AN INTEGRATION METHOD OF A HYBRID GENETIC ALGORITHM AND
THE LEVENBERG–MARQUARDT ALGORITHM FOR ULTRASONIC TESTING

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Abstract

The accurate measurement of time-of-flight (TOF) is essential in ultrasonic testing. Further, noise interference is the key factor affecting the measurement accuracy. Therefore, to develop a reliable computational method of TOF for test pieces working in noisy environments, an integration method of a hybrid genetic algorithm and the Levenberg–Marquardt algorithm (GA–LM) for ultrasonic thickness measurement is proposed in the present research. A Gaussian model is first established for an echo signal. Further, the model-based parameter estimation is converted into a nonlinear optimization problem by applying the least square method. As the parameter estimation methods are easily affected by the initial value, an integrating innovation of the GA–LM algorithm is proposed. The initial values of the model parameters are selected by GA to obtain an approximate global optimal solution. Subsequently, this approximate solution is used as the initial value for the LM algorithm to perform iterations. The accurate global optimal solution of the Gaussian model is obtained through these iterations. Finally, the measuring accuracy and robustness of the GA–LM algorithm for TOF computation are verified by both numerical simulation and experiment data.

Keywords: time-of-flight, parameter estimation, Genetic algorithm, Levenberg–Marquardt algorithm, measuring accuracy.

1. Introduction

Online monitoring of high-temperature pressure vessels and piping using ultrasonic testing is crucial for ensuring their safe operation. The online monitoring technology using waveguide transducer provides a new approach to solve the problem of long-term online monitoring of equipment in high-temperature environments [1]. However, the signal received from test pieces working in noisy environments often contains interference noise, resulting in a low signal-to-noise ratio (SNR) of the useful ultrasonic signal and limiting the computational accuracy of time-of-flight (TOF) [2]. Therefore, the computational accuracy of TOF is a major issue that should be addressed to improve the accuracy of ultrasonic testing. Generally, TOF is determined when the received
signal exceeds a given threshold value for the first time, which is called the threshold method [3]. It can be calculated as the difference of peak time between two adjacent echoes [4]. In addition, it can also be estimated as the difference in the propagation time between the maximums of the envelopes of adjacent echoes [5]. However, for an ultrasonic signal including a sufficiently low SNR, these approaches may lead to great errors in TOF calculation [6].

Previous research discovered that a model-based parameter estimation method with a strong anti-interference ability could be adopted to address this issue [7, 8]. In this method, an ultrasonic echo model is first established, and then, the model parameters are fitted using the collected discrete signal data points. Characteristic information of a waveform could be obtained from the model parameters, which contain highly accurate TOF information. Jiao et al. proposed a novel scheme named ABIDE, which combines complementary ensemble empirical mode decomposition and the synchrosqueezed wavelet transform as well as the expectation maximization algorithm for parameter estimation of ultrasonic echo signals. Simulation and experimental results show that the scheme can maintain the low-frequency characteristics of an echo signal and reduce the influence of noise interference on the parameter estimation accuracy [9]. Lu et al. proposed a method for TOF estimation based on envelopes. In this method, a mathematical function model of the envelop of an echo signal is established. Then a modified Gauss–Newton (GN) algorithm is used to iteratively solve the parameters of envelope function model. However, iterations lead to local convergence or nonconvergence problems [10]. Wang et al. proposed an ultrasonic TOF calculation method based on a nonlinear echo envelope model. In this method, an objective function with optimized least squares is first established by modelling an ultrasonic echo signal, and then, signal parameters are optimized using an improved particle swarm optimization algorithm to obtain accurate TOF [11].

The parameter-estimation method involves, in essence, a global optimization problem wherein a suitable search algorithm is utilized to obtain the global optimal solution of the model parameters. Generally, the optimization algorithms can be roughly divided into two groups: classical gradient-based methods and relatively modern non-gradient-based methods [12, 13]. Gradient-based methods, such as the Levenberg–Marquardt (LM) algorithm, are more sensitive to initial values. It is easy to fall into a local minimum when the initial value is too far away from the optimal solution. The latter category, such as the genetic algorithm, offers great robustness in finding the global minimum. However, it obtains high-precision solutions by setting high search conditions, which means that more computing time is sacrificed. Jin et al. proposed an automated optimization method based on the LM algorithm for electron-optical systems, which combines an adaptive optimal value function switching process to enhance minimum convergence. The robustness of the method is demonstrated, and the procedure can greatly enhance efficiency in the design process of electron-optical systems [14]. Gholami et al. proposed a hybrid particle swarm optimization (PSO) and LM algorithms for estimating the parameters of the asymmetric Gaussian chirplet model used for modelling echoes. PSO first provides a rough estimate, and the output is consequently inputted to the LM algorithm for more accurate estimation. The accuracy and reliability of the combined algorithm are corroborated by the experiment [8]. Compared to several other TOF measurement methods [15], the parameter estimation method demonstrates a high measurement accuracy and anti-interference ability [16]. Considering the noise interference in industrial environments and disregarding the real-time calculation requirement, an accurate TOF calculation method is established in this study based on parameter estimation. In addition, the innovative integration of a genetic algorithm and the Levenberg–Marquardt algorithm (GA–LM) is proposed to improve estimation results affected by initial values and leading to a local optimum.
2. Echo signal model and parameter estimation

