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NUMERICAL DETERMINATION OF THE STATE OF STRESS IN A WALL OF A BUILDING UNDER THE INFLUENCE OF DISCONTINUOUS GROUND DEFORMATIONS

NUMERYCZNE WYZNACZANIE STANU NAPRĘŻENIA W ŚCIANIE BUDYNKU PODDANEGO WPŁYWOWI NIECIĄGŁYCH DEFORMACJI POWIERZCHNI

The structures situated in the areas influenced by mining exploitation are subjected to a very complex system of loads. Some of those loads are caused by ground deformations. It is possible to predict the deformations quantity but only if trough of subsidence is of regular shape, however, due to different reasons (geological or connected with exploitation system) discontinuous deformations or irregularities in the shape of trough may appear.

In the paper, the following problem was discussed: how would the answer of the building look like if the ground was subjected to discontinuous deformation? In particular: how would stress distribution in the wall change?

Assumed physical model was presented on Fig. 3. It consisted of two elastic shields: upper shield represented wall of the building and a lower one represented the ground. Both shields were totally fastened together. Suitable material constants were set in the Table.

Two different schemes were discussed:

— with the edge of the wall not supported and forming something like a cantilever of "a"-reach (Fig. 2a, 4a),

— with a cavern of "k"-width appearing under the wall (Fig. 2b, 4b).

The mathematical model consisted of linear elasticity equations system with suitable boundary conditions. Analytical solution of the system was abandoned in favour of numerical solution by means of Finite Element Method (FEM). The problem was formulated according to calculus of variation rules.

The numerical calculations were made by means of FEM-program ALGOR.

T h e f i r s t s c h e m e

Normal stress σ_{yy} distribution change as a function of increase of cantilever reach "a" was presented on Fig. 6. On Fig. 7, changes in σ_{zz} due to changes in "a"-value were presented. The propagation of tensioned zone in the wall with the increase of length of unsupported wall edge was presented on Fig. 8. The calculations were stopped, when tensions appeared in connection between wall and building, on the side of building opposite to landslide. That phenomenon testified loss of contact between the wall and the ground.

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T h e s e c o n d s c h e m e

Changes in σ_{yy} and σ_{zz} stresses as functions of increase of cavern width "k" were presented on Fig. 10 and 11. Fig. 12 showed distribution of compressed and tensioned zones for consecutive values of "k". It was visible, that those changes were insignificant.

Key words: discontinuous deformation, a wall, a shield.

W procesach projektowania budynków usytuowanych na terenach górniczych jeden z podstawowych składników kombinacji obciążeń stanowią obciążenia spowodowane zniekształceniem podłoża. Istnieją metody przewidywania wielkości wpływów ciągłych deformacji powierzchni na obiekty budowlane. W przypadku deformacji nieciągłych teren często klasyfikuje się jako nieprzydatny dla celów budowlanych.

W pracy prowadzono rozważania nad wpływem wystąpienia deformacji nieciągłej pod istniejącym budynkiem na stan naprężenia w jego ścianach. Przedmiotem analizy była ściana oparta na sprężystym podłożu. Zagadnienie przedstawiono w postaci dwóch sprężystych tarcz, różniących się parametrami mechanicznymi. Wartości stałych materiałowych zestawiono w tablicy. Wymiary tarczy reprezentującej podłoże dobrano tak, by poza nimi wpływ zjawiska był znikomy. Model matematyczny stanowiło zagadnienie brzegowe liniowej teorii sprężystości.

Rozważono dwie teoretyczne sytuacje lokalizacji zapadliska pod ścianą budynku (rys. 2). Odpowiadają im dwa schematy obliczeniowe pokazane na rys. 4. Przeprowadzono obliczenia numeryczne Metodą Elementów Skończonych. Posłużono się programem MES-ALGOR. Zmieniając w kolejnych krokach obliczeniowych wielkość odcinka, na którym podłoże ściany objęte zostaje zapadliskiem, obserwowano zmiany następujące w rozkładach oraz w wartościach naprężeń. Uzyskane wyniki w postaci map naprężeń przedstawiają rysunki: 6 i 7 — dla schematu pierwszego oraz 10 i 11 — dla schematu drugiego. Na rysunku 8 i 12 pokazano propagację stref rozciąganych (ciemne pola) w ścianie budynku odpowiednio dla schematów pierwszego i drugiego.

