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MATHEMATICAL DESCRIPTION OF THE OPERATION OF MINING ELECTRIC AIR REFRIGERATOR

MATEMATYCZNY OPIS DZIAŁANIA GÓRNICZEJ SPRĘŻARKOWEJ CHŁODZIARKI POWIETRZA

This article attempts to describe mathematically the operation of a mining refrigerating system of air cooling, which consists of an evaporator functioning as a proper air cooler, a compressor of cooling factor, a condenser and an expansion valve regulator.

Surface heat exchangers (an evaporator, a condenser) were considered as units with variable parameters of media along their axis; heat exchange (air, freon, water) takes place between these media. Descriptions of the operation of a counter-current evaporator and also of a co- and counter-current condenser were presented by means of sets of ordinary differential equations, algebraic equations and boundary conditions for differential equations. How a compressor affects parameters of a cooling factor was described by means of an equation of isentrope (isentropic compression of freon) and the influence of an expansion valve regulator by means of an equation of isenthalpe (isenthalpic expansion of freon).

While describing an evaporator the possibility of water outdropping from air during the cooling process and also the existence of a zone in an evaporator, where freon is present in an overheated state, were taken into consideration. For this reason an evaporator was divided into two zones from the side of air and two zones from the side of freon, obtaining in a general case three zones different from each other because of equations describing them. Similarly, when describing a condenser the existence of three zones was taken into consideration: cooling of gaseous freon, freon condensation and cooling of liquid freon.

Theoretical considerations were illustrated by a calculation example, whose results were presented in the form of graphs.

Key words: mining aerology, air-conditioning of mines, surface coolers

W artykule podjęto próbę matematycznego opisu pracy górniczego sprężarkowego układu chłodzenia powietrza, w skład którego wchodzi stanowiący właściwą chłodnicę powietrza bezpośredniego działania parownik, sprężarka czynnika chłodniczego, skraplacz i zawór rozprężny. Przepływ powietrza przez parownik wymusza współpracujący z nim wentylator lutniowy.

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Przeponowe wymienniki ciepła (parownik, skraplacz) uznano za obiekty o zmiennych wzdłuż ich osi parametrach mediów, między którymi zachodzi wymiana ciepła (powietrze, freon, woda). Za pomocą układów równań różniczkowych zwyczajnych i równań algebraicznych oraz warunków brzegowych do równań różniczkowych podane zostały opisy działania przeciwprądowego parownika oraz współprądowego i przeciwprądowego skraplacza. Oddziaływanie sprężarki na parametry czynnika chłodniczego opisano równaniem izentropy (izentropowe sprężanie freonu) — zależność (25), zaś oddziaływanie zaworu rozprężnego — równaniem izentalpy (izentalpowe rozprężanie freonu) — zależność (43).

W opisie parownika uwzględniono możliwość wykraplania wody z powietrza w czasie jego chłodzenia, jak też istnienie w parowniku strefy, w której freon znajduje się w postaci pary przegrzanej. W tym celu podzielono parownik na dwie strefy od strony powietrza i na dwie strefy od strony freonu, otrzymując w przypadku ogólnym trzy strefy różniące się opisującymi je równaniami (4)–(7). Podobnie w opisie skraplacza uwzględniono istnienie trzech stref — chłodzenia freonu gazowego, skraplania freonu i chłodzenia freonu ciekłego. Pracę skraplacza współprądowego dla wymienionych trzech stref opisują równania (26)–(28), a skraplacza przeciwprądowego równania (29)–(31).

Wywody teoretyczne zilustrowano przykładem obliczeniowym. Dotyczy on chłodzenia powietrza w wyrobisku górniczym przewietrzanym prądem opływowym chłodziarką typu DV-290 z freonem R22 jako czynnikiem chłodniczym. Skraplacz tej chłodziarki potraktowano jako szeregowe połączenie od strony wody dwóch skraplaczy — współprądowego i przeciwprądowego. Dla czynnika chłodniczego skraplacze te połączone są równolegle. Wykorzystując program komputerowy utworzony do rozwiązania równań różniczkowych i algebraicznych, stanowiących model matematyczny rozważanej chłodziarki, otrzymano wyniki obliczeń pozwalające wyznaczyć rozkłady parametrów termodynamicznych wszystkich mediów biorących udział w wymianie ciepła. Są to: temperatura i wilgotność właściwa chłodzonego powietrza, temperatura i stopień suchości freonu oraz temperatura chłodzącej skraplacz wody. Rozkłady te przedstawiono graficznie w formie wykresów na rysunkach 4–7.

Słowa kluczowe: aerologia gónicza, klimatyzacja kopalń, chłodnice przeponowe

1. Introduction

Air coolers are a kind of equipment that is used more and more frequently in excavations of deep mines in order to combat heat hazards. A mining electric refrigerator usually consists of two separate sets. The former one is an aggregate containing a refrigerator of primary refrigerant (most frequently one of the freons) together with an electric motor propelling it, a condenser, an expansion valve regulator, control and monitoring systems; the latter one is an evaporator, which is a factual air cooler. Both sets are linked by pipes in which freon flows between an evaporator and a condenser. An operating refrigerator makes it move around. The stream of intake air used for direct cooling flows through an evaporator, cools itself as a result of heat exchange with evaporating freon, which while condensing itself in a condenser emits the heat to cooling water, by means of which it is taken out of the set under consideration. Water cooling a condenser usually flows in a closed system; then it is cooled in an evaporative water cooler located in the stream of return air. The heat exchangers mentioned above (an evaporator, a condenser, an evaporative water cooler) are surface heat exchangers in which media flowing through them are separated from one another by means of a surface (a pipe), usually a copper one, impermeable for mass but easily conducting heat. The scheme of flow of air, freon and water is shown in Fig. 1.

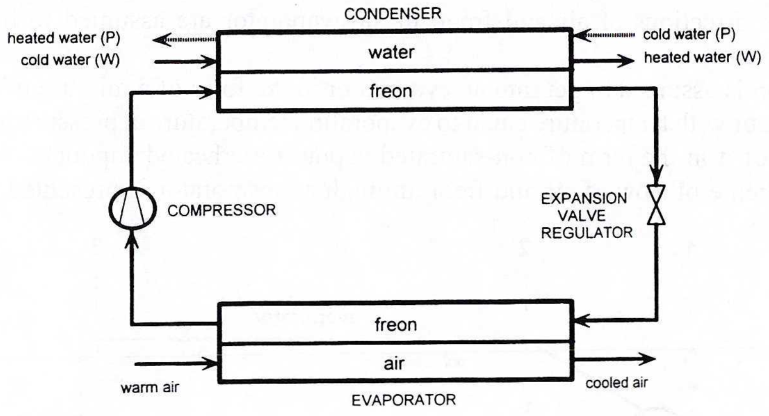


Fig. 1 Scheme of flow of air, freon and water through an electric air refrigerator
W — a co-current condenser, *P* — a counter-current condenser

Rys. 1. Schemat przepływu powietrza, freonu i wody przez sprężarkową chłodziarkę powietrza
W — skraplacz współprądowy, *P* — skraplacz przeciwprądowy

The mathematical description presented further on refers to a steady state of air cooling. It is assumed that mass flow rate of both air that is cooled (Q_m) and cooling water (Q_w), temperature (t) and specific humidity (x) of air at the entry to an evaporator, water temperature (t_w) at the entry to a condenser and pressures of freon in an evaporator and a condenser are known and steady in time. The changes in pressure of freon in an evaporator, a condenser and connecting pipes are neglected, that is it is assumed that it changes only in a refrigerator and an expansion valve regulator. The mathematical description does not take into consideration an evaporative water cooler.

