1. Introduction

Steam-based vacuum systems, employed within steam power plant cycles, hold tremendous significance in improving their energy efficiency. They are usually structured as a three-stage assembly, comprising a steam jet (steam ejector) and a mixing heat exchanger. Figure 1 presents a schematic diagram illustrating such a vacuum system.

The arrangement depicted above draws a vapour-air mixture from the condenser space in order to eliminate inert gases and maintain a vacuum during its operation. In such a system, the ejector draws vapours from the condenser space and compresses them into a mixing chamber to separate air from steam. The configuration of three such interconnected elements allows for a high vacuum to be achieved in the condenser, thereby improving heat exchange parameters and the overall efficiency of the energy system by enhancing the thermal cycle efficiency. In the literature [1–3] there exist vacuum system models based on thermodynamics and gas dynamics. In reference [2], thermodynamic functions are employed to calculate the enthalpy and entropy of superheated and saturated steam. The paper provides a detailed discussion of a single-stage vacuum cycle model, and then, for a larger number of stages, repeats the calculation sequence using this model. An important observation in this work is the statement that multi-zone heat exchangers can only be considered in the lowest stage of the condenser, which, in a cascading condensate flow configuration, means that sequentially cooled condensates achieve subcooling only in the final stage.
Matysko R.

**Nomenclature**

\( A \) – heat exchange surface area, m²
\( d_{out} \) – diameter of the outlet nozzle of the supply nozzle, m
\( d_{in} \) – diameter of the mixing chamber (cylindrical section), m
\( h_p \) – enthalpy of saturated steam, \( h_p = f(p_{sat}), J/(kg \cdot K) \)
\( h_t \) – enthalpy of condensate, \( h_t = f(p_{sat}), J/(kg \cdot K) \)
\( h_{w,in} \) – enthalpy of the cooling water at the inlet of the heat exchanger, \( J/(kg \cdot K) \)
\( h_{w,out} \) – enthalpy of the cooling water at the outlet of the heat exchanger, \( J/(kg \cdot K) \)
\( k \) – polytropic exponent for superheated steam
\( l_m \) – length of the mixing chamber, m
\( m_w \) – mass flow rate of the feed stream to the ejector, kg/s
\( m_{w,in} \) – mass flow rate of water vapour at the inlet of the mixing heat exchanger (excluding the feed steam to the ejector), kg/s
\( m_{w,out} \) – mass flow rate of water vapour at the outlet of the mixing heat exchanger, kg/s
\( m_p \) – mass flow rate of inert gas, kg/s
\( m_s \) – mass flow rate of condensate stream, kg/s
\( \dot{m}_{s,op} \) – mass flow rate at the ejector suction, kg/s
\( \dot{m}_{t,op} \) – mass flow rate at the ejector outlet, kg/s
\( \dot{m}_{w} \) – mass flow rate of the cooling water, kg/s
\( P_{atm} \) – atmospheric pressure, Pa
\( P_r \) – suction pressure in the ejector, Pa
\( P_d \) – discharge pressure of the ejector, Pa
\( P_{stat} \) – steam pressure in the heat exchanger, Pa
\( q_{H,CO} \) – heat flow carried by the exiting cooling water, W
\( t_s \) – saturation temperature, \( t_s = f(p_{sat}), °C \)
\( t_{w,in} \) – inlet water temperature to the heat exchanger, °C
\( t_{w,out} \) – temperature of the cooling water, °C
\( t_{w,cool} \) – outlet water temperature from the heat exchanger, °C

**Greek symbols**

\( \Delta P_{dpp} \) – measured difference of pressure at orifice system, Pa
\( \Delta T_{log} \) – logarithmic temperature difference, °C
\( \eta_{e} \) – efficiency of the ejector diffuser
\( \pi_{e} \) – ejector compression ratio
\( \chi_{m} = \frac{m_{e}}{m_{w}} \) – ejection ratio