Analiza otrzymanych wyników pozwala na sformułowanie następujących wniosków.

1. W odniesieniu do s c h e m a t u p i e r w s z e g o.

1.1. Wyraźne zmiany rozkładów naprężeń normalnych, zarówno w kierunku poziomym, jak i pionowym (Y i Z) występują w przypadku, gdy powstałe zapadlisko obejmuje boczną część ściany (schemat 1).

1.2. Wielkość zmian postępuje ze wzrostem długości odcinka ściany, którego podłoże objęte jest zapadliskiem.

1.3. Obszarami narażonymi na powstanie zarysowań są okolice otworów oraz obszar dolnego brzegu ściany pozostający bez podparcia.

1.4. Gdy zapadlisko obejmuje zasięgiem 1/3 szerokości ściany, dochodzi do pojawienia się po stronie przeciwnej do zapadliska naprężeń rozciągających w gruncie, co informuje o utracie kontaktu między ścianą a podłożem.

2. W odniesieniu do s c h e m a t u d r u g i e g o.

2.1. W przypadku, gdy zapadlisko zlokalizowane jest pod wewnętrzną (środkową) częścią ściany budynku (schemat 2), zmiany w uzyskanych kolejno rozkładach naprężeń są stosunkowo nieduże. Również wartości naprężeń zmieniają się nieznacznie.

2.2. Dodatkowe naprężenia rozciągające pojawiają się przy dolnym brzegu na odcinku objętym zapadliskiem przy rozpiętości zapadliska wynoszącej ok. 1/6 szerokości ściany.

Ograniczenia zastosowanego modelu, polegające głównie na pominięciu faktu współpracy ściany z pozostałymi elementami konstrukcji budynku oraz przyjęciu bardzo uproszczonego modelu materiałowego ściany oraz podłoża powodują, że na podstawie uzyskanych wyników nie można formułować wniosków zbyt daleko idących. Tworzyć mogą one jednak podstawę i wstęp do dalszych badań, których kierunek stanowić ma

zastosowanie bardziej skomplikowanych modeli materiałowych opisujących materiał ściany i podłoże, jak również uwzględnienie współpracy poszczególnych elementów ustroju budynku.

Słowa kluczowe: szkody górnicze, deformacje nieciągłe, ściana, tarcza, stan naprężenia.

1. Introduction

The designing of buildings which are supposed to be located in the areas affected by mining exploitation always involves the consideration of a possible negative influence exerted on the building structure. It is usually a major factor in the combination of loads. It especially refers to the dimensioning of foundations and lower floors. The methods of the prediction of the subsidence trough parameters make it possible to determine the internal forces that would appear in the structural elements.

Apart from continuous deformations, in the mining areas there are also discontinuous ones, which can take the form of craters, irregular collapse holes, rock sills, rift valleys, landslides, flexures as well as cracks and fractures. There appearance does not necessarily follow every instance of mining exploitation and, moreover, take place either immediately after it or even several years later.

Those phenomena are usually difficult to predict, as they are not preceded by any early symptoms. However, there are some factors indicating that a given area may be endangered by a possible occurrence of discontinuous deformations or some anomalies in the formation of the trough. The danger of the appearance of discontinuous deformations is mostly caused by:

— shallow excavation (less than 80 m);

— mining in the vicinity of tectonic faults;

— stoppage of the longwall face of subsequent seams along a single vertical plane;

— mining at high speed (more than 7 m/day).

Since discontinuous surface deformations are caused by the activation of oldworks that have appeared in the rock mass (due to both natural processes and the exploitation of minerals), the kind of rock constituting the overlay must also be taken into account (K o w a l s k i, K a s z o w s k a in: K w i a t e k et al., 1998).

The occurrence of discontinuous deformations in the ground supporting a building can result in serious damages. The possible danger especially refers to the foundations, basement or cellar walls and to the walls and ceilings of lower floors. Since the interactions of the ground discontinuous deformation are regarded as exceptional in a load combination, the ground conditions in the potentially endangered areas ought to be analysed with particular attention. According to the obvervational results (K a w u l o k et al., 1999), the damages may not endanger the safety of the whole system but only affect single elements, on the condition that the building has been designed and constructed in compliance with the designing recommendations concerning buildings placed in mining areas. Any negligence in this respect, especially concerning the dilatation of long buildings into segments, the bracing and reinforcing of the foundations, the construction of cellar walls and ceilings, the reinforcement of the areas circumjacent to the holes as well as the bracing of the building with suitable curbs and roofs may lead to disastrous conditions if, additionally, some discontinuous deformations appear in the ground (A d j u k i e w i c z et al., 1999).

