

Analysis of the impact of the method of manufacturing the rotor on the parameters of an AC IPM machine

Mariusz Korkosz^{1*}, Adrian Młot², Elżbieta Sztajmec¹, Karol Ryłko¹

¹ Rzeszow University of Technology, The Faculty of Electrical and Computer Engineering, Rzeszow, Poland

² Opole University of Technology, Faculty of Electrical Engineering, Automatic Control and Informatics, Opole, Poland

Abstract. This article presents the technological problem related to the production of rotor with internal permanent magnet. Most often, the magnetic circuits of such rotors used in alternating current synchronous motors (AC IPM) are made of isotropic magnetic sheets. At this point, it should be noted that it is often not taken into account that each isotropic magnetic sheet exhibits some anisotropy. This significantly affects the operational parameters of the brushless permanent magnet (PM) motor such as a cogging torque, electromagnetic torque ripples and an increase in induced voltage harmonics. To illustrate how important it is to properly design the rotor core, two rotors of the IPM motor were analysed in this work. In the first rotor solution, the minimisation of the anisotropy of the magnetic sheet was not taken into account, and the skew of the magnets was not used. In the second case of the IPM motor, the problem of anisotropy of the rotor magnetic circuit was minimised and an additional skew of the PMs was used. The obtained measurements and calculations of selected useful parameters of both rotor designs were compared with each other. Importantly, the conclusions and comments are extremely important for designers, assemblers and manufacturers of electrical machine components.

Key words: anisotropy; cogging torque; IPM motor; electrical steel sheet; torque ripple reduction

1. THE IMPORTANCE OF THE ANISOTROPY OF ELECTRICAL STEEL SHEET

Certain technological operations performed during the manufacture of electrical machine cores have a great impact on their magnetic properties. Therefore, selection of appropriate oriented and non-oriented electrical steel sheet types will affect the working efficiency of the electric machine. By using appropriate technologies for the production of magnetic circuits, while ensuring technical precision, it is possible to minimise their detrimental effects on the quality of the electrical steel [1-3]. Designers and developers of electrical machines must pay significant attention to selecting the appropriate lamination method, thus producing final versions of magnetic cores of stators and rotors of various sizes and shapes. An appropriately designed electromagnetic circuit, which allows the device to achieve high efficiency, reduces operating costs. So, one of the main aspects is selection of appropriate magnetic materials. When considering ferromagnetic materials, the materials used are expected to have: high relative magnetic permeability, high saturation induction value, low loss in a wide frequency range. The most important parameters of electrical steel sheets are: magnetisation, hysteresis and eddy current losses, magnetic permeability, residual induction, coercivity, anisotropy and magnetostriction.

However, the quality of the magnetic cores of electrical machines is determined by the magnetisation and power loss characteristics [4, 5]. The anisotropy of the magnetisation characteristics can be observed both for highly textured electrical steel sheets, i.e. with oriented grains, and for generator sheets. Therefore, anisotropy has a strong influence on functional features of electric machines and it is necessary to take it into account at the design stage. Consequently, magnetic anisotropy affects a number of material properties, such as iron loss, coercivity, magnetostriction and demagnetisation. In sheet metal production, anisotropic grains are oriented in such a way that the directions of easy magnetisation were consistent with the direction of sheet rolling. Changes in the properties of the electrical steel sheet alloy (e.g. iron-silicon) alter the anisotropy energy and also the flux density saturation e.g. in the range $0.2 \cdot 10^7 \div 0.5 \cdot 10^7 \text{ Jm}^3$ and $1.7 \div 2.2 \text{ T}$, respectively [6]. It is also worth noting that due to the smaller size of magnetic cores, their greater variety of shapes and sizes (electrical machines, transformers, etc.), the soft magnetic materials used in electrical engineering are also required to have low losses, high induction, and a low price. For this reason, the following are the most widely used: non-oriented and oriented electrical steel sheets with low and high silicon content as well as sheets from iron-nickel alloys and microcrystalline alloys [6-8].

*e-mail: mkosz@prz.edu.pl

Magnetic sheets used in the construction of magnetic circuits of electric motors are usually isotropic materials. It is assumed that their magnetic parameters are identical regardless of direction. In practice, typical isotropic magnetic sheets exhibit some anisotropic characteristics that depend on the thickness of the magnetic sheet. Typically, 0.5 mm sheets have an anisotropy of around 10%, and for 0.35 mm sheets the value increases above 15%. These are approximate values, as within each magnetic sheet thickness there is a so-called series of types that differ in magnetic parameters and anisotropy value. As mentioned earlier, one of the magnetic directions is the direction of rolling, which has the most favourable magnetic parameters. The second distinctive direction is the one in which the material in question has the worst magnetic parameters. The parameter characteristic of anisotropy is the angle φ_B between these two directions [9]. Figure 1 shows examples of $B=f(H)$ for different values of angle φ_B [10]. By analysing the data, one might notice that the value of φ_B has a very significant effect on the characteristic $B=f(H)$ and thus on the efficiency loss of the motor (not shown here). The only known and effective way to minimise the phenomenon of anisotropy of isotropic magnetic sheets is to use the rotation method. Rotating magnetic sheets involves the appropriate rotation of the preferred magnetic direction while packing the magnetic circuit. In the case of stators of induction motors, three layers are usually used (each with an identical arrangement of magnetic sheets with the same preferred magnetic direction), which are offset by an angle of 120 degrees. This approach allows to significantly reduce the effect of anisotropy. It should be noted that increasing the amount of work involved in producing an electric motor leads to an increase in production costs.

