

# Steady state and transient analysis of solid oxide fuel cell within the framework of thermodynamic energy cycles

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**Abstract** The provided article comprehensively explores the modelling and analysis of solid oxide fuel cell (SOFC) systems within the context of thermodynamic energy cycles. The paper provides insight into various applications of these cells, with a specific emphasis on their role as the primary source of electrical energy in systems that work with biogas and heat recovery. The technological structure of these systems is delineated, with a focus on their principal components and the chemical reactions occurring within SOFCs. Moreover, the article incorporates a mathematical model of SOFCs and presents calculation results that illustrate the influence of air and fuel temperature on the cells' efficiency. The research indicates that optimal SOFC efficiency is attained at higher temperatures of supplied air and fuel. The presentation of the results of calculations for the solid oxide fuel cell and its thermodynamic cycle, considering fuel supply and its thermodynamic parameters under both steady-state and transient conditions, is the main aim of the article.

**Keywords:** SOFC; Transient model; Thermal cycle

## Nomenclature

$c_p$	–	specific heat at constant pressure, J/kg/K
$E$	–	reversible potential of each cell, V
$h$	–	molar enthalpy, J/kmol
$\Delta H_f^\circ$	–	standard enthalpy of formation for the respective chemical species, J/kmol
$I$	–	electric current, A

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$K$	-	equilibrium reaction constant
$M$	-	mass, kg
$\dot{n}$	-	molar flow rate, kmol/s
$\dot{Q}$	-	heat flow, W
$p$	-	pressure, Pa
$P$	-	theoretical power, W
$T$	-	temperature, K
$t$	-	time, s
$U$	-	voltage, V
$W$	-	ORC turbine power, W

### Subscripts

act	-	activation
cat	-	cathode
CH <sub>4</sub>	-	methane
CO	-	carbon monoxide
CO <sub>2</sub>	-	carbon dioxide
con	-	conversion
conc	-	concentration
cond	-	ORC condenser
cpo	-	catalytic partial oxidation of methane
dir	-	direct internal reforming reaction
evap	-	ORC evaporator
$fc$	-	fuel cell
H <sub>2</sub>	-	hydrogen
H <sub>2</sub> O	-	water
in	-	inlet
N <sub>2</sub>	-	nitrogen
O <sub>2</sub>	-	oxygen
ohm	-	Ohmic
out	-	outlet
prod	-	production
ref	-	reforming process
th	-	theoretically
wgs	-	water gas shift reaction

### Acronyms

ORC	-	organic Rankine cycle
SOFC	-	solid oxide fuel cell

## 1 Introduction

Thermodynamic cycles that consider the introduction of fuel cells have various accompanying systems in their structure [1]. Below, among many proposed applications, systems are presented where fuel cells constitute the



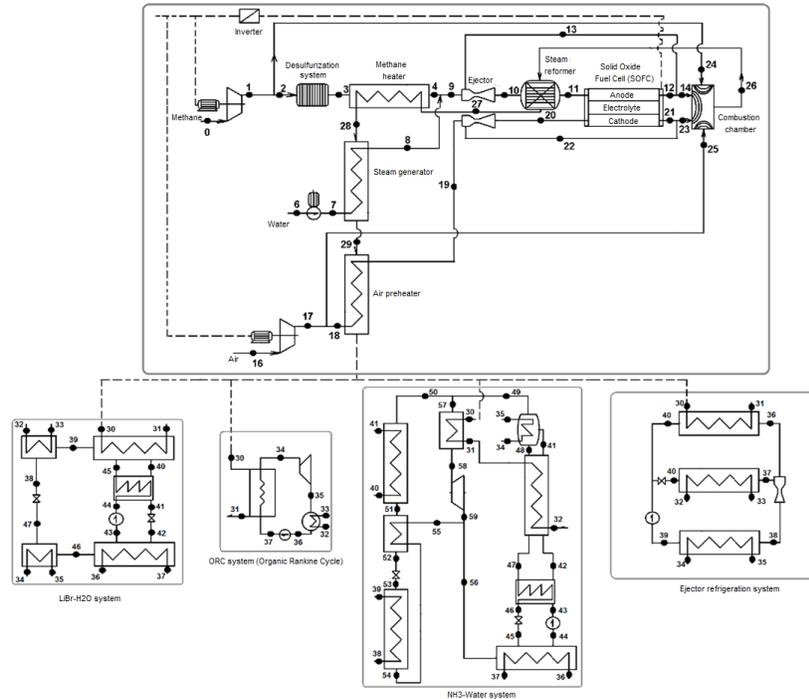


Figure 2: Trigeneration/polygeneration system with a fuel cell and its potential cooperation with absorption systems LiBr-H<sub>2</sub>O, NH<sub>3</sub>-H<sub>2</sub>O, ejector refrigeration cycle, and ORC.

