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Consensus Seeking: How Juror Exclusion Shapes Strategic Incentives in the Borda Count

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Abstract

We examine incentives and strategic behavior in a voting game using a new modification of the Borda count in which the score of the juror with the largest deviation from the mean score vector is excluded. We show that introducing juror exclusion has a strong effect on incentives. In particular, it motivates jurors to align with the mean. When jurors' preferences are closely aligned – that is, when the subjective component of the evaluation is small relative to the objective one – excluding the outlier's score is likely to reduce manipulation. However, when jurors' preferences differ significantly, the method may actually increase misreporting compared to the standard Borda count without juror exclusion.

Keywords: Borda method, strategic voting, manipulation, outliers

JEL Classification: D71, C72

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1 Introduction

The Borda rule is a simple election method, commonly used in many real-life decisions in music competitions, educational institutions, professional and technical societies, sports, and even political elections (Reilly, 2002; Fraenkel and Grofman, 2014). Its advantages are multiple. The rule is simple to understand and easy to apply. It guarantees a transitive ordering of preferences, thus eliminating voting paradoxes based on money pumps (for more theoretical properties, see Nitzan and Rubinstein, 1981). Also, Borda rule is likely to select an alternative that is acceptable for a substantial number of voters, or a Condorcet winner (see Fraenkel and Grofman, 2014; Van Newenhizen, 1992). Boutilier et al. (2012) show that Borda is the optimal social choice function that maximizes average expected utility of agents whose utilities are drawn independently from a neutral, that is, uniform distribution.

Borda count belongs to the class of positional voting methods that result not only in selecting a winner, but also provide a complete ranking of the candidates along with the size of the differences between them. As Marchant (2000) suggests, analyzing the magnitudes of the differences in Borda scores can provide additional insights into the relative strengths of candidates. All those favorable properties add to the popularity of the method, which is used not only in multiple real-life voting procedures, such as music competitions, but also for ranking and aggregating information from multiple sources, such as in information retrieval systems (Tsai et al., 2014), and even wine rankings (Barberà et al., 2023). Extensions of the Borda method were also used for ranking multivalued objects (Zhang et al., 2012; García-Lapresta and Llamazares, 2003).

However, the Borda method is susceptible to strategic voting, and a common strategy is to underrepresent true preferences for a potentially popular candidate who poses a threat to the preferred candidate. As Black (1998) puts it: "even to the unsophisticated voter the Borda count is an invitation to strategic voting; and the strategy employed tends to be for the voter to show accurately the candidate of his first preference, and apart from that to rank the remaining candidates in the reverse of what is believed to be their order of popularity".

This should come as no surprise, as no "voting procedure" (Satterthwaite, 1975), which selects three or more alternatives, is strategy-proof, that is, induces the jurors to truthfully reveal their preferences. As Sen (1984) proves, none of the other systems belonging to a more general class of "positional rules", is strategy-proof and the incentives for manipulation are influenced by the jurors' reward from the outcome Pérez et al., 2014). In fact, by Gibbard- Satterthwaite theorem, the only non-manipulable voting mechanism with more than three alternatives, is a dictatorship (Gibbard, 1973; Satterthwaite, 1975).

One approach to tackle the issue of manipulation is to modify the Borda count – typically, by excluding candidates who received the lowest scores – e.g. Nanson's and Baldwin's rules (Baldwin, 1926). Davies et al. (2014) and Rothe (2019) show that the computational complexity of manipulation in Nanson and Baldwin rules is higher than in standard Borda, which makes dishonest reporting cognitively more difficult. The issue of complexity of the strategic choice is also tackled by Bartholdi and Orlin (1991), who argue that, even in light of Gibbard-Satterthwaite impossibility results, "resistance to manipulation" can be effectively proxied by complexity of the manipulation process. The Borda modification we study in this paper follows a similar path, making it distinctly more difficult to vote strategically than the standard Borda count. This is because of the dynamic nature of excluding jurors, as well as a discrete effect of even a subtle change.