Rayleigh model could be applied. In papers [1,3], highly simplified models are presented, allowing only for the determination of pressure values in the prevailing heat exchangers, while neglecting heat exchange processes. Unfortunately, due to the complex phase separation process in intercooler condensers, the models discussed above do not delve into the physics of the heat exchange phenomenon in intercooler condensers. Publication [4] focuses on the analysis and assessment of the performance of steam power plants, including vacuum systems. The author introduces advanced methods for evaluating energy efficiency and optimizing thermal processes. Book [5] provides extensive information on the design and applications of steam turbines, including vacuum systems. It also discusses re-rating techniques that can help improve the efficiency of existing installations. The functioning of the condenser in the presence of inert gases has been discussed in prior studies [6,7] emphasizing the critical role played by vacuum maintenance systems utilised in steam power plant setups.

The topic related to the impact of the operation of vacuum systems remains relevant. Research is being conducted using new computational methods, such as in the work [8], where a model of a steam power plant's vacuum system was presented, and attempts were made to extrapolate and approximate operational parameters using a neural network. In the work [9], calculations of the vacuum system were presented using ejector characteristics obtained from a CFD model. There are also numerous other works related to vacuum systems themselves or focused on improving the performance of energy condensers due to the presence of inert gases. However, these are not the subject of this analysis.

The application of steam ejector systems in power plants is not limited to providing vacuum in condensers. In the following studies, other uses of steam ejector systems in steam power plants are presented. Two-phase ejector systems can also be employed as safety systems for nuclear reactors [10]. Additionally, a 660MW supercritical coal-fired power plant, combined with a steam ejector, is suggested to raise the feedwater temperature,
ensuring NOx reduction capability during low load operation [11].

The presented model allows for the prediction of the parameters of a power condenser system (pressure and saturation temperature) based on equations that describe the processes occurring in the inert gas extraction system.

2. Model of the air removal system for the power plant condenser

A model of the air removal system, as presented here, can be employed in the comparative analysis of component performance within the energy system. Through a differential analysis of theoretical calculations against measurement data, one can gather information to identify the causes of deteriorating performance in energy systems. This holds significant operational and economic importance due to the cost associated with replacing worn-out components (potential shutdown of the entire power block) and reducing fuel consumption to compensate for additional heat losses resulting from inefficient condenser operation. Given that the power block operates for extended periods in a steady-state condition, with transient states occurring only during start-up or shutdown, the focus of this study is on the analysis of the steady-state condenser air removal system. Figure 2 illustrates an example of such a system along with a description of the variables used in the presented theoretical model.

In the model, it was assumed that the initial state of variables for calculations is set as for the system operating under nominal working conditions. Known boundary conditions at the outlet of the final heat exchanger and the supply parameters of individual ejectors were used in the calculations.

To calculate the changes in system parameters during venting, a model consisting of fundamental equations was established.

Mathematical model of a three-stage vacuum ejector system

The equations were applied as follows:
- equations (1) and (2), were utilized to determine variables governing heat exchangers at I, II, III stage parameters,
- equations (3) and (4) were employed to determine variables defining the functioning of the ejectors at I, II, III stage.

The equations were formulated separately for each stage, taking into account boundary values characterizing the limits of each stage.

Input data required for the calculations of the heat exchanger (Fig. 3) includes the following parameters: $m_{in}$, $m_p$, $m_e$ and $P_{w,v,s}$. The calculations for this heat exchanger are intended to determine two specific data points: $m_{in}$ and $m_{op}$. These values are essential for assessing and understanding the performance of the heat exchanger. Additionally, there are parameters and variables that are derived from closure equations, with one of them being $m_{2}$. These derived values play a crucial role in the overall analysis of the heat exchanger’s behaviour and characteristics. In summary, this information provides a framework for conducting calculations and analysis related to the specified heat exchanger, with a focus on input data, sought data, and derived parameters and variables.