2. Evaporator

Mathematical equations of a model of an evaporator are presented in (Filek, Nowak 2001) while this work presents only their results and the assumptions on which they are based. Here are the assumptions for an evaporator:

- changes in pressure of air flowing through a fan and an evaporator are neglected,
- pressure of freon is assumed to be constant and equal to evaporation pressure p_0,
- along the axis of an evaporator the distribution of mass of surface, internal surface of heat exchange, external surface of heat exchange, volume occupied by air and volume occupied by freon is assumed to be uniform,
- heat exchange through external walls of an evaporator is neglected,
- thermal conduction in the direction parallel to the longitudinal axis of an evaporator in all the media (air, surface, freon) is neglected,
- flows of air and freon in an evaporator is assumed to be one-dimensional in space,

- flow directions of air and freon in an evaporator are assumed to be counter--current,
- freon is assumed to get into an evaporator in the form of a mixture of liquid and vapour with temperature equal to evaporation temperature at pressure p_0, and gets out of it in the form of non-saturated vapour (overheated vapour).

The scheme of flow of air and freon through an evaporator is presented in Fig. 2.

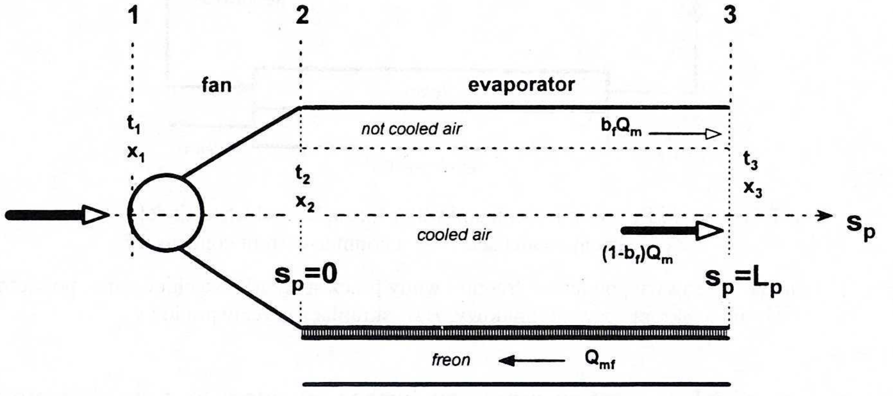


Fig. 2. Scheme of flow of air and freon through an evaporator

Rys. 2. Schemat przepływu powietrza i freonu przez parownik

A duct fan forcing airflow is installed at the entry to an evaporator. It is assumed that it raises air temperature by a known value Δt_{went} not changing its specific humidity. Air temperature in the entry cross-section of a fan is equal to t_1 and specific humidity x_1 in the entry cross-section of an evaporator ($s_p = 0$) equals respectively to t_2 and x_2, and it is equal to t_3 and x_3 in the outlet cross-section. Therefore the following equations are true:

$$t_2 = t_1 + \Delta t_{went} \tag{1}$$

$$x_2 = x_1 \tag{2}$$

While describing air cooling by an evaporator the concept of a diversity factor (Filek, Nowak 1999; Filek et. al. 1999; Wacławik 1992) (marked by symbol b_f), which conventionally divides the stream of air of mass flow rate Q_m into a cooled part of mass flow rate $(1 - b_f)Q_m$, whose temperature and humidity is further on marked by index c (t_c and x_c) and into a part of flow rate b_fQ_m conventionally flowing through a by-pass; therefore not taking part in heat and mass exchange — the parameters of this part of the stream are along the whole evaporator equal to t_2 and x_2.

Relative humidity (φ) of air heated in a fan is smaller than 100% in the cross-section of an evaporator $s_p = 0$. Therefore dry cooling, without outdropping of water, takes place in the zone closer to an evaporator (from $s_p = 0$ to $s_p = s_{pw}$). In this zone the change in temperature (t_c) of a cooled part of air from value t_2 to t_{pr} (where t_{pr} is the dew point temperature of intake air) takes place. However, specific humidity remains constant ($x_c = x_2$). In the cross-section of evaporator $s_p = s_{pw}$ cooled air achieves saturated state ($\varphi = 100\%$) and in the zone beyond this cross-section air is cooled to such an extent that outdropping of water vapour contained in it takes place. Along axis s_p, apart from the fall in temperature t_c from t_{pr} in $s_p = s_{pw}$ to t_{c3} in $s_p = L_p$ (as a result of condensation of vapour the fall is slower than in the first zone), there is also a fall in humidity x_c from x_2 in $s_p = s_{pw}$ to x_{c3} in $s_p = L_p$.

In a counter-flow evaporator freon flows in the direction opposite to the direction of airflow. As a result of expansion in an expansion valve installed in front of an evaporator, this medium entering an evaporator in cross-section $s_p = L_p$ is already in an evaporation state, being a mixture of liquid and vapour. Therefore its temperature (t_{fp}) is equal to evaporation temperature (t_{f0}) at pressure p_0 and vapour quality (χ_p) is defined as the ratio of vapour mass over the whole mass of all the mixture and is equal to χ_{pp}. Freon flowing out of an evaporator in cross-section $s_p = 0$ has temperature t_{fpr} and vapour quality χ_{pr}. Freon flows from an evaporator into a compressor. Therefore full evaporation ($\chi_{pr} = 1$) is required to take place before. Drops of liquid freon could damage a compressor. Monitoring consists in an automatic continuous control of temperature of freon flowing out of an evaporator (t_{fpr}); for safety reasons temperature should be a few °C higher than evaporation temperature.

$$\Delta t_{fp} = t_{fpr} - t_{f0} \tag{3}$$

where:

Δt_{fp} — freon overheating at the outlet from an evaporator [°C].

As far as freon is concerned two zones can be distinguished in an evaporator: an evaporation zone spreading from $s_p = s_{pg}$ to $s_p = L_p$, where freon takes the form of saturated wet vapour; then its temperature is constant and equal to t_{f0} and vapour quality rises from χ_{pp} in $s_p = L_p$ to $\chi_{pr} = 1$ in $s_p = s_{pg}$; and overheated vapour zone ranging from $s_p = 0$ to $s_p = s_{pg}$, where vapour quality is constant and equal to 1 and temperature rises by Δt_{fp} from t_{f0} in $s_p = s_{pg}$ to t_{fpr} in $s_p = 0$.

Therefore taking into consideration both a part of an evaporator outside pipes, occupied by air cooled by a dry or wet method and a also a part inside pipes, where freon either evaporates or takes the form of overheated vapour, three zones can be distinguished in an evaporator, where air and freon undergo different processes (in a special case $s_{pg} = s_{pw}$ the number of zones is limited to two). The zones are as follows:

- for $s_{pg} < s_{pw}$:
  - zone Ip_1 — dry cooling of air, without freon evaporation,
  - zone IIp_1 — dry cooling of air, with freon evaporation,
  - zone IIp_2 — wet cooling of air, with freon evaporation,
- for $s_{pg} > s_{pw}$:
  - zone Ip_1 — dry cooling of air, without freon evaporation,
  - zone Ip_2 — wet cooling of air, without freon evaporation,
  - zone IIp_2 — wet cooling of air, with freon evaporation.

Symbol I signifies dry overheated vapour of freon and II — its evaporation. Index 1 signifies dry and index 2 wet cooling of air.