We follow a similar approach of making manipulation more complex and cognitively difficult. However, we focus on a Borda modification that excludes complete *jurors'* scores rather than individual *candidate's* score. The idea of excluding complete juror scores to decrease manipulation was originally proposed for the Borda count by Kontek and Sosnowska (2020) and Kontek and Kenner (2023). This approach entails excluding the complete juror's score vector (rather than a score of a single candidate) if it diverges most from the average score vector. This modification significantly alters the incentive structure for manipulation. This works well when jurors' true preferences are strongly aligned. This may be the case in competitions with specialized juries, e.g. classical music competitions, where the objective component of the jurors' preferences is relatively large compared to the subjective taste. Importantly, the exclusion of the full score of a given juror may lead to failure of the monotonicity property (Felsenthal et al., 2017), which is satisfied by the standard Borda count. We observe that relaxing the monotonicity property results in a decrease in incentives to manipulate.

For unfair reporting to be profitable, it almost always involves a greater departure from the average score. This is because in order to increase the chance of a juror's preferred candidate, he or she must reduce the chance of a popular candidate who poses a threat to the preferred candidate. In the juror exclusion method, a juror who anticipates that his or her preferred candidate may lose due to deviation from the general consensus may be less willing to manipulate. Indeed, excluding a score that deviates from the consensus may adversely affect the juror's preferred candidate.

In this article, we address the problem of strategic manipulation using a game–theoretic setup. In doing this we follow the ideas of Myerson and Weber (1993), albeit taking a different approach to equilibria selection. As Myerson and Weber (1993) show, there is a plethora of Nash equilibria (NE) in scoring games. However, we focus explicitly on paths leading to NE when starting from true preferences. We analyze and compare equilibria of the standard Borda voting game and JE Borda game, i.e. the modified Borda game, in which the outlier juror is excluded. We show that juror exclusion increases the incentive to report results closer to social consensus. If jurors' preferences are highly aligned, then the social consensus is close to each juror's preferences. In such a case, the juror exclusion method will discourage manipulation. If juror preferences are more divergent, or if one juror's preferences differ significantly from those of others, the juror exclusion method will encourage sticking to the consensus even at the expense of moving away from truthful reporting. This article is structured as follows. We begin by formally introducing incentives within the Borda and modified Borda framework, with an analysis of the effects of manipulation. Second, through illustrative examples, we analyze the strategic considerations of a single player in the traditional Borda method and the juror exclusion (JE) Borda method. In Section 3, we discuss some basic properties of equilibria of the voting game under Borda and JE Borda by providing preference examples and their Nash equilibria. Finally, we discuss the applicability of JE Borda and draw directions for future research.

2 Game–theoretical approach to voting

Let $\mathcal{C} = \{1, ..., C\}$ be the set of candidates, $C \geq 2$, and let $\mathcal{J} = \{1, ..., J\}$ be the set of jurors (both finite), with generic elements c and j, respectively. Every juror has preferences over candidates represented by a complete and transitive binary relation $\succsim_j \subset \mathcal{C} \times \mathcal{C}$, $j \in \mathcal{J}$, which admits a utility representation $u_j : \mathcal{C} \to \mathbb{R}$, that is the following holds for all $c, c' \in \mathcal{C}$,

$$c \succsim_j c' \iff u_j(c) \geq u_j(c').$$

Let $s : \mathcal{J} \times \mathcal{C} \to \mathcal{C}$ be the reported score matrix. Its j-th row, denoted by $s(j, \cdot)$ or s_j, belongs to the set of all permutations of $\mathcal{C}$ (i.e. bijective mappings from $\mathcal{C}$ to itself), which we denote by $\mathfrak{S}_{\mathcal{C}}$. It is arranged in a vector and designates the score of juror j. With a slight abuse of notation, we use $s(j, \cdot)$ and s_j, depending on the context – the former is more useful for voting outcomes, while the latter is a standard game-theoretical notation.
In our examples for $\sigma \in \mathfrak{S}_{\mathcal{C}}$ we will often write

$$\sigma(1)\sigma(2)\sigma(3)\sigma(4)$$

to denote a juror's score vector instead of $[\sigma(1), \sigma(2), \sigma(3), \sigma(4)]$. The c-th column of the score matrix, denoted by $s(\cdot, c)$, is the vector of scores given to candidate c by each of the jurors. The total score given to candidate c by all jurors is $s(c) := \sum_{j \in \mathcal{J}} s(j, c)$, and by all jurors except juror j^* is $s_{-j^*}(c) := \sum_{j \in \mathcal{J} \setminus \{j^*\}} s(j, c)$.
In the Borda count, each juror's score is a permutation of the set of candidates. The winner is the candidate whose total score is the highest: $c^* \in \arg\max_{c \in \mathcal{C}} s(c)$. We assume that ties are resolved at random, with each candidate having a $\frac{1}{|\arg\max s(c)|}$ chance of winning. Note that in real-life applications, ties are usually resolved with a second stage of voting (possibly with a different voting scheme). To simplify the analysis, we abstract from such considerations but retain the basic intuition that the winning of any candidate becomes "uncertain".