As a results of the formation of a ground brace, a collapse sink hole or landslide, a building standing in the way of that formation will be deprived of its support in a certain segment (Fig. 1). Some additional internal forces will appear in the structural elements; they will be caused by the decrease or even disappearance of the reaction (passive ground pressure) below a part of the foundations (A n d e r m a n, 1966).

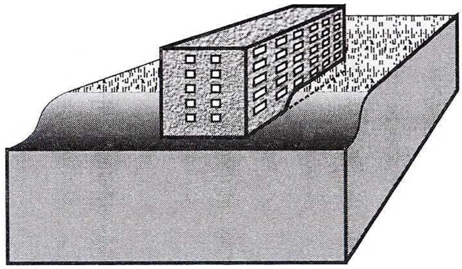


Fig. 1. A building on the ground with discontinuous deformation

This study presents an attempt at the analysis of the stress distributions in the wall of a building deprived of its support in a given segment. A single (isolated) wall treated as a structural element of a building was taken into consideration. The wall was modelled as a shield with two exemplary holes. Their dimensions do not correspond to actual dimensions of possible window-openings or doorways. They were only supposed to demonstrate the influence of an opening upon the stress distribution. The connection between the wall and other elements as well as its co-operation with the whole construction has been ignored.

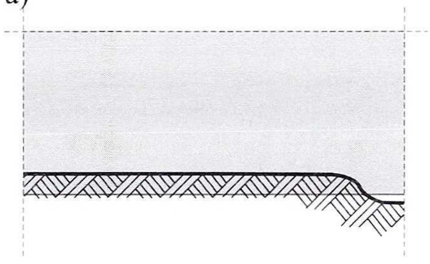
2. Subject, scope and purpose of the study

Subject

The study presents an analysis of an isolated wall of a building deprived of its support in a certain segment as a result of discontinuous deformation. Two schemes have been analysed (L u e t k e n s, 1951), which correspond to the following situations:

a) a part of the building has got within the range of the collapse area (Fig. 2a),
b) the collapse hole is situated below the central part of the building (Fig. 2b).

a)



b)

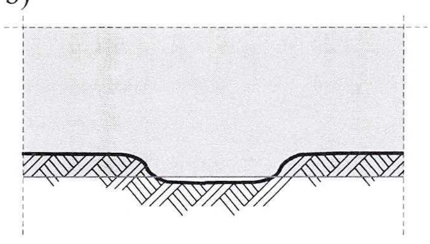


Fig. 2. The variants of the wall situation in relation to discontinuous ground deformations

Scope

In the study, the linear elasticity theory methods were used, with a formulation of the two-dimensional boundary value problem. Therefore, the following assumptions are obligatory:

— elasticity, homogeneity and isotropy of the material,

— small displacements and small displacement gradients.

Purpose

The purpose of the study is to determine the stress distribution in a wall of a building under which a discontinuous deformation has taken place with the result of depriving it, in a given segment, of its support. It was supposed to find out the relation between the changes in size of the non-supported area.

3. The formulation of the problem

The physical model

The physical model consists of two elastic shields totally fastened together and differing in their material properties (Fig. 3). The upper shield represents the wall of the building while the lower one represents the ground supporting it. The dimensions of the shield modelling the rock mass have been selected in such a way that the outside impact of the building should be negligibly small. The contact between the shields in a given segment (a or k, according to the particular scheme) gets broken (Fig. 4).

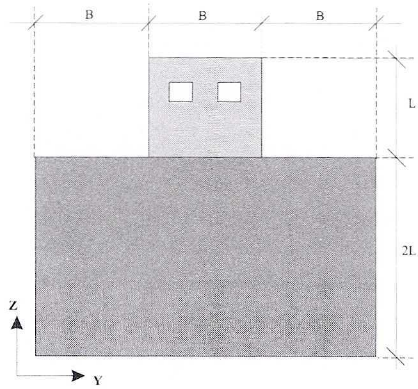
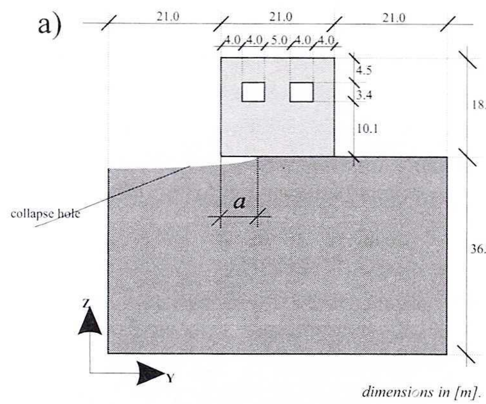