For uniformity reasons the further part of this work additionally presents top indexes in symbols of searched variables: of temperature and specific humidity of cooled part of air (t_c and x_c) and of temperature and freon vapour quality in an evaporator (t_{fp} and χ_p). In the first zone of an evaporator ($\mathrm{I}p_1$) these variables are marked by index $^{(1)}$ ($t_c^{(1)}$, $x_c^{(1)}$, $t_{fp}^{(1)}$ and $\chi_p^{(1)}$), in the second zone ($\mathrm{II}p_1$ or $\mathrm{I}p_2$) by index $^{(2)}$ ($t_c^{(2)}$, $x_c^{(2)}$, $t_{fp}^{(2)}$ and $\chi_p^{(2)}$) and in the third zone ($\mathrm{II}p_2$) by index $^{(3)}$ ($t_c^{(3)}$, $x_c^{(3)}$, $t_{fp}^{(3)}$ and $\chi_p^{(3)}$).

Sets of equations describing the operation of an evaporator take the following form (Filek, Nowak 1999; Filek, Nowak 2001):

- in zone $\mathrm{I}p_1$ (in this zone $x_c^{(1)}(s_p) = x_2 = \text{const}$, $\chi_p^{(1)}(s_p) = 1$):

$$\begin{cases} \dfrac{dt_c^{(1)}}{ds_p} = -\dfrac{\alpha_{pz}F_{pz}\alpha_{pw}F_{pw}}{Q_m L_p(1-b_f)(c_p + c_w x_2)(\alpha_{pz}F_{pz} + \alpha_{pw}F_{pw})}(t_c^{(1)} - t_{fp}^{(1)}) \\ \\ \dfrac{dt_{fp}^{(1)}}{ds_p} = -\dfrac{\alpha_{pz}F_{pz}\alpha_{pw}F_{pw}}{Q_{mf} L_p c_{pf0}(\alpha_{pz}F_{pz} + \alpha_{pw}F_{pw})}(t_c^{(1)} - t_{fp}^{(1)}) \end{cases} \tag{4}$$

- in zone $\mathrm{II}p_1$ (in this zone $x_c^{(2)}(s_p) = x_2 = \text{const}$, $t_{fp}^{(2)}(s_p) = t_{f0} = \text{const}$):

$$\begin{cases} \dfrac{dt_c^{(2)}}{ds_p} = -\dfrac{\alpha_{pz}F_{pz}\alpha_{pw}F_{pw}}{Q_m L_p(1-b_f)(c_p + c_w x_2)(\alpha_{pz}F_{pz} + \alpha_{pw}F_{pw})}(t_c^{(2)} - t_{f0}) \\ \\ \dfrac{d\chi_p^{(2)}}{ds_p} = -\dfrac{\alpha_{pz}F_{pz}\alpha_{pw}F_{pw}}{Q_{mf} L_p[r_{pf0} - (c_{cf0} - c_{pf0})t_{f0}](\alpha_{pz}F_{pz} + \alpha_{pw}F_{pw})}(t_c^{(2)} - t_{f0}) \end{cases} \tag{5}$$

- in zone $\mathrm{I}p_2$ (in this zone $x_c^{(2)}(s_p) = x_n(t_c^{(2)}), \chi_p^{(2)}(s_p) = 1$):

$$\begin{cases} \dfrac{dt_c^{(2)}}{ds_p} = -\dfrac{\alpha_{pz}F_{pz}\alpha_{pw}F_{pw}}{Q_m L_p(1-b_f)[c_p + c_w x_c^{(2)} + (r_p + c_w t_c^{(2)} - c_c t_c^{(2)})\Phi](\alpha_{pz}F_{pz} + \alpha_{pw}F_{pw})}(t_c^{(2)} - t_{fp}^{(2)}) \\ \\ \dfrac{dt_{fp}^{(2)}}{ds_p} = -\dfrac{\alpha_{pz}F_{pz}\alpha_{pw}F_{pw}}{Q_{mf} L_p c_{pf0}(\alpha_{pz}F_{pz} + \alpha_{pw}F_{pw})}(t_c^{(2)} - t_{fp}^{(2)}) \end{cases} \tag{6}$$

- in zone $\mathrm{II}p_2$ (in this zone $x_c^{(3)}(s_p) = x_n(t_c^{(3)}), t_{fp}^{(3)}(s_p) = t_{f0} = \text{const}$):

$$\begin{cases} \dfrac{dt_c^{(3)}}{ds_p} = -\dfrac{\alpha_{pz}F_{pz}\alpha_{pw}F_{pw}}{Q_m L_p(1-b_f)[c_p + c_w x_c^{(3)} + (r_p + c_w t_c^{(3)} - c_c t_c^{(3)})\Phi](\alpha_{pz}F_{pz} + \alpha_{pw}F_{pw})}(t_c^{(3)} - t_{f0}) \\ \dfrac{d\chi_p^{(3)}}{ds_p} = -\dfrac{\alpha_{pz}F_{pz}\alpha_{pw}F_{pw}}{Q_{mf} L_p[r_{pf0} - (c_{cf0} - c_{pf0})t_{f0}](\alpha_{pz}F_{pz} + \alpha_{pw}F_{pw})}(t_c^{(3)} - t_{f0}) \end{cases} \quad (7)$$

In a special case $s_{pg} = s_{pw}$ two zones can be distinguished in an evaporator: Ip_1 and IIp_2 with specific sets of equations (4) and (7).

Value Φ used above is defined by dependence

$$\Phi = \frac{379.8 \cdot 237.29 \cdot 7.5 \cdot \ln 10 \cdot b \cdot 10^u}{[(t_c + 237.29)(b - 610.6 \cdot 10^u)]^2} \quad (8)$$

where

$$u = \frac{7.5 t_c}{t_c + 237.29}$$

while in zone Ip_2 t_c should be replaced by $t_c^{(2)}$ in (8) and in zone IIp_2 by $t_c^{(3)}$. However $x_n(t_c)$ is specific humidity of air saturated by water vapour in temperature t_c, expressed by the following formula:

$$x_n(t_c) = \frac{379.8 \cdot 10^u}{b - 610.6 \cdot 10^u} \quad (9)$$

The symbols used above, which have not been explained so far, are as follows:

b — absolute air pressure [Pa],
c_c — specific heat of water (liquid) [J/(kgK)],
c_{cf0} — specific heat of liquid freon at evaporation pressure p_0 [J/(kgK)],
c_p — specific heat of dry air at constant pressure [J/(kgK)],
c_{pf0} — specific heat of freon vapour at constant pressure p_0 [J/(kgK)],
c_w — specific heat of water vapour at constant pressure [J/(kgK)],
F_{pw} — area of internal surface of pipes of an evaporator [m^2],
F_{pz} — area of external surface of pipes of an evaporator [m^2],
L_p — length of a heat exchanger in an evaporator [m],
Q_m — mass airflow rate in an evaporator [kg/s],
Q_{mf} — mass freon flow rate in an evaporator [kg/s],
r_p — latent heat of water evaporation/condensation [J/kg],
r_{pf0} — latent heat of freon evaporation at evaporation pressure p_0 [J/kg],
α_{pw} — co-efficient of heat absorption by freon on the internal surface of pipes of an evaporator [W/(m^{2}K)],
α_{pz} — co-efficient of heat absorption by air on the external surface of pipes of an evaporator [W/(m^{2}K)].