However, according to the authors, this will not provide a satisfactory minimisation with regard to the issue of anisotropy for other electric machine designs. This will be particularly important in the construction of synchronous motor, where the PMs are inserted inside the rotor. In this case, one should consider using a different angle φ_B for rotation between the preferred magnetic direction. Figure 2 shows an example of another displacement. The appropriate angle of rotation of the preferred magnetic direction of the magnetic sheets will depend on the type of electric machine being designed. This will minimise the anisotropy even when using an anisotropic magnetic sheet. In the analysed design of IPM motor, due to the number of rotor poles, a rotation angle of the rotor plates of 90

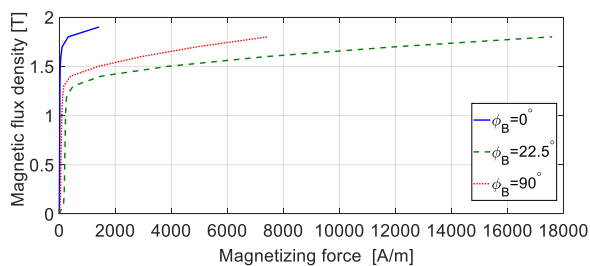


Fig.1. Magnetic flux density vs. magnetising force at $\varphi_B = \text{var}$

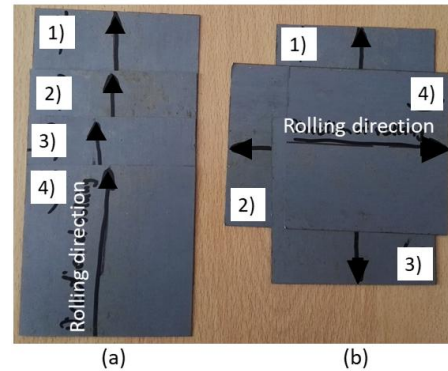


Fig.2. Arrangement of the steel sheets, with the rolling direction of a steel plate in the same direction (a), and with an angular shift of 90 degrees (b)

degrees was used (Fig. 2). The brushless synchronous motor prototype under consideration (presented in this article) uses a magnetic material with JHC coercivity < 100 A/m. The loss in anisotropy of the 0.5 mm thick electrical steel used in the rotor and stator core is about 8%. The importance of the arrangement of rolled electrical steel laminations employed for two different prepared rotors used in the synchronous machines has been investigated. It was discovered that they have a huge impact on the field quantities, torque ripples and the iron loss. Regardless of the motor design, decreasing the iron losses of the lamination steel sheets will improve the motor efficiency.

2. COGGING TORQUE REDUCTION IN IPM MOTORS

There are many methods for reducing torque ripple in electrical machines. In fact, those methods for cogging torque reduction were widely analysed and investigated. Optimisation of cogging torque can be accomplished by the following methods: skewing stator slots or rotor magnets, harmonic current injection, changing the embrace and offset of the magnet, slot-pole combination, pole-arc to pole-pitch ratio, Halbach array (used in surface mounted magnets), slot opening, designing a fractional slot, and adding additional slots or teeth [11-13]. In the case of IPM motors, there is less possibility of eliminating cogging torque. In addition to the appropriate selection of the stator design parameters at the air gap, in practice there is only one more option, i.e. the use of pseudo-skew PMs. This means that it is necessary to segment the rotor magnetic circuit. This design approach is presented in this paper, with angular shifts of the stator package sheets in each segment of the rotor magnetic circuit by an angle value that guarantees a minimum torque magnitude.

It should also be noted that magnetic anisotropy can also affect an electromagnetic torque ripple [14]. Improper arrangement of the stator or rotor core electrical steel sheet may have disastrous consequences. This leads to an increase in the amplitude of the cogging torque. As a consequence, this has an impact on a significant increase in the ripples of the generated electromagnetic torque. Changing the relative angle between electrical steel sheets affects the anisotropy and can be observed when the sheets are stacked with the rolling direction in each piece being parallel, and the effect can be cancelled out when they are stacked with the rolling direction

rotated. Where the magnetic anisotropy is dominant, the amplitude of cogging torque is maximum [14].

