j}\right)\right) d_{i_j}}{\sum_{i=1}^{n}\left(1+T(d_i)\right)}\right)^{\lambda_j}}{C_n^k}\right)^{\frac{1}{\lambda_1+\lambda_2+\ldots+\lambda_k}}, \tag{14}$$

where $T(d_i) = \sum_{i=1, i\neq j}^{n} Sup\left(d_i, d_j\right)$*, and* $Sup\left(d_i, d_j\right)$ *denotes the support degree for* d_i *from* d_j*, satisfying the following properties:*

1) $0 \leqslant Sup\left(d_i, d_j\right) \leqslant 1$*;*

2) $Sup\left(d_i, d_j\right) = Sup\left(d_j, d_i\right)$*;*

3) $Sup\left(d_i, d_j\right) \leqslant Sup\left(d_s, d_t\right)$*, if* $d\left(d_i, d_j\right) \geqslant d\left(d_s, d_t\right)$*.*

If we assume

$$\sigma_i = \frac{1+T(d_i)}{\sum_{i=1}^{n}(1+T(d_i))}, \tag{15}$$

then Eq. (14) can be written as

$$PMVLSFPGMSM^{(k)}(d_1,d_2,\ldots,d_n) = \left(\frac{\bigoplus_{1\leqslant i_1<\ldots<i_k\leqslant n}\bigotimes_{j=1}^{k}\left(n\sigma_i d_{i_j}\right)^{\lambda_j}}{C_n^k}\right)^{\frac{1}{\lambda_1+\lambda_2+\ldots+\lambda_k}},$$

where $\lambda_1,\lambda_2,\ldots,\lambda_k \geqslant 0$, $\lambda_1+\lambda_2+\ldots+\lambda_k > 0$, $k = 1,2,\ldots,n$ is an integer.

Theorem 2. *Let $d_i = \left(g_i\middle|p_{g_i}, t_i\middle|q_{t_i}, h_i\middle|m_{h_i}\right)$ $(i = 1,2,\ldots,n)$ be a collection of PMVLSFEs, the aggregated value by the PMVLSFPGMSM operator is still a PMVLSFE and*

$$PMVLSFPGMSM^{(k)}(d_1,d_2,\ldots,d_n) = \left(\frac{\bigoplus_{1\leqslant i_1<\ldots<i_k\leqslant n}\bigotimes_{j=1}^{k}\left(n\sigma_i d_{i_j}\right)^{\lambda_j}}{C_n^k}\right)^{\frac{1}{\lambda_1+\lambda_2+\ldots+\lambda_k}}$$

$$= \bigcup_{\substack{u_{i_j}\in g_{i_j},\\ v_{i_j}\in t_{i_j},\\ \eta_{i_j}\in h_{i_j}}} \left\{\left\{ s_{l\left(1-\prod_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}}\left(1-\prod_{j=1}^{k}\left(1-\left(1-\frac{u_{i_j}^2}{l^2}\right)^{n\sigma_i}\right)^{\lambda_j}\right)^{1/C_n^k}\right)^{\frac{1}{2(\lambda_1+\lambda_2+\ldots+\lambda_k)}}} \middle| \prod_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}}\prod_{j=1}^{k} p_{u_{i_j}} \right\},\right.$$

$$\left\{ s_{l\left(1-\left(1-\left(\prod_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}}\left(1-\prod_{j=1}^{k}\left(1-\left(\frac{v_{i_j}}{l}\right)^{2n\sigma_i}\right)^{\lambda_{i_j}}\right)\right)^{1/C_n^k}\right)^{\frac{1}{\lambda_1+\lambda_2+\ldots+\lambda_k}}\right)^{\frac{1}{2}}} \middle| \prod_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}}\prod_{j=1}^{k} q_{v_{i_j}} \right\},$$

$$\left.\left\{ s_{l\left(1-\left(1-\left(\prod_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}}\left(1-\prod_{j=1}^{k}\left(1-\left(\frac{\eta_{i_j}}{l}\right)^{2n\sigma_i}\right)^{\lambda_{i_j}}\right)\right)^{1/C_n^k}\right)^{\frac{1}{\lambda_1+\lambda_2+\ldots+\lambda_k}}\right)^{\frac{1}{2}}} \middle| \prod_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}}\prod_{j=1}^{k} m_{\eta_{i_j}} \right\}\right\}.$$

Proof. According to the operations of PMVLSF presented in Definition 8, we have

$$n\sigma_i d_{i_j} = \bigcup_{\substack{u_{i_j}\in g_{i_j},\\ v_{i_j}\in t_{i_j},\\ \eta_{i_j}\in h_{i_j}}} \left\{\left\{ s_{l\left(1-\left(1-\frac{u_{i_j}^2}{l^2}\right)^{n\sigma_i}\right)^{1/2}} \middle| p_{u_{i_j}} \right\}, \left\{ s_{l\left(\frac{v_{i_j}}{l}\right)^{n\sigma_i}} \middle| q_{v_{i_j}} \right\}, \left\{ s_{l\left(\frac{\eta_{i_j}}{l}\right)^{n\sigma_i}} \middle| m_{u_{i_j}} \right\}\right\}$$

$$\left(n\sigma_i d_{i_j}\right)^{\lambda_j} = \bigcup_{\substack{u_{i_j}\in g_{i_j},\\ v_{i_j}\in t_{i_j},\\ \eta_{i_j}\in h_{i_j}}} \left\{\left\{ s_{l\left(1-\left(1-\frac{u_{i_j}^2}{l^2}\right)^{n\sigma_i}\right)^{\lambda_j/2}} \middle| p_{u_{i_j}} \right\},\right.$$

$$\left.\left\{ s_{l\left(1-\left(1-\left(\frac{v_{i_j}}{l}\right)^{2n\sigma_i}\right)^{\lambda_{i_j}}\right)^{1/2}} \middle| q_{v_{i_j}} \right\}, \left\{ s_{l\left(1-\left(1-\left(\frac{u_{i_j}}{l}\right)^{2n\sigma_i}\right)^{\lambda_{i_j}}\right)^{1/2}} \middle| m_{u_{i_j}} \right\}\right\}$$

And

$$\bigotimes_{j=1}^{k} \left(n\sigma_i d_{i_j}\right)^{\lambda_j} = \bigcup_{\substack{u_{i_j}\in g_{i_j},\\ v_{i_j}\in t_{i_j},\\ \eta_{i_j}\in h_{i_j}}} \left\{\left\{ s_{l\prod_{j=1}^{k}\left(1-\left(1-\frac{u_{i_j}^2}{l^2}\right)^{n\sigma_i}\right)^{\lambda_j/2}} \middle| \prod_{j=1}^{k} p_{u_{i_j}} \right\},\right.$$

$$\left.\left\{ s_{l\left(1-\prod_{j=1}^{k}\left(1-\left(\frac{v_{i_j}}{l}\right)^{2n\sigma_i}\right)^{\lambda_{i_j}}\right)^{1/2}} \middle| \prod_{j=1}^{k} q_{v_{i_j}} \right\}, \left\{ s_{l\left(1-\prod_{j=1}^{k}\left(1-\left(\frac{u_{i_j}}{l}\right)^{2n\sigma_i}\right)^{\lambda_{i_j}}\right)^{1/2}} \middle| \prod_{j=1}^{k} m_{u_{i_j}} \right\}\right\}.$$

Further,

$$\bigoplus_{1\leqslant i_1<\ldots<i_k\leqslant n} \bigotimes_{j=1}^{k} \left(n\sigma_i d_{i_j}\right)^{\lambda_j}$$

$$= \bigcup_{\substack{u_{i_j}\in g_{i_j},\\ v_{i_j}\in t_{i_j},\\ \eta_{i_j}\in h_{i_j}}} \left\{\left\{ s_{l\left(1-\prod_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}}\left(1-\prod_{j=1}^{k}\left(1-\left(1-\frac{u_{i_j}^2}{l^2}\right)^{n\sigma_i}\right)^{\lambda_j}\right)\right)^{\frac{1}{2}}} \middle| \prod_{1\leqslant i_1<\ldots<i_k\leqslant n} \prod_{j=1}^{k} p_{u_{i_j}} \right\},\right.$$

$$\left\{ s_{l \prod\limits_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}} \left(1-\prod\limits_{j=1}^{k}\left(1-\left(\frac{v_{i_j}}{l}\right)^{2n\sigma_i}\right)^{\lambda_{i_j}}\right)^{1/2}} \middle| \prod_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}} \prod_{j=1}^{k} q_{v_{i_j}} \right\},$$

$$\left\{ s_{l \prod\limits_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}} \left(1-\prod\limits_{j=1}^{k}\left(1-\left(\frac{u_{i_j}}{l}\right)^{2n\sigma_i}\right)^{\lambda_{i_j}}\right)^{1/2}} \middle| \prod_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}} \prod_{j=1}^{k} m_{u_{i_j}} \right\}\Bigg\}$$

