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DOI: 10.24425/ame.2024.151336 Early Access; pp. 1–16

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# Couette flow of micropolar fluid in a channel filled with anisotropic porous medium

**Received** 13 August 2024, **Revised** 11 October 2024, **Accepted** 14 October 2024, **Published online** 20 October 2024

**Keywords:** Darcy-Brinkman equation, Couette flow, micropolar fluid, anisotropic porous medium, permeability ratio

This paper presents a mathematical model for the flow of micropolar fluid in a horizontal channel filled with an anisotropic porous medium, bounded by two parallel plates—where the upper plate is stationary, and the lower plate moves at a constant velocity. The flow, driven by both a constant pressure gradient and the movement of the lower plate, is governed by the Darcy-Brinkman equation. Using no-slip and no-spin boundary conditions, we analytically derive expressions for the velocity, microrotational velocity, and stress distributions. The study provides a graphical analysis of the flow behavior influenced by key parameters such as the Darcy number, porous medium anisotropy, anisotropy angle, and the micropolar fluid's material parameters. Furthermore, the effects of the material parameters and Darcy number on shear stress and couple stress are thoroughly investigated. The findings have applications in modeling fluid flow in striated or fractured rock formations.

## 1. Introduction

Fluid flow through porous media plays a critical role in various scientific and engineering applications, such as enhanced oil recovery, filtration systems, and groundwater hydrology. The Darcy-Brinkman model [1] is frequently used to account for both viscous and porous effects in these applications. For example, Chamkha [2] and Rahman et al. [3], focused on heat generation and slip conditions in complex flow regimes. However, for the low-velocity laminar flow regimes considered here, Darcy's law predominates, and the Forchheimer inertia effects,

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which are significant at higher velocities, can be neglected. Liu et al. [4] studied flow behavior in composite porous channels, while Hooshyar et al. [5] analyzed the stability of pressure-driven flow in porous structures. Rajesh et al. [6] investigated heat generation effects in magnetohydrodynamic (MHD) nanofluid flow over a stretching sheet. Other notable studies, such as Umavathi et al. [7] on flow behavior inside vertical porous channels, Krishna et al [8] on the problem of MHD free convective flow in a vertical porous plate, Gorla et al. [9] on nanofluid flow in natural convection, and Dogonchi et al. [10] on Joule heating effects in MHD flows, contribute to a deeper understanding of fluid dynamics in porous systems.

Micropolar fluids [11, 12], characterized by micro-rotational effects and couple stresses, introduce additional complexity in flow dynamics, especially within porous media. These fluids include polymeric suspensions, liquid crystals, and certain biological fluids, such as blood. Their microstructure leads to behaviors that classical fluid models cannot capture. Studies by Papautsky et al. [13], Ahmad et al. [14], and Jalili et al. [15] have explored the impact of magnetic fields and slip conditions on micropolar fluids, while Jat and Rajotia [16] and Salahuddin [17] emphasized their significance in specialized engineering applications. Researchers such as Nazar et al. [18] and Kelson and Desseaux [19] examined micropolar fluid flow in stretching sheets, while Rahman et al. [20] investigated variable fluid properties. Faltus et al. [21], Chamkha and Al-Mudhaf [22], and Olajuwon et al. [23] focused on various thermal effects on micropolar fluids. Nazar et al. [24] studied flow of micropolar fluid in a circular porous cylinder.

Anisotropy in porous media introduces further complexities, particularly due to directional variations in permeability. This is important in fields like geophysics and cardiovascular medicine, where anisotropic conditions influence fluid flow, as seen in blood flow through tissues. Wang [25] explored rotational effects in anisotropic porous channels, while Verma and Ansari [26], Degan et al. [27], Mobedi et al. [28], and Yovogan and Degan [29] studied the interplay between flow, heat transfer, and anisotropic media. Such studies have advanced our understanding of fluid dynamics in environments where anisotropy plays a critical role.

Despite these efforts, much of the existing research has focused on Newtonian fluids in anisotropic porous media or micropolar fluids in isotropic porous media. This study bridges the gap by analyzing micropolar fluid flow through anisotropic porous media, providing an analytical solution to the Darcy-Brinkman equation. Our focus is on key parameters, including the Darcy number, anisotropy angle, and micropolar material properties, offering new insights into optimizing complex fluid flow processes for both engineering and scientific applications.

The significance of micropolar fluid flow through anisotropic porous media extends to fields where microstructural effects, such as microrotations and couple stresses, are critical. This includes modeling biological systems (e.g., blood flow through anisotropic tissues), advanced filtration processes, and enhanced oil recovery in fractured reservoirs. Anisotropic porous media, such as bones or muscles, exhibit directional variations in permeability, which influence fluid flow.

Understanding these effects is essential for applications like drug delivery, tissue engineering, and biomechanics.

The developed mathematical model in this paper has significant applications across multiple fields, including enhanced oil recovery, filtration systems, biomedical engineering, lubrication technology, and groundwater hydrology. Specifically, it can optimize processes where understanding the behavior of complex fluids in porous structures is crucial. Moreover, insights into anisotropy and micropolar parameters can aid in designing more efficient porous media for biomedical applications and environmental engineering, such as in water resource management and pollution control.