The installation presented in Fig. 2 is constructed with main components such as:

- fermentation chamber and biogas desulfurization system; this chamber typically comprises a supply system, a heating system, a mixing system, a system for discharging post-fermentation mass, and a biogas disposal system;
- gas turbine containing an air compressor, combustion chamber, reforming system, fuel cell, and a turbine, where the expansion of the reaction products from the combustion chamber and fuel cell occurs. An additional system in this case is the compression system for biogas to the pressure required in the combustion chamber and fuel cell;
- ORC (organic Rankine cycle) with a steam generator, condenser, steam turbine with a generator, and circulating pump.

The arrangement in Fig. 2 consists of a fuel cell subsystem [2, 4] and/or, alternatively, subsystems for low-temperature heat recovery such as:

- ejector cooling system [3],
- bromide cooling system or heat pumps [5],
- hybrid ammonia cooling system [5],
- ORC system [2].

The fuel cell circuit subsystem comprises a sulfur removal, methane and air preheating, steam generation, steam reforming, solid oxide fuel cell, combustion chamber, and feed pumps and fans.

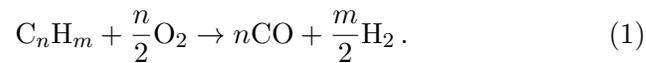
Additional systems for low-temperature heat recovery significantly enhance the energy efficiency of the entire SOFC-based system. To confirm this, calculations were performed for an SOFC cycle integrated with the ORC system. Below, the mathematical model of the fuel cell and the calculation results for the integrated cycle with the ORC system are presented.

## 2 Modelling of solid oxide fuel cell in steady state

The fuel cell cycle from a thermodynamic perspective is a flow system in which complex chemical reactions occur, leading through catalysis to the breakdown of fuel, generation of electric current, and chemical products of this reaction.

Thermodynamic processes that are essential for the creation of a fuel cell are described in more detail in references [6–9]. These processes include:

- Partial oxidation through catalysis (CPO - catalytic partial oxidation), which is realized according to the reaction [10]:



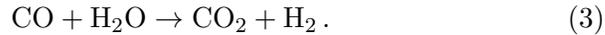
This reaction occurs within the temperature range of 800 to 900°C. Nickel can serve as a catalyst in this process. In this case, high purity gas without sulfur impurities is required, below 50 ppm.

- Steam reforming (DIR – direct internal reforming), also realized with the presence of a nickel catalyst within the temperature range of

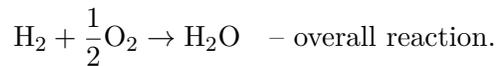
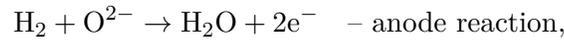
700 to 1100°C. During reforming, an endothermic reaction occurs, consuming heat [11]:



- Water gas shift reaction (WGS) – an exothermic reaction, producing heat [11]:



- Reaction in the solid oxide fuel cell (SOFC), which can be divided into local reactions occurring in the anode and cathode, yielding steam as the global product. In contrast to the proton-exchange membrane (PEM) fuel cell [8], in the SOFC, water is produced on the anode side according to the following reactions [7]:



In SOFCs,  $\text{Y}_2\text{O}_3$  yttria-stabilized  $\text{ZrO}_2$  is most commonly used as the electrolyte. The anode material consists of chemical compounds like  $\text{Co-ZrO}_2$  and  $\text{NiZrO}_2$ . The cathode material utilizes the chemical compound  $\text{Sr+LaMnO}_3$ .

To model the phenomena occurring in the SOFC, the following assumptions were made:

- The model describes a steady-state operation of the SOFC.
- Chemical reactions are modelled using molar mass and energy balances.
- The description of thermodynamic properties of reacting substances relies on Helmholtz equations.
- The processes are described in accordance with the isobaric potential.
- The systems are isolated.
- The reactions occur in an equilibrium manner.
- Issues related to the geometry and material properties of the fuel cell are omitted.