Further,

$$\frac{\bigoplus\limits_{1\leqslant i_1<\ldots<i_k\leqslant n} \bigotimes\limits_{j=1}^{k}\left(n\sigma_i d_{i_j}\right)^{\lambda_j}}{C_n^k}$$

$$= \bigcup_{\substack{u_{i_j}\in g_{i_j},\\ v_{i_j}\in t_{i_j},\\ \eta_{i_j}\in h_{i_j}}} \left\{ \left\{ s_{l\left(1-\prod\limits_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}}\left(1-\prod\limits_{j=1}^{k}\left(1-\left(1-\frac{u_{i_j}^2}{l^2}\right)^{n\sigma_i}\right)^{\lambda_j}\right)^{1/C_n^k}\right)^{\frac{1}{2}}} \middle| \prod_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}} \prod_{j=1}^{k} p_{u_{i_j}} \right\},\right.$$

$$\left\{ s_{l\left(\prod\limits_{1\leqslant i_1<\ldots<i_k\leqslant n}\left(1-\prod\limits_{j=1}^{k}\left(1-\left(\frac{v_{i_j}}{l}\right)^{2n\sigma_i}\right)^{\lambda_{i_j}}\right)^{1/2}\right)^{1/C_n^k}} \middle| \prod_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}} \prod_{j=1}^{k} q_{v_{i_j}} \right\},$$

$$\left\{ s_{l\left(\prod\limits_{1\leqslant i_1<\ldots<i_k\leqslant n}\left(1-\prod\limits_{j=1}^{k}\left(1-\left(\frac{u_{i_j}}{l}\right)^{2n\sigma_i}\right)^{\lambda_{i_j}}\right)^{1/2}\right)^{1/C_n^k}} \middle| \prod_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}} \prod_{j=1}^{k} m_{u_{i_j}} \right\}\Bigg\}$$

and

$$\left(\frac{\bigoplus\limits_{1\leqslant i_1<\ldots<i_k\leqslant n}\bigotimes\limits_{j=1}^{k} d_{i_j}^{\lambda_j}}{C_n^k}\right)^{\frac{1}{\lambda_1+\lambda_2+\ldots+\lambda_k}}$$

$$= \bigcup_{\substack{u_{i_j}\in g_{i_j},\\ v_{i_j}\in t_{i_j},\\ \eta_{i_j}\in h_{i_j}}} \left\{\left\{ s_{l\left(1-\prod\limits_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}}\left(1-\prod\limits_{j=1}^{k}\left(1-\left(1-\frac{u_{i_j}^2}{l^2}\right)^{n\sigma_i}\right)^{\lambda_j}\right)^{1/C_n^k}\right)^{\frac{1}{2(\lambda_1+\lambda_2+\ldots+\lambda_k)}}} \,\middle|\, \prod_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}}\prod_{j=1}^{k} p_{u_{i_j}} \right\},\right.$$

$$\left\{ s_{l\left(1-\left(1-\left(\prod\limits_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}}\left(1-\prod\limits_{j=1}^{k}\left(1-\left(\frac{v_{i_j}}{l}\right)^{2n\sigma_i}\right)^{\lambda_{i_j}}\right)\right)^{1/C_n^k}\right)^{\frac{1}{\lambda_1+\lambda_2+\ldots+\lambda_k}}\right)^{\frac{1}{2}}} \,\middle|\, \prod_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}}\prod_{j=1}^{k} q_{v_{i_j}} \right\},$$

$$\left.\left\{ s_{l\left(1-\left(1-\left(\prod\limits_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}}\left(1-\prod\limits_{j=1}^{k}\left(1-\left(\frac{\eta_{i_j}}{l}\right)^{2n\sigma_i}\right)^{\lambda_{i_j}}\right)\right)^{1/C_n^k}\right)^{\frac{1}{\lambda_1+\lambda_2+\ldots+\lambda_k}}\right)^{\frac{1}{2}}} \,\middle|\, \prod_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}}\prod_{j=1}^{k} m_{\eta_{i_j}} \right\}\right\}.$$

Then, we will discuss the special case of the PMVLSFPGMSM operator.

Case 1. If $k = 1$ and $\lambda_1 = 1$, the PMVLSFPGMSM operator will reduce to the probabilistic multi-valued linguistic special fuzzy power average (PMVLSFPA) operator.

$$PMVLSFPGMSM^{(k)}(d_1, d_2, \ldots, d_n) = \left(\frac{\bigoplus_{1\leqslant i_1<\ldots<i_k\leqslant n} \bigotimes_{j=1}^{k} \left(n\sigma_i d_{i_j}\right)^{\lambda_j}}{C_n^k} \right)^{\frac{1}{\lambda_1+\lambda_2+\ldots+\lambda_k}}$$

$$= \bigcup_{\substack{u_{i_j}\in g_{i_j},\\ v_{i_j}\in t_{i_j},\\ \eta_{i_j}\in h_{i_j}}} \left\{ \left\{ s_{l\left(1-\prod_{1\leqslant i_1\leqslant n}\left(1-\frac{u_{i_j}^2}{l^2}\right)^{\sigma_i}\right)^{\frac{1}{2}}} \middle| \prod_{1\leqslant i_1\leqslant n} p_{u_{i_j}} \right\}, \right.$$
$$\left. \left\{ s_{l\prod_{1\leqslant i_1\leqslant n}\left(\left(\frac{v_{i_j}}{l}\right)^{\sigma_i}\right)} \middle| \prod_{1\leqslant i_1\leqslant n} q_{v_{i_j}} \right\}, \left\{ s_{l\prod_{1\leqslant i_1\leqslant n}\left(\left(\frac{v_{i_j}}{l}\right)^{\sigma_i}\right)} \middle| \prod_{1\leqslant i_1\leqslant n} m_{\eta_{i_j}} \right\} \right\}$$
$$= \bigoplus_{i=1}^{n} \sigma_i d_i = PMVLSFPA(d_1, d_2, \ldots, d_n). \tag{16}$$

In addition, when the $\mathrm{Sup}(d_i, d_j) = t > 0$, the PMVLSFPGMSM operator will reduce to the probabilistic multi-valued linguistic special fuzzy average (PMVLSFA) operator.

$$PMVLSFPGMSM^{(k)}(d_1, d_2, \ldots, d_n) = \left(\frac{\bigoplus_{1\leqslant i_1<\ldots<i_k\leqslant n} \bigotimes_{j=1}^{k} \left(n\sigma_i d_{i_j}\right)^{\lambda_j}}{C_n^k} \right)^{\frac{1}{\lambda_1+\lambda_2+\ldots+\lambda_k}}$$

$$= \bigcup_{\substack{u_{i_j}\in g_{i_j},\\ v_{i_j}\in t_{i_j},\\ \eta_{i_j}\in h_{i_j}}} \left\{ \left\{ s_{l\left(1-\prod_{1\leqslant i_1<\ldots<i_k\leqslant n}\left(1-\frac{u_{i_j}^2}{l^2}\right)^{\frac{1}{n}}\right)^{\frac{1}{2}}} \middle| \prod_{1\leqslant i_1\leqslant n} p_{u_{i_j}} \right\}, \right.$$
$$\left. \left\{ s_{l\prod_{1\leqslant i_1\leqslant n}\left(\left(\frac{v_{i_j}}{l}\right)^{\frac{1}{n}}\right)} \middle| \prod_{1\leqslant i_1\leqslant n} q_{v_{i_j}} \right\}, \left\{ s_{l\prod_{1\leqslant i_1\leqslant n}\left(\left(\frac{v_{i_j}}{l}\right)^{\frac{1}{n}}\right)} \middle| \prod_{1\leqslant i_1\leqslant n} m_{\eta_{i_j}} \right\} \right\}$$
$$= \frac{1}{n} \bigoplus_{i=1}^{n} d_i = PMVLSFPA(d_1, d_2, \ldots, d_n). \tag{17}$$

Case 2. If $k = 2$ and the parameter λ_n will transfer to λ_1 and λ_2, the PMVLSF-PGMSM operator will reduce to the probabilistic multi-valued linguistic special fuzzy power Bonferroni mean (PMVLSFPBM) operator.