The novelty of this research lies in its investigation of micropolar fluid flow in anisotropic porous media—a scenario that has received limited attention. Previous works have explored either micropolar fluids in isotropic porous environments or Newtonian fluids in anisotropic media, but not the combined influence of anisotropy and microrotation effects. By employing the Darcy-Brinkman equation within the context of Couette flow, this study fills a critical gap, offering a comprehensive parametric analysis of factors like the Darcy number, anisotropy angle, and micropolar material properties. The findings contribute to our understanding of fluid flow in stratified or fractured porous formations, with important implications for hydrology, petroleum engineering, and biomedical applications.

## 2. Formulation of the problem

In this study, the following assumptions are made for the analysis of the flow of a viscous micropolar fluid through a horizontal anisotropic porous channel:

- The flow is assumed to be steady, meaning that fluid properties and flow variables do not vary with time.
- The fluid is considered incompressible, implying that its density remains constant throughout the flow.
- The flow is laminar, characterized by smooth and orderly fluid motion in parallel layers without mixing between them.
- The flow is one-dimensional, with variations in fluid properties occurring solely along the $y'$-direction.
- The fluid is a viscous micropolar fluid, incorporating both viscosity and microrotational effects that influence its flow behavior.
- The channel is horizontal, and the porous medium is anisotropic, characterized by differing permeabilities along two principal axes.
- The channel is bounded by two plates located at $y' = 0$ and $y' = h$.
- The plate at $y' = 0$ moves with a uniform velocity $U$, driving the fluid flow (as shown in Fig. 1).
- The porous medium is anisotropic, with permeabilities $k_1$ and $k_2$ along the principal axes, and an angle $\phi$ between the axis of $k_2$ and the horizontal axis.

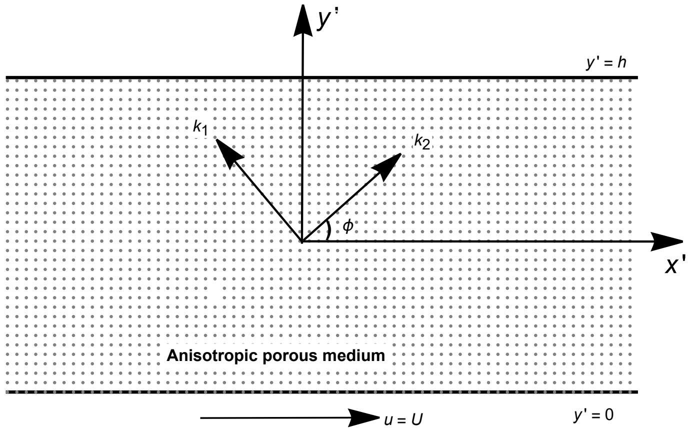


Fig. 1. Schematic diagram of considered problem

- The flow is driven by a constant pressure gradient in addition to the movement of the lower plate.
- The velocity of the micropolar fluid is represented as $\mathbf{u} = (u'(y), 0, 0)$, varying only in the $y'$-direction with no components in the $x'$- and $z'$-directions.
- The micro rotational velocity of the fluid is given by $\mathbf{w} = (0, 0, w'(y))$, varying only in the $y'$-direction with no components in the $x'$- and $z'$-directions.

Since the permeability of porous medium is taken anisotropic rather than isotropic, then that permeability in the form of the symmetrical second-order tensor is given by [27, 29–31],

$$\begin{bmatrix} k_1 \sin^2\phi + k_2 \cos^2\phi & -(k_1 - k_2)\sin\phi\cos\phi \\ -(k_1 - k_2)\sin\phi\cos\phi & k_1 \sin^2\phi + k_2\cos^2\phi \end{bmatrix}.$$

Also we have assumed the viscosity of fluid is identically same as the effective viscosity of fluid [25, 26]. The governing equations for the flow of micropolar fluids through anisotropic porous channel under the above assumptions are [27, 29–32],

$$\frac{\partial p'}{\partial x'} = (\mu + \lambda)\frac{d^2u'}{dy'^2} - \frac{\alpha_1\mu}{k_1}u' + \lambda\frac{dw'}{dy'}, \tag{1}$$

$$\frac{\partial p'}{\partial y'} = -\frac{\alpha_2\mu}{k_1}u', \tag{2}$$

$$\gamma\frac{d^2w'}{dy'^2} - 2\lambda w' - \lambda\frac{du'}{dy'} = 0, \tag{3}$$

where $\mu$, $\gamma$ and $\lambda$ are viscosity, spin gradient viscosity and vortex viscosity, respectively. Also $\alpha_1 = \sin^2\phi + r\cos^2\phi$, $\alpha_2 = (r-1)\sin\phi\cos\phi$ and $r = \dfrac{k_1}{k_2}$ is permeability ratio. The no-slip and no-spin boundary conditions on the plates of the channel are $u'(0) = 1, u'(1) = 0, w'(0) =$ and $w'(1) = 0$.