The equations describing the processes occurring in SOFC are presented below (with the symbols explained in the Nomenclature section):

- Molar nitrogen flux on the anode side

$$\dot{n}_{N_2,fc,out} = \dot{n}_{N_2,cpo,out} \cdot \quad (4)$$

- Molar nitrogen flux on the cathode side

$$\dot{n}_{N_2,cat,out} = \dot{n}_{N_2,cat,in} \cdot \quad (5)$$

- Molar methane flux during reforming

$$\dot{n}_{CH_4,ref} = \dot{n}_{CH_4,cpo,out} K_{fc,CH_4-H_2} \cdot \quad (6)$$

- Equilibrium constant for the reaction  $CH_4-H_2$

$$K_{fc,CH_4-H_2} = 0.95. \quad (7)$$

- Molar hydrogen flux during reforming

$$\dot{n}_{H_2,fc,ref} = 3\dot{n}_{CH_4,ref} \cdot \quad (8)$$

- Molar carbon monoxide flux during reforming

$$\dot{n}_{CO,fc,ref} = \dot{n}_{CH_4,ref} \cdot \quad (9)$$

- Molar water flux during reforming

$$\dot{n}_{H_2O,fc,ref} = \dot{n}_{CH_4,ref} \cdot \quad (10)$$

- Molar flow of carbon monoxide during steam conversion

$$\dot{n}_{CO,fc,wgs} = (\dot{n}_{CO,cpo,out} + \dot{n}_{CO,fc,ref}) K_{fc,CO-H_2} \cdot \quad (11)$$

- Equilibrium constant for the reaction  $CO-H_2$

$$K_{fc,CO-H_2} = 0.643. \quad (12)$$

- Molar flow of water vapour during carbon monoxide conversion

$$\dot{n}_{H_2O,fc,wgs} = \dot{n}_{CO,fc,wgs} \cdot \quad (13)$$

- Molar flow of hydrogen during carbon monoxide conversion with water vapour

$$\dot{n}_{\text{H}_2,fc,wgs} = \dot{n}_{\text{CO},fc,wgs} \cdot \quad (14)$$

- Molar flow of carbon dioxide during carbon monoxide conversion with water vapour

$$\dot{n}_{\text{CO}_2,fc,wgs} = \dot{n}_{\text{CO},fc,wgs} \cdot \quad (15)$$

- Total molar flow of hydrogen resulting from the conducted chemical reactions

$$\dot{n}_{\text{H}_2,\text{dir},\text{out}} = \dot{n}_{\text{H}_2,\text{cpo},\text{out}} + \dot{n}_{\text{H}_2,fc,\text{ref}} + \dot{n}_{\text{H}_2,fc,wgs} \cdot \quad (16)$$

- Total molar flow of carbon monoxide resulting from the conducted chemical reactions

$$\dot{n}_{\text{CO},\text{dir},\text{out}} = \dot{n}_{\text{CO},\text{cpo},\text{out}} + \dot{n}_{\text{CO},fc,\text{ref}} - \dot{n}_{\text{CO},fc,wgs} \cdot \quad (17)$$

- Total molar flow of water vapour resulting from the conducted chemical reactions

$$\dot{n}_{\text{H}_2\text{O},\text{dir},\text{out}} = \dot{n}_{\text{H}_2\text{O},\text{cpo},\text{out}} - \dot{n}_{\text{H}_2\text{O},fc,\text{ref}} - \dot{n}_{\text{H}_2\text{O},fc,wgs} \cdot \quad (18)$$

- Total molar flow of carbon dioxide resulting from the conducted chemical reactions

$$\dot{n}_{\text{CO}_2,\text{dir},\text{out}} = \dot{n}_{\text{CO}_2,\text{cpo},\text{out}} + \dot{n}_{\text{CO}_2,fc,wgs} \cdot \quad (19)$$

- Molar flow of methane resulting from the conducted chemical reactions

$$\dot{n}_{\text{CH}_4,\text{dir},\text{out}} = \dot{n}_{\text{CH}_4,\text{cpo},\text{out}} (1 - K_{fc,\text{CH}_4-\text{H}_2}) \cdot \quad (20)$$

- Outlet molar flow of methane from SOFC

$$\dot{n}_{\text{CH}_4,fc,\text{out}} = \dot{n}_{\text{CH}_4,\text{dir},\text{out}} \cdot \quad (21)$$

- Outlet molar flow of carbon monoxide from SOFC

$$\dot{n}_{\text{CO},fc,\text{out}} = \dot{n}_{\text{CO},\text{dir},\text{out}} \cdot \quad (22)$$

- Outlet molar flow of carbon dioxide from SOFC

$$\dot{n}_{\text{CO}_2,fc,\text{out}} = \dot{n}_{\text{CO}_2,\text{dir},\text{out}} \cdot \quad (23)$$