It is known that the magnetic flux generated by PMs through the air gap is closed by the stator magnetic circuit. Even assuming that the magnetic circuits of the stator and rotor are free from the problem of the magnetic sheet anisotropy, the phenomenon of minimising the reluctance for a given magnetic path will occur. This is the reason why the rotor occupies characteristic positions forced by the phenomenon of anisotropy. Cogging torque caused by the alternation between slots and teeth can be analytically examined by considering the derivative of magnetic energy (stored in the air gap) with respect to φ , which represents the angular position of the rotor [15], and the cogging torque which can be expressed as Eq. (1). Here the magnetic energy is defined as an integral of the square of a magnetic flux density B_m over a volume V of the region with a magnetic permeability of air μ_0 . B_m is dependent on the magnetic permeance which varies with the position of the slot openings to φ , and the magnetomotive force produced by the PMs.

$$T_{cogg} = \frac{d\left(\frac{1}{2\mu_0}\right)_V \int B_m^2(\varphi) dV}{d\varphi} \quad (1)$$

The number of cycles of changing the cogging torque amplitude (peak-to-peak) N_{cogg} is determined by the following relationship [27]:

$$N_{cogg} = \frac{NCM(N_s, 2p)}{2p} \quad (2)$$

where: $NCM(N_s, 2p)$ is the least common multiple of its arguments, N_s is slot numbers, $2p$ is rotor pole numbers.

The cogging torque of the synchronous motor can be regarded as the sum of the interaction between each edge of the PM and the slot opening. In order to completely eliminate the cogging torque, the skew angle of the PM must be equal to the cycle of the cogging torque [16]. Therefore, it is important to know the number of fundamental cogging torque cycles γ in each revolution of the PM brushless motor rotor, which has the following relationship with the greatest common divisor GCD of the number of stator slots N_s and the number of rotor poles $2p$ [17]:

$$\gamma = \frac{2pN_s}{GCD} \quad (3)$$

Below is a short review of the most important selected methods for reducing electromagnetic torque ripple, in particular cogging torque.

2.1. Rotor /stator skew design

The basic idea behind skewing is to influence the interaction between the stator slots and the rotor magnets. Rotor/stator skew design is a common approach for reducing torque ripple and cogging torque. The study [18-20] examines the effects of

different rotor skew patterns on the cogging torque and extensively investigates the excitation torque ripple. These works demonstrated that the skewed stator slots and offset poles can significantly reduce cogging torque. Moreover, they showed a slight decrease in the mean value of the electromagnetic torque. IPM motors in traction applications often employ discrete rotor skewing constructions to reduce torsional excitations and back-EMF harmonics. It should be noted that as the number of steps and the complexity of the skew pattern increase, the manufacturing cost also rises.

2.2. Influence of winding arrangement on torque pulsations.

Some of research investigations shows that a number of layers combined with coil side shift to other layer can achieve significant reduction of torque ripple while maintaining the torque average. It is also important to pay attention to the choice of the coil pitch. Research results shows that some winding combinations reduce the torque average while decreasing the torque ripple [21]. It is also possible to use synchronous machines with the alternate teeth wound, which have larger torque ripple compared to all teeth wound machines having the same dimensions. This is mainly due to the adjacent stator teeth asymmetric saturation caused by the armature reaction [22].

2.3. Stator slot openings, tooth profile and slot/pole number.

The choice of the slot opening width and tooth profile is critical in reducing cogging torque in synchronous PM machines. Furthermore, when a parallel profile is adopted, cogging torque can be reduced by 24% compared to a trapezoidal profile [20, 23, 24]. Another method of reducing cogging torque is by fractional slot/pole numbers, because the magnets align differently with respect to the stator teeth, and then produce cogging torques in opposite directions that tend to cancel each other out. So, the right combination of slots and poles leads to a substantial reduction in cogging torque [25].

3. TEST RIG AND PROTOTYPES OF IPM MOTORS

The structure of the AC IPM motor has been designed for multi-channel operation, i.e. it can operate with power from four independent power electronic systems (the problem considered

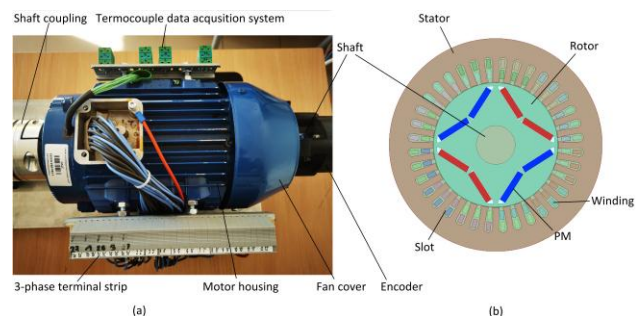


Fig.3. IPM motor prototype (a) and cross-section of the stator /rotor core (b)

in this work is not related to multi-channel work). Too large an

amplitude of the cogging torque (caused mainly by the anisotropy phenomenon) is a problem for the electric motor operation regardless of the number of channels currently used. However, as the number of active channels decreases, the influence of cogging torque becomes more and more problematic. Figure 3 shows a cross-section of the geometry of the analysed structure of the tested electric machine.