Fig. 3. The physical model

a)



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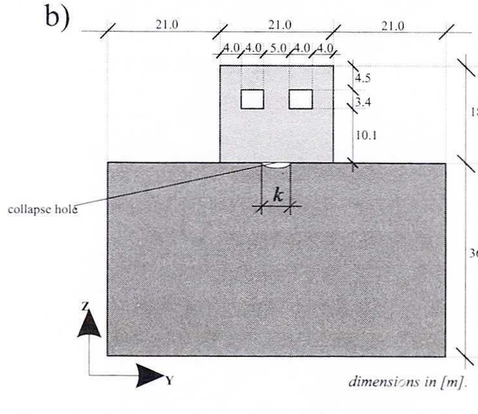

Fig. 4. The physical model accepted for numerical calculations

The mathematical model

The mathematical model includes a system of equations of the linear elasticity theory describing the two-dimensional boundary value problem (Szefer, 1964; Timoszenko, Goodier, 1962), which consist of:

1. two equations of equilibrium:

$$\begin{aligned} \sigma_{11,1}+\sigma_{12,2} &= 0 \\ \sigma_{21,1}+\sigma_{22,2} &= 0 \end{aligned} \tag{1}$$

2. three geometrical equations:

$$\begin{aligned} \varepsilon_{11} &= u_{1,1} \\ \varepsilon_{22} &= u_{2,2} \\ \varepsilon_{12} &= \frac{1}{2}\left(u_{1,2}+u_{2,1}\right) \end{aligned} \tag{2}$$

3. three physical equations:

$$\begin{aligned} \varepsilon_{11} &= \frac{1}{E}\left(\sigma_{11}-v\sigma_{22}\right) \\ \varepsilon_{22} &= \frac{1}{E}\left(\sigma_{22}-v\sigma_{11}\right) \\ \varepsilon_{12} &= \frac{1}{2G}\sigma_{12} \end{aligned} \tag{3}$$

4. with the boundary conditions:

$$\begin{aligned} \sigma_{ij}n_j &= p_i \qquad \forall p_i \in S\sigma \\ u_i &= u_{0i} \qquad \forall u_i \in S_u \end{aligned} \tag{4}$$

where:

v — Poisson's ratio,

G — non-dilatational strain modulus $G = \dfrac{E}{2(1+v)}$,

λ — material constant $\lambda = \dfrac{Ev}{(1+v)(1-2v)}$,

S_σ — the part of the boundary where the load was applied (where the boundary value conditions were formulated);

S_u — the part of the boundary where external constraints were applied (where the displacement boundary values were formulated).

The solution of these eight equations will provide us with the values of eight unknown quantities:

— three components of the state of stress,
— three components of the state of strain,
— two components of the vector of displacement.

Since solving the problem by means of analysis is arduous and does not guarantee arriving at the right result, it is more convenient to use approximate methods. In this case the Finite Element Method will be applied. In order to pass from the set of equations of the linear elasticity theory to numerical formulation, the calculus of variation will be used.

4. The process of solving the problem

In order to satisfy the whole system of equations of the linear elasticity theory, including the boundary value conditions, it is also necessary to satisfy the equation of virtual displacement rule. According to this rule, *the necessary and sufficient condition for the kinematically permissible displacement to be real is that the virtual load work δL in this displacement must be equal to the stress work at proper virtual strains.*

$$\int_{\Omega} \sigma_{ij} \{\tilde{\varepsilon}(\tilde{u})\} \delta\varepsilon_{ij} dV = \int_{\Omega} \varrho b_i \delta u_i dV + \int_{S_\sigma} p_i \delta u_i dS. \tag{5}$$

Now let us replace the continuous model of the problem with a discrete one. We are looking for the vector of displacement in the form,

$$u_i(x) = \sum_{k=1}^{N} A_k^{(i)} \varphi_k(\tilde{x}), \tag{6}$$

where:

A_k — unknown numerical coefficients;

φ_k — so-called base functions, treated as known ones; they are required to be such as to make the whole expression kinematically permissible.

