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H. STÜWE*

LOCALIZATION OF LIMITED STRAINS

LOKALIZACJA ODKSZTAŁCEŃ GRANICZNYCH

The Considère criterion for plastic instabilities in the tensile test is extended to include strain rate sensitivity.

In analogy, a stability criterion for torsion tests on cylindrical specimens is formulated. It takes into account the temperature dependence of flow stress in addition to strain hardening and strain rate sensitivity. In both experiments the localization of limited strains may lead to new stages of homogeneous deformation.

W pracy przedstawiono rozszerzenie kryterium Considère'a dla niestabilności plastycznej w próbie rozciągania w materiałach czułych na prędkość odkształcenia.

Analogiczne kryterium stabilności zostało sformułowane dla prób skręcania próbek cylindrycznych.

Powyższe kryterium, oprócz wpływu umocnienia oraz czułości na prędkość odkształcenia, bierze również pod uwagę wpływ temperatury na naprężenie płynięcia.

Stwierdzono, że w obydwu przypadkach ograniczenie wielkości odkształcenia zlokalizowanego może prowadzić do nowych stadiów deformacji jednorodnej.

1. Introduction. Strain localization in the tensile test

A bar tested in tension can bear the load

$$P = A \cdot \sigma_y, \tag{1}$$

where A is the cross-section and σ_y the flow stress. P, A and σ_y will change with increasing logarithmic plastic strain φ. If the uniform strain is exceeded locally by a small increment $\Delta\varphi$ then the strength o the bar is changed in that section by

$$\Delta P = A\Delta\sigma_y + \sigma_y \Delta A. \tag{2}$$

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* ERICH SCHMID INSTITUT DER ÖSTERREICHISCHEN AKADEMIE DER WISSENSCHAFTEN, A-8700 LEOBEN, AUSTRIA

If $\Delta P > 0$ the fluctuation will be smoothed out with further strain. Rewriting equ. (2) we obtain as condition for strain localisation

$$\frac{\Delta P}{P} = \frac{\Delta\sigma_y}{\sigma_y} - \Delta\varphi < 0 . \tag{3}$$

2. Considère and Lüders

In this classical book [1] Considère considers the case where the flow stress is a function of strain alone, i. e.

$$\sigma_y = \sigma_y(\varphi) . \tag{4}$$

The condition (3) is then reduced to

$$\frac{d\sigma_y}{d\varphi} = \Theta < \sigma_y . \tag{5}$$

In a typical metal σ_y will go up with increasing strain and the hardening coefficient Θ will go down towards zero. So eventually the condition (5) will be met and the specimen will begin to neck. Unless the specimen breaks beforehand the cross section of the neck will be reduced to zero which means that in principle the localised strain is unlimited.

Condition (5) will, of course, also be met when Θ is negative as e. g. in the transition from an upper to a lower yield point. Fig. 1. shows that in this case the localised strain is limited to a finite "Lüders strain" by the subsequent hardening of the material. Further strain will now occur somewhere else.

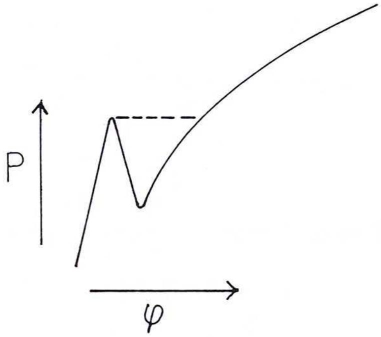

Fig. 1. Lüders strain at transition from upper to lower yield point (schematic)

In the case considered here this will most probably be in the material adjacent to the deformed volume because of the misfit stresses between both regions. As a consequence, a "Lüders band" is spreading through the specimen until all of it has passed the Lüders strain. At this point strain is not localised anymore.

3. The role of other variables and of strain rate sensitivity

Equ. (4), of course, is oversimplified. σ_y depends on more variables that may change with strain such as orientation in single crystals, texture in polycrystals and temperature.

Each of them can contribute to plastic instabilities as has been discussed in more detail in [2].

In the present paper we shall look especially at the role of strain rate sensitivity m. If

$$\sigma_y = \sigma_y(\varphi, \dot{\varphi}) \tag{6}$$

then the instability condition (3) expands to

$$\frac{\Delta\varphi}{\sigma_y}\frac{\partial\sigma_y}{\partial\varphi} + \frac{\Delta\dot{\varphi}}{\sigma_y}\frac{\partial\sigma_y}{\partial\dot{\varphi}} - \Delta\varphi < 0 \tag{7}$$

and, using the definition of m:

$$m = \frac{d\log\sigma_y}{d\log\dot{\varphi}} \tag{8}$$

we obtain

$$m + \Delta\varphi\left(\frac{\Theta}{\sigma_y} - 1\right) < 0. \tag{9}$$

Four cases may be distinguished. The tensile test will be

for	$m > o$	$m < o$
$\sigma_y < \Theta$	stable $\boxed{A}$	unstable for $\boxed{B}$ $\Delta\varphi < \dfrac{m}{1 - \dfrac{\Theta}{\sigma_y}}$
$\sigma_y > \Theta$	unstable for $\boxed{C}$ $\Delta\varphi > \dfrac{m}{1 - \dfrac{\Theta}{\sigma_y}}$	unstable $\boxed{D}$

In typical metals m is small and positive ($o < m < o.\,l.$). Its dependence on strain and strain rate is discussed in [3]. Tensile specimens made of such metals will neck and break after the Considère condition is fulfilled, but this requires a local fluctuation of a finite size (Case *C*). In certain ranges of strain rate and temperature fine-grained materials may have *m*-values > 0.3 and small values of Θ.

