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On Behavior of Sampled Signal when Value of Sampling Period Tends to Zero and Analog-to-Digital Conversion in Microwave Photonic Systems

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***Abstract*—In this paper, we compare behaviors of two possible descriptions of the sampled signal in the case when the sampling period tends to zero, but remains all the time greater than zero. Note that this is the case we are dealing with in analog-to-digital conversion in microwave photonic systems. From this comparison, it follows that the description with the weighted step function is superior to the description with the weighted Dirac comb. A couple of useful comments and remarks associated with the results presented are also provided in the context of microwave photonic systems.**

***Keywords*—Modelling of sampled signal in general and in microwave photonic systems; behavior of this signal for sampling period going to zero description of signals occurring in analog-to-digital microwave photonic systems.**

I. Introduction

IT is well known that measured 'images' of signals that we consider to be functions (signals) of a continuous time t are not exact copies (in 100 percent) of the latter ones [1], [2]. This is so for many reasons. One of them are inertia of physical measuring and recording devices. (In what follows, we focus only on this distorting factor.) The above causes that the values of a waveform cannot be recorded continuously. They must be seen as recorded at intervals, which most often are so short that a waveform interpolated on them can be assumed to be identical to the 'true' signal. Whereby the latter term, we understand as that signal through which we 'see' a physical object or its physical property in time space (described by a timeline t that is a set of real numbers $\mathbf{R}$). In practice, it means that we identify this signal with a given physical object, and its representation are measurements (which are more or less accurate). In other words, this can be also expressed in such a way: the 'true' signal represents some physical object, but the measured one is a more or less accurate representation of the former [3]. And this is a commonly used convention.

Complementing the above observations, let us share one more observation. See that in general a relation between the 'true' signal and its measured version is not known (in fact, this is always the case in reality). What we do in practice is to make certain assumptions about measurements – among them, most importantly, to assume that we make measurements with very accurate instruments – which allows us to assume that practically the relation mentioned is one to one. (That is the relation between the 'true' signal and its measured version.) And, possibly, some measurement errors (among them also those following from the measuring devices) should be taken into account (if this point is important). But usually we assume that they are small and do not obscure the nature of the 'true' signal. Therefore, within the accuracy determined by small measurement errors, we identify the 'true' signal with its measured version. Moreover, because the 'true' signal is always a continuous one in time t, then its measured version should imitate this property and it does.

We hope that this presented above brief reminder of the basic assumptions (axioms) underlying the interpretation of signals acquired in measurements will allow us to better understand the context of our primary issue addressed in this paper, namely the study of how descriptions of sampled signals behave in the extreme case of sampling period values going to zero. The possibility of interpreting the measurement process by means of the signal sampling process has been pointed out in publications [4] and [5]. Here, we will use the conceptual relations between these processes discussed there in detail. They will also prove to be useful here.

Now, let us present shortly the two possible descriptions of the sampled signal (sampled in an ideal way) that are used in the literature for describing the continuous time signal at the output of an A/D converter, exploited in calculations of its spectrum. The first one is via a weighted Dirac comb [6–17], but the second through a weighted step function [18], [19].

The description that uses the weighted Dirac comb, and which we denote here by $x_D(t)$, has the following form:

$$x_D(t) = \delta_T(t) \cdot x(t) = \sum_{k=-\infty}^{\infty} x(kT)\delta(t-kT), \qquad (1)$$

where $x(t)$ means an analog signal to be sampled and the $x(kT)$'s are values of its samples at the corresponding discrete time instants kT's with T standing up for a sampling

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period. Furthermore, the k's there belong to the set of integers, i.e. $k \in \mathrm{Z}$. Moreover, the time-dependent object $\delta_T(t)$ in (1) is called the Dirac comb. It is defined as

$$\delta_T(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT), \tag{2}$$

where the $\delta(t-kT)$'s, $k \in \mathrm{Z}$, mean the time-shifted Dirac deltas.

The second representation for sampled signals via the weighted step function, which is denoted here by $x_{STEP}(t)$, possesses the following form:

$$x_{STEP}(t) = \sum_{k=-\infty}^{\infty} x(kT)\,\mathrm{rect}(t-kT), \tag{3}$$

In (3), $\mathrm{rect}(\cdot)$ means the standard rectangular function, which is defined as

$$\mathrm{rect}(t) = 1 \text{ for } t \in \langle 0,T) \text{ and } 0 \text{ otherwise}. \tag{4}$$

We illustrate here with Fig. 1 the description given by (1).