The rotor of the electric motor is designed with PMs embedded in the rotor core, where the PMs are formed in the shape of the letter "V". This solution ensures the creation of an additional component of the generated electromagnetic moment, i.e. the reluctance moment.

TABLE 1. Main IPM motor parameters

Parameter	Description
Rated power	1,500 (W)
Rated speed	1,500 (rpm)
Winding type	Distributed
Rotor pole number	4
Slot number of stator	36
Active length of rotor / stator	140 (mm)
Outer /inner radius of stator	140 (mm) /82.5 (mm)
Outer /inner radius of rotor	81.5 (mm) /25 (mm)
Air gap between stator and rotor	0.5 (mm)
Steel sheet of stator /rotor	M400-50A

As part of the research issue, two identical rotors with internally mounted PMs were designed and manufactured. In both cases, an isotropic magnetic sheet was used to build the magnetic circuit. Version I of the rotor with PMs (IPM I) was practically made of isotropic magnetic sheet metal without minimising the problem of anisotropy. Moreover, none of the cogging torque reduction methods discussed in Chapter II were used. The magnet was divided into 4 segments (to reduce eddy currents). This rotor is shown in Fig. 4a. The second version of the rotor (marked here as IPM II) was made using the method of minimising the anisotropy of an isotropic magnetic sheet. One of the most effective methods of reducing the cogging torque was also used, i.e. pseudo-skew of the magnet. Dividing the magnet into four segments and using a pseudo-skew should theoretically ensure a large reduction in the cogging torque. It is important to decide on the number of segments and pseudo-skew at the initial stage of the design of the electric machine.

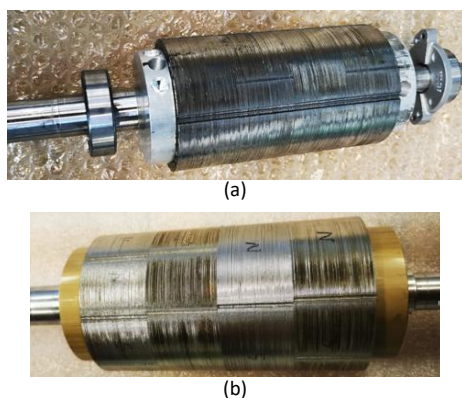


Fig.4. Rotor with laminated magnetic core with un-skewed magnets comprising version I (IPM I) (a), and with skewed magnets comprising version II (IPM II) (b)

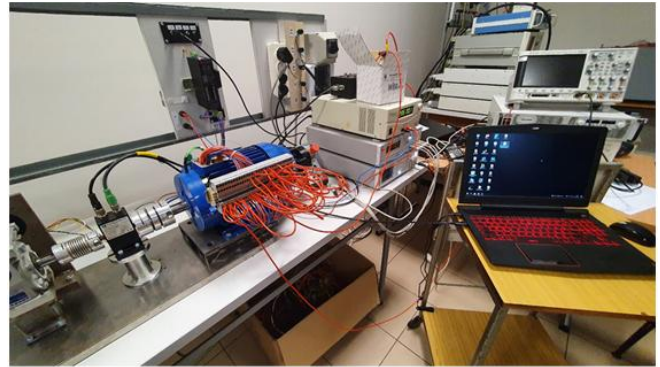


Fig.5. Test rig to measure static AC IPM motor's parameters

Otherwise, it will be impossible or very difficult to make any changes and interventions to the structure of the rotor's magnetic circuit. Therefore, the decision to divide the rotor's magnetic circuit into segments is a critical decision. A version of this rotor is shown in Fig. 4b.

The measuring test rig for static tests is shown in Fig. 5, where electric motor with IPM I and IPM II rotors were tested. The station includes a stepper motor with a controller, gear, torque transducer, clutches and the tested AC IPM motor.

Additionally, the test rig includes a function generator and a PC with data acquisition software.

4. MEASUREMENTS AND NUMERICAL ANALYSIS

Numerical calculations were performed in the Ansys Electronics program environment (base on the finite element method) [26]. Three cases were analysed in the calculations. In the first case (Case 1), a magnetic sheet was used without taking into account the phenomenon of anisotropy and the pseudo-skew of the rotor. In the second case (Case 2 – IPM I), a magnetic sheet was used taking into account the anisotropy phenomenon, but excluding the rotor pseudo-skew. In the last case (Case 3 – IPM II), a magnetic sheet was used taking no account of the anisotropy phenomenon but including the rotor pseudo-skew. Since in Case 3 the rotation of the rotor sheet layers occurs with the preferred magnetic direction, it was assumed that the phenomenon of anisotropy was completely

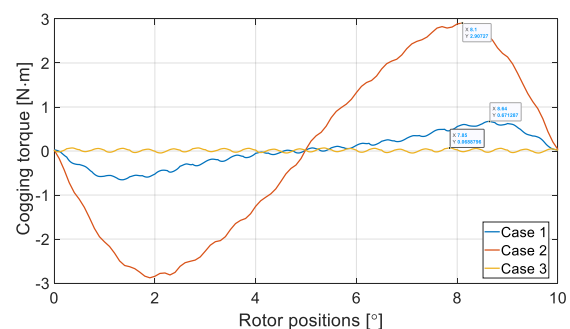


Fig.6. Numerical calculation of cogging torque vs. rotor position for each considered case of rotor design

eliminated. In each of the above cases, the stator magnetic circuit was assumed to be free from the influence of anisotropy due to the shift of the preferred magnetic direction by an angle

of 120 degrees. Figure 6 shows the calculation of the cogging torque for one period, which represents 10 mechanical degrees.