Now differentiating the equations (1) and (2) with respect to $x'$ and using the fact that velocity $u$ is a function of $y'$ only, we have,

$$\frac{\partial^2 p'}{\partial x'^2} = 0 \quad \text{and} \quad \frac{\partial^2 p'}{\partial x' \partial y'} = 0, \tag{4}$$

implies the applied pressure gradient $\dfrac{\partial p'}{\partial x'}$ is constant.
Now consider the dimensionless quantities as follows:

$$x = \frac{x'}{h}, \; y = \frac{y'}{h}, \;\; u = \frac{u'}{U}, \; w = \frac{w'h}{U} \;\text{ and }\; p = \frac{p'h}{U\mu}.$$

The vortex viscosity $\lambda$ represents the additional resistance to the rotation of fluid particles in micropolar fluids. This parameter is crucial for capturing the micro-rotational effects and is experimentally determined based on the fluid's microstructure. As for the spin gradient viscosity $\gamma$, its dependence on the channel geometry arises because, in micropolar fluid theory, the micro-rotational effects vary with the spatial configuration of the flow domain. This parameter is particularly significant in simplified geometries like channels but could become more complex in other flow domains. In more intricate geometries, $\gamma$ would likely vary spatially, necessitating numerical methods for its determination. In the present article, the spin gradient viscosity $\gamma$ is given by the mathematical formula, $\gamma = (\mu + \frac{\lambda}{2})h^2$ [33]. Therefore the equations (1) and (3) become,

$$(1 + K)\frac{d^2u}{dy^2} - \frac{\alpha_1}{\alpha}u + K\frac{dw}{dy} = P, \tag{5}$$

$$(1 + \frac{K}{2})\frac{d^2w}{dy^2} - 2Kw - K\frac{du}{dy} = 0, \tag{6}$$

where $P = \dfrac{\partial p}{\partial x}$, $K = \dfrac{\lambda}{\mu}$ is material parameter of micropolar fluid and $\alpha = \dfrac{k_1}{h^2}$ is Darcy number. Also after applying non-dimensional quantities, the boundary conditions become $u(0) = 1, u(1) = 0, w(0) =$ and $w(1) = 0$.

## 3. Solution of the problem

The equations (5) and (6) are coupled equations hence can be evaluated analytically by direct method. Therefore, the solutions of equations (5) and (6) are given by,

$$u(y) = C_1 \exp(\eta_1 y) + C_2 \exp(-\eta_1 y) + C_3 \exp(\eta_2 y) + C_4 \exp(-\eta_2 y) - \frac{P\alpha}{\alpha_1}, \tag{7}$$

$$w(y) = \delta_1(C_2\exp(-\eta_1 y) - C_1\exp(\eta_1 y)) + \delta_2(C_4\exp(-\eta_2 y) - C_3\exp(\eta_2 y)), \quad (8)$$

where

$$\eta_1 = \sqrt{\frac{M+\sqrt{(M^2-4N)}}{2}}, \quad \eta_2 = \sqrt{\frac{M-\sqrt{(M^2-4N)}}{2}},$$

$$M = \frac{\alpha_1}{\alpha(1+K)} + \frac{2K}{(1+K)}, \quad N = \frac{4\alpha_1}{\alpha(2+K)(1+K)},$$

$$\delta_1 = \frac{\eta_1^3(1+K)(2+K)}{4K^2} - \frac{\eta_1\alpha_1(2+K)}{4\alpha K^2} + \frac{\eta_1}{2},$$

$$\delta_2 = \frac{\eta_2^3(1+K)(2+K)}{4K^2} - \frac{\eta_2\alpha_1(2+K)}{4\alpha K^2} + \frac{\eta_2}{2}.$$

Using boundary conditions $u(0) = 1, u(1) = 0, w(0) =$ and $w(1) = 0$ on the equations (7) and (8), we get the system of linear equations as follows,

$$C_1 + C_2 + C_3 + C_4 = \frac{P\alpha}{\alpha_1} + 1, \quad (9)$$

$$C_1\exp(\eta_1) + C_2\exp(-\eta_1) + C_3\exp(\eta_2) + C_4\exp(-\eta_2) = \frac{P\alpha}{\alpha_1}, \quad (10)$$

$$\delta_1(C_2 - C_1) + \delta_2(C_4 - C_3) = 0, \quad (11)$$

$$\delta_1(C_2\exp(-\eta_1) - C_1\exp(\eta_1)) + \delta_2(C_4\exp(-\eta_2) - C_3\exp(\eta_2)) = 0. \quad (12)$$

Solving the above system of linear equations by software MATHEMATICA 12.3, we obtained the arbitrary constants $C_1, C_2, C_3$ and $C_4$.
The non-dimensional shear stress $\tau$ and couple stress $m_f$ acting on the upper plate $y = 1$ of the channel are given by

$$\tau = -\left[(1+K)\frac{du}{dy} + w\right]_{y=1} \quad \text{and} \quad m_f = -\left[(1+\frac{K}{2})\frac{dw}{dy}\right]_{y=1}. \quad (13)$$

**Remark:** The analytical solution of the ODE system (equations (5) and (6)) is possible because the parameters (such as permeability, viscosity) are considered constant. Otherwise if these parameters were functions of spatial variable, a numerical solution would be required.