Boundary conditions for equations (4)–(7) are as follows:
For a set of equations (4):

$$t_c^{(1)}(s_p = 0) = t_2 \tag{10}$$

$$t_{fp}^{(1)}(s_p = s_{pg}) = t_{f0} \qquad \text{for } s_{pg} < s_{pw} \tag{11}$$

$$t_{fp}^{(1)}(s_p = s_{pw}) = t_{fp}^{(2)}(s_p = s_{pw}) \qquad \text{for } s_{pg} > s_{pw} \tag{12}$$

For a set of equations (5):

$$t_c^{(2)}(s_p = s_{pg}) = t_c^{(1)}(s_p = s_{pg}) \tag{13}$$

$$\chi_p^{(2)}(s_p = s_{pw}) = \chi_p^{(3)}(s_p = s_{pw}) \tag{14}$$

For a set of equations (6):

$$t_c^{(2)}(s_p = s_{pw}) = t_{pr} \tag{15}$$

$$t_{fp}^{(2)}(s_p = s_{pg}) = t_{f0} \tag{16}$$

For a set of equations (7):

$$t_c^{(3)}(s_p = s_{pw}) = t_{pr} \qquad \text{for } s_{pg} < s_{pw} \tag{17}$$

$$t_c^{(3)}(s_p = s_{pg}) = t_c^{(2)}(s_p = s_{pg}) \qquad \text{for } s_{pg} > s_{pw} \tag{18}$$

$$\chi_p^{(3)}(s_p = L_p) = \chi_{pp} \tag{19}$$

Co-ordinates s_{pg} and s_{pw} are determined from conditions:

$$\chi_p^{(2)}(s_p = s_{pg}) = 1 \qquad \text{for } s_{pg} < s_{pw} \tag{20}$$

$$\chi_p^{(3)}(s_p = s_{pg}) = 1 \qquad \text{for } s_{pg} > s_{pw} \tag{21}$$

$$t_c^{(2)}(s_p = s_{pw}) = t_{pr} \qquad \text{for } s_{pg} < s_{pw} \tag{22}$$

$$t_c^{(1)}(s_p = s_{pw}) = t_{pr} \qquad \text{for } s_{pg} > s_{pw} \tag{23}$$

The distribution of temperature of pipes of an evaporator (t_{tp}) can be determined from the following dependence:

$$t_{tp}(s_p) = \frac{\alpha_{pz} F_{pz} t_c(s_p) + \alpha_{pw} F_{pw} t_{fp}(s_p)}{\alpha_{pz} F_{pz} + \alpha_{pw} F_{pw}} \tag{24}$$

3. Compressor

In a compressor isentropic compression of freon vapour from pressure p_0 in an evaporator to pressure p_k in a condenser takes place. This process is described by an equation, from which the temperature of compressed unsaturated freon vapour at the outlet from a compressor and at the same time at the entry to a condenser t_{fss} can be calculated (as it was mentioned before, changes in freon temperature in connecting pipes are neglected)

$$t_{fss} = (t_{fpr} + 273)\left(\frac{p_k}{p_0}\right)^{\frac{\kappa_s - 1}{\kappa_s}} - 273 \tag{25}$$

where:

t_{fpr} — temperature of overheated freon vapour at the outlet from an evaporator and at the entry to a compressor [°C],

κ_s — isentropic exponent.

4. Condenser

It is assumed that the construction of a condenser enables either its co-current or counter-current operation, that is the directions of freon and water flow are, as a matter of fact, parallel and their senses are respectively either consistent or opposite. The derivation of equations of a mathematical model of a condenser is presented in (Filek, Nowak 2002) but this work presents only, similarly as in the case of an evaporator, the results obtained in the work mentioned above and valid assumptions. It is suggested that:

- freon pressure in a condenser is constant and equal to condensation pressure p_k,
- distributions along the longitudinal axis of a condenser,
  - of mass of pipes,
  - of internal surface of heat exchange,
  - of external area of heat exchange,
  - volume occupied by freon,
  - volume occupied by water,

  are uniform.

The following are neglected:

- heat exchange through external walls of a condenser,

- thermal conduction in a parallel way to the longitudinal axis of a condenser in all the media (freon, water, pipes).

The following are assumed:

- water and freon flows in a condenser are one-dimensional,
- freon enters a condenser in the form of vapour with temperature higher than condensation temperature at pressure p_k and leaves it in the form of liquid with temperature lower than that temperature.

The scheme of freon and water flow through a condenser is presented in Fig. 3. Freon vapour compressed to pressure p_k and heated to temperature higher than condensation temperature at pressure p_k enters a condenser (in a co-current condenser from the side of the water inlet and in a counter-current condenser from the side of its outlet). In the part of a condenser comprising the range of current co-ordinate $0 < s_s < s_{sd}$ vapour flowing around the external surface of pipes of a condenser gets cooled as a result of heat exchange through the surface of a condenser with cooling water flowing in pipes (the walls of pipes are the surface). In the cross-section of a condenser with co-ordinate $s_s = s_{sd}$ gaseous freon reaches condensation temperature t_{fk} and its condensation begins, during which temperature remains constant, emitted latent heat is given away to cooling water. In the cross-section of a condenser with co-ordinate $s_s = s_{sg} > s_{sd}$ condensation is over. In the remaining part of a condenser ($s_{sg} < s_s < L_s$) liquid freon is cooled. Therefore three zones can be distinguished in a condenser:

- zone I*s* ($0 < s_s < s_{sd}$) where freon vapour is cooled,
- zone II*s* ($s_{sd} < s_s < s_{sg}$) where condensation of freon takes place,
- zone III*s* ($s_{sg} < s_s < L_s$) where liquid freon is cooled.

Similarly as in the case of an evaporator, additional symbols were used for searched values: of temperature and of vapour quality of freon in a condenser (t_{fs} and χ_s) and water temperature (t_w). In the first zone of a condenser (I*s*) these variables were marked by index $^{(1)}$ ($t_{fs}^{(1)}, \chi_s^{(1)}$ and $t_w^{(1)}$); in the second zone (II*s*) by index $^{(2)}$ ($t_{fs}^{(2)}, \chi_s^{(2)}$ and $t_w^{(2)}$) and in the third zone (III*s*) by index $^{(3)}$ ($t_{fs}^{(3)}, \chi_s^{(3)}$ and $t_w^{(3)}$).

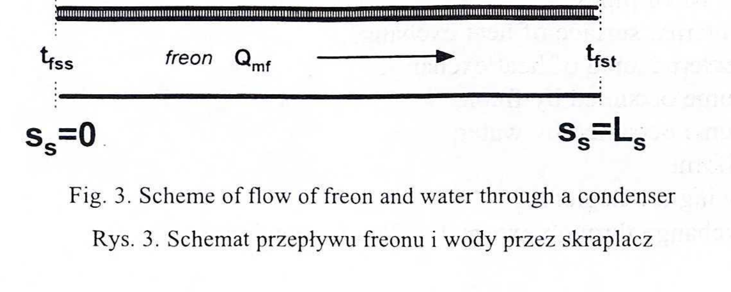


Fig. 3. Scheme of flow of freon and water through a condenser

Rys. 3. Schemat przepływu freonu i wody przez skraplacz

Sets of equations describing the operation of a co-current condenser take the following form:

- In zone Is (in this zone $\chi_s^{(1)}(s_s) = 1$):

$$\begin{cases} \dfrac{dt_{fs}^{(1)}}{ds_s} = -\dfrac{\alpha_{sz}F_{sz}\alpha_{sw}F_{sw}}{Q_{mf}L_s c_{pfk}(\alpha_{sz}F_{sz} + \alpha_{sw}F_{sw})}(t_{fs}^{(1)} - t_w^{(1)}) \\ \\ \dfrac{dt_w^{(1)}}{ds_s} = \dfrac{\alpha_{sz}F_{sz}\alpha_{sw}F_{sw}}{Q_w L_s c_c(\alpha_{sz}F_{sz} + \alpha_{sw}F_{sw})}(t_{fs}^{(1)} - t_w^{(1)}) \end{cases} \tag{26}$$