2.1. Gaussian echo model for thickness measurement

As regards online monitoring systems using waveguide transducers, the characteristics of the ultrasonic transducer and the structure of the test pieces have a great influence on the ultrasonic echo. Furthermore, a significant nonlinear phenomenon is observed in the distortion and attenuation of ultrasonic echo due to the influence of noise and materials. Therefore, a non-linear model can be established for the echo signal [7]. The Gaussian model is commonly used [17], which could be expressed as:

\[ s(\theta; t) = \beta e^{-\alpha(t-\tau)^2} \cos[2\pi f_c(t - \tau) + \phi], \]

(1)

where \( \theta = [\alpha, \beta, \tau, f_c, \phi] \) is the eigenvector in the ultrasonic echo, \( \alpha \) is the bandwidth factor, \( \beta \) is the amplitude factor, \( \tau \) is the arrival time, \( f_c \) is the centre frequency and \( \phi \) is the phase. The Gaussian model describes the envelope of the echo signal \( z(\Theta; t) \) as:

\[ z(\Theta; t) = \beta e^{-\alpha(t-\tau)^2}, \]

(2)

where \( \Theta = [\alpha, \beta, \tau] \) is the envelop parameter.

In the process of practical measurement, the echo signal is inevitably affected by additive noise, and thus, the ultrasonic model could be rewritten as [18]:

\[ x(t) = s(\theta; t) + n(t), \]

(3)

where \( n(t) \) is the additive white Gaussian noise (WGN) component, and \( x(t) \) denotes the model corrupted by noise.

Equation (3) could be further extended to a multiple echo model to represent echoes from a known number of reflectors. Assuming a Gaussian echo for each reflector, the received signals could be modelled by \( M \)-superimposed Gaussian echoes:

\[ y(t) = \sum_{i=1}^{M} s(\theta_i; t) + n(t), \]

(4)

where \( \theta_i = [\alpha_i, \beta_i, \tau_i, f_{ci}, \phi_i] \) defines the shape and location of the corresponding echo.

By using (4), TOF could be obtained by taking the difference among the arrival time parameter \( \tau_i \) of any two adjacent echoes. Thereafter, the precise TOF calculation is transformed into a problem of accurately estimating parameter vector \( \theta \), given observed vector \( x(t) \). The schematic of the Gaussian echo model and envelope when \( M = 3 \) is shown in Fig. 1, where \( TOF \) equals to \( \tau_3 \) minus \( \tau_2 \).

The transformation from signal to parameter space is a multi-dimensional parameter estimation problem. Owing to the non-linear functional relationship between the observation and parameter vectors, an analytical solution cannot be determined easily. Moreover, the presence of noise interference in real environments further complicates the process of parameter estimation [19]. According to the maximum likelihood criterion, the problem can be solved by constructing a least squares function as follows:

\[ f(\theta) = \sum_{n=1}^{N} [s(\theta; nT_s) - x(nT_s)]^2, \]

(5)

where \( s(\theta; nT_s) \) is the discrete expression of \( s(\theta; t) \), \( T_s \) is the sampling interval, \( nT_s \) is the actual measured echo signal and \( N \) is the number of samples fitted to the model.
From (5), the parameter estimation is transformed into a nonlinear optimization problem. That is, the parameter vector $\theta$ that minimizes the objective function $f(\theta)$ must be determined. For this, function approximation algorithms in optimization search, such as the gradient descent and GN methods, can be used. The estimated vector is obtained by providing an initial value and using the iterative algorithm. The minimum value of the objective function provides the optimal solution. However, local minima may also exist. Depending on the initial value, the iteration may remain in one of the local minima, resulting in a suboptimal solution. To address this problem, we propose the use of the GA–LM algorithm. First, GA is used to find the global rough solution of the optimization problem in the entire parameter domain. Thereafter, the obtained search result is taken as the initial value, and the LM algorithm, with its extraordinary local search capability, is utilized to find the exact optimal solution of the model [20]. The GA–LM algorithm integrates the global search capability of GA and the local rapid search capability of LM, thus effectively reducing the impact of initial-value selection on the optimal solution.