These are then superplastic because for strain localization they would need accidental fluctuations of strain of an unprobably high amplitude.

On the other hand certain alloys have negative values of *m*. (Well explored examples are a number of commercial aluminium alloys). They will show limited localized strains before the Considère criterion is fulfilled (Case *B*). This is known as "serrated yield" or as "Portevin-Le Chatelier-Effect".

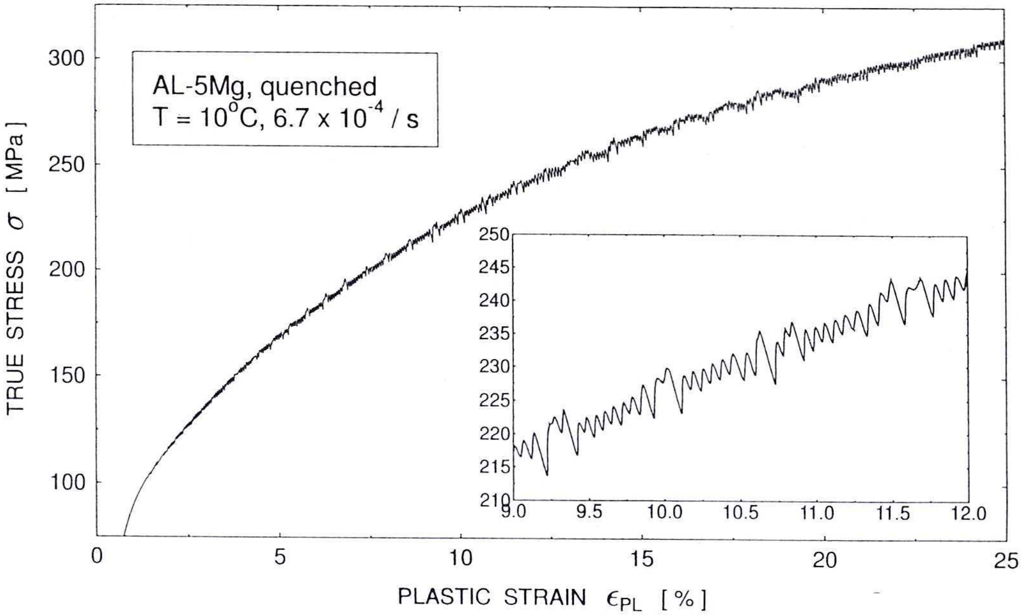

Fig. 2. Serrated yielding in Al-5 Mg_2

Fig. 2. shows an example. One can see that the height of the local strain bursts $\Delta\varphi$ (characterised by the height of the serrations) is increasing as the Considère condition is approached — without, however, exceeding all limits because of the concurrent drop in stress. After the localized event the tensile machine will again raise the stress until the next localized strain burst happens in some other part of the specimen: this kind of strain localization does not spread in the form of a Lüders band.

4. Torsion test

Torsion tests can be evaluated (see, e.g., [2]) as

$$\tau_{ya} = \frac{3M}{2\pi a^3} - \frac{\gamma_a}{2\pi}\frac{d(M/a^3)}{d\gamma_a} \tag{10}$$

where M is the moment, a the radius of the solid cylindrical specimen and γ_a and τ_{ya} are shear strain and shear stress at the surface of the specimen. At high strains (where torsion tests are most useful) it is often a good approximation to omit the second term so that in analogy to equ. (3) we obtain as condition for strain localization

$$\frac{\varDelta M_y}{M_y} = \frac{\varDelta \tau_{ya}}{\tau_{ya}} + 3\frac{\varDelta a}{a} < 0. \tag{11}$$

The second ("geometrical") term allows for the possibility that a free-end torsion specimen may change its length and, hence, its diameter. Such length changes reflect the anisotropy of the material [4]. They may be positive or negative and they are usually small.

On the other hand, changes in temperature are important in torsion tests and cannot be neglected as they were above for slow tensile tests. With

$$\vartheta = \frac{\partial \tau_y}{\partial y} \tag{12}$$

and

$$m = \frac{d\log\tau}{d\log\gamma} \tag{13}$$

equ. (11) — in analogy to equ. (9) — expands to

$$m + \varDelta\gamma_a X < 0, \tag{14}$$

where

$$X = \frac{\vartheta}{\tau_{ya}} + \frac{I}{\varrho c_p}\frac{\partial \tau_{ya}}{\partial T} + \frac{3}{a}\frac{da}{d\gamma_a}. \tag{15}$$

Again, four cases can be distinguished in analogy to the table following equ. (9). It is just more difficult to make general statements on the term X than on the Considère criterion. (Note that neither equ. (9) nor equ. (15) contain a term on the influence of changes in texture. This term and effects caused by it will be discussed in a later paper).

A film on torsion of alloys with serrated yield [5] will be shown at the conference. The first part shows strain localization spreading in analogy to Lüders bands. The second part shows random localization of limited strain. Both processes finally lead to uniform strain.

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