- In zone IIs (in this zone $t_{fs}^{(2)}(s_s) = t_{fk}$):

$$\begin{cases} \dfrac{d\chi_s^{(2)}}{ds_s} = -\dfrac{\alpha_{sz}F_{sz}\alpha_{sw}F_{sw}}{Q_{mf}L_s[r_{pfk} - (c_{cfk} - c_{pfk})t_{fk}](\alpha_{sz}F_{sz} + \alpha_{sw}F_{sw})}(t_{fk} - t_w^{(2)}) \\ \\ \dfrac{dt_w^{(2)}}{ds_s} = \dfrac{\alpha_{sz}F_{sz}\alpha_{sw}F_{sw}}{Q_w L_s c_c(\alpha_{sz}F_{sz} + \alpha_{sw}F_{sw})}(t_{fk} - t_w^{(2)}) \end{cases} \tag{27}$$

- In zone IIIs (in this zone $\chi_s^{(3)}(s_s) = 0$):

$$\begin{cases} \dfrac{dt_{fs}^{(3)}}{ds_s} = -\dfrac{\alpha_{sz}F_{sz}\alpha_{sw}F_{sw}}{Q_{mf}L_s c_{cfk}(\alpha_{sz}F_{sz} + \alpha_{sw}F_{sw})}(t_{fs}^{(3)} - t_w^{(3)}) \\ \\ \dfrac{dt_w^{(3)}}{ds_s} = \dfrac{\alpha_{sz}F_{sz}\alpha_{sw}F_{sw}}{Q_w L_s c_c(\alpha_{sz}F_{sz} + \alpha_{sw}F_{sw})}(t_{fs}^{(3)} - t_w^{(3)}) \end{cases} \tag{28}$$

Analogical sets of equations describing the operation of a counter-current condenser take the following form:

- In zone Is (in this zone $\chi_s^{(1)}(s_s) = 1$):

$$\begin{cases} \dfrac{dt_{fs}^{(1)}}{ds_s} = -\dfrac{\alpha_{sz}F_{sz}\alpha_{sw}F_{sw}}{Q_{mf}L_s c_{pfk}(\alpha_{sz}F_{sz} + \alpha_{sw}F_{sw})}(t_{fs}^{(1)} - t_w^{(1)}) \\ \\ \dfrac{dt_w^{(1)}}{ds_s} = -\dfrac{\alpha_{sz}F_{sz}\alpha_{sw}F_{sw}}{Q_w L_s c_c(\alpha_{sz}F_{sz} + \alpha_{sw}F_{sw})}(t_{fs}^{(1)} - t_w^{(1)}) \end{cases} \tag{29}$$

- In zone II*s* (in this zone $t_{fs}^{(2)}(s_s) = t_{fk}$):

$$\begin{cases} \dfrac{d\chi_s^{(2)}}{ds_s} = -\dfrac{\alpha_{sz}F_{sz}\alpha_{sw}F_{sw}}{Q_{mf}L_s[r_{pfk} - (c_{cfk} - c_{pfk})t_{fk}](\alpha_{sz}F_{sz} + \alpha_{sw}F_{sw})}(t_{fk} - t_w^{(2)}) \\ \\ \dfrac{dt_w^{(2)}}{ds_s} = -\dfrac{\alpha_{sz}F_{sz}\alpha_{sw}F_{sw}}{Q_wL_sc_c(\alpha_{sz}F_{sz} + \alpha_{sw}F_{sw})}(t_{fk} - t_w^{(2)}) \end{cases} \tag{30}$$

- In zone III*s* (in this zone $\chi_s^{(3)}(s_s) = 0$):

$$\begin{cases} \dfrac{dt_{fs}^{(3)}}{ds_s} = -\dfrac{\alpha_{sz}F_{sz}\alpha_{sw}F_{sw}}{Q_{mf}L_sc_{cfk}(\alpha_{sz}F_{sz} + \alpha_{sw}F_{sw})}(t_{fs}^{(3)} - t_w^{(3)}) \\ \\ \dfrac{dt_w^{(3)}}{ds_s} = -\dfrac{\alpha_{sz}F_{sz}\alpha_{sw}F_{sw}}{Q_wL_sc_c(\alpha_{sz}F_{sz} + \alpha_{sw}F_{sw})}(t_{fs}^{(3)} - t_w^{(3)}) \end{cases} \tag{31}$$

Here is the explanation for the symbols used in equations (26)–(31):

c_{cfk} — specific heat of liquid freon at condensation pressure p_k [J/(kgK)],
c_{pfk} — specific heat of freon vapour at constant pressure p_k [J/(kgK)],
F_{sw} — area of internal surface of pipes of a condenser [m^2],
F_{sz} — area of external surface of pipes of a condenser [m^2],
L_s — length of a heat exchanger in a condenser [m],
Q_w — mass flow rate of water in a condenser [kg/s],
r_{pfk} — latent heat of freon evaporation at condensation pressure p_k [J/kg],
α_{sw} — co-efficient of heat absorption by water on the internal surface of pipes of a condenser [W/(m^{2}K)],
α_{sz} — co-efficient of heat absorption from freon on the external surface of pipes of a condenser [W/(m^{2}K)].

Boundary conditions for equations (26)–(31) take the following form:

A co-current condenser

- for a set of equations (26) in zone I*s*:

$$t_{fs}^{(1)}(s_s = 0) = t_{fss} \tag{32}$$

$$t_w^{(1)}(s_s = 0) = t_{w0} \tag{33}$$

- for a set of equations (27) in zone II*s*:

$$\chi_s^{(2)}(s_s = s_{sd}) = 1 \tag{34}$$

$$t_w^{(2)}(s_s = s_{sd}) = t_w^{(1)}(s_s = s_{sd}) \tag{35}$$

- for a set of equations (28) in zone III*s*:

$$t_{fs}^{(3)}(s_s = s_{sg}) = t_{fk} \tag{36}$$

$$t_w^{(3)}(s_s = s_{sg}) = t_w^{(2)}(s_s = s_{sg}) \tag{37}$$

Values of co-ordinates s_{sd} and s_{sg} are determined on the basis of the following conditions:

$$t_{fs}^{(1)}(s_s = s_{sd}) = t_{fk} \tag{38}$$

$$\chi_s^{(2)}(s_s = s_{sg}) = 0 \tag{39}$$

A counter-current condenser

- for a set of equations (29) in zone I*s*:

$$t_{fs}^{(1)}(s_s = 0) = t_{fss} \tag{32}$$

$$t_w^{(1)}(s_s = s_{sd}) = t_w^{(2)}(s_s = s_{sd}) \tag{40}$$

- for a set of equations (30) in zone II*s*:

$$\chi_s^{(2)}(s_s = s_{sd}) = 1 \tag{34}$$

$$t_w^{(2)}(s_s = s_{sg}) = t_w^{(3)}(s_s = s_{sg}) \tag{41}$$

- for a set of equations (31) in zone III*s*:

$$t_{fs}^{(3)}(s_s = s_{sg}) = t_{fk} \tag{36}$$

$$t_w^{(3)}(s_s = L_s) = t_{w0} \tag{42}$$

Values of co-ordinates s_{sd} and s_{sg}, similarly as in the case of a co-current condenser, are determined on the basis of conditions (38) and (39).

5. Expansion valve regulator

After leaving a condenser, liquid freon with temperature t_{fst} undergoes isenthalpic throttling in an expansion valve regulator. Its pressure falls rapidly without a change in enthalpy and initially without any significant change in temperature (as long as the medium is in liquid state). After reaching the left boundary line (equivalent to zero value of vapour quality) freon begins to evaporate while pressure is falling down, there is no change in enthalpy, but with rapid fall in its temperature. It becomes clear that in an expansion valve regulator, apart from the fall in freon pressure, there is an increase in vapour quality to value χ_{pp} and fall in its temperature. It was assumed that behind the valve temperature of freon is equal to its evaporation temperature t_{f0} at pressure p_0. Vapour quality of the mixture of liquid freon and its saturated vapour entering an evaporator can be calculated on the basis of an equation of stability of enthalpy (Ochęduszko 1970; Staniszewski 1982)

$$\chi_{pp} = \frac{t_{fst}c_{cfk} - t_{f0}c_{cf0}}{t_{f0}(c_{pf0} - c_{cf0}) + r_{pf0}} \quad . \tag{43}$$

where:

t_{fst} — temperature of liquid freon behind the condenser (in front of an expansion valve regulator).