$$PMVLSFPGMSM^{(k)}(d_1, d_2, \dots, d_n) = \left(\frac{\bigoplus_{1 \leqslant i_1 < \dots < i_k \leqslant n} \bigotimes_{j=1}^{k} \left(n\sigma_i d_{i_j} \right)^{\lambda_j}}{C_n^k} \right)^{\frac{1}{\lambda_1 + \lambda_2 + \dots + \lambda_k}}$$

$$= \bigcup_{\substack{u_{i_j} \in g_{i_j}, \\ v_{i_j} \in t_{i_j}, \\ \eta_{i_j} \in h_{i_j}}} \left\{ \left\{ s_{\left(1 - \prod_{\substack{1 \leqslant i_1 < \dots \\ < i_2 \leqslant n}} \left(1 - \prod_{j=1}^{2} \left(1 - \left(1 - \frac{u_{i_j}^2}{l^2}\right)^{n\sigma_i}\right)^{\lambda_j}\right)^{1/n(n-1)}\right)^{\frac{1}{2(\lambda_1+\lambda_2)}}} \middle| \prod_{\substack{1 \leqslant i_1 < \dots \\ < i_2 \leqslant n}} \prod_{j=1}^{2} p_{u_{i_j}} \right\}, \right.$$

$$\left\{ s_{l \left(1 - \left(1 - \left(\prod_{\substack{1 \leqslant i_1 < \dots \\ < i_2 \leqslant n}} \left(1 - \prod_{j=1}^{2} \left(1 - \left(\frac{v_{i_j}}{l}\right)^{2n\sigma_i}\right)^{\lambda_{i_j}}\right)\right)^{1/n(n-1)}\right)^{\frac{1}{\lambda_1+\lambda_2}}\right)^{\frac{1}{2}}} \middle| \prod_{\substack{1 \leqslant i_1 < \dots \\ < i_2 \leqslant n}} \prod_{j=1}^{2} q_{v_{i_j}} \right\},$$

$$\left. \left\{ s_{l \left(1 - \left(1 - \left(\prod_{\substack{1 \leqslant i_1 < \dots \\ < i_2 \leqslant n}} \left(1 - \prod_{j=1}^{2} \left(1 - \left(\frac{\eta_{i_j}}{l}\right)^{2n\sigma_i}\right)^{\lambda_{i_j}}\right)\right)^{1/n(n-1)}\right)^{\frac{1}{\lambda_1+\lambda_2}}\right)^{\frac{1}{2}}} \middle| \prod_{\substack{1 \leqslant i_1 < \dots \\ < i_2 \leqslant n}} \prod_{j=1}^{2} m_{\eta_{i_j}} \right\} \right\}$$

$$= \left(\frac{\bigoplus_{1 \leqslant i_1 < \dots < i_2 \leqslant n} \bigotimes_{j=1}^{k} \left(n\sigma_i d_{i_j} \right)^{\lambda_j}}{n(n-1)} \right)^{\frac{1}{\lambda_1 + \lambda_2}} = PMVLSFPBM(d_1, d_2, \dots, d_n). \quad (18)$$

In addition, when the $\mathrm{Sup}(d_i, d_j) = t > 0$, the PMVLSFPGMSM operator will reduce to the probabilistic multi-valued linguistic special fuzzy Bonferroni mean (PMVLSFBM) operator.

$$PMVLSFPGMSM^{(k)}(d_1, d_2, \ldots, d_n) = \left(\frac{\bigoplus_{1 \leqslant i_1 < \ldots < i_k \leqslant n} \bigotimes_{j=1}^{k} \left(n\sigma_i d_{i_j} \right)^{\lambda_j}}{C_n^k} \right)^{\frac{1}{\lambda_1 + \lambda_2 + \ldots + \lambda_k}}$$

$$= \bigcup_{\substack{u_{i_j} \in g_{i_j}, \\ v_{i_j} \in t_{i_j}, \\ \eta_{i_j} \in h_{i_j}}} \left\{ \left\{ s_{\left(1 - \prod_{1 \leqslant i_1 < \ldots < i_2 \leqslant n} \left(1 - \prod_{j=1}^{2} \left(\frac{u_{i_j}^2}{l^2}\right)^{\lambda_j}\right)^{1/n(n-1)}\right)^{\frac{1}{2(\lambda_1+\lambda_2)}}} \middle| \prod_{\substack{1 \leqslant i_1 < \ldots \\ < i_2 \leqslant n}} \prod_{j=1}^{2} p_{u_{i_j}} \right\}, \right.$$

$$\left\{ s_{l\left(1 - \left(1 - \left(\prod_{\substack{1 \leqslant i_1 < \ldots \\ < i_2 \leqslant n}} \left(1 - \prod_{j=1}^{2} \left(1 - \left(\frac{v_{i_j}}{l}\right)^2\right)^{\lambda_{i_j}}\right)\right)^{1/n(n-1)}\right)^{\frac{1}{\lambda_1+\lambda_2}}\right)^{\frac{1}{2}}} \middle| \prod_{\substack{1 \leqslant i_1 < \ldots \\ < i_2 \leqslant n}} \prod_{j=1}^{2} q_{v_{i_j}} \right\},$$

$$\left. \left\{ s_{l\left(1 - \left(1 - \left(\prod_{\substack{1 \leqslant i_1 < \ldots \\ < i_2 \leqslant n}} \left(1 - \prod_{j=1}^{2} \left(1 - \left(\frac{\eta_{i_j}}{l}\right)^2\right)^{\lambda_{i_j}}\right)\right)^{1/n(n-1)}\right)^{\frac{1}{\lambda_1+\lambda_2}}\right)^{\frac{1}{2}}} \middle| \prod_{\substack{1 \leqslant i_1 < \ldots \\ < i_2 \leqslant n}} \prod_{j=1}^{2} m_{\eta_{i_j}} \right\} \right\}$$

$$= \left(\frac{\bigoplus_{1 \leqslant i_1 < \ldots < i_2 \leqslant n} \bigotimes_{j=1}^{k} \left(d_{i_j} \right)^{\lambda_j}}{n(n-1)} \right)^{\frac{1}{\lambda_1 + \lambda_2}} = PMVLSFBM(d_1, d_2, \ldots, d_n). \tag{19}$$

Case 3. If $\lambda_1 = \lambda_2 = \ldots = \lambda_k = 1$, the PMVLSFPGMSM operator will reduce to the probabilistic multi-valued linguistic special fuzzy power Maclaurin symmetric mean (PMVLSFPMSM) operator.

$$PMVLSFPGMSM^{(k)}(d_1, d_2, \ldots, d_n) = \left(\frac{\bigoplus_{1\leqslant i_1<\ldots<i_k\leqslant n} \bigotimes_{j=1}^{k}}{C_n^k} \left(n\sigma_i d_{i_j}\right)^{\lambda_j} \right)^{\frac{1}{\lambda_1+\lambda_2+\ldots+\lambda_k}}$$

$$= \bigcup_{\substack{u_{i_j}\in g_{i_j},\\ v_{i_j}\in t_{i_j},\\ \eta_{i_j}\in h_{i_j}}} \left\{ \left\{ s_{l\left(1-\prod_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}}\left(1-\prod_{j=1}^{k}\left(1-\left(1-\frac{u_{i_j}^2}{l^2}\right)^{n\sigma_i}\right)^{\lambda_j}\right)^{1/C_n^k}\right)^{\frac{1}{2k}}} \middle| \prod_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}} \prod_{j=1}^{k} p_{u_{i_j}} \right\}, \right.$$

$$\left\{ s_{l\left(1-\left(1-\left(\prod_{1\leqslant i_1<\ldots<i_k\leqslant n}\left(1-\prod_{j=1}^{k}\left(1-\left(\frac{v_{i_j}}{l}\right)^{2n\sigma_i}\right)^{\lambda_{i_j}}\right)\right)^{1/C_n^k}\right)^{\frac{1}{k}}\right)^{\frac{1}{2}}} \middle| \prod_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}} \prod_{j=1}^{k} q_{v_{i_j}} \right\},$$

$$\left. \left\{ s_{l\left(1-\left(1-\left(\prod_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}}\left(1-\prod_{j=1}^{k}\left(1-\left(\frac{\eta_{i_j}}{l}\right)^{2n\sigma_i}\right)^{\lambda_{i_j}}\right)\right)^{1/C_n^k}\right)^{\frac{1}{k}}\right)^{\frac{1}{2}}} \middle| \prod_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}} \prod_{j=1}^{k} m_{\eta_{i_j}} \right\} \right\}$$

$$= \left(\frac{\bigoplus_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}} \bigotimes_{j=1}^{k}}{C_n^k} \left(n\sigma_i d_{i_j}\right)^{\lambda_j} \right)^{\frac{1}{k}} = PMVLSFPMSM^{(k)}(d_1, d_2, \ldots, d_n). \tag{20}$$

In addition, when the $\mathrm{Sup}(d_i, d_j) = t > 0$, the PMVLSFPGMSM operator will reduce to the probabilistic multi-valued linguistic special fuzzy Maclaurin symmetric mean (PMVLSFMSM) operator.