When anisotropy occurs, the amplitude of the cogging torque increases rapidly (over 4 times compared to Case 1). The simultaneous use of pseudo-skew PMs, assuming that the phenomenon of anisotropy does not occur, practically eliminates the problem of cogging torque. Figure 7 shows the

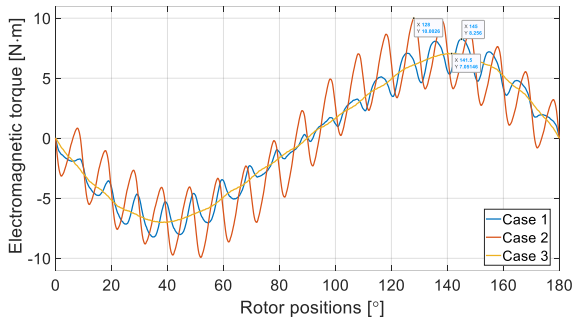


Fig.7. Numerical calculation of static electromagnetic torque at load $I = 6A$

variability of the static electromagnetic torque with a period of 180 mechanical degrees (two pairs of poles). The analysed electric motor was designed for a sinusoidal power supply, the static electromagnetic torque was computed when the stator winding phases were supplied with constant current values, i.e. $I_1 = I$, $I_2 = I_3 = -I/2$, where $I = 6 A$.

As one might see, cogging torque affects the shape of the generated static electromagnetic torque. Cogging torque is a pulsating, parasitic, and undesired torque ripple inherent in the design of a AC IPM motor. It can be seen that for each of the analysed rotor cases, the maximum amplitude of the

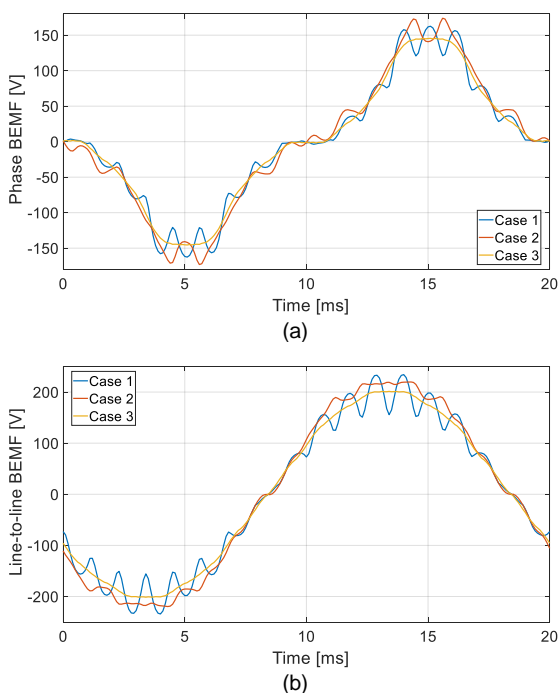


Fig.8. Numerical calculation of line-neutral (a) and line-line (b) BEMF voltage waveforms at 1,500 rpm for each case of rotor design

electromagnetic torque occurs at a different value of the rotor position angle. This is a very unfavorable effect because it makes it difficult to control the operation of the machine in the field of sinusoidal power supply. Moreover, the influence of the reluctance torque is clearly noticeable. This is clearly visible for the Case 3 of the rotor, where the maximum torque occurs at 141.5 mechanical degrees. For PM surface mounting, the maximum should occur at an angle of 135 degrees.

Another important parameter of the AC IMP motor is the shape of the induced voltage and its content of higher harmonics. Induced voltages can be determined using only the field model or the field-circuit model. When analysing the AC IPM motor, a field-circuit model was used because it allows analysing not only symmetrical operating conditions, but also asymmetry

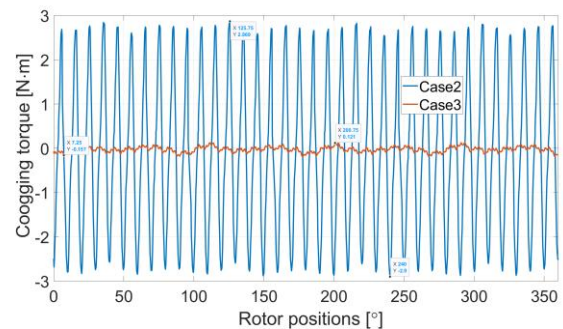


Fig.9. Cogging torque measurements at no-load operation for analysed rotor from Case 2 and Case 3

conditions. Figure 8 shows the line-neutral BEMF and line-line BEMF voltage waveforms, which were determined at the rated motor speed of 1,500 rpm.