### 3.1. Particular cases

1. If $K = 0$, that is, vortex viscosity of micropolar fluid is zero, then micropolar fluid which is non-Newtonian fluid becomes Newtonian fluid and governing equation (5) reduce to,

$$\frac{d^2u}{dy^2} - \frac{\alpha_1}{\alpha}u = P. \quad (14)$$

This problem becomes the Couette flow of Newtonian fluid through anisotropic porous medium. The velocity in this case is given by,

$$u(y) = C_1^* \exp(\sqrt{\frac{\alpha_1}{\alpha}} y) + C_2^* \exp(-\sqrt{\frac{\alpha_1}{\alpha}} y) - \frac{P\alpha}{\alpha_1}, \tag{15}$$

where $C_1^*$ and $C_2^*$ are arbitrary constants, which can be found by applying boundary condiions $u(0) = 1$ and $u(1) = 0$. We found that the results are well matched with Degan et al [27], Karmakar [31], and Karmakar et al [29] with given suitable boundary conditions in each cases.

2. If $K \neq 0$ and $k_1 = k_2$, that is, in the case of isotropic porous medium then the governing equations (5) and (6) reduce to,

$$(1 + K)\frac{d^2u}{dy^2} - \frac{1}{\alpha}u + K\frac{dw}{dy} = P, \tag{16}$$

$$(1 + \frac{K}{2})\frac{d^2w}{dy^2} - 2Kw - K\frac{du}{dy} = 0. \tag{17}$$

Therefore, the present problem reduces to Couette flow of micropolar fluids through isotropic porous channel.

## 4. Discussion and results

The behavior of velocity profiles $u(z)$, microrotational velocity $w(y)$, shear stress $\tau$, and couple stress $m_f$ on the upper plate of the channel is analyzed in this section under the influence of various parameters, including the Darcy number $\alpha$, permeability ratio $r$, anisotropy angle $\phi$, and material parameter $K$. The applied pressure gradient, $P = -1$, remains constant throughout the analysis.

Fig. 2 illustrates the effect of the material parameter $K$ on the micropolar fluid velocity $u(y)$ across the channel, with fixed values of $\alpha = 0.01$, permeability ratio $k = 0.5$, and anisotropy angle $\phi = \frac{\pi}{4}$. The results show that the material parameter $K$ significantly influences the velocity $u(y)$. As $K$ increases, the velocity $u(y)$ decreases, which can be attributed to the fact that higher vortex viscosity $\lambda$ increases rotational resistance, thereby enhancing the effect of micro-rotations on the flow and reducing the overall velocity. This behavior is consistent with findings reported in [32]. Additionally, when $K$ approaches zero, the micropolar fluid approximates the behavior of a Newtonian fluid.

Fig. 3 presents the effect of the permeability ratio $r$ on the micropolar fluid velocity $u(y)$ across the channel, with fixed values of $\alpha = 0.01$, $K = 1$, and $\phi = \frac{\pi}{4}$. As $r = \frac{k_1}{k_2}$ increases, which corresponds to a decrease in the permeability $k_2$, the velocity $u(y)$ decreases. This trend has also been observed in [27, 29].

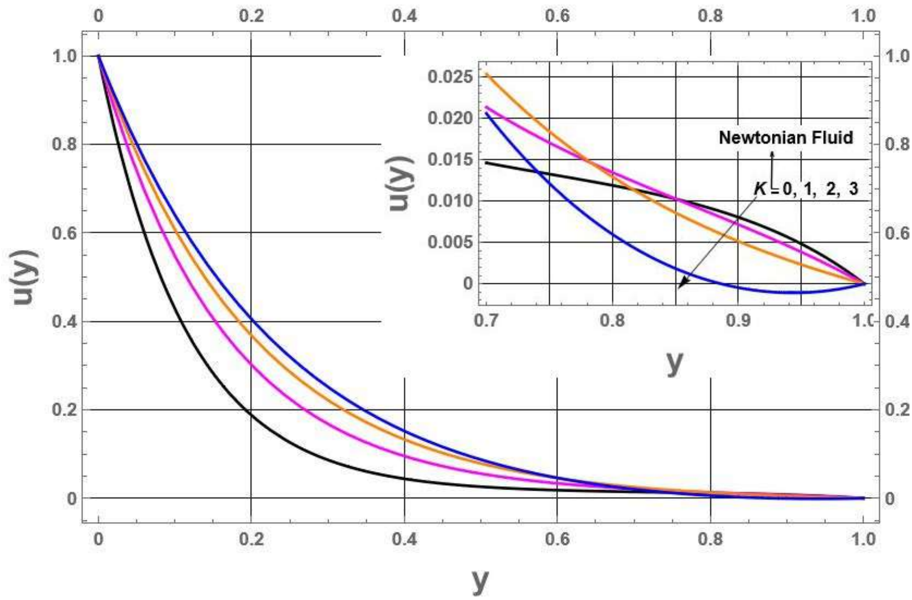


Fig. 2. Effect of material parameter $K$ on velocity $u(y)$ as a function of $y$, with $\alpha = 0.01$, $r = 0.5$, $\phi = \frac{\pi}{4}$, and $P = -1$ fixed