$$PMVLSFPGMSM^{(k)}(d_1, d_2, \ldots, d_n) = \left(\frac{\bigoplus_{1\leqslant i_1<\ldots<i_k\leqslant n} \bigotimes_{j=1}^{k}}{C_n^k} \left(n\sigma_i d_{i_j}\right)^{\lambda_j} \right)^{\frac{1}{\lambda_1+\lambda_2+\ldots+\lambda_k}}$$

$$= \bigcup_{\substack{u_{i_j}\in g_{i_j},\\ v_{i_j}\in t_{i_j},\\ \eta_{i_j}\in h_{i_j}}} \left\{ \left\{ s_{l\left(1-\prod_{1\leqslant i_1<\ldots<i_k\leqslant n}\left(1-\prod_{j=1}^{k}\left(\frac{u_{i_j}^2}{l^2}\right)^{\lambda_j}\right)^{1/C_n^k}\right)^{\frac{1}{2k}}} \middle| \prod_{1\leqslant i_1<\ldots<i_k\leqslant n} \prod_{j=1}^{k} p_{u_{i_j}} \right\}, \right.$$

$$\left\{ s_{l\left(1-\left(1-\left(\prod_{1\leqslant i_1<\ldots<i_k\leqslant n}\left(1-\prod_{j=1}^{k}\left(1-\left(\frac{v_{i_j}}{l}\right)^2\right)^{\lambda_{i_j}}\right)\right)^{1/C_n^k}\right)^{\frac{1}{k}}\right)^{\frac{1}{2}}} \middle| \prod_{1\leqslant i_1<\ldots<i_k\leqslant n} \prod_{j=1}^{k} q_{v_{i_j}} \right\},$$

$$\left. \left\{ s_{l\left(1-\left(1-\left(\prod_{1\leqslant i_1<\ldots<i_k\leqslant n}\left(1-\prod_{j=1}^{k}\left(1-\left(\frac{\eta_{i_j}}{l}\right)^2\right)^{\lambda_{i_j}}\right)\right)^{1/C_n^k}\right)^{\frac{1}{k}}\right)^{\frac{1}{2}}} \middle| \prod_{1\leqslant i_1<\ldots<i_k\leqslant n} \prod_{j=1}^{k} m_{\eta_{i_j}} \right\} \right\}.$$

$$= \left(\frac{\bigoplus_{1\leqslant i_1<\ldots<i_k\leqslant n} \bigotimes_{j=1}^{k}}{C_n^k} \left(d_{i_j}\right)^{\lambda_j} \right)^{\frac{1}{k}} = PMVLSFMSM^{(k)}\left(d_1, d_2, \ldots, d_n\right). \qquad (21)$$

4.2. The Probabilistic Multi-valued Linguistic Special Fuzzy Power Weight Generalized Maclaurin Symmetric Mean (PMVLSFPWGMSM) Operator

Definition 12. *Let* $d_i = (g_i|p_{g_i}, t_i|q_{t_i}, h_i|m_{h_i})$ $(i = 1, 2, \ldots, n)$ *be a collection of PMVLSFEs, and* $w = (w_1, w_2, \ldots, w_n)^T$ *be the weight vector, such that* $0 \leqslant w_i \leqslant 1$ *and* $\sum_{i=1}^{n} w_i = 1$*. Then, the probabilistic multi-valued linguistic special fuzzy power weight generalized Maclaurin symmetric mean (PMVLSFPGMSM) operator is defined as*

$$PMVLSFPWGMSM^{(k)}(d_1, d_2, \ldots, d_n) = \left(\frac{\bigoplus_{\substack{1\leqslant i_1<\ldots \\ <i_k\leqslant n}} \bigotimes_{j=1}^{k} \left(\frac{nw_i\left(1+T\left(d_{i_j}\right)\right) d_{i_j}}{\sum_{i=1}^{n} w_i\,(1+T(d_i))} \right)^{\lambda_j}}{C_n^k} \right)^{\frac{1}{\lambda_1+\lambda_2+\ldots+\lambda_k}}. \tag{22}$$

where $T(d_i) = \sum_{i=1, i\neq j}^{n} Sup(d_i, d_j)$*, and* $Sup(d_i, d_j)$ *denotes the support degree for* d_i *from* d_j*, satisfying the properties proposed in Definition 10. If we assume*

$$\psi_i = \frac{1+T(d_i)}{\sum_{i=1}^{n} (1+T\,(d_i))}, \tag{23}$$

then Eq. (22) can be written as

$$PMVLSFPGMSM^{(k)}\,(d_1, d_2, \ldots, d_n) = \left(\frac{\bigoplus_{1\leqslant i_1<\ldots<i_k\leqslant n} \bigotimes_{j=1}^{k} \left(n\psi_i d_{i_j}\right)^{\lambda_j}}{C_n^k} \right)^{\frac{1}{\lambda_1+\lambda_2+\ldots+\lambda_k}},$$

where $\lambda_1, \lambda_2, \ldots, \lambda_k \geqslant 0$*,* $\lambda_1 + \lambda_2 + \ldots + \lambda_k > 0$*,* $k = 1, 2, \ldots, n$ *is an integer.*

Theorem 3. *Let* $d_i = (g_i|p_{g_i}, t_i|q_{t_i}, h_i|m_{h_i})$ $(i = 1, 2, \ldots, n)$ *be a collection of PMVLSFEs, the aggregated value by the PMVLSFPWGMSM operator is still a*

PMVLSFE and

$$PMVLSFPGMSM^{(k)}\left(d_1, d_2, \ldots, d_n\right)=\left(\frac{\bigoplus_{1\leqslant i_1<\ldots<i_k\leqslant n}\bigotimes_{j=1}^{k}}{C_n^k}\left(n\psi_i d_{i_j}\right)^{\lambda_j}\right)^{\frac{1}{\lambda_1+\lambda_2+\ldots+\lambda_k}}$$

$$\bigcup_{\substack{u_{i_j}\in g_{i_j},\\ v_{i_j}\in t_{i_j},\\ \eta_{i_j}\in h_{i_j}}}\left\{\left\{s_{l\left(1-\prod_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}}\left(1-\prod_{j=1}^{k}\left(1-\left(1-\frac{u_{i_j}^2}{l^2}\right)^{n\psi_i}\right)^{\lambda_j}\right)^{1/C_n^k}\right)^{\frac{1}{2(\lambda_1+\lambda_2+\ldots+\lambda_k)}}}\left|\prod_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}}\prod_{j=1}^{k}p_{u_{i_j}}\right.\right\},\right.$$

$$\left\{s_{l\left(1-\left(1-\left(\prod_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}}\left(1-\prod_{j=1}^{k}\left(1-\left(\frac{v_{i_j}}{l}\right)^{2n\psi_i}\right)^{\lambda_{i_j}}\right)\right)^{1/C_n^k}\right)^{\frac{1}{\lambda_1+\lambda_2+\ldots+\lambda_k}}\right)^{\frac{1}{2}}}\left|\prod_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}}\prod_{j=1}^{k}q_{v_{i_j}}\right.\right\},$$

$$\left.\left\{s_{l\left(1-\left(1-\left(\prod_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}}\left(1-\prod_{j=1}^{k}\left(1-\left(\frac{\eta_{i_j}}{l}\right)^{2n\psi_i}\right)^{\lambda_{i_j}}\right)\right)^{1/C_n^k}\right)^{\frac{1}{\lambda_1+\lambda_2+\ldots+\lambda_k}}\right)^{\frac{1}{2}}}\left|\prod_{\substack{1\leqslant i_1<\ldots\\<i_k\leqslant n}}\prod_{j=1}^{k}m_{\eta_{i_j}}\right.\right\}\right\}. \quad (24)$$

The proof of the Theorem 3 is similar to the process of the Theorem 2.

5. The method based on PMVLSFPWGMSM operator for MAGDM

In this part, we introduce a new decision-making method based on PMVLSF-PWGMSM operator and apply the method to solve the problem of site selection of charging pile.

5.1. A new method based on PMVLSFPWGMSM operator

In this part, we proposed the the method based on PMVLSFGMSM operator for MAGDM. In the proposed method, a novel procedure is presented to solve the decision-making method. For a selection problems, assume that $A = \{A_1, A_2, \ldots, A_m\}$ are the sets of the alternative and $C = \{C_1, C_2, \ldots, C_n\}$ are the sets of the attributes which can be evaluated in the process of the decision-making. The weight of the attributes is defined as $W = (w_1, w_2, \ldots, w_n)^T$ which satisfied $0 \leqslant w_j \leqslant 1$ and $\sum_{j=1}^{n} w_j = 1$. Assume that the experts choose PMVLSFEs to express their personal preferences. After the evaluation, we can obtain a decision matrix, denoted by $D = \left[d_{ij}\right]_{m\times n}$. The procedure of the proposed method are described as follows:

Step 1. Define the alternatives which will be evaluated by experts.

Step 2. Define the attributes and the type of the attributes. In the next step, we should normalize the decision matrix according to the type of the attributes.

Step 3. Collect the evlauation information denoted by PMVLSFEs and obtain the original decision matrix.