2.2. The genetic algorithm and the Levenberg–Marquardt algorithm (GA–LM)

The GA simulates the process of biological evolution in nature through artificial evolution, thus achieving a global search of the target space. In addition, the solutions in GA termed as chromosomes are updated using the selection, crossover, and mutation operations to achieve the optimal solution [21,22]. By using this algorithm, the selection operation is applied to find the best candidate according to the fitness value, and then the crossover and mutation operations are applied to update the solution. Thereafter, the optimal solution is obtained through continuous iteration. The basic process of GA is as follows:

Step 1: Choose an encoding strategy. The parameters of the problem space need to be converted to genetic space solutions through real-number coding, Gray coding, binary coding, etc., as GA cannot directly use the value of an individual. Therefore, considering the simplicity of subsequent genetic operators, real number coding is adopted.

Step 2: Initializing the population. A random method is used to generate a specified number of individuals as the initial population in the feasible domain of the parameters. The population size is set to $M$, and population $P(t)$ comprises $M$ individuals:

$$P^{(t)} = \{\alpha(i), \tau(i), \beta(i), f_c(i), \varphi(i)\}_{i=1,\ldots,M}, \quad (6)$$

where $t$ is the population algebra, $P^{(1)}$ is the initial population. Each parameter in (6) is defined in (1), and obtained by taking a random value in the respective feasible domain.
Step 3: Calculating the fitness function. For the objective function shown in (5), the fitness value depends on the variance sum of the echo signal and the reconstructed signal. The fitness function suitable for this problem is constructed as follows:

\[ F(i) = \frac{1}{f(i)}, \quad 1 \leq i \leq M, \quad (7) \]

where \( M \) is the number of populations and \( f(i) \) is the sum of variances obtained for the first group of individuals by (5).

Step 4: Execution of the selection, crossover, and mutation operations. In this study, the roulette wheel selection method is adopted, which is a drop-in random sampling method. Two individuals are first randomly selected from the population and their combined chromosomes are exchanged. Then the excellent individuals are produced by inheriting the excellent characteristics of the parents to the offspring [23]. Furthermore, another group of individuals is randomly selected from the population, and a point mutation is applied to one of the selected individuals to introduce further improvements and enhance population diversity.

Step 5: Perform iterative calculation. New individuals of the population are continuously generated through Step 4 until the population performance satisfies a certain index or the iteration termination condition is met.

However, due to the randomness introduced by each genetic operator, the results of each execution have a certain deviation. To improve the local convergence, the LM algorithm is introduced to achieve an optimal solution. It is a non-linear least squares algorithm in the gradient search method. It combines the advantages of gradient descent and the Gauss–Newton method [24]. The iteration formula is:

\[ [H(\theta^{(k)}) + \mu I] \Delta \theta^{(k)} = -J(\theta^{(k)})^T \varepsilon_k, \quad (8) \]

where \( \mu \) is the penalty factor, \( I \) is the unit matrix, \( \theta^{(k)} \) is the parameter vector for the \( k \)-th iteration, \( \varepsilon_k = x - s(\theta^{(k)}) \) is the estimation error, where \( x \) is the actual measured echo signal and \( s(\theta^{(k)}) \) is the estimated echo signal, \( J(\theta^{(k)}) \) is the Jacobi matrix of the ultrasonic echo waveform:

\[ J(\theta^{(k)}) = \left( \frac{\partial s_\theta(t)}{\partial \alpha} \frac{\partial s_\theta(t)}{\partial \beta} \frac{\partial s_\theta(t)}{\partial \tau} \frac{\partial s_\theta(t)}{\partial f_c} \frac{\partial s_\theta(t)}{\partial \phi} \right), \quad (9) \]

\( H(\theta^{(k)}) \) is the Hessian matrix, which is composed of second-order partial derivatives with respect to the parameters. To simplify the operation, the second partial derivatives are omitted and solved directly using the following equation:

\[ H(\theta^{(k)}) = J(\theta^{(k)})^T J(\theta^{(k)}). \quad (10) \]

In this study, the GA–LM algorithm is used for iterative calculations, as detailed in the following steps:

Step 1: Set the population size \( M \), population algebra \( gen = 1 \), and randomly generate the initial population \( P^{(1)} = \{ \alpha(1), \beta(1), \tau(1), f_c(1), \phi(1) \} \).