6. Calculation example

The following calculation example was solved in order to illustrate the method of calculations based on the equations presented. This calculation example refers to cooling of air in an excavation of an underground mine by means of refrigerator DV-290 with freon R22 as a cooling medium. A refrigerator of this type is equipped with a counter-current evaporator; therefore relevant equations were applied straightforward in the form presented in section 2. However the construction of a condenser, different from the one presented in section 4, made it necessary to replace it in calculations by two condensers, one — co-current and the other — counter--current connected in series from the side of water and in parallel from the side of freon. Calculations were done by the use of a computer programme written especially for this purpose.

The solution to the example presented below determines distributions of the following values along an evaporator or both parts of a condenser:

- temperature of cooled part of air along an evaporator — $t_c(s_p)$,
- specific humidity of cooled part of air along an evaporator — $x_c(s_p)$,
- freon temperature along an evaporator — $t_{fp}(s_p)$,
- vapour quality of freon along an evaporator — $\chi_p(s_p)$,
- freon temperature along a co-current part of a condenser — $t_{fs}(s_s)$,
- vapour quality of freon along a co-current part of a condenser — $\chi_s(s_s)$,
- temperature of cooling water along a co-current part of a condenser — $t_w(s_s)$,
- freon temperature along a counter-current part of a condenser — $t_{fs}(s_s)$,
- vapour quality of freon along a counter-current part of a condenser — $\chi_s(s_s)$,
- temperature of cooling water along a counter-current part of a condenser — $t_w(s_s)$.

Moreover the following were calculated:

- mass stream of freon enabling overheating of its vapour in an evaporator — Q_{mf},
- total power of an evaporator — N_p,
- cooling power of air in an evaporator — N_{ps},
- drying power of air in an evaporator — N_{pw},
- boundary co-ordinate of the cross-section of an evaporator, in which outdropping of water from cooled air begins — s_{pw},
- boundary co-ordinate of the cross-section of an evaporator, in which evaporation of freon finishes — s_{pg},
- temperature of cooled air flowing out of an evaporator — t_3,
- specific humidity of cooled air flowing out of an evaporator — x_3,
- relative humidity of cooled air flowing out of an evaporator — φ_3,
- total power of a condenser — N_s,
- power of a co-current part of a condenser — N_{sw},
- power of a counter-current part of a condenser — N_{sp},
- boundary co-ordinate of the cross-section of a co-current part of a condenser, in which condensation of freon begins — s_{sdw},
- boundary co-ordinate of the cross-section of a co-current part of a condenser, in which condensation of freon finishes — s_{sgw},
- boundary co-ordinate of the cross-section of a counter-current part of a condenser, in which condensation of freon begins — s_{sdp},
- boundary co-ordinate of the cross-section of a counter-current part of a condenser, in which condensation of freon finishes — s_{sgp},
- temperature of freon at the outlet from a condenser — t_{fst},
- temperature of water at the outlet from a condenser — t_{ws},
- vapour quality of freon vapour at the entry to an evaporator — χ_{pp}.

The following data were assumed (numerical values of freon parameters in conditions of evaporation and condensation were determined on the basis of (Bonca et al. 1998; Platzer et. al. 1990); values characterising a refrigerator were determined on the basis of technical and motion documentation):

With regard to an evaporator, air and freon:

• absolute air pressure	$b = 105$ kPa
• air temperature at the entry to a fan	$t_1 = 30°C$
• relative humidity of air at the entry to a fan	$\varphi_1 = 80\%$
• increase in air temperature in a fan	$\Delta t_{went} = 2°C$
• volumetric airflow rate in an evaporator	$Q = 8\ m^3/s$
• diversity factor of an evaporator	$b_f = 0.1$
• area of internal surface of the pipe of an evaporator	$F_{pw} = 67.5\ m^2$
• area of external surface of the pipe of an evaporator	$F_{pz} = 84.5\ m^2$
• length of a heat exchanger in an evaporator	$L_p = 2$ m
• freon pressure in an evaporator	$p_0 = 5 \cdot 10^5$ Pa
• overheating of freon vapour leaving an evaporator	$\Delta t_{fp} = 8°C$

Moreover on the basis of the data above the following were calculated:

- air temperature at the entry to an evaporator $t_2 = 32°C$
- air dew point temperature at the entry to an evaporator $t_{pr} = 26.17°C$
- specific humidity of air at the entry to an evaporator $x_2 = 20.773$ g/kg
- relative humidity of air at the entry to an evaporator $\varphi_2 = 71.39\%$
- mass stream of air in an evaporator $Q_m = 9.34$ kg/s
- temperature of freon evaporation $t_{f0} = 0.22°C$
- specific heat of liquid freon in evaporation temperature $c_{cf0} = 1.176$ kJ/(kgK)
- specific heat of freon vapour in evaporation temperature $c_{pf0} = 0.746$ kJ/(kgK)
- heat of freon evaporation at pressure p_0 $r_{pf0} = 202.583$ kJ/kg

With regard to a condenser, water and freon:

- temperature of water at the entry to a condenser $t_{w0} = 25°C$
- mass stream of water in a condenser $Q_w = 6.21$ kg/s
- temperature of freon at the entry to a condenser $t_{fss} = 70.11°C$
- pressure of freon in a condenser $p_k = 17{\cdot}10^5$ Pa
- area of internal surface of the pipe of one part of a condenser $F_{sw} = 15.28$ m^2
- area of external surface of the pipe of one part of a condenser $F_{sz} = 51$ m^2
- length of a heat exchanger in one part of a condenser $L_s = 3$ m

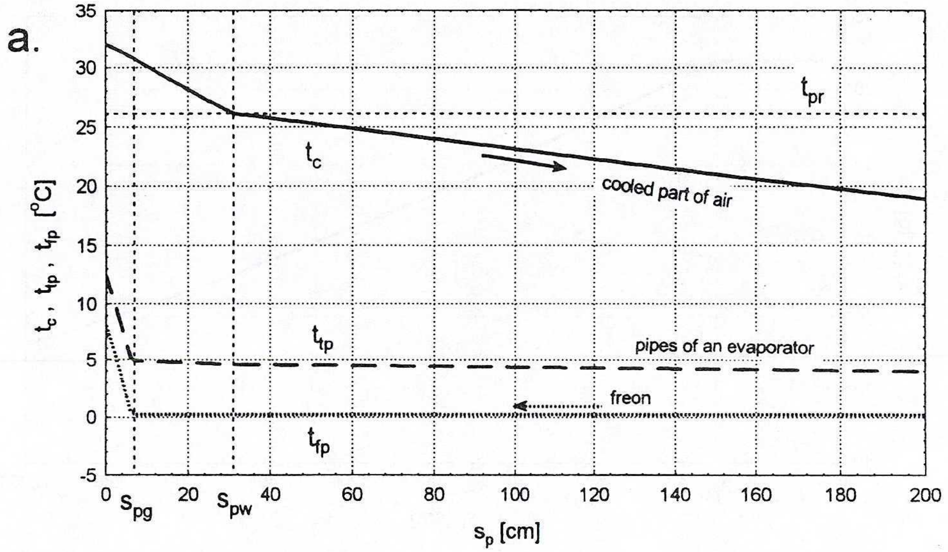
Moreover on the basis of the data above the following were calculated:

- temperature of freon condensation $t_{fk} = 44.29°C$
- specific heat of liquid freon in condensation temperature $c_{cfk} = 1.369$ kJ/(kgK)
- specific heat of freon vapour in condensation temperature $c_{pfk} = 1.059$ kJ/(kgK)
- heat of freon condensation at pressure p_k $r_{pfk} = 158.762$ kJ/kg

The following parameters were assumed for air, water and water vapour:

- specific heat of air at constant pressure $c_p = 1.005$ kJ/(kgK)
- specific heat of water $c_c = 4.190$ kJ/(kgK)
- specific heat of water vapour $c_w = 1.926$ kJ/(kgK)
- heat of water vapour condensation $r_p = 2500$ kJ/kg

Distributions of parameters of air, pipes and freon along an evaporator are presented in the form of graphs in Fig. 4 (Fig. 4a — temperature of cooled part of air t_c, temperature of the pipes of an evaporator t_{tp} and temperature of freon in an evaporator t_{fp}; Fig. 4b — vapour quality of freon vapour in an evaporator χ_p) and in Fig. 5 (specific humidity of cooled part of air x_c). Similarly, distributions of parameters of freon, pipes and water along a co-current part of a condenser are presented in Fig. 6 (Fig. 6a — temperature of freon t_{fs}, temperature of the pipe of a condenser t_{ts} and temperature of water t_w; Fig. 6b — vapour quality of freon vapour χ_s) and along a counter-current part of a condenser in Fig. 7 (Fig.7a — temperature of freon t_{fs}, temperature of the pipe of a condenser t_{ts} and temperature of water t_w; Fig. 7b — vapour quality of freon vapour χ_s). The remaining calculated values are presented below:



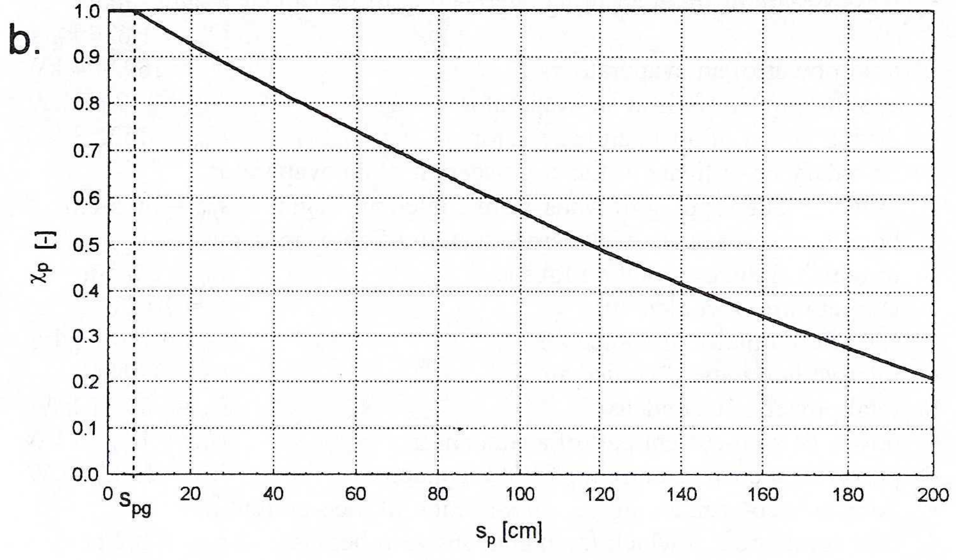


Fig. 4. Distributions along an evaporator
a — of temperature of cooled part of air t_c, temperature of pipe t_{tp} and temperature of freon t_{fp};
b — of vapour quality of freon vapour χ_p

Rys. 4. Rozkłady wzdłuż parownika
a — temperatury chłodzonej części powietrza t_c, temperatury przepony t_{tp} i temperatury freonu t_{fp};
b — stopnia suchości pary freonu χ_p

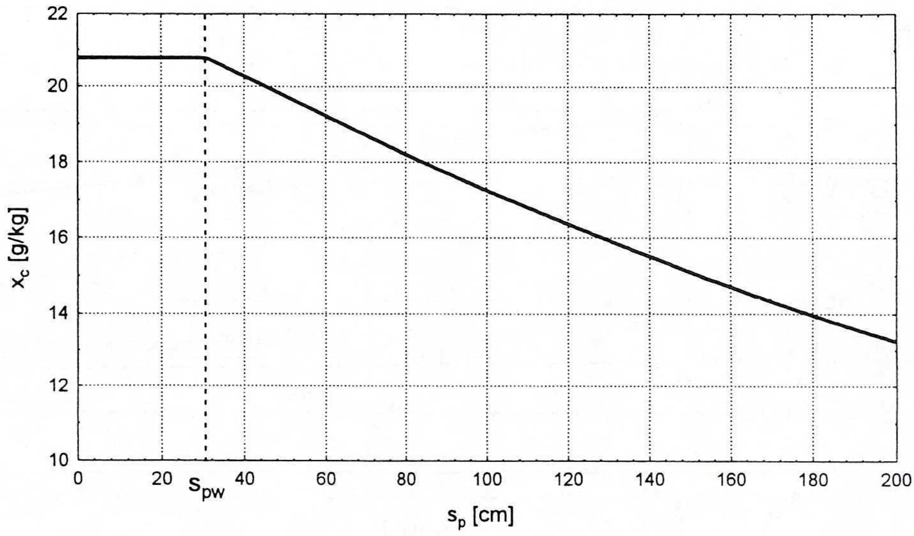
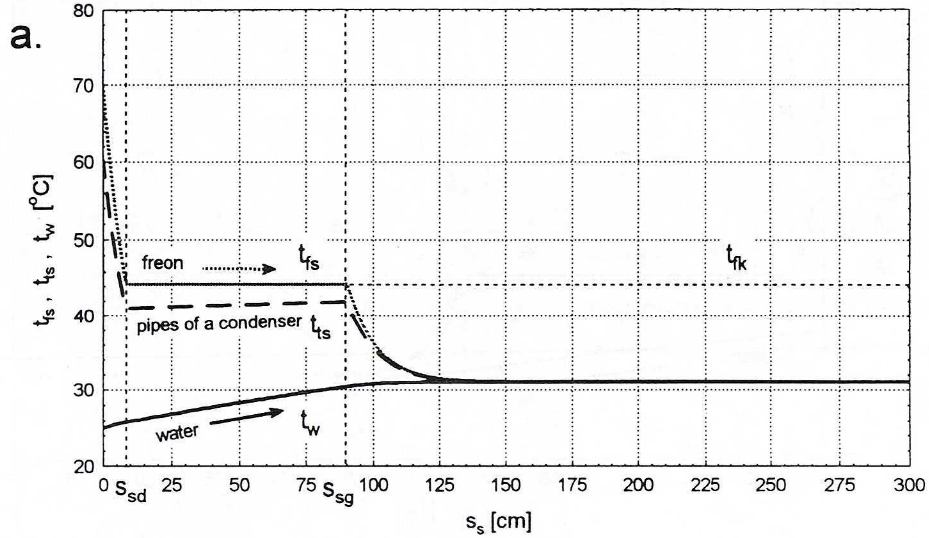


Fig. 5. Distribution of specific humidity of cooled part of air x_c along an evaporator

Rys. 5. Rozkład wilgotności właściwej chłodzonej części powietrza x_c wzdłuż parownika