- Molar flow of hydrogen undergoing conversion in SOFC

$$\dot{n}_{\text{H}_2,fc,\text{con}} = \dot{n}_{\text{H}_2,\text{dir},\text{out}} \cdot \quad (24)$$

- Molar flow of hydrogen at the outlet of SOFC

$$\dot{n}_{\text{H}_2,fc,\text{out}} = \dot{n}_{\text{H}_2,\text{dir},\text{out}} - \dot{n}_{\text{H}_2,fc,\text{con}} \cdot \quad (25)$$

- Molar flow of water produced in SOFC

$$\dot{n}_{\text{H}_2\text{O},fc,\text{prod}} = \dot{n}_{\text{H}_2,fc,\text{con}} \cdot \quad (26)$$

- Molar demand for oxygen to facilitate the conversion of hydrogen fuel in SOFC

$$\dot{n}_{\text{O}_2,\text{catode},\text{con}} = 0.5\dot{n}_{\text{H}_2,fc,\text{con}} \cdot \quad (27)$$

- Molar flow at the cathode outlet of SOFC

$$\dot{n}_{\text{O}_2,\text{out},\text{catode}} = \dot{n}_{\text{O}_2,\text{catode},\text{in}} + \dot{n}_{\text{O}_2,\text{catode},\text{con}} \cdot \quad (28)$$

- Molar flow of produced water vapour at the anode of SOFC

$$\dot{n}_{\text{H}_2\text{O},\text{out}} = \dot{n}_{\text{H}_2\text{O},\text{dir},\text{out}} + \dot{n}_{\text{H}_2\text{O},\text{prod},\text{out}} \cdot \quad (29)$$

- Thermal potential equation describing the substances supplied to SOFC

$$\begin{aligned} \dot{Q}_{fc,\text{in}} = & \left[ \dot{n}_{\text{CH}_4,\text{in}} \left( \Delta H_{f,\text{CH}_4}^o + h_{\text{CH}_4,\text{in}} - h_{\text{CH}_4,\text{amb}} \right) \right. \\ & + \dot{n}_{\text{H}_2\text{O},\text{in}} \left( \Delta H_{f,\text{H}_2\text{O}}^o + h_{\text{H}_2\text{O},\text{in}} - h_{\text{H}_2\text{O},\text{amb}} \right) \\ & + \dot{n}_{\text{CO},\text{in}} \left( \Delta H_{f,\text{CO}}^o + h_{\text{CO},\text{in}} - h_{\text{CO},\text{amb}} \right) \\ & + \dot{n}_{\text{CO}_2,\text{in}} \left( \Delta H_{f,\text{CO}_2}^o + h_{\text{CO}_2,\text{in}} - h_{\text{CO}_2,\text{amb}} \right) \\ & + \dot{n}_{\text{H}_2,\text{in}} \left( \Delta H_{f,\text{H}_2}^o + h_{\text{H}_2,\text{in}} - h_{\text{H}_2,\text{amb}} \right) \\ & \left. + \dot{n}_{\text{N}_2,\text{in}} \left( \Delta H_{f,\text{N}_2}^o + h_{\text{N}_2,\text{in}} - h_{\text{N}_2,\text{out}} \right) \right]_{\text{anode}} \\ & + \left[ \dot{n}_{\text{O}_2,\text{in}} \left( \Delta H_{f,\text{O}_2}^o + h_{\text{O}_2,\text{in}} - h_{\text{O}_2,\text{amb}} \right) \right. \\ & \left. + \dot{n}_{\text{N}_2,\text{in}} \left( \Delta H_{f,\text{N}_2}^o + h_{\text{N}_2,\text{in}} - h_{\text{N}_2,\text{out}} \right) \right]_{\text{catode}} \cdot \quad (30) \end{aligned}$$

- Equation describing the thermal potential of substances exiting SOFC

$$\begin{aligned}
\dot{Q}_{fc,out} = & \left[ \dot{n}_{CH_4,out} \left( \Delta H_{f,CH_4}^o + h_{CH_4,out} - h_{CH_4,amb} \right) \right. \\
& + \dot{n}_{H_2O,out} \left( \Delta H_{f,H_2O}^o + h_{H_2O,out} - h_{H_2O,amb} \right) \\
& + \dot{n}_{CO,out} \left( \Delta H_{f,CO}^o + h_{CO,out} - h_{CO,amb} \right) \\
& + \dot{n}_{CO_2,out} \left( \Delta H_{f,CO_2}^o + h_{CO_2,out} - h_{CO_2,amb} \right) \\
& + \dot{n}_{H_2,out} \left( \Delta H_{f,H_2}^o + h_{H_2,out} - h_{H_2,amb} \right) \\
& + \dot{n}_{N_2,out} \left( \Delta H_{f,N_2}^o + h_{N_2,out} - h_{N_2,amb} \right) \Big]_{anode} \\
& + \left[ \dot{n}_{O_2,out} \left( \Delta H_{f,O_2}^o + h_{O_2,out} - h_{O_2,amb} \right) \right. \\
& + \dot{n}_{N_2,out} \left( \Delta H_{f,N_2}^o + h_{N_2,out} - h_{N_2,amb} \right) \Big]_{catode} . \quad (31)
\end{aligned}$$