The area of two rectangular shields under consideration has been divided into quadrangular shields under consideration has been divided into quadrangular and quadrinodal finite elements (Fig. 5) and, consequently, the vector of nodal displacements

$$\{\delta\}^e = \begin{Bmatrix} \delta_i \\ \delta_j \\ \delta_k \\ \delta_l \end{Bmatrix}, \tag{7}$$

where:

$$\{\delta_m\} = \begin{Bmatrix} u_m \\ v_m \end{Bmatrix}$$

as well as the form of the displacement function explicitly define the displacements of an element.

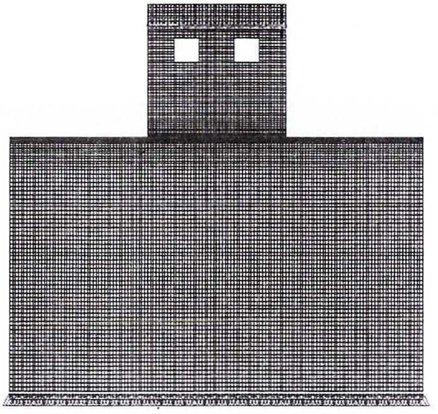

Fig. 5. The numerical model

The displacements of the elements at given point are described by the vector f:

$$\{f\} = \begin{Bmatrix} u(x, y) \\ v(x, y) \end{Bmatrix} = [N]\{\delta\}^e \tag{8}$$

N refers to the functions binding the displacements. They are called functions of shape and play an important part in the Finite Element Method. They should meet certain requirements securing the convergence towards the correct result (Zienkiewicz, 1972).

With the assumption that the investigated object is two-dimensional and made of Hooke's material, the relation between stresses and strains is as follows,

$$\{T_\sigma\} = [D]\{T\varepsilon\}, \tag{9}$$

where the elasticity matrix D has the form:

$$[D] = \frac{E}{1-v^2}\begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \frac{1-v}{2} \end{bmatrix}. \tag{10}$$

The relation between the state of strain with the displacement area,

$$\{T_\varepsilon\} = \begin{Bmatrix} u_{,x} \\ v_{,y} \\ u_{,y} + v_{,x} \end{Bmatrix} \tag{11}$$

makes it possible, with taking into consideration both the function of shape and the formula (8), to explicity determine the intra-elemental strains as a function of displacements in the nodes,

$$\{T_{\varepsilon}\} = [B]\{\delta\}^{e}, \tag{12}$$

where B is the strain matrix binding the strain area inside the element with the node displacements. The formulae (9) and (11) can also make it possible to express the state of stress at a given point of the element by the vector of displacements in nodes:

$$\{T_{\sigma}\} = [D][B]\{\delta\}^{e}. \tag{13}$$

In order to obtain the solution for th displacements in nodes the nodal forces $\{\boldsymbol{F}\}^{e}$, caused by the displacements of nodes, can be bound with the vector of nodal displacement through the stiffness matrix of the element $[\boldsymbol{k}]^{e}$.

$$\{F\} = [k]^{e}\{\delta\}^{e}, \tag{14}$$

where:

$$[k]^{e} = \int_{S}[B]^{T}[D][B]\,tdxdy. \tag{15}$$

The relations (14) and (15) make is possible to find the stiffness matrix $[\boldsymbol{k}]^{e}$ and obtain the solution for the displacements. The matrix $[\boldsymbol{k}]$ characterizes the deformability of the element.

By substituting the assumed form of displacements and performing several transformations (Krzykowska, 1999), we can now formulate the equation (5) as:

$$\boldsymbol{KA} = \boldsymbol{F}. \tag{16}$$

where:

$\boldsymbol{K}$ — stiffness matrix (symmetrical, band matrix),
$\boldsymbol{A}$ — matrix of unknown numerical coefficients,
$\boldsymbol{F}$ — load matrix.

5. The results of numerical calculations

The model accepted for the consideration of the problem consists of two shields, closely fastened together. Both shields have been made of Hooke's material. They differ in density, elasticity moduli and Poisson's ratio. The upper shield represents the wall (concrete B-20), the lower one represents the ground (middle-cohesive sand-clay soil), that is, the close-to-surface layer of the rock mass. The dimensions of the shield are selected in such a way that the outside influence of the phenomenon is negligible. The material constants have been juxtaposed in table.