Mass balance of the steam-gas mixture at the inlet to the heat exchanger (see Fig. 3) is expressed by:

$$m_{in} + m_p + m_e = m_\gamma + m_{out} + m_p,$$  

$$m_{op} = m_{in} + m_p + m_e,$$  

where $m_{op}$ is the total mass flow rate of the steam-gas mixture at the inlet to the heat exchanger.

Ejectors, as illustrated in Fig. 4, involve the following key details:
- Input data essential for performing calculations for the ejector consists of the following parameters: $P_t$ (specifically equal to $P_{w,v}$), $m_p$, $m_e$, and $m_{op}$. These parameters are fundamental for conducting accurate assessments and analyses.
- The calculations for the ejector aim to determine two specific pieces of data: $P_t$ and $m_{in}$. These values are sought to gain a comprehensive understanding of the ejector’s operational characteristics.
Furthermore, within the context of closure equations, there are parameters and variables that result from these equations. One such variable is \( \pi_t \). These derived variables play a significant role in the overall analysis of the ejector’s performance.

\[ \dot{m}_{op} = \dot{m}_e + \dot{m}_n^\text{in}. \]  

The labels used in Fig. 4 and the symbols in Eqs. (3) and (4) represent the following, respectively (see also Nomenclature): \( P_s \) – suction pressure in the ejector, \( P_t \) – discharge pressure of the ejector, \( P_e \) – supply pressure in the ejector, \( \pi_t \) – ejector compression ratio.

The remaining stages of the vacuum system and their basic components are described by similar equations as in the previously described systems. It is evident that the condenser pressure is \( P_{sk} = P_s \) at the first stage of ejector system, and the mass flow rate of vapour sucked from the condenser is \( \dot{m}_{sk,op} = \dot{m}_{sk,op} \).

The system of equations provided earlier has been augmented with additional closure equations that describe the flow characteristics of the ejector and heat exchanger. This comprehensive description includes equations that characterize the behaviour of the analysed components of the vacuum system.

### 2.1. Supplementary equations for the ejector

In Fig. 5, the symbols adopted in the ejector calculation are presented. The figure also presents proprietary software for determining the parameters of the designed steam ejector. The software was developed based on the mathematical description of the steam ejector as presented in [3] with all the necessary symbols provided to determine the ejector’s parameters.
The general equations utilized are:

\[ P_s = \frac{\rho_s}{\nu_s} \]  
\[ \dot{m}_{s,op} = \dot{m}_{t,op} - \dot{m}_e, \]

where: \( \dot{m}_{s,op} \) - mass flow rate at the ejector suction, \( \dot{m}_{t,op} \) - mass flow rate at the ejector outlet.

The parameter \( \pi_t \) is determined based on the equation:

\[ \pi_t = \frac{\rho_s \pi_t}{1 + \frac{\rho_s}{\nu_s}} \left[ 1 + \frac{\rho_s}{\nu_s}\left| \frac{c_e}{c_m} \right|^2 \right] \]

where each parameter is determined from the following relationships:

\[ \pi_d = \left( 1 + \frac{\rho_s}{\nu_s} \right)^{\frac{k}{k-1}}, \]
\[ \beta_s = \frac{P_s}{\nu_s} \approx \left( \frac{\pi_d}{\pi_t} \right)^{\frac{k-1}{k}}. \]

For an ejector with a conical-cylindrical geometry of the mixing chamber, additional auxiliary equations are:

\[ B_o = \left[ 1 - \frac{\pi_d}{\nu_s} \left( \frac{\rho_s}{\nu_s} \right)^{\frac{k}{k-1}} \right] - \left( 1 + \frac{\pi_d}{\nu_s} \right) \left( \frac{\rho_s}{\nu_s} \right)^{\frac{1}{k-1}} \]
\[ E_o = 1 + \frac{\lambda}{\nu_s} \left( \frac{\rho_s}{\nu_s} \right)^{\frac{1}{k-1}} \left( 1 + \frac{\rho_s}{\nu_s} \right) \left( \frac{\rho_s}{\nu_s} \right)^{\frac{1}{k-1}} \]