Step 2: Algebraically increase the population, randomly initialize the number of the \( gen \)-th generation population.

Step 3: Calculate the fitness values of all individuals in the previous generation population. The chromosomes of each population are then selected, crossed, and mutated to produce the next generation, according to the fitness value.
Step 4: Calculate the fitness value $F(i)$ of the $i$-th individual to find the optimal individual.
Step 5: Determine whether the iteration ends. If not, return to Step 2.
Step 6: Use the optimal value obtained by GA as the iteration initial value $\theta^{(0)}$ of the LM algorithm and set the number of iterations $k = 0$.
Step 7: Calculate the Hessian matrix $H(\theta^{(k)})$ and model parameters $s(\theta^{(k)})$, and perform iterative operations using (8).
Step 8: $k = k + 1$, judge the convergence condition, if $\|\epsilon^{k+1} - \epsilon^k\| \leq$ error, the iteration ends, otherwise return to Step 7. The essential steps of the GA–LM algorithm are summarized as the pseudo code given in Algorithm 1.

Algorithm 1. Pseudocode of GA–LM Algorithm

1. Set the population size and randomly initialize population.
2. Evaluate the fitness value.
3. While the convergence condition is not attained
4. Perform crossover and mutation operations to produce the next generation.
5. Evaluate the fitness values of all individuals in the previous generation.
6. Select the optimal individual.
7. End
8. Use the optimal value obtained by GA as the initial value of the LM algorithm.
9. While the convergence condition is not attained
10. Calculate the Hessian matrix and model parameters.
11. Perform iterative operations using (8).
12. End

Through the iteration of the GA–LM algorithm, the characteristic parameters of the ultrasonic echo, including the arrival time $\tau$, could be obtained directly from the optimal solution $\theta_{\text{best}} = [\alpha, \beta, \tau, f_c, \phi]$. Meanwhile, the ultrasonic signal $s(\theta_{\text{best}}; t)$ is reconstructed according to (1) using the characteristic parameters.

3. Reliability verification of the GA–LM algorithm using numerical simulation

To verify the parameter-estimation accuracy of the GA–LM algorithm, the following numerical simulations are performed. They are executed in MATLAB 2020a. The two ultrasonic echo signals are first set to simulate the adjacent echoes received by the transducer, then the actual echo signals are simulated by adding white Gaussian noise of different sizes, and finally, the initial parameters and convergence conditions are set for the GA–LM algorithm. The parameters of the first echo are as follows: the bandwidth factor $\alpha_1 = 2.5$ (MHz)$^2$, the amplitude $\beta_1 = 1$, the arrival time $\tau_1 = 2.5$ $\mu$s, the centre frequency $f_{c1} = 2$ MHz and the phase $\phi_1 = 1$ rad, the parameters of the second echo are set as follows: the bandwidth factor $\alpha_2 = 2.5$ (MHz)$^2$, the amplitude $\beta_2 = 0.5$, the arrival time $\tau_2 = 7.5$ $\mu$s, the centre frequency $f_{c2} = 2$ MHz and the phase $\phi_2 = 1$ rad. Therefore, the parameter vector $\theta_1 = [2.5, 1, 2.5, 2, 1]$, $\theta_2 = [2.5, 0.5, 7.5, 2, 1]$, TOF = $\tau_2 - \tau_1 = 5$ $\mu$s. The signal is sampled at the sampling frequency $f_s = 200$ MHz. Moreover, white Gaussian noise of
different sizes is added to the echo signal with SNRs of 30, 20, and 10 dB. The initial parameter settings of GA are as follows: the population size is 600, the maximum genetic algebra is 100, the crossover probability is 0.9, the mutation probability is 0.1 and the convergence condition is that the error is less than $10^{-6}$. In addition, the convergence condition of the LM algorithm is that the difference between the objective functions calculated in the previous and subsequent iterations is less than $10^{-5}$. The simulated and estimated echoes are shown in Fig. 2.

![Fig. 2. Time domain waveforms of the simulated and estimated echo, (a) Noise-free echo, (b) Echo with a SNR of 30 dB, (c) Echo with a SNR of 20 dB, (d) Echo with a SNR of 10 dB.](image)

Figure 2 shows that the simulated signal is basically restored by the estimated signal. The original signal under noise-free conditions could be accurately estimated using the GA–LM algorithm. As SNR decreases, the waveform of the echo signal significantly oscillates, resulting in a decrease in the accuracy of parameter estimation. Overall, the proposed algorithm demonstrates a strong anti-interference ability. The echo signal in Fig. 2d can be estimated accurately even when considerable noise is present.