The numerical model has been constructed by dividing the area with the use of a rectangular mesh of quadrinodal elements, congested in the contact area of both shields (Fig. 5). The numerical calculations were made by means of the Finite Element Method — program ALGOR.

TABLE

Material	Elasticity modulus E [kPa]	Specific gravity γ [kN/m^3]	Poisson's ratio ν	Thickness of element t [m]
Soil (sand-clay)	33 600	22	0.32	0.4
Concrete	27 000 000	18	0.1667	0.4

5.1. The first calculation scheme
(Refers to a collapse hole partly located under the wall edge)

Figures 6 and 7 present the changes in normal stress distribution in the direction Y (horizontal) and Z (vertical) with the increase in length of the segment "a", where the wall remains unsupported. A considerable propagation of the tensioned zones in the horizontal direction close to the holes can be observed as well as the appearance of a tensioned zone in the vertical direction at the lower edge of the wall, where it no longer rests on the ground. There is also a noticeable increase of the areas compressed by internal forces of great values in both directions. Those values grow close to the edge of the area affected by the collapse and decrease on the opposite side. Before the length of the cantilever "a" arrives at the value equal to 1/3 of the total wall length, the stresses σ_{zz} in the corner opposite to the collapse change their sings and become tensions. That means a loss of contact between the wall and the ground at that point.

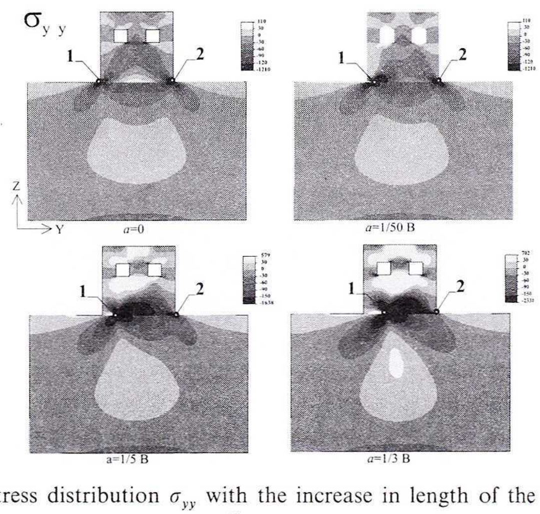

Fig. 6. The change in stress distribution σ_{yy} with the increase in length of the wall segment "a" in the collapse area

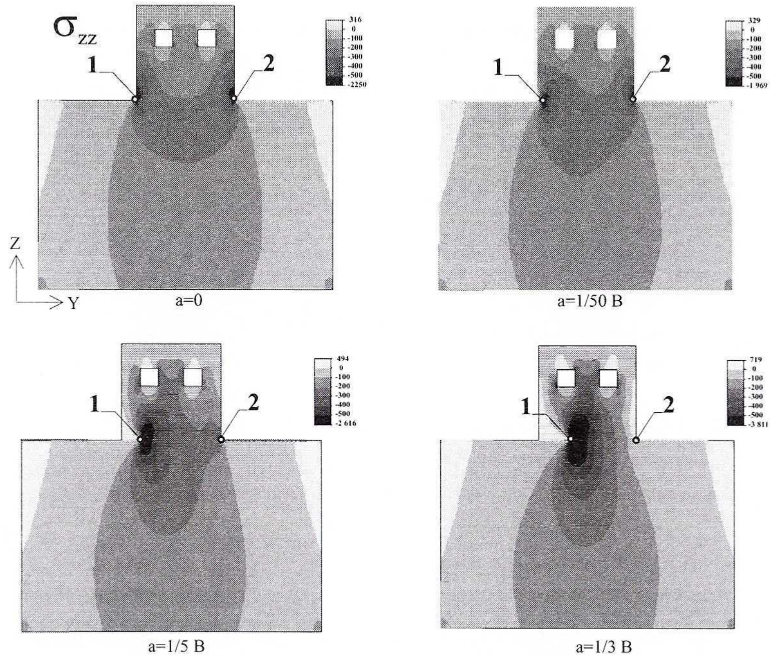

Fig. 7. The change in stress distribution σ_{zz} with the increase in length of the wall segment "*a*" in the collapse area

Figure 8 shows which parts of the wall are extended and how these zones expand with the progression of the collapse reach. This propagation is especially noticeable in the horizontal direction. The diagrams in Fig. 9 present the relationships between the stress values at point "1" (i. e. at the beginning of the collapse area) and "2" (in the corner of the wall opposite to the collapse). These diagrams indicate the non-linear character of the changes.