where (see also Nomenclature): \( \zeta_{ex} \) - hydraulic resistance coefficient (\( \zeta_{ex}/2 \approx 0.05 \)), \( \lambda \) - coefficient of friction of the fluid against the chamber wall (\( \lambda = 0.004 \)), \( c \) - fluid velocity for a given ejector geometry (Fig. 5),

\[ c_m = \frac{d_m}{d_{eo}}^2, \]
\[ \frac{\zeta_{ex}}{\zeta_{scr}} = \phi e \sqrt{\frac{k+1}{k-1}} \left[ \left( \frac{\rho_s}{\nu_s} \right)^{\frac{1}{k-1}} \left( \frac{\rho_s}{\nu_s} \right)^{\frac{1}{k-1}} \right], \]
\[ \frac{\zeta_{scr}}{\zeta_{scr}} = \phi s \sqrt{\frac{k+1}{k-1}} \left[ \left( \frac{\rho_s}{\nu_s} \right)^{\frac{1}{k-1}} \left( \frac{\rho_s}{\nu_s} \right)^{\frac{1}{k-1}} \right], \]
\[ n = \frac{k-1}{k}, \]
\[ \frac{\zeta_{ex}}{\zeta_{scr}} = \frac{\zeta_{ex}}{\zeta_{scr}} \sqrt{\frac{k+1}{k-1}}. \]

In the initial stage of calculations, preliminary values for the compression ratio as a function of the ejection ratio should be assumed (\( \beta_s \)), as well as the geometry of the ejector and parameters resulting from gas transformations, i.e., the enthalpy ratios (pressure) during expansion in the suction chamber and the driving nozzle:

\[ k, \frac{i_{ex}}{i_{eo}} = \frac{i_{ex}}{i_{eo}} \approx \left( \frac{P_{ex}}{P_{eo}} \right)^n, \frac{i_{eo}}{i_{ex}} = \left( \frac{P_{eo}}{P_{ex}} \right)^n, \phi e, \phi s. \]

Mathematical model of a three-stage vacuum ejector system

where: \( i \) - fluid enthalpy for a given ejector geometry (Fig. 5), \( \phi_s \) - coefficient for velocity determining the influence of fluid friction against the suction chamber wall, \( \phi e \) - coefficient for velocity determining the influence of fluid friction against the nozzle wall, \( k \) - isentropic exponent.

2.2. Additional equations for heat exchange in the condensate separator

The equations describing the heat transfer from condensing steam to cooling water, assuming that the heat fluxes are equal, are presented, i.e. \( q_{H_2O} = q_k = q_s \)

\[ q_{H_2O} = \dot{m}_w \left( h_{in}^{H_2O} - h_{out}^{H_2O} \right), \]
\[ q_k = kA \Delta t_{log}, \]
\[ q_s = \dot{m}_s \left( h_p - h_s \right). \]

From the above equations, variables resulting from additional closure equations were determined, describing parameters such as: \( t_{w,out} \) - temperature of the cooling water heated by the condensing steam stream at the outlet of the heat exchanger (where \( t_{w,out} = f (h_{out}^{H_2O}) \)), \( h_{H_2O} \) - enthalpy of the cooling water at the outlet of the heat exchanger, \( \dot{m}_w \) - mass flow rate of the cooling water, \( h_{H_2O} \) - enthalpy of the cooling water at the inlet of the heat exchanger, \( q_{H_2O} \) - heat flow carried by the exiting cooling water, \( \dot{m}_s \) - mass flow rate of condensate at the outlet of the heat exchanger, \( t_s \) - saturation temperature (\( t_s = f (p_{vs}) \)), \( t_{w,in} \) - inlet water temperature to the heat exchanger, \( \Delta t_{log} \) - logarithmic temperature difference, and:

\[ h_{H_2O} = \frac{q_{H_2O}}{\dot{m}_w}, \]
\[ \dot{m}_s = \frac{h_{H_2O}}{h_{H_2O} - h_{H_2O}}, \]
\[ \Delta t_{log} = \frac{\left( t_{w,in} - t_s \right) - \left( t_{w,cool} - t_s \right)}{\ln \left( \frac{t_{w,in}}{t_{w,cool}} \right)}. \]