In our examples, we will state jurors' true preferences by means of a Borda score. To keep things simple, we exclude preference indifferences to ensure that a given Borda score uniquely determines preferences. Note that if we allowed for indifferences, there would be more than one Borda score associated with a given preference over the set of candidates: for example, $c_1 \sim_j c_2 \succ_j c_3 \succ_j c_4 \succ_j c_5$ would have two Borda scores consistent with it, namely 54321 and 45321.
A distinguishing feature of the Borda count is that the sum of scores of each juror is constant:

$$\sum_{c=1}^{C} s(j,c) = \tfrac{(C+1)C}{2}.$$

Consequently, if a juror aims to increase the total score of a specific group of candidates by a total of x points, she must offset this by decreasing the scores of the remaining candidates by the same total of x points. Indeed, for two scoring matrices s, s' and a subset of candidates A, the following holds:

$$\sum_{c\in A}[s(j,c) - s'(j,c)] = -\sum_{c\in\mathcal{C}\setminus A}[s(j,c) - s'(j,c)].$$

We are interested in strategic, or manipulative voting, when the reported scores might differ from the underlying true preferences of the agents. We implicitly assume that jurors have some underlying preferences, that the truthful ranking must obey.

Definition 1. *A truthful scoring for juror j is a scoring, denoted by $\hat{s}(j,\cdot)$, that obeys juror j's preference order i.e. if $\succ_j$ is an asymmetric part of $\succsim_j$, then $c \succ_j c'$ implies $\hat{s}(j,c) > \hat{s}(j,c')$.*

To simplify our analysis, we shall employ an assumption that jurors only care about the winner of the competition and that their preferences are lexicographic. This assumption is often made and reflects the idea that "the winner takes it all".

Assumption 1. *Jurors' preferences are lexicographic. That is, let*

$$W = \arg\max_{c\in\mathcal{C}} s(c)$$

be the set of winners and the k-th preferred candidate of juror j, denoted by c_{j_k}, satisfies $c_{j_k} \in W$. Then juror j receives a utility payoff of

$$u_j(s) = \sum_{k: c_{j_k}\in W} \frac{1}{(C+1)^{k-1}} \frac{1}{|W|}.$$

.

For example, a juror's utility is equal to 1 if her top candidate wins outright and a utility of $\frac{1}{|W|}$ if there is a tie between $|W|$ winners, including her top candidate.

This assumption implies, in particular, that jurors value even a small chance of their favorite candidate winning more than the sure victory of their second-best candidate. For instance, in a scenario with five candidates, the sure victory of her second-best candidate is valued at most at $\frac{1}{6}$, which is lower than $\frac{1}{5}$, the utility she receives from a tie among all candidates, including her most preferred one.

The three elements – players, feasible strategies and payoffs for each juror – allow us to fully characterize the game $\left(J, (\mathfrak{S}_{\mathcal{C}})^J, (u_j)_{j\in\mathcal{J}}\right)$ between jurors. Notice that the result depends on the voting method only by its impact on the payoffs u_j. This is a common feature of voting games. The voting scheme is therefore crucial for incentives. In the next two subsections, we will discuss jurors' incentives under the standard Borda and the JE Borda games.

2.1 Incentives – Borda

Consider the classic Borda count under jurors' preferences consistent with Assumption 1. Focus on the case of a single juror j^*, whose most preferred candidate is c^*. We assume that, given the reports of all other jurors, juror j^* has the potential to influence the final outcome, specifically, to determine whether c^* emerges as the sure winner.