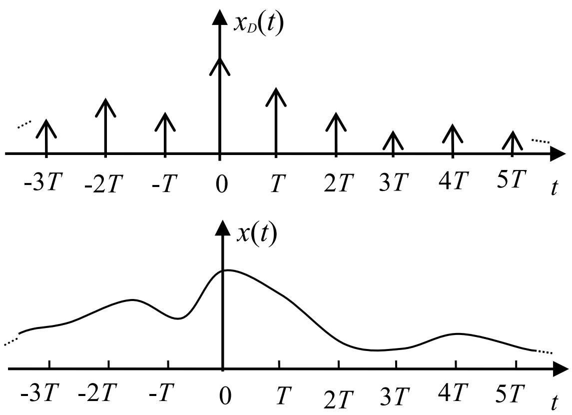

Fig. 1. Example sampled signal representation (upper curve) in form of a weighted Dirac comb and its un-sampled version (lower curve). Figure taken from [3].

Whereas the second description, given by (3), is visualized via Fig. 2.

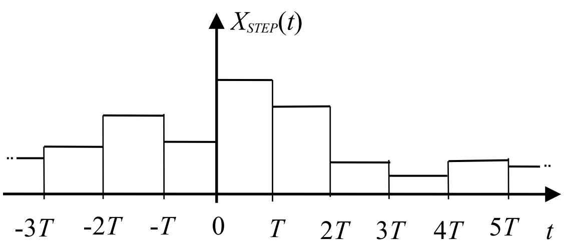

Fig. 2. Example sampled signal representation in form of a weighted step function that corresponds with its un-sampled version shown in Fig. 1 (lower curve). Figure taken from [18].

Note that a usual way [6–17] of visualization of Dirac deltas of a weighted Dirac comb, as in Fig. 1 (upper curve), is via the use of arrows of different heights equal to the values of the successive signal samples $x(kT)$'s. Whereby all these values are finite numbers. But, on the other hand, according to the naive theory of Dirac distributions [20], they represent at the same time infinities. This is so because any $x(kT)$ times infinity gives infinity. We get thus a contradiction.

As an aside, let us note that the visualization of the sampled signal shown in Fig. 1, which is commonly used in the literature [6–17], is an attempt to use the Dirac delta description according to the naive Dirac distribution theory [20]. Because if this was not the case, it would mean that the Dirac delta understood in a strictly mathematical, distributional sense could be somehow visualized, yet this is not possible [20]. The Dirac delta is characterized by a functional, not by providing a waveform of a continuous time. So, concluding, we say that because of the reasons given the above attempt does not lead to a satisfactory result. Or, having said it more clearly: for the reasons given, this description should not be used at all. However, since it is used, let us examine (to satisfy our pure curiosity) what one obtains from it in the case when the sampling period T tends to zero. (That is in the case when any signal sampling becomes invisible to us, where we expect the signal to equal approximately $x(t)$.) We will consider this case in the next section.

And finally here a remark to Fig. 2: the waveform shown there represents a sampled signal according to (3), whose an un-sampled version is illustrated in Fig. 1 (lower curve).

The remainder of the paper is organized as follows. In the next section, we compare with each other the descriptions given by the formulas (1) and (3) for an extreme case, when the value of the sampling period T tends to zero. Additionally, this analytical comparison is illustrated by Figs. 4 and 5. Section 3 is devoted to discussion of the relevance of different general models of signal sampling proposed in the literature in correct description of the signals occurring in analog-to-digital conversion in microwave photonic systems. The paper ends with a conclusion.

II. Comparison of Sampled Signal Descriptions via (1) and (3) when Sampling Period Goes to Zero

Let us note at the beginning of this section that in older sampling models, as for example in Bracewell's [7] (see Fig. 10.2 (c) and (d), page 221 there), we have the distinction between a signal in the form of a function consisting of isolated discrete points (i.e. denoting finite sample values of a sampled signal) and this sampled signal modelled with the use of a weighted Dirac comb, as in Fig. 1 (upper curve). In contrast, such a distinction is no longer made by the authors of a younger generation, for instance in the textbook of Ingle and Proakis [15]. Bracewell speaks about an equivalence of the two representations (although it is not clear what this equivalence would mean precisely), while in the work of Ingle and Proakis [15] it is said only that such a signal as in Fig. 1 (upper curve) is generated in the path: A/D converter – signal processor – D/A converter (as an aside, it is not clear why in their work this generation takes place at the input of the D/A converter, see [15], page 87 there).