Table 1 lists the estimation results of parameters $\tau_1$ and $\tau_2$, and TOF can be obtained by calculating their difference. The GA–LM algorithm yields considerably low errors under high SNR conditions; in addition, under low SNR conditions (SNR = 10 dB), the estimation errors can be kept at less than 0.5%.

Owing to the randomness of noise, the results of each estimation may not necessarily be the same. To further verify the accuracy of TOF calculated using the GA–LM algorithm, four groups of samples with different thicknesses are simulated. The TOF of each sample is respectively
simulated as 5 µs, 7.5 µs, 10 µs, and 12.5 µs. Further, noisy signals at the SNR from 5 dB to 30 dB (step size of 5 dB) are considered. This procedure is repeated 30 times, and the error values are compared with the threshold (TH) method, the envelope peak (EP) method, and GA. The results are shown in Fig. 3.

As shown in the figure, all four algorithms demonstrate a high calculation accuracy under high SNR conditions. However, the errors of EP and TH increase with decreasing SNR, whereas GA and GA–LM can maintain high calculation accuracies. In addition, the GA–LM algorithm overcame the shortcoming that GA cannot accurately iterate locally; thus, the results obtained
using GA–LM are more accurate than those obtained using only GA. The average running time of TH, EP, GA, and GA–LM are 0.42 s, 0.51 s, 0.98 s and 1.21 s, respectively. While the GA–LM algorithm achieves higher accuracy, it also takes more time to be executed. Because there is no real-time computing requirement in long-term online monitoring, the GA–LM algorithm is an optimal choice.

4. Reliability verification of the GA–LM algorithm using experiments

To verify the practicability of the GA–LM algorithm, a thickness measurement experiment is conducted using a 45-steel plate (10 mm thick). The experimental system comprised a signal generator (AFG3021C), power amplifier (AG1006), digital oscilloscope (MDO3012), duplexer (RDX-6), and 45-steel plate (140 mm × 60 mm × 10 mm). A waveguide bar (450 mm × 18 mm × 1 mm) is fixed on the 45-steel plate using a clamp, and the top of the waveguide bar is encapsulated with a d35 type piezoelectric chip with a frequency of 2 MHz (waveguide transducer). The waveguide transducer is vertically dry coupled onto the plate by the clamping force of the fixture, and the fixed transducer is then connected to the experimental system, as shown in Fig. 4.

In the experiment, a five-cycle sine signal modulated with a Hanning window is generated at 2 MHz using the experimental system, and the signal is amplified by 20% using the power amplifier. The signal data are collected using the oscilloscope with a sampling frequency of 250 MHz. As the sound velocity of ultrasonic waves in 45-steel is about 3193 m/s at room temperature (25°C), TOF is calculated to be 6.26 µs.

The GA–LM algorithm is used to estimate the experimental echo. The original echo and the estimated echo measured in the experiment are shown in Fig. 5. The measured echo and the estimated echo with noise are shown in Fig. 6.

The calculation results obtained with noise is summarized in Table 2. The experimental results are consistent with those obtained in the previous simulations. Under high SNR conditions with less noise, all the algorithms demonstrated high calculation accuracy. The relative error of TH, EP, GA, and GA–LM is 1.12%, 1.28%, 0.79% and 0.64%, respectively. However, the waveform is severely disturbed and distorted with noise. Currently, TH and EP generate significant errors, the relative error is up to 4.79% and 3.35%, respectively. In contrast, the proposed algorithm can maintain high accuracy, with the relative error of 0.96%. In addition, the results obtained using the GA–LM algorithm are more accurate compared with those obtained using GA.
5. Conclusions

To solve the problem of noise interference in the accurate calculation of TOF in ultrasonic testing, this study proposes a novel method integrating GA and the LM algorithms. The developed algorithm fully combines the favourable characteristics of GA being good at global search and LM being good at fast local search. GA is first used to search for an approximate global solution of the optimization problem in the full parameter domain. Thereafter, this search result was used as the initial value, and the LM algorithm is utilized to find the exact optimal solution of the
model. Finally, the accurate TOF value was determined by subtracting the arrival time of adjacent echoes. The simulation and experimental results showed that the proposed algorithm exhibits extremely high anti-interference performance. TOF can be accurately calculated even under low SNR conditions. Compared with several typical methods, the GA–LM algorithm shows a generally better estimation of TOF and can be effectively applied in ultrasonic testing.

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References


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