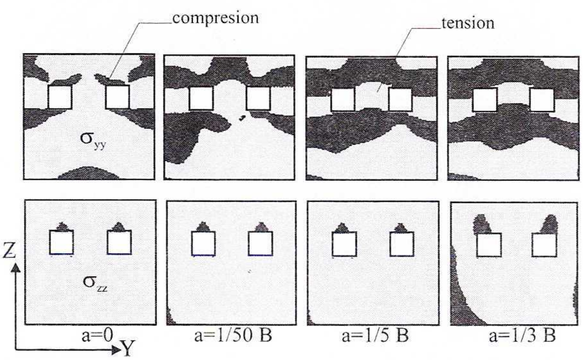

Fig. 8. The propagation of the tensioned walls with the increase in length of the segment "*a*"

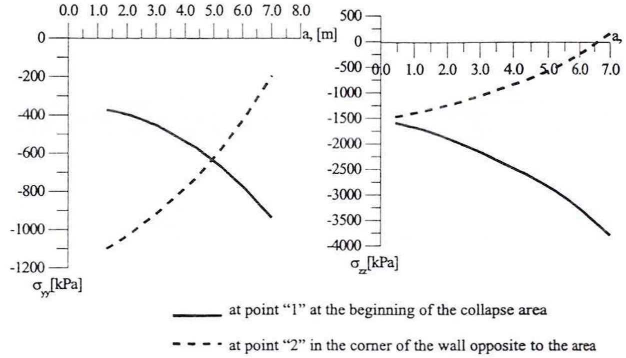

Fig. 9. The change in stress values at points "1" and "2" in relation to the changes in the length of the segment "*a*"

5.2. The second calculation scheme

(Refers to the situation when the whole of collapse has occurred under the wall)

In the second scheme, which describes the change in stresses in the wall of a building under which a collapse has taken place, the changes of the internal forces distribution with the increase of the collapse area width are relatively small (Fig. 10, 11).

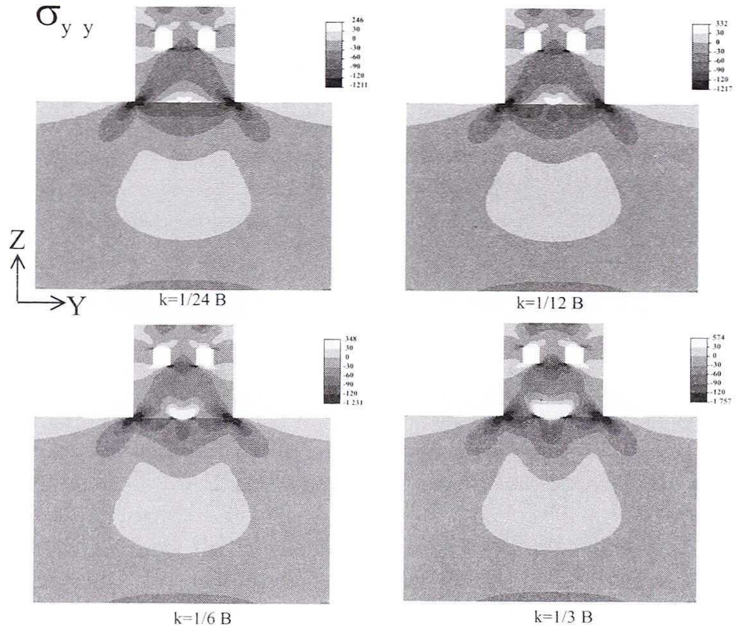

Fig. 10. The change in stress distributions σ_{yy} with the increase of the collapse width "*k*"