Figure 9 shows the measurements of cogging torques for electric machines with IPM I (Case 2) and IPM II (Case 3) rotors.

The obtained reduction in the cogging torque is satisfactory, with the amplitude limited to a value of approximately 0.15 N.m. For the IPM I rotor, the high value of the cogging torque was mainly caused by the imprecise arrangement of

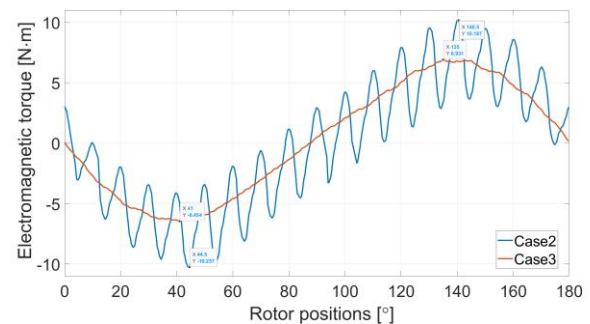


Fig.10. Static electromagnetic torque measurements at load operation for analysed rotor from Case 2 and Case 3

individual layers of the rotor sheets and small differences between the positions of the PMs.

Figure 10 shows the electromagnetic torque characteristics as a function of rotor position on the same test rig. The test was carried out assuming identical conditions to those in the numerical calculations.

The influence of excessive cogging torque on the static electromagnetic torque is difficult to accept. After significant minimisation of the torque ripple, a characteristic without any fluctuations related to the cogging torque was obtained. The electromagnetic torque can be expressed as:

$$T_e = \frac{dW(\theta, i)}{d\theta} \Big|_{i=const} = \frac{\partial}{\partial \theta} \left[\int_V \left(\int_0^H \mathbf{B} \cdot d\mathbf{H} \right) dV \right] \quad (4)$$

where: \mathbf{B} – magnetic flux density, \mathbf{H} – magnetic field, i – current, $dW(\theta, i)$ – magnetic coenergy of the system, θ – rotor position.

The resulting induced voltages under laboratory conditions at the rated machine speed (1,500 rpm) for Case 2 and Case 3 are shown in Figs. 11-12.

In both cases of line-neutral and line-line voltage waveforms, the voltage distortion that negatively affects the operation of the AC IPM motor can be observed. In this case, this is due to the increased losses occurring not only in the stator windings, but also in the rotor. In extreme cases, this may lead to a reduction in efficiency of up to 1%. This will also result in an increase in the operating temperature of the machine by up to several degrees Celsius, which negatively affects the durability of the winding insulation [27].

Minimising the cogging torque also significantly affects the shape of the obtained induced voltage. After minimising the anisotropy phenomenon and introducing rotor segmentation, the measured induced voltages (Figs. 11-12) are similar to those indicated by the numerical test results. Laboratory test results show differences between individual phases.

5. CONCLUSIONS AND RECOMMENDATIONS

The design of the rotor with internally mounted PMs is very important and has a significant impact on the efficiency of the electric motor. This motor technology is very sensitive to the problem of anisotropy of the isotropic magnetic sheet. Not only the stator magnetic circuit, but also the anisotropy of the rotor magnetic sheet may contribute to the generation of excessive cogging torque. Therefore, the authors of the article recommend that it is good practice to limit not only the phenomenon of anisotropy of the magnetic circuit on the stator side, but also on the rotor side. This article proves that the combination of the method of minimising the phenomenon of anisotropy of the magnetic sheet and the use of a pseudo-skew of the rotor provides very good results in relation to, among other things, limiting the pulsation of the electromagnetic torque. Moreover, the cogging torque was limited to a value allowing free manual rotation of the rotor. This resulted in the elimination of fluctuations in the generated electromagnetic torque, and a reduction in the content of higher harmonics in the induced phase and line voltage.

REFERENCES

- [1] A. Schoppa, J. Schneider, C.D. Wuppermann, "Influence of the manufacturing process on the magnetic properties of non-oriented electrical steels", *Journal of Magnetism and Magnetic Materials*, vol. 215–216, pp. 100–102, 2000, doi: 10.1016/S0304-8853(00)00070-6.
- [2] T. Chevalier, A. Kedous-Lebouc, B. Cornut "Influence of electrical sheet with on dynamic magnetic properties", *Journal of Magnetism and Magnetic Materials*, vol. 215–216, pp. 623–625, 2000, doi: 10.1016/S0304-8853(00)00244-4.
- [3] M. Dems, Z. Gmyrek, K. Komenza, "The influence of cutting technology on magnetic properties of non-oriented electrical steel-review state of the art", *Energies*, vol. 16, no. 11, pp. 1–26, 2023, doi: 10.3390/en16114299.