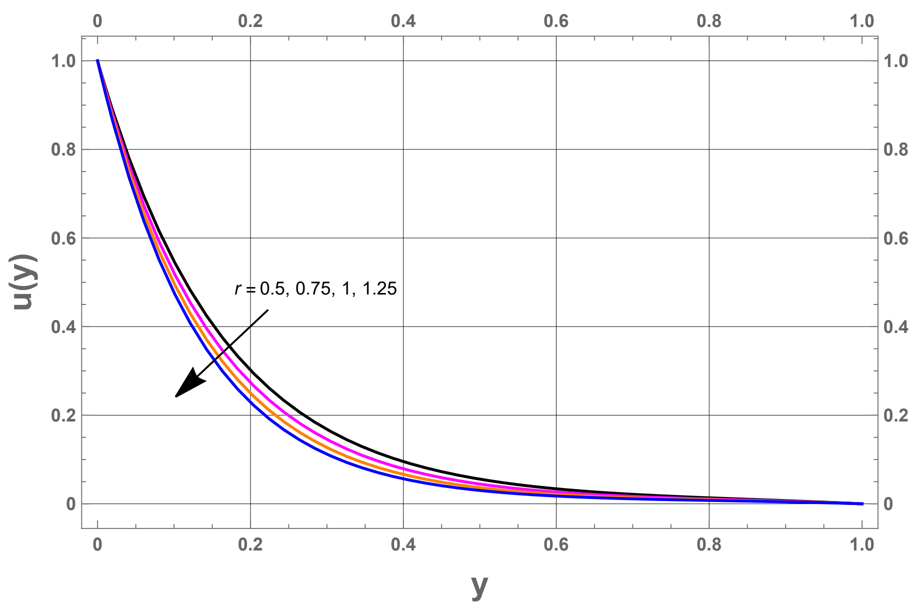


Fig. 3. Effect of permeability ratio $r$ on velocity $u(y)$ as a function of $y$, with $\alpha = 0.01$, $K = 1$, $\phi = \frac{\pi}{4}$, and $P = -1$ fixed

Fig. 4 demonstrates the effect of the Darcy number $\alpha$ on the velocity $u(y)$ across the channel, keeping the permeability ratio $r = 0.5$, material parameter $K = 1$, and anisotropy angle $\phi = \dfrac{\pi}{4}$ constant. As the Darcy number $\alpha$ increases, the velocity $u(y)$ also increases due to the enhanced permeability along the flow direction.

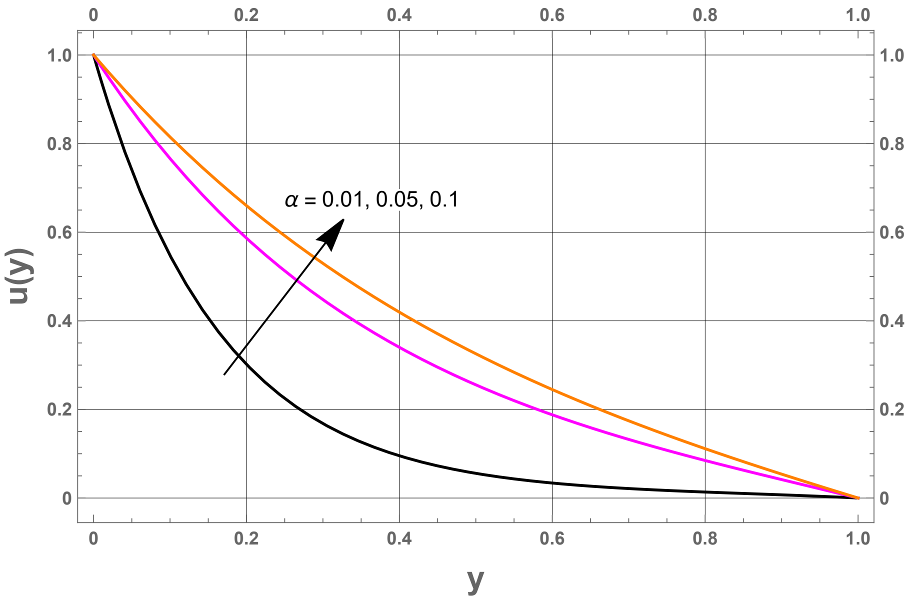


Fig. 4. Effect of Darcy number $\alpha$ on velocity $u(y)$ as a function of $y$, with $\phi = \pi/4$, $K = 1$, $r = 0.5$, and $P = -1$ fixed

Fig. 5 shows the influence of the anisotropy angle $\phi$ on the velocity $u(y)$, with $r = 0.5$, $K = 1$, and $\alpha = 0.01$ held constant. As the anisotropy angle $\phi$ increases, the velocity $u(y)$ decreases. Across Figs. 2, 3, 4, and 5, it is consistently observed that the velocity $u(y)$ reaches its maximum at the lower plate and gradually reduces to zero at the upper plate, in accordance with the boundary conditions.

Fig. 6 shows the effect of the material parameter $K$ on the microrotational velocity $w(y)$ across the channel, with fixed values of $\alpha = 0.01$, $r = 0.5$, and $\phi = \dfrac{\pi}{4}$. As the material parameter $K$ increases, the microrotational velocity $w(y)$ decreases. Similar behavior is also noted in [32].

Fig. 7 depicts the effect of the permeability ratio $r$ on the microrotational velocity $w(y)$, with $\alpha = 0.01$, $K = 1$, and $\phi = \dfrac{\pi}{4}$ held constant. The results show that as $r$ increases, the microrotational velocity $w(y)$ decreases.