- mass stream of freon ensuring overheating of its vapour in an evaporator $Q_{mf} = 1.626$ kg/s
- total power of an evaporator $N_p = 269.904$ kW
- cooling power of air in an evaporator $N_{ps} = 117.37$ kW
- drying power of air in an evaporator $N_{pw} = 152.53$ kW
- boundary co-ordinate of the cross-section of an evaporator, in which outdropping of water from cooled air begins $s_{pw} = 30.5$ cm
- boundary co-ordinate of the cross-section of an evaporator, in which freon evaporation finishes $s_{pg} = 6.5$ cm
- temperature of cooled air $t_3 = 20.23$°C
- specific humidity of cooled air $x_3 = 14.011$ g/kg
- relative humidity of cooled air $\varphi_3 = 97.58\%$
- total power of a condenser $N_s = 309.51$ kW
- power of a co-current part of a condenser $N_{sw} = 154.75$ kW
- power of a counter-current part of a condenser $N_{sp} = 154.76$ kW
- boundary co-ordinate of the cross-section of a co-current part of a condenser, in which freon condensation begins $s_{sdw} = 8.2$ cm
- boundary co-ordinate of the cross-section of a co-current part of a condenser, in which freon condensation finishes $s_{sgw} = 89.8$ cm
- boundary co-ordinate of the cross-section of a counter-current part of a condenser, in which freon condensation begins $s_{sdp} = 12.0$ cm
- boundary co-ordinate of the cross-section of a counter-current part of a condenser, in which freon condensation finishes $s_{sgp} = 132.9$ cm



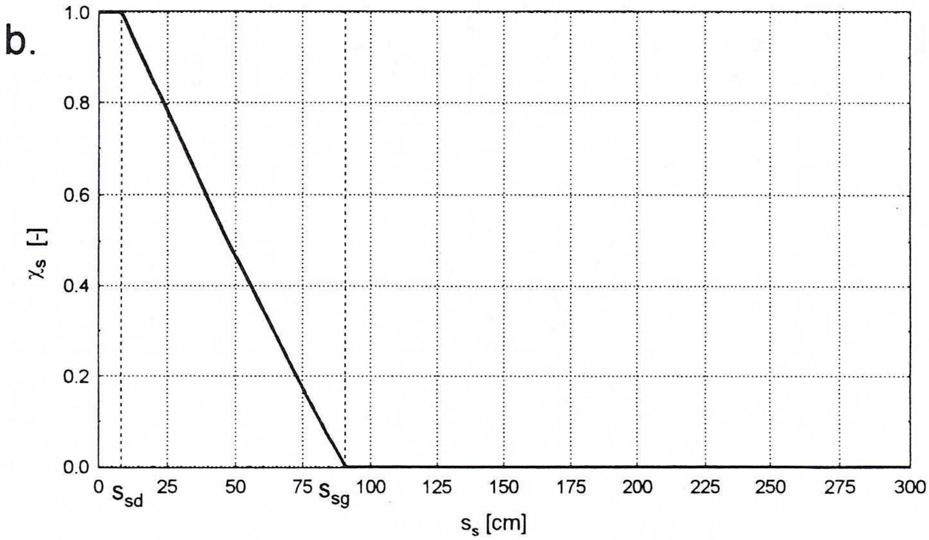
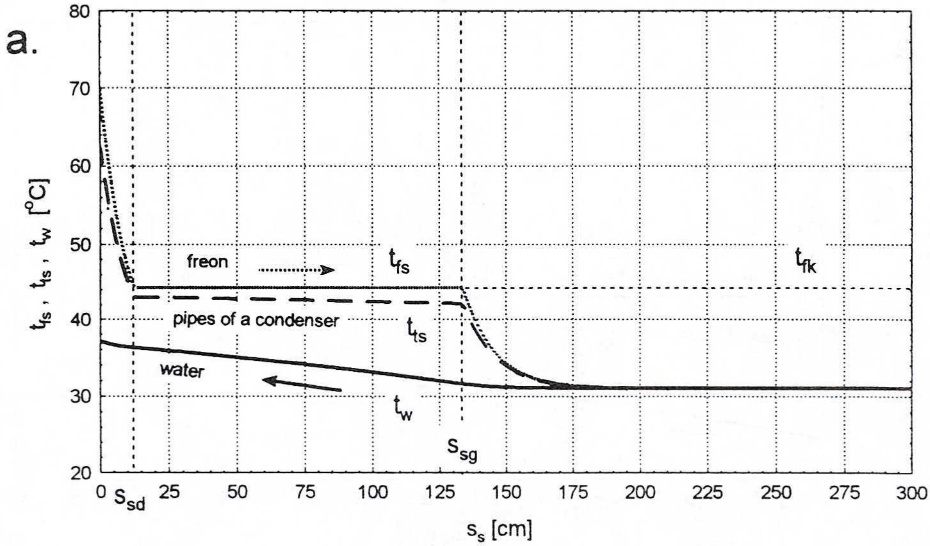


Fig. 6. Distributions along a co-current part of a condenser
a — of temperature of freon t_{fs}, temperature of pipe t_{ts} and temperature of water t_w;
b — of vapour quality of freon vapour — χ_s

Rys. 6. Rozkłady wzdłuż współprądowej części skraplacza
a — temperatury freonu t_{fs}, temperatury przepony t_{ts} i temperatury wody t_w;
b — stopnia suchości pary freonu χ_s



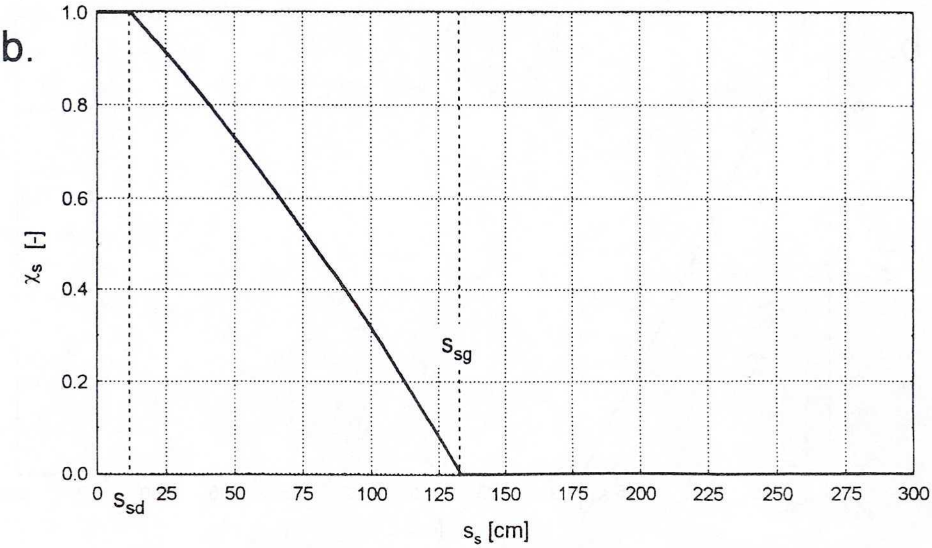


Fig. 7. Distributions along a counter-current part of a condenser
a — of temperature of freon t_{fs}, temperature of pipe t_{ts} and temperature of water t_w,
b — of vapour quality of freon vapour — χ_s

Rys. 7. Rozkłady wzdłuż przeciwprądowej części skraplacza
a — temperatury freonu t_{fs}, temperatury przepony t_{ts} i temperatury wody t_w,
b — stopnia suchości pary freonu χ_s

- temperature of freon at the outlet from a condenser $t_{fst} = 31.15°C$
- temperature of water at the outlet from a condenser $t_{ws} = 36.91°C$
- vapour quality of freon vapour at the entry to an evaporator $\chi_{pp} = 0.21$

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