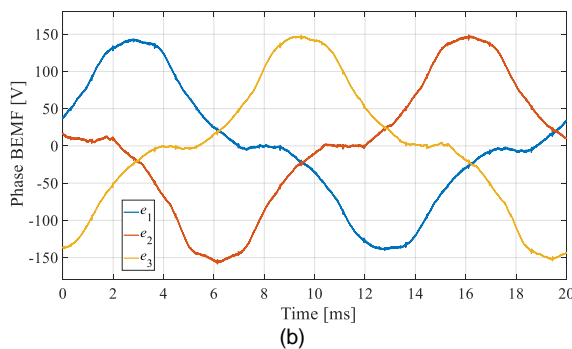
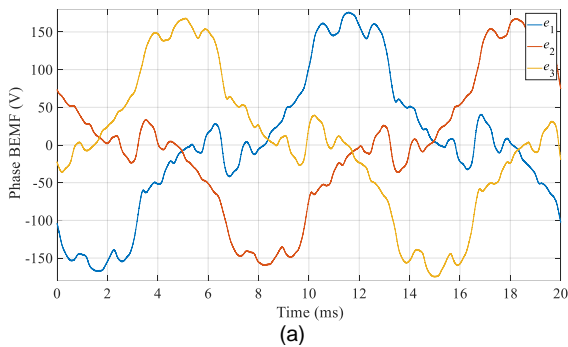


Fig.11. Measurements of line-neutral BEMF voltage waveforms at 1,500 rpm for rotor version from Case 2 (a) and from Case 3 (b)

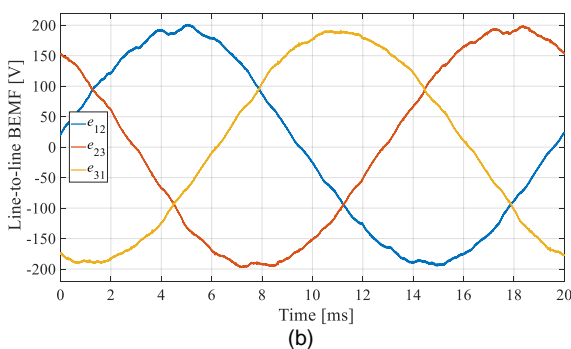
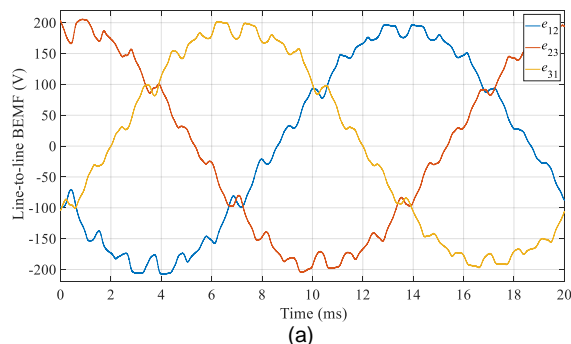


Fig.12. Measurements of line-line BEMF voltage waveforms at 1,500 rpm for rotor version from Case 2 (a) and from Case 3 (b)