Fig. 8 highlights the influence of the Darcy number $\alpha$ on the microrotational velocity $w(y)$. With $r = 0.5$, $K = 1$, and $\phi = \dfrac{\pi}{4}$ held constant, it is observed that

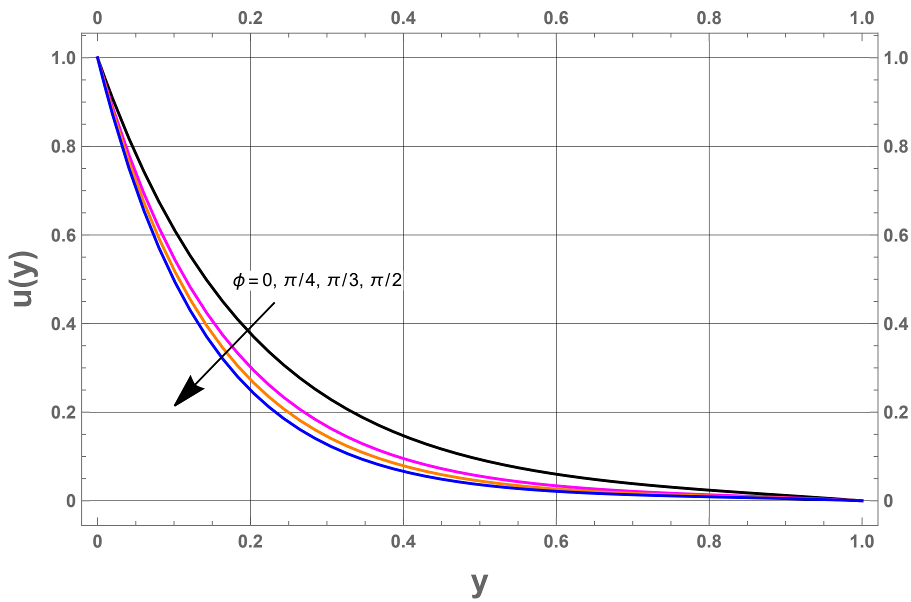


Fig. 5. Effect of anisotropy angle $\phi$ on velocity $u(y)$ as a function of $y$, with $\alpha = 0.01$, $K = 1$, $r = 0.5$, and $P = -1$ fixed

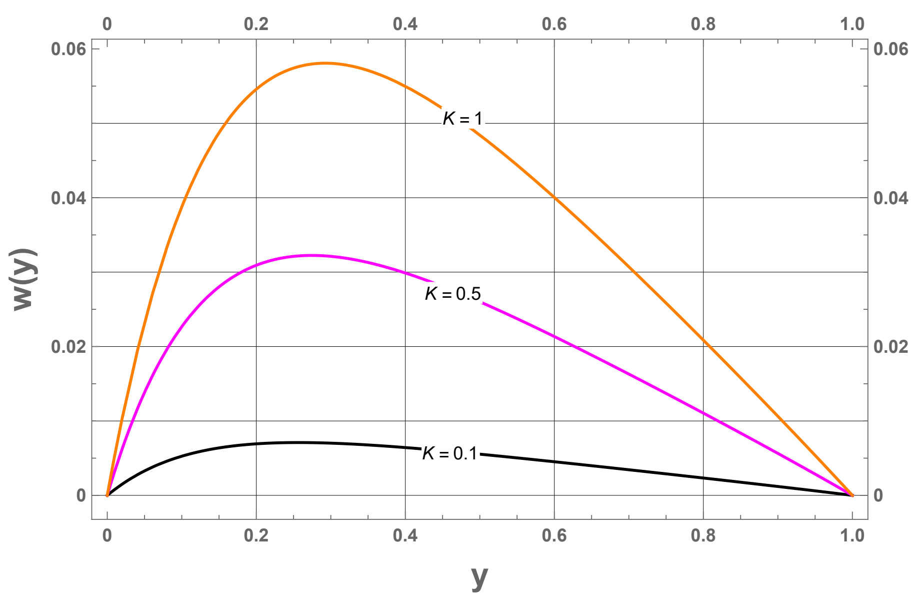


Fig. 6. Effect of material parameter $K$ on microrotational velocity $w(y)$ as a function of $y$, with $\alpha = 0.01$, $r = 0.5$, $\phi = \dfrac{\pi}{4}$, and $P = -1$ fixed.