3. Methodology and calculation results

The computational algorithm for a steam power plant vacuum system is presented in Fig. 6. The diagram illustrates the sequence of initializing individual calculation steps along with boundary conditions for the specific heat exchangers and ejectors. Table 1 presents the constants used in the calculations, which allow for the reproduction of the calculations presented in this paper. The key system in the vacuum system of a steam power plant is the ejector system, for which the constants used in the calculations are presented in Fig. 5. The mass flow rate from the feed stream to the ejector determines the suction and delivery flows and pressure to the heat exchangers according to the algorithm shown in Fig. 6.
The preliminary calculation results for the pressure in the condenser and individual heat exchangers (condensate separators) are presented. The calculation results show the influence of an additional element, namely the measurement cross, on the pressure in the condenser. It is evident that the vacuum is deteriorated due to the additional resistance posed by the measurement cross, contributing to the overall efficiency reduction of the thermal circuit. This suggests that additional measurement elements placed at the outlet of the vacuum system should have low flow resistance. Please note that the model used has not been verified with the actual vacuum system of the power generation system. Figure 7 shows the calculation results of the pressure in the individual elements of the vacuum system. Figure 8 presents the characteristic of the jet considered in the vacuum system calculations. The characteristic describes the behaviour of three jets installed in the vacuum system.

In Figs. 9, 10, and 11, characteristics describing the operation and thermodynamic parameters of heat exchangers associated with the ejector system of each vacuum stage are presented. It is evident that the influence of the ejection ratio on the saturation pressure in the individual heat exchangers is a nonlinear function dependent on mass flow rates. The logarithmic temperature difference is also related to the mass flow rates of condensate extracted from the inter-stage heat exchangers. It can be observed that for a constant difference in cooling water temperature, the logarithmic difference depends on the saturation temperature value, which is linked through the thermodynamic properties of water vapour with the saturation pressure.

### Table 1. Assumed input parameters for calculations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass flow rate of the feed stream to the ejector.</td>
<td>$\dot{m}_e = \text{loop (0.8:0.2:24)}$ kg/h</td>
</tr>
<tr>
<td>Atmospheric pressure.</td>
<td>$P_{at} = 101.325$ kPa</td>
</tr>
<tr>
<td>Measured difference of pressure at orifice system.</td>
<td>$\Delta P_{dpp} = 10$ kPa</td>
</tr>
<tr>
<td>Assumed nominal inlet temperature of the cooling water.</td>
<td>$t_{w,in} = 20$ °C</td>
</tr>
<tr>
<td>Assumed nominal outlet temperature of the cooling water.</td>
<td>$t_{w,out} = 25$ °C</td>
</tr>
<tr>
<td>Flow rate of inert gases measured in orifice system.</td>
<td>$\dot{m}_{t,op3} = 1$ kg/h</td>
</tr>
</tbody>
</table>
Mathematical model of a three-stage vacuum ejector system

4. Conclusions

In the paper, a vacuum cycle model has been presented, which was developed for the purpose of diagnosing elements of the vacuum system in the energy system.

The results of preliminary pressure calculations in various elements of the vacuum system revealed the influence of additional measurement probe resistance on the vacuum drop in the condenser, which may reduce the overall system efficiency. The presented results draw attention to the magnitude of the vacuum drop in the condenser resulting from the installation of the measuring system at the outlet of the vacuum system chimney. The results also allow for the determination of the pressure value in the water separators at different stages of the vacuum system. However, it is worth noting that the presented model serves as a diagnostic tool that can be used to analyse elements of the vacuum system in energy systems. Nevertheless, this model still requires verification with real data and further research.
The paper provides a significant contribution to understanding and improving vacuum systems in steam power plants, which may contribute to increasing the energy efficiency of such systems in the future.

References