This is, in brief, the current state of understanding as well as the way of describing the sampling process of analog signals commonly used within the signal processing society. The model and description that uses the formula (3) has, so far, only one advocate [18], [19]. And we would like to highlight just these two facts, at this moment, before going to the analysis mentioned in the title of this paper.

Consider now a timeline t with Dirac deltas connected with each of the time instants $t \in \mathbf{R}$ of this axis. Or, in other words, let us perceive this as the time instants t's that possess attributes that are Dirac deltas. Further, let us denote such an object (set) as Δ_t, compactly defined by

$$\Delta_t = \{\text{all } \delta(t-d)\text{'s with } d \in \mathbf{R} \text{ going from } -\infty \text{ to } +\infty\}. \tag{5}$$

The object Δ_t is visualized in Fig. 3.

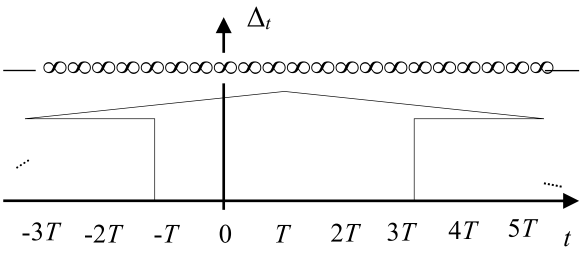

Fig. 3. Visualization of the object given by (5). The upper half-plane is tightly filled with Dirac deltas, which, according to their naive, illustrative model (see [20]), 'run' at all time instants to plus infinities.

The author of this paper would like to emphasize at this point that he did not invent this strange object defined by (5) and illustrated in Fig. 3. It was invented by serious scientists working in the field called Network Calculus [21] (see Fig. 3.1, page 107 there). And, since it is, and it is used in engineering analyses, we will also use it here.

More precisely, we will show that by using the object given by (5) we are able to describe abstractly the process of sampling an analog signal, as it is described by (1). So approaching this issue in a completely abstract way (embodied in (5), i.e. fully detached from the electronics of any A/D converter that performs this process), we can explain it as follows. The sampling process takes place against the background of an object Δ_t, which however actively participates in it. A sampling device selects values of the function $x(t)$ for the prescribed time instants of sampling and multiplies them successively by those $\delta(t-d)$'s, which are connected with these sampling time instants. More precisely, for an example instant of sampling kT, the value of d occurring in $\delta(t-d)$ is chosen to be equal to kT. This results in the signal given by (1), which consists of the values of the signal samples of $x(t)$ connected with the background elements (via operation of multiplication). And the latter ones are obviously usual Dirac deltas shifted on the time axis. That is, according to this picture, the sampling operation (let us denote it here by the symbol SAMP) can be compactly expressed as follows:

$$\mathrm{SAMP}\left(x(t),\Delta_t\right) = x_D(t) = \sum_{k=-\infty}^{\infty} x(kT)\,\delta(t-kT)\ . \tag{6}$$

Now, let us see what a sampled signal looks like in this abstract signal sampling model outlined above when the sampling period T goes to zero. In our reasoning, we will use graphical aids to describe this process. And, let us start with Fig. 1 (upper curve). See that when the sampling period T tends to zero, but remains positive all the time, the arrows in the upper curve of Fig. 1 are getting denser. (As an aside, we treat then T as an infinitesimal number [20].) Eventually it gets to the point where they begin to be indistinguishable from one another due to the passage of a resolution threshold (below which we are no longer able to distinguish between them). So at this point, we can say that we reached such a state as shown in Fig. 4.

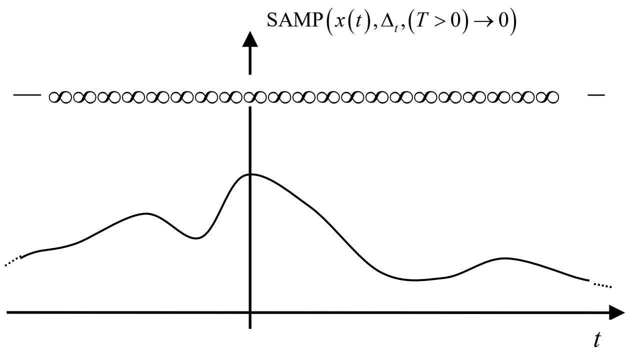

Fig. 4. Visualization of the upper curve of Figure 1 for the sampling period T tending to zero, but remaining positive all the time. Note that for the sake of transparency of this figure, a large arrow in Figure 3 is not repeated here.