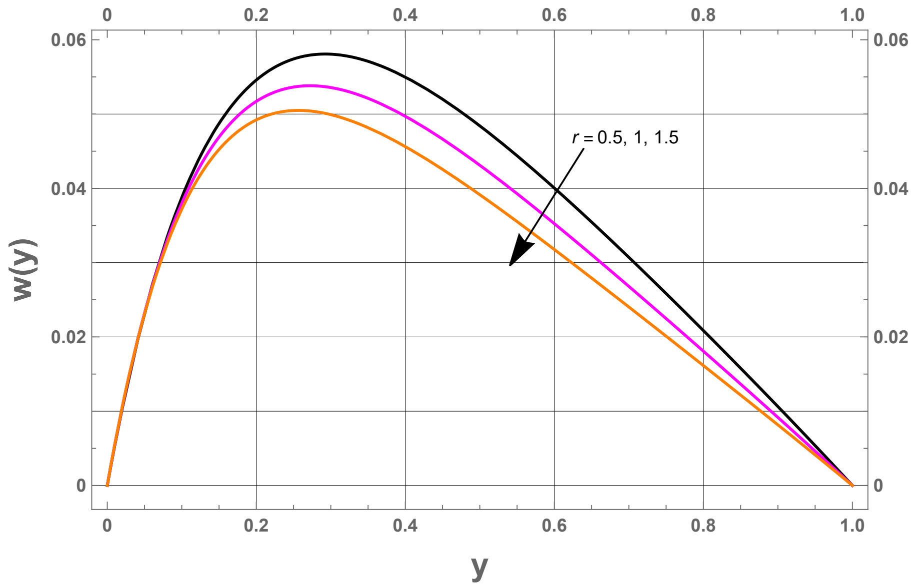


Fig. 7. Effect of permeability ratio $r$ on microrotational velocity $w(y)$ as a function of $y$, with $\alpha = 0.01$, $K = 1$, $\phi = \frac{\pi}{4}$, and $P = -1$ fixed

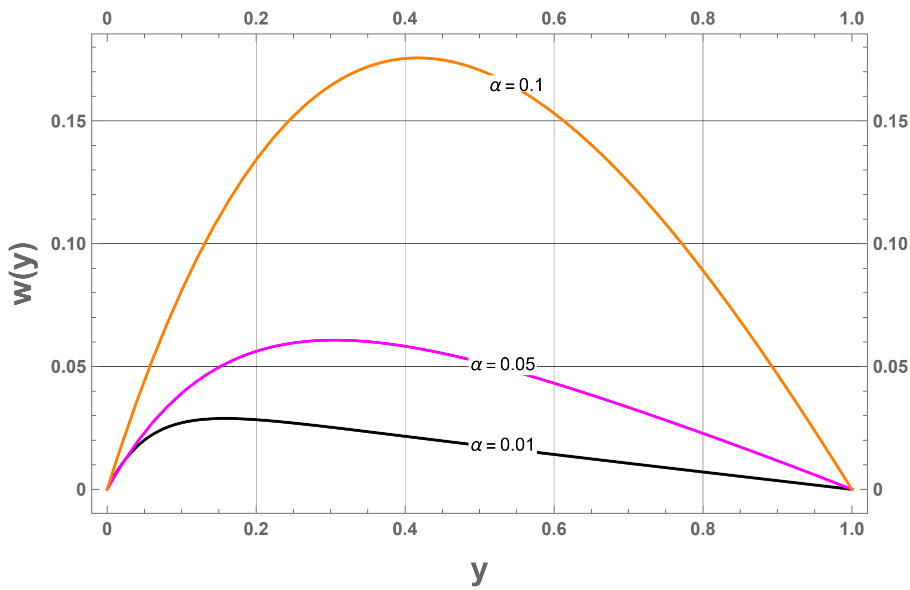


Fig. 8. Effect of Darcy number $\alpha$ on microrotational velocity $w(y)$ as a function of $y$, with $\phi = \pi/4$, $K = 1$, $r = 0.5$, and $P = -1$ fixed

$w(y)$ increases above $y = 0$ and decreases below $y = 0$ as $\alpha$ increases, due to the higher permeability along the flow direction.

Fig. 9 shows the effect of the anisotropy angle $\phi$ on the microrotational velocity $w(y)$ across the channel. With $r = 0.5$, $K = 1$, and $\alpha = 0.01$ held constant, the results indicate that the microrotational velocity $w(y)$ decreases as the anisotropy angle $\phi$ increases.

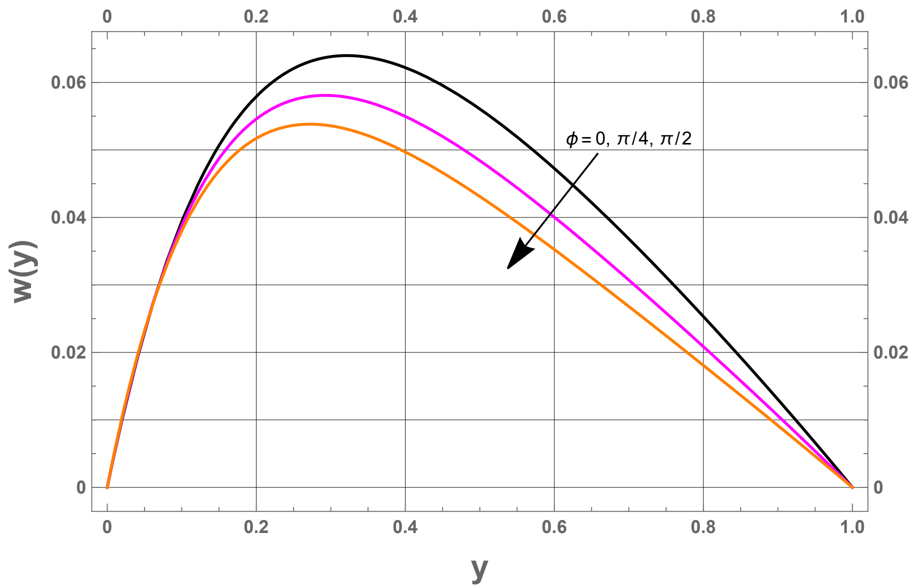


Fig. 9. Effect of anisotropy angle $\phi$ on microrotational velocity $w(y)$ as a function of $y$, with $\alpha = 0.01$, $K = 1$, $r = 0.5$, and $P = -1$ fixed

Figs. 10 and 11 depict the effect of the Darcy number $\alpha$ on the shear stress $\tau$ and couple stress $m_f$ acting on the upper wall, respectively, for various material parameters $K$, with $r = 0.5$ and $\phi = \pi/4$ held constant. The results indicate that both $\tau$ and $m_f$ increase with higher values of $K$ and $\alpha$.