Step 4. Normalize the original decision matrix. When the attribute is benefit, the PMVLSFEs remain static; when the attribute is cost, the PMVLSFEs are normalized according to the described rules. The rules as follows:

$$D = \begin{cases} d_{ij} = \left(g_{ij}\middle|p_{ij},\ t_{ij}\middle|q_{ij},\ h_{ij}\middle|m_{ij}\right) & \mathrm{C}_j \text{ is benefit,} \\ d_{ij} = \left(h_{ij}\middle|m_{ij},\ t_{ij}\middle|q_{ij},\ g_{ij}\middle|p_{ij}\right) & \mathrm{C}_j \text{ is cost.} \end{cases} \tag{25}$$

Step 5. Calculate the Sup (d_{il}, d_{in}) according to

$$\mathrm{Sup}\,(d_{il}, d_{im}) = 1 - d\,(d_{il}, d_{im})\,, \tag{26}$$

where $l, m = 1, 2, \ldots, n$; $l \neq m$

Step 6. Compute $T(d_{ij})$ by

$$T(d_{ij}) = \sum_{l,m=1, l\neq m}^{n} \mathrm{Sup}\,(d_{il}, d_{im})\,. \tag{27}$$

Step 7. Compute the power weight ψ_{ij} associated with PMVLSFEs according to the following formula

$$\psi_{ij} = \frac{w_i\left(1 + T(d_{ij})\right)}{\sum_{i=1}^{n} w_i\left(1 + T(d_{ij})\right)}\,. \tag{28}$$

Step 8. Obtain the values of the alternative $A = \{A_1, A_2, \ldots, A_m\}$ which aggregate the decision matrix by the proposed PMVLSFPWGMSM operator.

$$PMVLSFPWGMSM^{(k,\lambda_1,\lambda_2,\ldots,\lambda_n)}(d_1, d_2, \ldots, d_n) = \left(\frac{\bigoplus_{1\leqslant i_1<\ldots<i_k\leqslant n} \bigotimes_{j=1}^{k}}{C_n^k} \left(n\psi_i d_{i_j}\right)^{\lambda_j} \right)^{\frac{1}{\lambda_1+\lambda_2+\ldots+\lambda_n}}. \tag{29}$$

Step 9. Calculate the score values of the alternative $A = \{A_1, A_2, \ldots, A_m\}$ using the

$$S(d) = s_{\sqrt{\left(l^2 + \sum_{i=1,u\in g}^{\#g} u_i^2 p_i - \sum_{i=1,\eta\in h}^{\#h} \eta_i^2 m_i\right)/2}}. \tag{30}$$

Step 10. Obtain the rank of the alternatives according to the score values produced by Eq. (30) and choose the best alternative.

To better introduce the procedure of our proposed MAGDM method, a flowchart of this method is shown in Fig. 1.

5.2. An evaluation method for site selection of charging pile

With the continuous development, environmental issues have attracted the attention of scholars. With its characteristics of environmental protection and energy saving, electric vehicles gradually play an important role in future life. However, the limited charging piles have a negative impact on the electric vehicles industry. Therefore, it is urgent to increase the number of charging piles in the city. Choosing a suitable charging pile location is a problem worthy of study. In other words, if the site selection of charging piles is suitable, it will bring long-term benefits. Based on it, the site selection should convene many experts to evaluate the alternative sites and select the optimal site for construction. When considering whether the site selection is appropriate, the experts will analyze it from several factors such as the distance from the living area, the construction cost, etc. Therefore, the site selection of charging piles is a multi-attribute decision-making problem, which can be solved based on the evaluation method proposed in the previous section shown in Fig. 1. There are many researchers on evaluation methods of the site of charging station [1, 45, 46]. Lin et al. [45] introduced that social benefit (land utilization) should be considered into the site of the charging station. In 2020, Zhou et al. [46] establish an evaluation index system about electric vehicle (EV) charging station composed with natural, economic, technical, and social. Dang et al. [2] propose a multi-attribute decision-making method for determining the site of the island photovoltaic charging station.

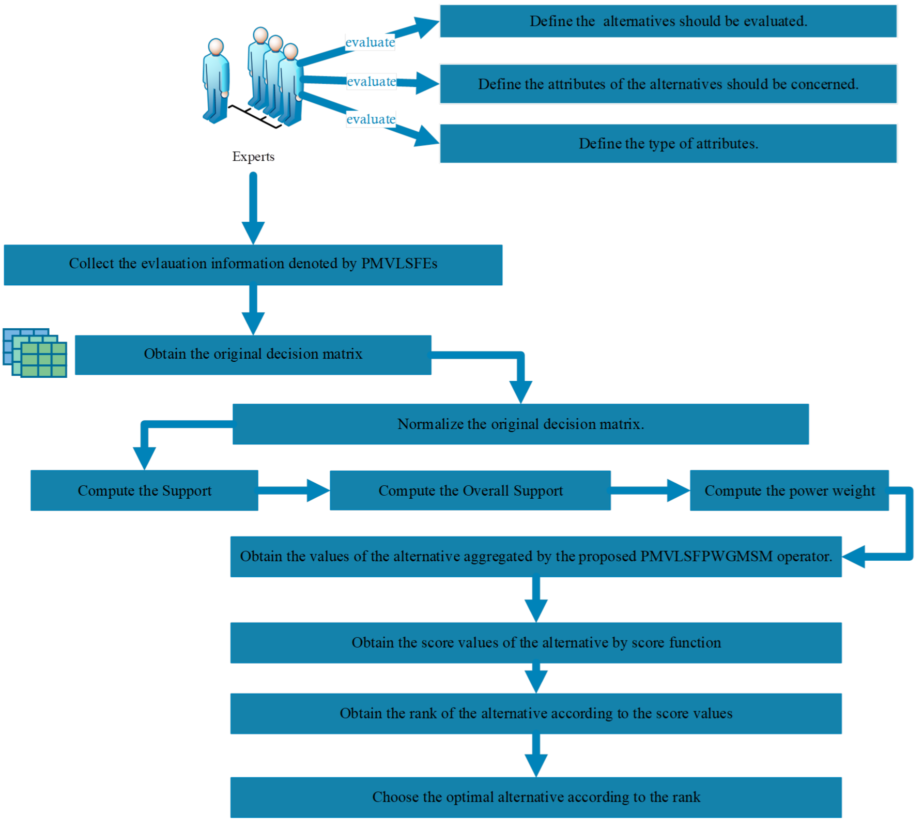

Figure 1: The flowchart of our proposed method

After reading the literature, we find that the site selection evaluation index usually composed by nature, social, construction and economy. For the aspect of the nature, the number of electric vehicles which can be supported by charging pile is an important factor in determining whether the site selection is appropriate. Because the electric vehicles charge at stations different from the electric ships. From a social perspective, the location of the charging pile needs to be supported by the local government. In addition to the government support, the convenient transportation and public acceptance are also important components of social factor. In the evaluation process, the availability of the charging pile construction condition and impact on grid connection are also issues considered by experts. Not important but the factors that must be considered are economic factors, which include the cost of occupying land, labor costs and electricity.

Based on the methods mentioned above, this article constructs an evaluation method for the address selection of charging piles. In this evaluation method, we have summarized four attributes to evaluate the site of the charging pile: C_1 represents the nature factors, C_2 represents the social factor, C_3 represents the construction factor and C_4 represents the economic factor. The explanation and the type of the attributes are shown in Table 2. In addition, the experts are required to use PMVLSFEs when expressing their own evaluation information. The specific solutions for the site selection of charging piles are described in the next part with specific data cases.

Table 2: The attributes to evaluate the site of the charging pile

Attributes	Explanation	The type of attributes
Nature factors C_1	The social factors contain the capacity and scalability that the charging pile can support.	Benefit
Social factors C_2	The social factors contain the policy support and transport convenience and public acceptance.	Benefit
Construction factors C_3	The construction factors contain the impact on grid connection and the construction conditions	Cost
Economic factors C_4	The economic factors contain the cost of land, labor, and the electricity.	Cost

6. A numerical example for site selection of charging pile

In this subsection, we use a numerical example to depict the effectiveness about the evaluation method of the site selection of charging pile based on PMVLSFPWGMSM operators. Sensitivity analysis and comparative analysis of this method have also been carried out to prove the flexibility and practicability of our method.

6.1. A case description of the site selection of charging pile

In this part, suppose there are m alternative sites of the charging pile A_m $(m = 1, 2, 3, 4)$. The experts evaluate the sites according to the n attributes C_n $(n = 1, 2, 3, 4)$, C_1 represents the nature factors, C_2 represents the social factor, C_3 represents the construction factor and C_4 represents the economic factor. The weight of the attributes is $W = \left(0.1,\ 0.3,\ 0.3,\ 0.3\right)^T$ and the LTSs are $S = \left\{s_0 = \textit{extremly bad}, s_1 = \textit{bad},\ s_2 = \textit{little bad},\ s_3 = \textit{medium},\ s_4 = \textit{little good},\ s_5 = \textit{good},\ s_6 = \textit{perfect}\right\}$. The experts are invited to use the PMVLSFEs to express their evaluation the evaluation of experts are shown in Table 3.