Note now that it follows from Fig. 4 that the sampled signal in the model via the weighted Dirac comb tends to be indistinguishable from the un-sampled signal $x(t)$, but it occurs against the background of an object Δ_t that is inextricably connected to it. This is, of course, a somewhat bizarre result, because in this case ($(T>0)\to 0$) one would expect only $x(t)$. Because this case mimics in fact the case of a lack of any sampling, where we have only $x(t)$, and nothing more. There occurs then no background in the form of this strange non-physical object Δ_t. We just do not see it or measure it.

Note that using (6) we can express the above compactly as

$$\mathrm{SAMP}\left(x(t),\Delta_t,(T>0)\to 0\right) = x_D\left(t,(T>0)\to 0\right) \to \\ \to x(t) \text{ on the background of } \Delta_t\ . \tag{7}$$

Now, let us check how the second possible description of the sampled signal, given by (3), behaves when the sampling period T goes to zero, but remains all the time greater than zero. For this purpose, we also use a graphical method, as before, to show the changes taking place in the waveform of Fig. 2. And note that in this case the rectangles occurring in this figure become thinner and thinner as $(T>0)\to 0$, and their density of occurrence at a given time interval becomes greater and greater. But an envelope of these rectangles becomes then closer and closer to an un-sampled waveform $x(t)$. This is illustrated in Fig. 5.

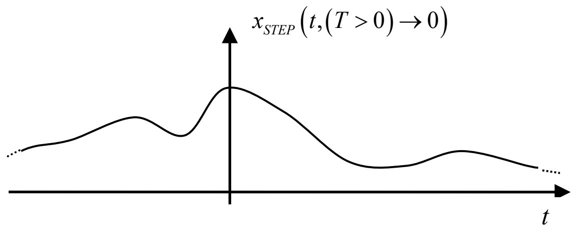

Fig. 5. Visualization of the waveform $x_{STEP}(t)$ given by (3) for the sampling period T tending to zero, but remaining positive all the time, with the use of an example un-sampled signal presented in Fig. 1 (lower curve).

Note further that we can describe compactly the above behavior as follows:

$$x_{STEP}(t,(T>0)\to 0)\to x(t). \tag{8}$$

III. Relevance of Models Analyzed Here in Correct Description of Signals Occurring in Analog-to-Digital Conversion in Microwave Photonic Systems

Many people involved in the field of microwave photonic systems probably doubt that the issues and models of signals in analog-to-digital conversion, which were considered in the previous two sections, have any relevance to them. If they think so, they are mistaken. We will try to demonstrate this briefly with an example discussed below.

In [22], the ideal sampling of analog signals is described by the following equation:

$$x_D(t)=x(t)\cdot\left(\sum_{k=-\infty}^{\infty}\delta(t-kT)\right)=\begin{cases}x(kT),\ t=kT,\ k\in Z\\ x(kT),\ \text{else}\end{cases}. \tag{9}$$

(Remarks: In (9), the same notation as in (1) above is applied. Furthermore, equation (9) is a re-written equation (2.3.1) from Section 2.3. Electro-optic sampling in [22] on page 7.)

Note now that the right side of (9) does not really equal to the right side of (1), that is to $\sum_{k=-\infty}^{\infty}x(kT)\delta(t-kT)$ – opposite to what is suggested in [22]. The latter is a weighted (by samples of the analog signal sampled) Dirac comb, but the right side of (9) is correspondingly a weighted Kronecker comb (in this way this function is named in [23] and [24]). And obviously, these are two different objects. The Dirac comb cannot be considered to be a function, while the Kronecker comb is a function (maybe a special one, but a function). So they cannot be identified with each other and assumed to be the same thing. However, many do so, as for example in [22]. That is they consider both of them as the same thing. And this, obviously, can lead to computational and design errors.

In other words, the middle part of (9) and its right-hand side are analytical descriptions of two different models of the sampled signal. In [22], they were unconsciously (we suppose so) identified with each other. But in our opinion, when doing so, it is better to know what it means and what are the consequences. Therefore, concluding this example, we can say that problems underlying correct modelling analog-digital conversion signals are also relevant for engineers working in the field of microwave photonic systems.

Complementing: note that the identification of the models mentioned above means taking the successive weighted Dirac impulses (distributions) $x(kT)\delta(t-kT)$, $k\in Z$ to be equal to the successive values of the analog signal samples $x(kT)$, $k\in Z$. And this is akin to assuming that a given number, e.g., 5 is the same as 5j, where $\sqrt{-1}$ denotes an imaginary unit. But this, as we well know, is rather unacceptable operation.