- Theoretical power generated in the fuel cell

$$P_{fc} = \dot{Q}_{fc,in} - \dot{Q}_{fc,out} . \quad (32)$$

In Table 1, the input data for the calculations of the fuel cell are presented. The mathematical model of the fuel cell, expressed by Eqs. (4)–(32) was used to compute the fuel cell parameters. When calculating the fuel cell parameters, the inlet temperature of the working fluids to the fuel cell was assumed as the variable parameter.

The results of the calculations based on the model using input data are presented in Fig. 3. As a result of the conducted calculations, it was observed that the power produced in the fuel cell doubles, with an increase in the inlet temperature of the reacting substances by 200°C. This leads to the conclusion that both, the fuel and the oxidant need to be supplied to the SOFC at a high temperature in order to generate electrical power more efficiently. The high thermal parameters impose special requirements on the materials used for pipelines in the installation due to safety concerns and sealing at a pressure of 1.23 MPa.

In Fig. 3, the theoretical power achievable in the fuel cell, resulting from the first law of thermodynamics (specifically from Eqs. (29) to (34)) is presented. the electric current produced in the fuel cell is not directly and strictly related to the theoretical power because it depends on the molar hydrogen flow rate, Faraday's constant, and the material and geometric

Table 1: Input data for fuel cell calculations.

Molar inflow rates of components at the fuel cell anode for the calculations presented in Fig. 3			
Substance	Symbol	Unit	Value
CH <sub>4</sub>	$\dot{n}_{\text{CH}_4,\text{in}}$	kmol/s	0.000001110
H <sub>2</sub> O	$\dot{n}_{\text{H}_2\text{O},\text{in}}$	kmol/s	0.000007844
CO	$\dot{n}_{\text{CO},\text{in}}$	kmol/s	0.000002848
H <sub>2</sub>	$\dot{n}_{\text{H}_2,\text{in}}$	kmol/s	0.000032700
N <sub>2</sub>	$\dot{n}_{\text{N}_2,\text{in}}$	kmol/s	0.000010610
CO <sub>2</sub>	$\dot{n}_{\text{CO}_2,\text{in}}$	kmol/s	0.000003474
Molar inflow rates of components at the cathode inlet of the fuel cell during calculations			
N <sub>2</sub>	$\dot{n}_{\text{N}_2,\text{in}}$	kmol/s	0.0002324
O <sub>2</sub>	$\dot{n}_{\text{O}_2,\text{in}}$	kmol/s	0.0003850

Thermodynamic parameters of SOFC			
Parameter	Symbol	Unit	Value
Inlet and outlet pressure on the cathode and anode sides	$p_{\text{in,K}}, p_{\text{in,A}}, p_{\text{out,K}}, p_{\text{out,A}}$	Pa	1231000
Temperature of the fuel and oxidant at the inlet to the fuel cell	$T_{\text{in,A}}, T_{\text{in,K}}$	K	800–1000
Temperature of the substances at the outlet from the fuel cell	$T_{\text{out,K}}, T_{\text{out,A}}$	K	1023
Standard molar enthalpy of formation CO	$\Delta H_{f,\text{CO}}^\circ$	J/kmol	-110 530 000
Standard molar enthalpy of formation CO <sub>2</sub>	$\Delta H_{f,\text{CO}_2}^\circ$	J/kmol	-393 520 000
Standard molar enthalpy of formation H <sub>2</sub> O	$\Delta H_{f,\text{H}_2\text{O}}^\circ$	J/kmol	-285 900 000
Standard molar enthalpy of formation CH <sub>4</sub>	$\Delta H_{f,\text{CH}_4}^\circ$	J/kmol	-74 850 000

properties of the fuel cell itself. Both material properties and geometric parameters of the fuel cell have an impact on the actual power obtained in the fuel cell. However, a detailed description of these issues significantly exceeds the scope of thermodynamics and falls within the realm of materials engineering and electrochemistry [12–14].