Table 3: The evaluation values of the experts by PMVLSFEs

	A_1	A_2
C_1	$((s_2, s_3, s_4)\vert(0.3, 0.3, 0.4), (s_2, s_3)\vert(0.5, 0.5), (s_3, s_6)\vert(0.2, 0.8))$	$((s_2, s_3)\vert(0.5, 0.5), (s_1, s_3)\vert(0.2, 0.8), (s_3, s_4)\vert(0.3, 0.7))$
C_2	$((s_3, s_4)\vert(0.4, 0.6), (s_3, s_4)\vert(0.2, 0.8), (s_2, s_3)\vert(0.5, 0.5))$	$((s_3, s_4)\vert(0.3, 0.7), (s_3, s_4)\vert(0.1, 0.9), (s_5, s_6)\vert(0.4, 0.6))$
C_3	$((s_3, s_4)\vert(0.4, 0.6), (s_4, s_5)\vert(0.3, 0.7), (s_1, s_2, s_3)\vert(0.3, 0.3, 0.4))$	$((s_0, s_1)\vert(0.6, 0.4), (s_3, s_5)\vert(0.6, 0.4), (s_1, s_3)\vert(0.3, 0.7))$
C_4	$((s_1, s_3, s_4)\vert(0.2, 0.4, 0.4), (s_3, s_5)\vert(0.5, 0.5), (s_1, s_2, s_3)\vert(0.2, 0.3, 0.5))$	$((s_0, s_2)\vert(0.6, 0.4), (s_0, s_1)\vert(0.6, 0.4), (s_2, s_5)\vert(0.3, 0.7))$
	A_3	A_4
C_1	$((s_2, s_3)\vert(0.1, 0.9), (s_4, s_3)\vert(0.4, 0.6), (s_5, s_6)\vert(0.3, 0.7))$	$((s_1, s_2, s_3)\vert(0.3, 0.3, 0.4), (s_2, s_3)\vert(0.4, 0.6), (s_3, s_5)\vert(0.5, 0.5))$
C_2	$((s_5, s_4)\vert(0.2, 0.8), (s_1, s_2)\vert(0.3, 0.7), (s_3, s_5)\vert(0.1, 0.9))$	$((s_3, s_5)\vert(0.4, 0.6), (s_1, s_2)\vert(0.2, 0.8), (s_5)\vert(1))$
C_3	$((s_3, s_5, s_6)\vert(0.2, 0.3, 0.5), (s_1, s_5)\vert(0.3, 0.7), (s_4, s_5)\vert(0.3, 0.7))$	$((s_0, s_1)\vert(0.7, 0.3), (s_3, s_4)\vert(0.5, 0.5), (s_2, s_3)\vert(0.4, 0.6))$
C_4	$((s_3, s_4, s_5)\vert(0.2, 0.4, 0.4), (s_1, s_3)\vert(0.7, 0.3), (s_2, s_3)\vert(0.3, 0.7))$	$((s_4, s_6)\vert(0.4, 0.6), (s_4, s_5)\vert(0.5, 0.5), (s_2, s_3)\vert(0.2, 0.8))$

Table 4: The normalized decision matric of the experts by PMVLSFEs

	A_1	A_2
C_1	$((s_2, s_3, s_4)\vert(0.3, 0.3, 0.4), (s_2, s_3)\vert(0.5, 0.5), (s_3, s_6)\vert(0.2, 0.8))$	$((s_2, s_3)\vert(0.5, 0.5), (s_1, s_3)\vert(0.2, 0.8), (s_3, s_4)\vert(0.3, 0.7))$
C_2	$((s_3, s_4)\vert(0.4, 0.6), (s_3, s_4)\vert(0.2, 0.8), (s_2, s_3)\vert(0.5, 0.5))$	$((s_3, s_4)\vert(0.3, 0.7), (s_3, s_4)\vert(0.1, 0.9), (s_5, s_6)\vert(0.4, 0.6))$
C_3	$((s_3, s_4)\vert(0.4, 0.6), (s_4, s_5)\vert(0.3, 0.7), (s_1, s_2, s_3)\vert(0.3, 0.3, 0.4))$	$((s_0, s_1)\vert(0.6, 0.4), (s_3, s_5)\vert(0.6, 0.4), (s_1, s_3)\vert(0.3, 0.7))$
C_4	$((s_1, s_3, s_4)\vert(0.2, 0.4, 0.4), (s_3, s_5)\vert(0.5, 0.5), (s_1, s_2, s_3)\vert(0.2, 0.3, 0.5))$	$((s_0, s_2)\vert(0.6, 0.4), (s_0, s_1)\vert(0.6, 0.4), (s_2, s_5)\vert(0.3, 0.7))$
	A_3	A_4
C_1	$((s_5, s_6)\vert(0.3, 0.7), (s_4, s_3)\vert(0.4, 0.6), (s_2, s_3)\vert(0.1, 0.9))$	$((s_3, s_5)\vert(0.5, 0.5), (s_2, s_3)\vert(0.4, 0.6), (s_1, s_2, s_3)\vert(0.3, 0.3, 0.4))$
C_2	$((s_3, s_5)\vert(0.1, 0.9), (s_1, s_2)\vert(0.3, 0.7), (s_5, s_4)\vert(0.2, 0.8))$	$((s_5)\vert(1), (s_1, s_2)\vert(0.2, 0.8), (s_3, s_5)\vert(0.4, 0.6))$
C_3	$((s_4, s_5)\vert(0.3, 0.7), (s_1, s_5)\vert(0.3, 0.7), (s_3, s_5, s_6)\vert(0.2, 0.3, 0.5))$	$((s_2, s_3)\vert(0.4, 0.6), (s_3, s_4)\vert(0.5, 0.5), (s_0, s_1)\vert(0.7, 0.3))$
C_4	$((s_2, s_3)\vert(0.3, 0.7), (s_1, s_3)\vert(0.7, 0.3), (s_3, s_4, s_5)\vert(0.2, 0.4, 0.4))$	$((s_2, s_3)\vert(0.2, 0.8), (s_4, s_5)\vert(0.5, 0.5), (s_4, s_6)\vert(0.4, 0.6))$

The relevant information of step 1–3 has been given, so we will omit it here, and directly start step 4.

Step 4. According to the type of attributes in Table 2, we normalize the original matrix and obtain the Table 4.

Step 5. Calculate the Sup (d_{il}, d_{in}) between attributes C_n $(n = 1, 2, 3, 4)$ according to Eq. (26), we can get

$$
\begin{aligned}
\mathrm{Sup}_{12} &= \mathrm{Sup}_{21} = (0.8210,\ 0.8472,\ 0.8403,\ 0.8528)\,,\\
\mathrm{Sup}_{13} &= \mathrm{Sup}_{31} = (0.8146,\ 0.9009,\ 0.8119,\ 0.9116)\,,\\
\mathrm{Sup}_{14} &= \mathrm{Sup}_{41} = (0.8712,\ 0.9088,\ 0.8282,\ 0.7843)\,,\\
\mathrm{Sup}_{23} &= \mathrm{Sup}_{32} = (0.9532,\ 0.7806,\ 0.8520,\ 0.7644)\,,\\
\mathrm{Sup}_{24} &= \mathrm{Sup}_{42} = (0.9427,\ 0.8144,\ 0.9056,\ 0.7731)\,,\\
\mathrm{Sup}_{34} &= \mathrm{Sup}_{43} = (0.9490,\ 0.8690,\ 0.8448,\ 0.8227)
\end{aligned}
$$

Step 6. Compute $T(d_{ij})$ according to Eq. (27) and have

$$
T = \begin{pmatrix} 2.6318 & 2.7076 & 2.7167 & 2.7628 \\ 2.6569 & 2.4421 & 2.5505 & 2.5921 \\ 2.4212 & 2.5272 & 2.5087 & 2.5786 \\ 2.5486 & 2.3903 & 2.4986 & 2.3801 \end{pmatrix}.
$$

Step 7. Compute the power weight ψ_{ij} according to Eq. (28) and obtain

$$
\psi = \begin{pmatrix} 0.0976 & 0.2991 & 0.2998 & 0.3035 \\ 0.1033 & 0.2916 & 0.3008 & 0.3043 \\ 0.0970 & 0.3001 & 0.2985 & 0.3004 \\ 0.1033 & 0.2960 & 0.3055 & 0.2952 \end{pmatrix}.
$$

Step 8. Obtain the values of the site of charging pile $A = \{A_1, A_2, \ldots, A_m\}$ according to Eq. (29). The results of the aggregation are so complex that we omit it here.

Step 9. Obtain the score values of the site of charging pile $A = \{A_1, A_2, \ldots, A_m\}$ according to Eq. (30) and the results are shown as:

$$
S(d_1) = 4.2426; \quad S(d_1) = 4.2424; \quad S(d_1) = 4.2429; \quad S(d_1) = 4.2428.
$$

Step 10. According to the score values in Step 9, $A_3 > A_4 > A_1 > A_2$ can be obtained. Therefore, the optimal site of charging pile is A_4.

6.2. Sensitivity analysis

From the aggregation method, it is obvious that the parameter λ_k and k have an important effect on the results. Therefore, we will discuss the parameter λ_k and k how to affect the final decision results in detail.

6.2.1. The impact of the parameter λ_k

In this part, we discuss the impact of the parameter λ_k on the result and ranks of the alternative site of the charging pile. To do this, we change the value of the λ_k, and keep the parameter k unchanged. The results obtained by different λ_k are shown in Table 5. The analysis of λ_k is divided into three group to explore. The results of three group are presented in Figs. 2–4.