## 5. Conclusion

The study investigates the flow of a micropolar fluid through a horizontal channel filled with anisotropic porous material, where the lower plate moves with a uniform velocity. Analytical evaluations of fluid velocity, microrotational velocity, and shear stresses on the channel plates are conducted under no-slip and no-spin boundary conditions. The results indicate that fluid velocity decreases with increasing permeability ratio, material parameter, and anisotropy angle, while it increases with a higher Darcy number. Additionally, microrotational velocity increases with higher material parameter and Darcy number but decreases with

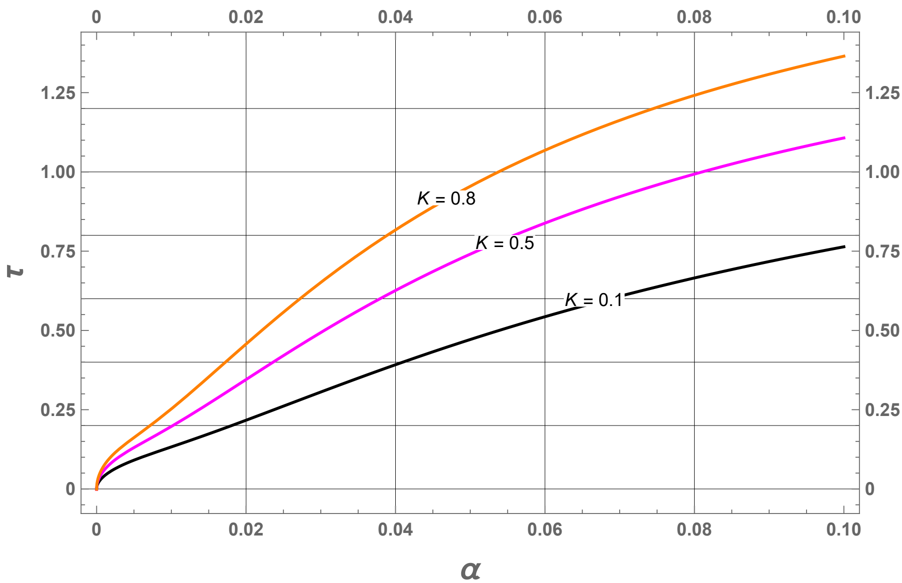


Fig. 10. Effect of material parameter $K$ on shear stress (on upper wall of the channel) $\tau$ with Darcy number $\alpha$ when $k = 0.5$, $\phi = \pi/4$ and $P = -1$ are fixed

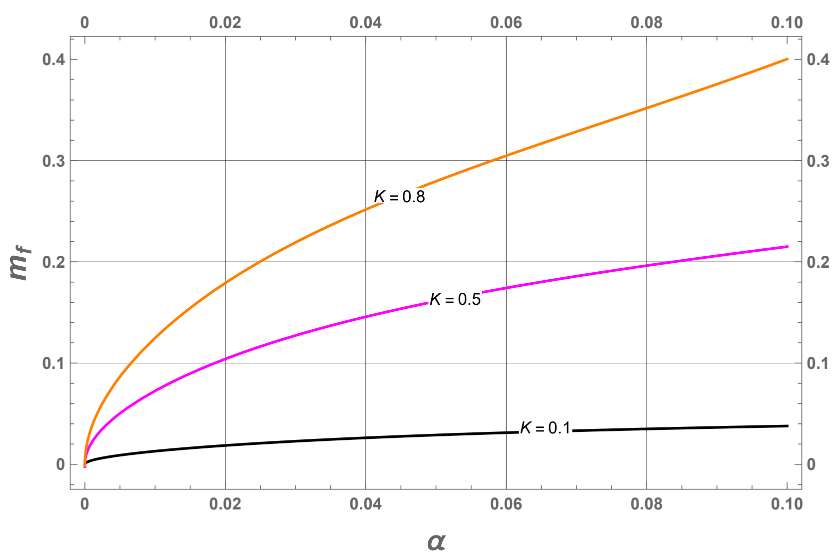


Fig. 11. Effect of material parameter $K$ on couple stress (on upper wall of the channel) $m_f$ with Darcy number $\alpha$ when $k = 0.5$, $\phi = \pi/4$ and $P = -1$ are fixed

increasing permeability ratio and anisotropy angle. Shear stress and couple stress on the upper plate are observed to increase with higher material parameter and Darcy number.

## Acknowledgements

We would like to express our sincere gratitude to the three anonymous reviewers for their constructive and insightful suggestions, which greatly helped to improve the quality of this manuscript.

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