Theoretically, the total electrical current can be determined by the equation

$$\dot{n}_{\text{H}_2}2F = 4\dot{n}_{\text{O}_2}F = I. \quad (33)$$

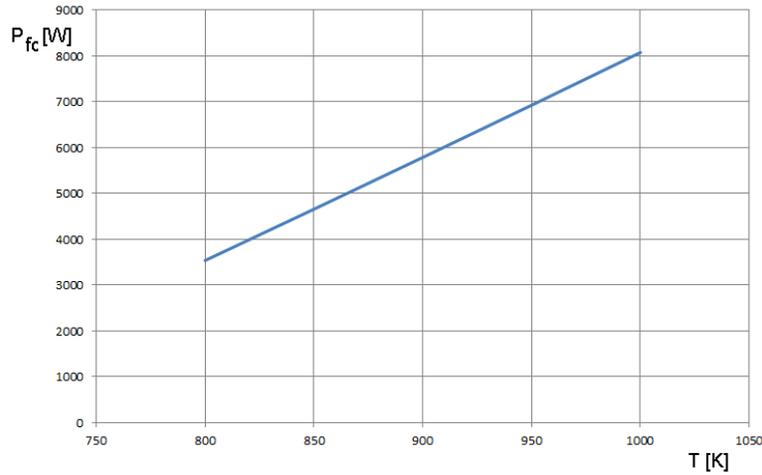


Figure 3: Calculations of the theoretical power obtained in the fuel cell due to the change in the inlet temperature of the working fluids.

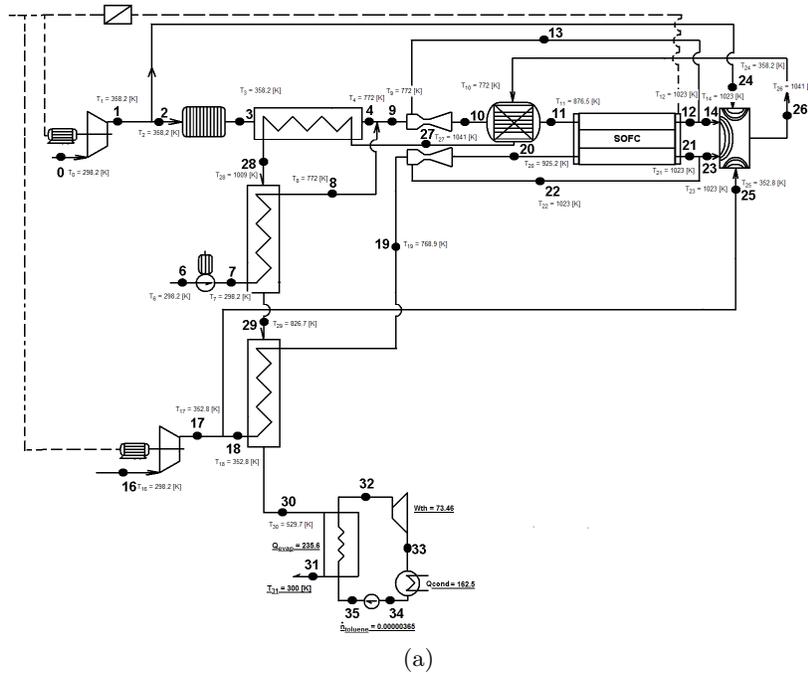
The maximum theoretical voltage that can be achieved from the equation

$$UI\eta = P_{fc}, \quad (34)$$

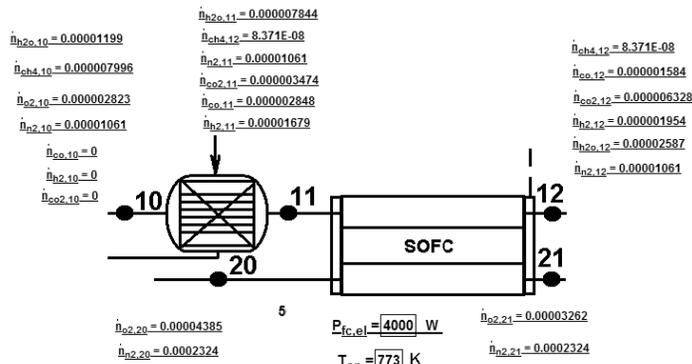
where  $\eta$  represents the overall efficiency of the SOFC cell, incorporating material properties, electrochemical effects, and more.

In addition to the calculations based on the previously presented mathematical model of the SOFC, the overall system depicted in Fig. 2 was also balanced. In this case, thermal parameters and molar flow rates of reactants at the outlets of the reforming systems and the SOFC were obtained (Fig. 4). As an illustrative solution for heat recovery, an ORC system was employed in this cycle.