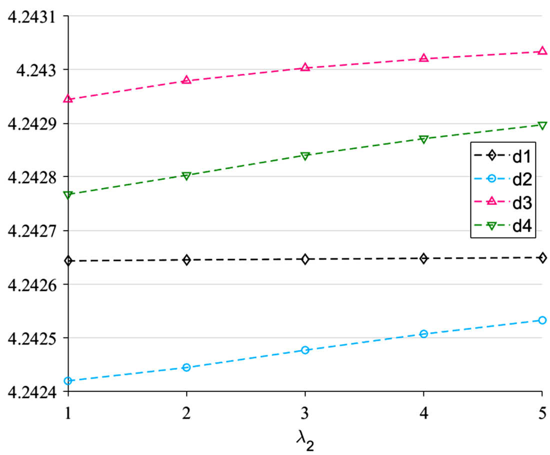

Figure 2: The ranking orders by proposed method when $k = 2$ for different λ_2

In the first group, we assume that the value of λ_1 is unchanged and discuss the impact of parameter λ_2 on the results. From the Fig. 2, it is obvious that different values of λ_2 can lead to different results. For detailed, the overall trend of the results is increase with the increase of λ_2 although the increasing trend is slow. However, the ranking orders of the alternative site of the charging pile is always $A_3 > A_4 > A_1 > A_2$. Therefore, the parameter λ_2 has little effect on the final result of the decision which indicate that our method is robustness.

In the two groups, we investigate the influence of the parameter λ_1 on the results. For convenience, we assume the value of λ_2 is always $\lambda_2 = 1$. As we can see from the Fig. 3, we find the same trend of the parameter λ_2. The final decision result changes with the increase of λ_1, and its trend is also increasing and slow. The ranking orders of the decision making is always $A_3 > A_4 > A_1 > A_2$. In

Table 5: The ranking orders by proposed method when $k = 2$ for different λ_1 and λ_2

λ_1	λ_2	Score values of $S(d_i)$ $(i = 1, 2, 3, 4)$	Ranking orders
$\lambda_1 = 1$	$\lambda_2 = 1$	$S(d_1) = 4.2426$; $S(d_2) = 4.2424$; $S(d_3) = 4.2429$; $S(d_4) = 4.2428$	$A_3 > A_4 > A_1 > A_2$
$\lambda_1 = 1$	$\lambda_2 = 2$	$S(d_1) = 4.2426$; $S(d_2) = 4.2425$; $S(d_3) = 4.2429$; $S(d_4) = 4.2428$	$A_3 > A_4 > A_1 > A_2$
$\lambda_1 = 1$	$\lambda_2 = 3$	$S(d_1) = 4.2426$; $S(d_3) = 4.2425$; $S(d_3) = 4.2429$; $S(d_4) = 4.2428$	$A_3 > A_4 > A_1 > A_2$
$\lambda_1 = 1$	$\lambda_2 = 4$	$S(d_1) = 4.2427$; $S(d_2) = 4.2426$; $S(d_3) = 4.2430$; $S(d_4) = 4.2428$	$A_3 > A_4 > A_1 > A_2$
$\lambda_1 = 1$	$\lambda_2 = 5$	$S(d_1) = 4.2427$; $S(d_2) = 4.2426$; $S(d_3) = 4.2430$; $S(d_4) = 4.2429$	$A_3 > A_4 > A_1 > A_2$
$\lambda_1 = 2$	$\lambda_2 = 1$	$S(d_1) = 4.2426$; $S(d_2) = 4.2424$; $S(d_3) = 4.2430$; $S(d_4) = 4.2428$	$A_3 > A_4 > A_1 > A_2$
$\lambda_1 = 3$	$\lambda_2 = 1$	$S(d_1) = 4.2426$; $S(d_2) = 4.2425$; $S(d_3) = 4.2430$; $S(d_4) = 4.2428$	$A_3 > A_4 > A_1 > A_2$
$\lambda_1 = 4$	$\lambda_2 = 1$	$S(d_1) = 4.2426$; $S(d_2) = 4.2425$; $S(d_3) = 4.2430$; $S(d_4) = 4.2429$	$A_3 > A_4 > A_1 > A_2$
$\lambda_1 = 5$	$\lambda_2 = 1$	$S(d_1) = 4.2426$; $S(d_2) = 4.2425$; $S(d_3) = 4.2430$; $S(d_4) = 4.2429$	$A_3 > A_4 > A_1 > A_2$
$\lambda_1 = 2$	$\lambda_2 = 2$	$S(d_1) = 4.2426$; $S(d_2) = 4.2425$; $S(d_3) = 4.2430$; $S(d_4) = 4.2428$	$A_3 > A_4 > A_1 > A_2$
$\lambda_1 = 3$	$\lambda_2 = 3$	$S(d_1) = 4.2426$; $S(d_2) = 4.2425$; $S(d_3) = 4.2430$; $S(d_4) = 4.2428$	$A_3 > A_4 > A_1 > A_2$
$\lambda_1 = 4$	$\lambda_2 = 4$	$S(d_1) = 4.2427$; $S(d_2) = 4.2425$; $S(d_3) = 4.2430$; $S(d_4) = 4.2428$	$A_3 > A_4 > A_1 > A_2$
$\lambda_1 = 5$	$\lambda_2 = 5$	$S(d_1) = 4.2427$; $S(d_2) = 4.2425$; $S(d_3) = 4.2430$; $S(d_4) = 4.2428$	$A_3 > A_4 > A_1 > A_2$

other words, it is proved that our proposed method is stable because of the little influence of the parameter λ_1 on the final decision results.

In the three groups, we study the parameter λ_1 and λ_2 how to affect the results (assume $\lambda_1 = \lambda_2$). From the Fig. 4, it is obvious that the score values of the alternative charging pile are different calculated by different λ_1 and λ_2. In addition, the final ranking orders are the same. After the above analysis, it's a good proof that the parameters λ_1 and λ_2 have little influence on the result, and it illustrates the stability and effectiveness of our proposed method on solving the selection of the site of charging pile.

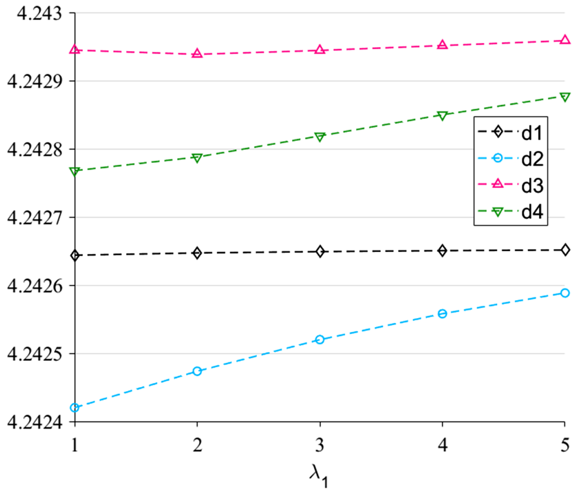

Figure 3: The ranking orders by proposed method when $k = 2$ for different λ_1

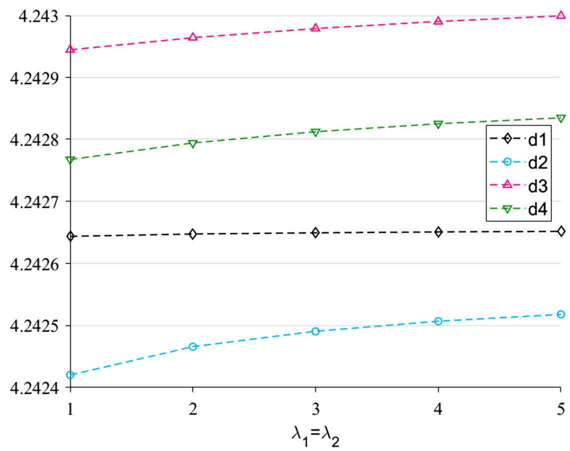

Figure 4: The ranking orders by proposed method when $k = 2$ for different λ_1 and λ_2

6.2.2. The impact of the parameter k and λ_k

In the following, we continue to explore the impact of the parameter k on the results. To do this, we assume different values of k and calculated the results which are depicted in Table 6. For convenience, we assume that $\lambda_1 = \lambda_2 = \ldots = \lambda_k = 1$. It is easy to find that different values k lead to different score values and different final ranking orders. From the Table 6, the ranking order of $k = 1$ is different form that of $k = 2, 3, 4$. However, the difference in the final ranking result is that the best choice is A_3 or A_4. Therefore, how to choose a appreciate value of k is a problem.

In our proposed method, when $k = 1$, we assume that the attributes (Scalability, Social factors, Construction factors, Economic factors) are independent. When $k = 2$, our method captures the interrelationship among any two attributes. When $k = 3$, the interrelationship among three attributes is taken into consideration in our method. When $k = 4$, our method considers the interrelationship among all four attributes. In reality, the attributes are always interrelated when solving the MADM problems. Therefore, our method provides a way to decide whether to consider the correlation between attributes according to actual needs through determine different values of k.