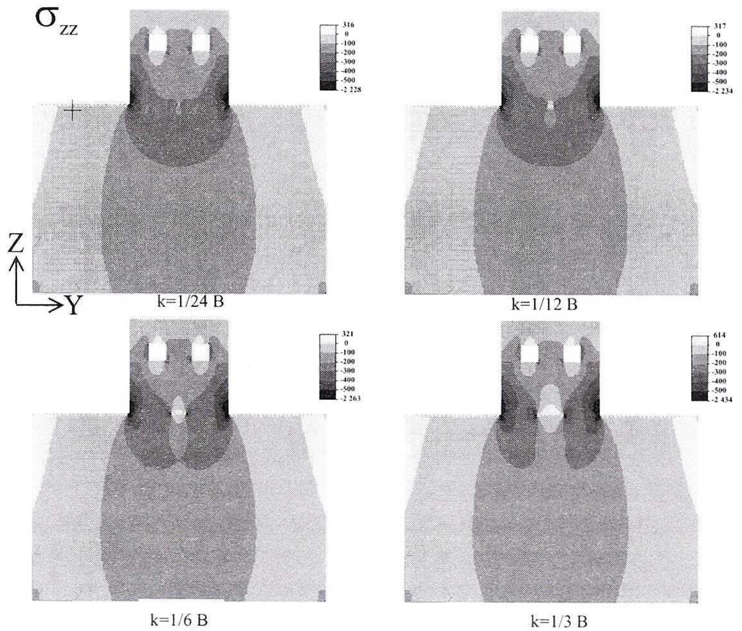

Fig. 11. The change in stress distributions σ_{zz} with the increase of the collapse width "k"

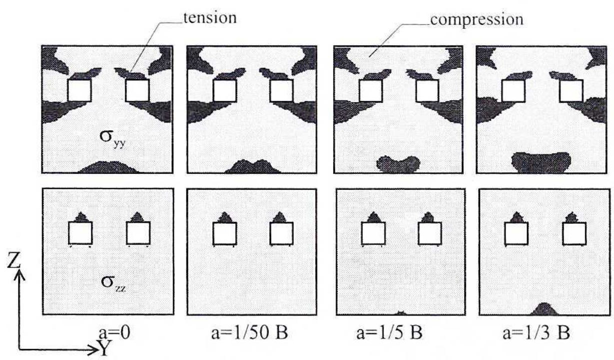

Fig. 12. The propagation of the tensioned zones in the wall in relation to the collapse width "k"

The propagation of the tensioned zones is also unnoticeable in the areas circumjacent to the hole. There occur only some small changes at the lower edge of the hollow (Fig. 12). The increase in the maximal stresses σ_{yy} and σ_{zz}, both compressing and tensioning ones, is also rather small. It can be seen that the cracks in the wall resulting from the formation of collapse hole under it can be expected only in the closest vicinity of the unsupported wall segment but they have practically no influence on the tensioned close-to-hole zones in the shield.

6. Summing up

In the process of numerical calculations two schemes, described in Chapter 2, were analysed. Those schemes refer to two theoretical situations, which may occur when a discontinuous deformation of the ground directly touches the building. As a result of changing, in consecutive calculation steps, the size of the segment where the ground beneath the wall had been affected by the collapse, the changes in stress distributions and values were observed. The obtained results (normal stress map) were used to determine the areas endangered by a possible appearance of scratches.

Those results lead to the following conclusions:

1. As regards the first scheme:

1.1. Distinct changes in normal stress distributions, in both horizontal and vertical directions (respectively Y and Z), occur when the collapse affects the lateral part of the wall.

1.2. Those changes progress with the increase in length of the wall segment within the collapsed area.

1.3. The areas most endangered by a possible occurrence of scratches are placed around the holes or at the lower (unsupported) wall edge.

1.4. If the collapse area extend to 1/3 of the wall width, it leads to the appearance of tensioning stresses in the ground opposite to the collapse hole. This indicates a loss of contact between the wall and the ground.

2. As regards the second scheme:

2.1. When the collapse hole is situated below the central part of the building wall, the changes in the consecutive stress distributions are relatively small. The changes in the stress values are also insignificant.

2.2. Additional tensioning stresses appear at the lower borderline of the segment within the collapse area, at the span equal to about 1/6 of the wall width.

Because of the limitations of the applied model, which mostly consist in ignoring the co-operation of the wall with other structural elements of the building as well as in accepting a largely simplified material model of both the wall and the ground, the obtained results do not permit to formulate any far-reaching conclusions. Nevertheless, those results can serve as an introduction and basis for further research, determined by the application of more complex material models as well as by taking into account the co-operation between particular elements of the building structural system.

The prospective research of the behaviour of a building affected by discontinuous ground deformation should involve the elaboration of a model more satisfactorily reflecting the physical properties of the building material and the ground and also including the problem of the co-operation between the structural elements of the building.

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