The high material requirements for constructing installations with SOFCs contribute to high investment costs. The challenges in designing such installations are not only related to the SOFC system itself but also to its accompanying sub-systems (reformer, heat exchangers, purification and desulfurization systems). These challenges and the associated high costs may arise during the next stages of research, but ultimately, a highly efficient source of electricity production can be achieved. Due to the significant potential for using SOFCs in Europe, implementation-oriented research is already being conducted for SOFC installations with capacities up to 5 kW [15].



(a)



(b)

Figure 4: Exemplary thermal parameters in the fuel cell circuit working with a waste heat recovery system in the form of an ORC system: (a) solid oxide fuel cell (SOFC) circuit; (b) fuel cell parameters:  $T_0 = 298.2 \text{ K}$ ,  $T_1 = 358.2 \text{ K}$ ,  $T_2 = 358.2 \text{ K}$ ,  $T_3 = 358.2 \text{ K}$ ,  $T_4 = 772 \text{ K}$ ,  $T_6 = 358.2 \text{ K}$ ,  $T_7 = 298.2 \text{ K}$ ,  $T_8 = 772 \text{ K}$ ,  $T_9 = 772 \text{ K}$ ,  $T_{10} = 772 \text{ K}$ ,  $T_{11} = 876.5 \text{ K}$ ,  $T_{12} = 1023 \text{ K}$ ,  $T_{14} = 1023 \text{ K}$ ,  $T_{16} = 298.2 \text{ K}$ ,  $T_{17} = 352.8 \text{ K}$ ,  $T_{18} = 352.8 \text{ K}$ ,  $T_{19} = 768.9 \text{ K}$ ,  $T_{20} = 925.2 \text{ K}$ ,  $T_{21} = 1023 \text{ K}$ ,  $T_{22} = 1023 \text{ K}$ ,  $T_{23} = 1023 \text{ K}$ ,  $T_{24} = 358.2 \text{ K}$ ,  $T_{25} = 352.8 \text{ K}$ ,  $T_{26} = 1041 \text{ K}$ ,  $T_{27} = 1041 \text{ K}$ ,  $T_{28} = 1009 \text{ K}$ ,  $T_{29} = 826.7 \text{ K}$ ,  $T_{30} = 529.7 \text{ K}$ ,  $T_{31} = 300 \text{ K}$ ,  $Q_{evap} = 235.6 \text{ W}$ ,  $W_{th} = 73.45 \text{ W}$ ,  $Q_{cond} = 162.5 \text{ W}$ ,  $\dot{m}_{toluene} = 0.00000365 \text{ kmol/s}$ .

### 3 Solid oxide fuel cell modelling under transient conditions

In the previous chapter, we discussed the steady-state model of the SOFC. In this chapter, we will describe the mathematical model of the SOFC and its computations for unsteady-state conditions. The mathematical model is similar to the one presented in the previous chapter. However, in contrast to the model described in the first part of this article, we have relaxed the assumption that the model is time-invariant in order to model the phenomena occurring in the SOFC. In the mathematical model presented in this chapter, we assume that SOFC operates under unsteady-state conditions over time.

For the SOFC, the general dynamics equation takes the following form:

$$\frac{d(TM c_p)}{dt} = \dot{Q}_{fc,in} - P_{fc} - \dot{Q}_{fc,out} . \quad (35)$$

If we assume that reacting masses and their specific heats do not change over time, the equation takes the form [7]:

$$M c_p \frac{dT}{dt} = \dot{Q}_{fc,in} - P_{fc} - \dot{Q}_{fc,out} . \quad (36)$$

Furthermore, to ascertain the electrical power of the fuel cell at a specific current density ( $I = 80$  A), an equation describing the cell voltage has been defined [7]:

$$U_{out} = E - U_{act} - U_{ohm} - U_{conc} . \quad (37)$$

Mathematical relationships describing individual components of Eq. (37) have been presented in [7]. Below, the results of calculations for the fuel cell operating temperature and the assumed temperature of gases supplied to the SOFC are shown (Fig. 5), as well as the values of voltage and current in the fuel cell (Fig. 6), and the power generated in the fuel cell (Fig. 7).

From the presented calculation results in Figs. 5 to 7, it is evident that the variation in the temperature of air supplied to the SOFC system has an impact on the generated power in the SOFC. Altering the temperature of supplied air and fuel affects the electrical current voltage and, consequently, the power generated by the cell. The calculations indicate that a lower air temperature leads to a significant deterioration in the electrical power output of the SOFC. The best operational parameters for the SOFC, as indicated by the theoretical analysis, are achieved when both the temperature of the supplied fuel and air are high and do not deviate significantly from each other.