Table 6: The final results by the proposed method when $\lambda_1 = \lambda_2 = \ldots = \lambda_k = 1$ for different values of k

k	Score values of $S(d_i)$ $(i = 1, 2, 3, 4)$	Ranking orders
$k = 1$	$S(d_1) = 4.2531$; $S(d_2) = 4.2142$; $S(d_3) = 4.3236$; $S(d_4) = 4.3313$	$A_4 > A_3 > A_1 > A_2$
$k = 2$	$S(d_1) = 4.2426$; $S(d_2) = 4.2424$; $S(d_3) = 4.2429$; $S(d_4) = 4.2428$	$A_3 > A_4 > A_1 > A_2$
$k = 3$	$S(d_1) = 4.2426$; $S(d_2) = 4.2424$; $S(d_3) = 4.2429$; $S(d_4) = 4.2427$	$A_3 > A_4 > A_1 > A_2$
$k = 4$	$S(d_1) = 4.2079$; $S(d_2) = 4.1358$; $S(d_3) = 4.2684$; $S(d_4) = 4.2160$	$A_3 > A_4 > A_1 > A_2$

6.3. Comparative analysis

In this subsection, we introduce a comparative analysis between our method based on PMVLSFPWGMSM operator and methods proposed by Jin et al. [23] based on linguistic spherical fuzzy weight aggregation (LSFWA) operator and linguistic spherical fuzzy weight geometric aggregation (LSFWG) operator, the method proposed by Wang et al. [47] based on dual hesitant linguistic Pythagorean fuzzy weight average (DHFPWA) operator.

6.3.1. Compared with the LSFWA operator and LSFWG operator by Jin et al. [23]

In this part, we make a comparison between the proposed method based on PMVLSFPWGMSM operator and the methods based on LSFWA operator and LSFWG operator by Jin et al. [23]. The comparative analysis is divided into two parts as follows.

First, we use the three methods to solve the MAGDM problems presented by Jin et al. [23]. The original evaluation information is shown in the reference [23] and we omit it here. The score values and the final results obtained by the three methods are depicted in Table 7. From the Table 7, we can find that the score values obtained by different methods are different. Then, the ranking order calculated by the LSFWA operator and LSFWG operator are the same $R_3 > R_1 > R_4 > R_2$, and the ranking orders obtained by our proposed method is different. However, the optimal alternative is always A_3. As we all know, WA and WG operator lost the ability to capture the interrelationship among attributes. From the results shown in Table 7, we can find the results are different when the values of k are different. In our method, we can choose appreciate value of k to decide whether take the interrelation among attributes into consideration. Therefore, our method based on PMVLSFPWGMSM operator is more flexibility than the LSFWA and LSFWG operator.

Table 7: Final results of the MAGDM problem presented by Jin et al. [23] by different methods

Method	Score values	Ranking orders
LSFWA proposed by Jin et al. [23]	$S(R_1) = 5.306$ $S(R_2) = 4.890$ $S(R_3) = 5.393$ $S(R_4) = 5.139$	$R_3 > R_1 > R_4 > R_2$
LSFWG proposed by Jin et al. [23]	$S(R_1) = 4.477$ $S(R_2) = 4.073$ $S(R_3) = 4.489$ $S(R_4) = 4.310$	$R_3 > R_1 > R_4 > R_2$
Our proposed method ($k = 1, \lambda_1 = 1$)	$S(R_1) = 5.6959$ $S(R_2) = 5.6935$ $S(R_3) = 5.6975$ $S(R_4) = 5.6732$	$R_3 > R_1 > R_2 > R_4$
Our proposed method ($k = 2, \lambda_1 = \lambda_2 = 1$)	$S(R_1) = 5.6570$ $S(R_2) = 5.65694$ $S(R_3) = 5.65692$ $S(R_4) = 5.6568$	$R_1 > R_2 > R_3 > R_4$
Our proposed method ($k = 3, \lambda_1 = \lambda_2 = \lambda_3 = 1$)	$S(R_1) = 5.65692$ $S(R_2) = 5.65689$ $S(R_3) = 5.65686$ $S(R_4) = 5.6568$	$R_1 > R_2 > R_3 > R_4$
Our proposed method ($k = 4, \lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 1$)	$S(R_1) = 5.6668$ $S(R_2) = 5.6574$ $S(R_3) = 5.6497$ $S(R_4) = 5.6362$	$R_1 > R_2 > R_3 > R_4$

Second, we use the three methods to solve the MAGDM problems presented in our paper. The final results and ranking orders computed by the different methods are shown in Table 8. From the results in Table 8, it is easy to find that the LSFWA operator and LSFWG operator fail to solve our problems. It is because that the LSFWA operator and LSFWG operator can aggregate LSFSs but cannot to aggregation the PMVLSFSs. However, our method has the capability to aggregate the two fuzzy sets. In addition, the PMVLSFSs can express a wider range of evaluation information than LSFSs and the degree of hesitation and uncertainty will be better expressed. Therefore, our method is more effective than other method.

In summary, our method has a strong ability to aggregate fuzzy information and has strong flexibility and practicability when dealing with MAGDM problems.

Table 8: Final results of the MAGDM problem presented in this paper by different methods

Method	Score values	Ranking orders
LSFWA proposed by Jin et al. [23]	–	–
LSFWG proposed by Jin et al. [23]	–	–
Our proposed method ($k = 2, \lambda_1 = \lambda_2 = 1$)	$S(d_1) = 4.2426;\ S(d_1) = 4.2424;$ $S(d_1) = 4.2429;\ S(d_1) = 4.2428$	$A_3 > A_4 > A_1 > A_2$

6.3.2. Compared with the DHLPFWA operator by Wang et al. [47]

In the following, we use the two methods to solve the MAGDM problem introduced by Wang et al. [47]. The description and original evaluation information of the MAGDM problems are omitted here, which can be shown in original article. Then, the final results obtained by different methods are shown in Table 9. Although the results in Table 9 tells us that the score values obtained by the two different methods are different, and the final ranking orders are the same $A_3 > A_1 > A_2 > A_4$. The results show that our method can not only process the evaluation information expressed by PMVLSFs but also the evaluation information expressed by DHLPFs. Therefore, our method is more flexible in dealing with MADM problems than DHLPFWA proposed by Wang et al. [47].

Table 9: Final results of the MAGDM problem presented by Wang et al. [47] by different methods

Method	Score values	Ranking orders
DHLPFWA proposed by Wang et al. (2021)	$S(d_1) = 6.4920;\ S(d_2) = 6.0067;$ $S(d_3) = 7.6187;\ S(d_4) = 4.5884$	$A_3 > A_1 > A_2 > A_4$
Our proposed method ($k = 2, \lambda_1 = \lambda_2 = 1$)	$S(d_1) = 5.6569;\ S(d_2) = 5.6567;$ $S(d_3) = 5.6571;\ S(d_4) = 5.6564$	$A_3 > A_1 > A_2 > A_4$

6.3.3. Summary of characteristics of MAGDM methods

After the above analysis, it is proved that our proposed method based on PMVLSFPWGMSM operator is useful than existing methods such as LSFWA, LEFWG and DHLPFWA. On the one hand, the PMVLSFs we proposed can well describe the hesitation and uncertainty of decision makers in evaluation because it combines the characteristics of probabilistic fuzzy sets, linguistic spherical fuzzy sets. In addition, our method can not only be used in PMVLSFSs fuzzy environments, but also in LSFSs and DHFLPFSs fuzzy environments. This shows that our method has strong flexibility in aggregating evaluation information. On the other hand, our method based on the PMVLSFPWGMSM operator can handle the correlation among attributes and adjust the influence between related attributes. Because of the existence of the power average operator, our method can also weaken the influence of extreme values on the final result.

Table 10: The characteristic of different methods

Methods	Whether it depict the abstinence degree	Whether it depict the probability of the evaluation	Whether it permits the multiple MDs AMDs and NMDs	Whether it can eliminate the influence of extreme values
LSFWA proposed by Jin et al. [23]	Yes	No	No	No
LSFWG proposed by Jin et al. [23]	Yes	No	No	No
DHLPFWA proposed by Wang et al. [47]	No	No	No	No
Our proposed method based on PMVLSFPWGMSM operator	Yes	Yes	Yes	Yes

7. Conclusion

This paper introduces a novel MAGDM method under the environment of fuzzy sets which provides a new tool for the site selection of charging piles. The contribution of this paper can be divided into five parts. Firstly, a new fuzzy sets PMVLSFSs is proposed in this paper, which can provide a novel manner to express the experts' hesitation, vagueness, and personal preference of alternative. The existence of probability value can also describe the degree of recognition of decision-making experts for their own evaluation. Secondly, we explore new aggregation operators to calculate the information by using PMVLSFSs. The PMVLSFPGMSM and PMVLSFPWGMSM operators can eliminate the influence of extreme values, consider the interrelationship among attributes and perform an adjustment about the impact among related attributes. Thirdly, we put forward a procedure of MAGDM in PMVLSFSs environment based on PMVLSFPWGMSM operators. Fourthly, we discuss the elements which can affect the site selection of charging pile and apply the proposed MAGDM method to solve it.

Finally, a numerical case and a series of analysis of our method are performed to prove the effectiveness and robustness when solving problem.

In the future, our main research directions are mainly from three aspects. In the first aspect, we will continue to explore the application of linguistic fuzzy sets in traditional decision-making methods (Liao and Wu 2020; Liao et al, 2020) and apply them to actual MAGDM problems make the decision more reasonable. In the second aspect, we also will explore the group decision-making method in social network and analysis the relationship between the trust and decision results. Finally, we try to add big data into the decision-making process to promote the rationality and objectivity of the decision-making.

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