Now, after this initial remark on the truth of the relationship (9), it is reasonable to ask which of these two models mentioned above would describe better signals we are dealing with in analog-to-digital conversion in microwave-photonics systems. Whether the one that uses the weighted Dirac comb or the second, which is based on the weighted Kronecker comb?

We will try to answer this question in what follows. And to this end, we begin with an observation, namely that the literature [22, 25–35] on microwave photonic systems shows that a model differing from those mentioned above is used in this area. This is a model based on a comb of very narrow pulses, but always of finite width (that is differing from zero). It can be derived from the two discussed above as an effect of "smearing" $x(kT)\delta(t-kT)$ objects via their filtering by a low-band filter with a very short impulse response, or as a result of "smearing" pulses of zero duration width of a weighted Kronecker comb.

Recently, the author of this paper showed in a series of articles [23, 24, 36, 37] that the descriptions of the signal sampled: the one with a weighted Dirac comb and the second which uses a Kronecker comb, are not the best models (for various reasons; details of this fact are analyzed in the works cited above). It has been proved by him [18, 19] that in the frequency range below microwave frequencies, the best description of the reality of the signal sampling process can be performed through the use of a step function, as given analytically by (3) and illustrated graphically in Fig. 2 in Section 1.

So, knowing this, let us ask the following question: Can be the description via a weighted step function also used in modelling of the sampled signal in microwave photonic systems? In the context of the today's practice in this area, where, as mentioned above, exclusively the description with the use of weighted "smeared" ("blurred") Dirac impulses or "smeared" ("blurred") Kronecker impulses is used, an answer to this question seems to be evident. That is it cannot.

Of course, this is a correct answer, because the actual photonic impulses, used in sampling the signal at microwave frequencies, are very narrow. However, their duration (let us denote it as t_r) is always greater than zero. And further, between the successive (weighted) impulses in their comb (which constitutes the sampled signal), we always have "holes" (larger or smaller). This is so because the time t_r is always smaller than the sampling period T (more or less), i.e. always $t_r < T$ or even $t_r \ll T$ holds. Thus, precisely because of these "holes" in the comb described above, it would be incorrect to approximate it with a weighted step function.

But, is modelling of a sampled signal with a weighted step function completely useless in the area of microwave photonic systems? It is not fully true. This means can be useful, for example, in modelling the operation of a receiver in a transmission path of a microwave photonic system. To show this, suppose a receiver in such a system receives a photonic signal with "superimposed" samples of a microwave signal on it. The values of the microwave signal samples are "recovered" from electrical pulses at the output of the receiver photodiode and then stored in a buffer, with a given sample held at a given address in the buffer only for a time equal to the sampling time *T*. Hence, it follows that what happens in the buffer can be accurately described using a weighted step function with the values of steps equal to the successive values of the samples. And this is the signal that the next part (the processing one) of the receiver sees on its input.

In summary of our above discussion on the usefulness of describing the sampled signal with a weighted Dirac comb or a weighted Kronecker one, we can say so: these models are not suitable for describing the sampled signal either before or after the receiver photodiode.

Let us now go back to the results presented in Section 2 of this paper. Behavior of the model based on a weighted Dirac comb for very small values of the sampling period T (tending to zero) is presented there. On the other hand, see that we deal with just very small values of sampling periods in the case of analog-to-digital conversion in microwave photonic systems. They are of order of picoseconds. So, because of this reason, the behavior of the above model as shown in Fig. 4 with the object Δ_t (defined analytically by (5)) occurring in the background seems to be well suited to describe this conversion, if, of course, we would want to apply it in this case. But the background in form of a strange and superfluous object Δ_t disqualifies it immediately to be used for modelling sampled signals in real microwave-photonics systems.

Note now that the model based on the use of a weighted step function differs significantly from that given by (1). Its behavior, visualized in Fig. 5, corresponds very well with that what we have in the processing part of the receiver, beginning at the aforementioned buffer. See that Fig. 5 suggests even that in this part of the receiver it is enough to merely filter out the microwave signal from interference, i.e. no any complicated digital-to-analog conversion is needed here.

IV. Conclusions

In the case when the effects of the sampling operation of a signal 'become invisible', which is the case when the sampling period T is very small, we would expect that our description of the sampled signal mimics the fact that then the latter signal is almost indistinguishable from its un-sampled version. In this paper, we have shown that the above condition is satisfied only by a description that uses a weighted step function, i.e. the description given by (3). Moreover, it is shown that this result is directly applicable to the description of analog-to-digital conversion in microwave photonic systems, where sampling periods have very small values. There are picosecond values.

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