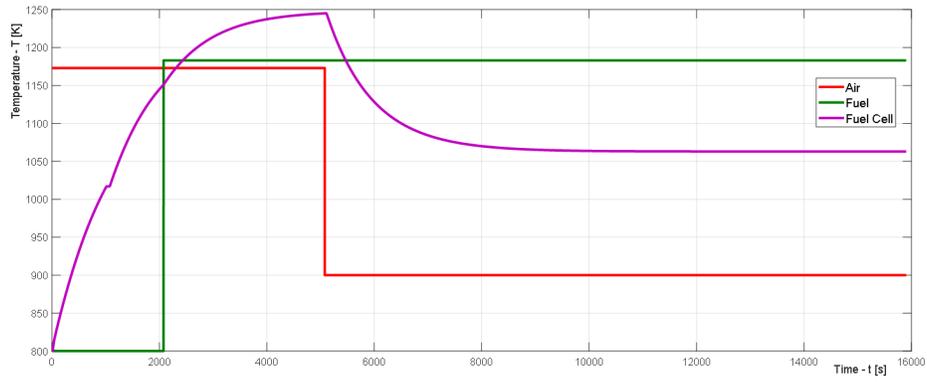


Figure 5: Temperatures of the fuel cell and gases supplied to the SOFC: Air – input air temperature, Fuel – input of fuel temperature, Fuel Cell – the result of the fuel cell temperature calculations.

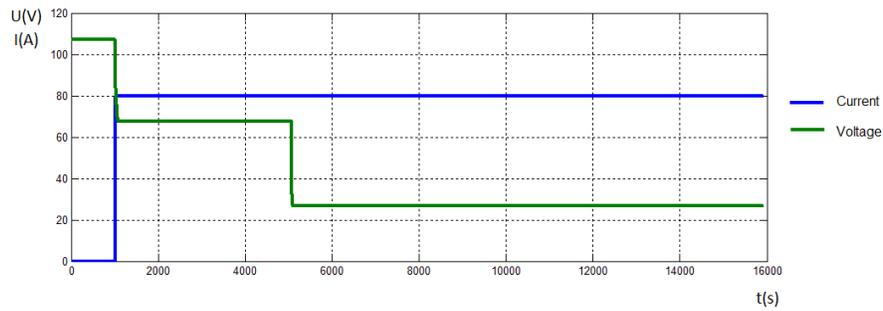


Figure 6: Voltage and current produced in the fuel cell.

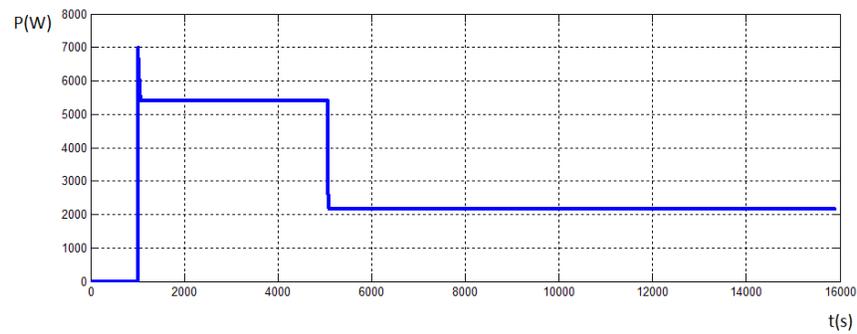


Figure 7: Power produced in the SOFC.

## 4 Summary

The article provides information on the modelling and analysis of a system based on solid oxide fuel cells (SOFCs) in the context of thermodynamic energy cycles. The paper discusses various aspects of systems related to fuel cells, including the use of biogas and heat recovery. It describes system configurations, major components, and chemical reactions occurring in SOFCs.

The paper also presents a mathematical model of SOFCs and provides calculation results regarding the electrical power generated by these cells depending on the temperature of the air and fuel supplied to the system. These results suggest that the optimal performance of SOFCs is achieved at high temperatures for both air and fuel.

It is important to emphasize that investments in SOFC installations can be costly due to high material requirements and the need to consider various subsystems. However, in the long term, they can be an efficient way to produce electrical energy.

The paper presents a comprehensive analysis of SOFCs and their role in thermodynamic energy cycles, taking into account various factors affecting their performance and their potential as a source of electrical energy.

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