- [4] S. Żurek, P. Borowik, K. Chwastek, "Anizotropia stratności wybranych bach elektrotechnicznych", *Przegląd Elektrotechniczny*, in polish, no. 2, pp. 96–99, 2018, doi: 10.15199/48.2018.02.23.
- [5] W. Mazgaj, Z. Szular, A. Warzecha, "Influence of magnetic anisotropy on flux density changes in dynamo steel sheets", *Archives of Electrical Engineering*, vol. 64, no. 1, pp. 81–88, 2015, doi: 10.1515/aee-2015-0008.
- [6] E. Napieralska - Juszcak, K. Komęza, "Modelowanie pola elektromagnetycznego w rdzeniach anizotropowych", Monografia Politechniki Łódzkiej, in polish, Poland, 2012.
- [7] M. Soiński, "Nowoczesne materiały magnetyczne miękkie w technice", *Przegląd Elektrotechniczny*, in polish, no. 9, pp. 219–223, 1999.
- [8] M. Mnich, M. Wilk, W.A. Pluta, "Określenie relacji pomiędzy anizotropią i parametrami modelu ODF strat mocy w blachach elektrotechnicznych", *Przegląd Elektrotechniczny*, in polish, no.5, pp. 66–63, 2020, doi: 10.15199/48.2020.05.13.
- [9] K.R. Chwastek, A.P.S. Baghel, M.F. de Campos, S.V. Kulkarni, J. Szczygłowski, "A Description for the Anisotropy of Magnetic Properties of Grain-Oriented Steels", *IEEE Trans. Magn.*, vol. 51, no. 12, pp. 1–6, 2015, doi: 10.1109/TMAG.2015.2449775.
- [10] M. Korkosz, P. Bogusz, J. Prokop, "Complex Performance Analysis and Comparative Study of Very High-Speed Switched Reluctance Motors", *IEEE Trans. Magn.*, vol. 55, no. 8, pp. 1–14, 2019, doi: 10.1109/TMAG.2019.2910492.
- [11] X. Zhu, W. Hua, Z. Wu, W. Huang, H. Zhang, M. Cheng, "Analytical approach for cogging torque reduction in flux-switching permanent magnet machines based on magnetomotive force-permeance model", *IEEE Trans. Ind. Electr.*, vol. 65, pp. 1965–1979, 2018, doi: 10.1109/TIE.2017.2739688.
- [12] L. Zhu, S. Z. Jiang, Z. Q. Zhu, C. C. Chan, "Analytical methods for minimizing cogging torque in permanent-magnet machines", *IEEE Trans. On Magn.*, vol. 45, no. 4, pp. 2023–2031, 2009, doi: 10.1109/TMAG.2008.2011363.
- [13] A. Daikoku, S. Yamaguchi, Y. Toide, K. Fujiwara, N. Takahashi, "Cogging torque estimation of permanent magnet motors resulting from magnetic anisotropy of non-oriented electrical steel sheets", *IEEE Trans. Ind. Electr.*, vol. IE-34, no. 2, pp. 205–215, 1987, doi: 10.1541/ieejias.126.1712.
- [14] M. Caruso, A.O. Di Tommaso, R. Miceli, F. Viola "A cogging torque minimization procedure for interior permanent magnet synchronous motors based on a progressive modification of the rotor lamination geometry", *Energies*, vol. 15, no. 14, pp. 2–19, 2022, doi: doi.org/10.3390/en15144956.
- [15] Z. Zhang, Z. Wang, J. Yu, Y. Luo, W. Yan, T. Hu, D. Yu, L. Wang, "Research on the influence of trapezoidal magnetization of bonded magnetic ring on cogging torque", *Open Physics*, vol. 21, no. 1, pp. 1–8, 2022, doi.org/10.1515/phys-2022-0223.
- [16] Y.B. Yang, X.H. Wang, C.Q. Zhu, "Effect of permanent magnet segmentation on the cogging torque of surface mounted permanent magnet motors", *Trans. China Electrotech soc.*, vol. 27, no. 3, pp. 73–78, 2012.
- [17] J. W. Jiang, B. Bilgin, Y. Yang, A. Sathyan, H. Dadkhah, A. Emadi, "Rotor skew pattern design and optimisation for cogging torque reduction", *IET Electrical System in Transportation*, vol. 6, no. 2, pp. 126–135, 2016, doi: 10.1049/iet-est.2015.0021.
- [18] M. Korkosz, A. Mlot, "Torque ripple reduction by using multi-slice FE modelling of brushless DC motor with skewed magnets", *Zeszyty Problemowe – Maszyny Elektryczne*, in polish, no. 86, pp. 105–108, 2010.
- [19] K. Abbaszadeh, F. Rezaee Alam, M. Teshnehlab, "Slot opening optimization of surface mounted permanent magnet motor for cogging torque reduction", *Energy Conversion and Management*, vol. 55, pp. 108–115, 2012, doi: 10.1016/j.enconman.2011.10.014.
- [20] M. Muteba, D.V. Nicolae, "Influence of mixed winding arrangements on torque ripples of five-phase induction machines", *Electric Power Systems Research*, vol. 151, pp. 154–165, 2017, doi: 10.1016/j.epsr.2017.05.027.
- [21] Y.X. Li, Z.Q. Zhu, G. Li, "Influence of stator topologies on average torque and torque ripple of fractional-slot SPM machines with fully closed slots", *IEEE Transactions on Industry Applications*, vol. 54, no. 3, pp. 2151–2164, 2018, doi: 10.1109/TIA.2018.2799178.
- [22] T. Nur, L. E. Joe, M. Siregar, "Novel of cogging torque reduction technique for permanent magnet generator by compounding of magnet edge shaping and dummy slotting in stator core", *International Journal on Advanced Science Engineering Information Technology*, vol. 10, no. 3, pp. 1191–1199, 2020, doi: 10.18517/ijaseit.10.3.10372.
- [23] J. Wanjiku, M.A. Khan, P.S. Barendse, P. Pillay "Influence of slot openings and tooth profile on cogging torque in axial-flux PM machines", *IEEE Transactions on Industry Applications*, vol. 62, no. 12, pp. 7578–7589, 2015, doi: 10.1109/TIE.2015.2458959.
- [24] S. Wang, J. Hong, Y. Sun, H. Cao, "Effect comparison of zigzag skew PM pole and straight skew slot for vibration mitigation of PM brush DC motors", *IEEE Trans. Ind. Electron.*, vol. 67, pp. 4752–4761, 2020, doi: 10.1109/TIE.2019.2927175.
- [25] Ansys, www.ansys.com/products/electronics, 2024.
- [26] Sz. Lipiński, J. Zawilak, "Influence of voltage harmonic distortion on temperature distribution in line-start permanent magnet synchronous motor", *Napędy i Sterowanie*, in polish, no. 10, pp. 102–107, 2017.
- [27] D. Hanselman, *Brushless permanent magnet motor design*, Second edition, Magna Physics Publishing, Orono, USA, 2006, pp. 3-379.