By Assumption 1, if only juror j^* has the potential to make c^* the sure winner, then doing so is strictly preferred. To simplify the analysis, we focus exclusively on this case. Following standard game-theoretic principles, we isolate juror j^*'s strategy from the strategies of all other players. The objective of juror j^* can thus be written as:

$$\max_{s(j^*,\cdot)} E(\mathbb{1}_{\{c^* \text{ is a sure winner}\}}) = \max_{s(j^*,\cdot)} P\left(\sum_j s(j,c^*) > \sum_j s(j,c)\ \forall c\right) =$$

$$= \max_{s(j^*,\cdot)} P\left(s(j^*,c^*) - s(j^*,c) > \sum_{j\neq j^*} s(j,c) - \sum_{j\neq j^*} s(j,c^*)\ \forall c\right), \quad (1)$$

where the maximum is taken over all score vectors of juror j^*. Here, $E(\cdot)$ denotes expectation with respect to beliefs over the strategies of all jurors except j^*, and P the corresponding probability measure. Note that we analyze a perfect information game, so the probabilities are binary. It follows from (1) that juror j^*'s objective is to maximize the score difference between their preferred candidate and any other. Crucially, the scores must adhere to Borda rules. If c^* has a positive probability of winning, the optimal strategy of j^* is to assign the highest possible score to c^* – a strategy is optimal if, for all strategies of other jurors, its payoff is at least as high as that of any alternative strategy. While the rest of the optimal strategy depends on the distribution of the scores of other jurors, the basic incentive is straightforward: *if* juror j^* can influence the winner, he is incentivized to assign the lowest scores to the candidates posing the greatest threat to c^*.

2.2 Incentives – juror exclusion Borda

We now define a modification of the Borda count in which the vote of one juror – specifically, the most extreme score vector – is excluded. This contrasts with more common truncation methods that remove candidates with the lowest support or scores deviating most from the average (Kontek and Sosnowska, 2020, Sections 2.3-2.5).
Excluding a juror's vote is functionally equivalent to abstention. While it alters the incentives of individual jurors, the juror exclusion (JE) Borda preserves the structural properties of the Borda count. In particular, removing a juror affects only the number of contributing jurors, not the score matrix itself. This feature is a key advantage of juror exclusion over alternative vote modification methods such as score exclusion or candidate removal.
Importantly, JE Borda results maintain the desirable features of Borda: they yield a full ranking of candidates, reflect "distances" between them, and keep the total score assigned fixed. We assume that exactly one most deviating juror is excluded. If the number of most deviating jurors is greater than 1, the excluded juror is selected at random.
Let us consider a strategy profile s which and – associated with it – a vector of final candidate scores $(s(c))_{c\in\mathcal{C}}$. The average report $\left(\frac{1}{J}s(c)\right)_{c\in\mathcal{C}}$ serves as a benchmark. For each juror, one may calculate the Euclidean distance between s_j and the average report. Let $\hat{j}(s)$ denote the juror whose score vector deviates most under strategy profile s. Then, the optimization problem of juror j^*, who can secure the win of his most preferred candidate is analogous to (1):

$$\max_{s(j^*,\cdot)} E\mathbb{1}_{\{c^* \text{ is a winner}\}} = \max_{s(j^*,\cdot)} P\left(\sum_{j\neq\hat{j}(s)} s(j,c^*) > \sum_{j\neq\hat{j}(s)} s(j,c)\ \forall c\right). \tag{2}$$

Formally, the new game among jurors is defined on the same set of players and strategies. The players' payoffs still depend on the probability of their favorite candidate winning, but this probability now implicitly depends on the voting system. Crucially, juror exclusion influences payoffs, but not the set of players, which remains to be $\mathcal{J}$.
Given our assumptions on jurors' utilities, the decision process for any juror becomes significantly more complex than under the standard Borda rule. First, if a juror casts a vote that is most deviating, they are excluded and thus have no influence on the outcome. Assuming a juror only deviates from their truthful ranking when pivotal, they would *never* want to be excluded, as that implies their preferred candidate will likely lose. Thus, the juror is incentivized to submit a score vector that avoids removal. Second, juror i's score vector may influence the identity of the excluded juror $\hat{j}(s)$. Therefore, the juror must consider both their direct influence on candidates' scores via $s(i,c)$, and their indirect influence via the potential removal of $s(\hat{j}(s),c)$.

More precisely, a juror's deviation can be decomposed into two components:

(a) change in the ranking of candidates,

(b) change in the average score, which may change the excluded juror.

In what follows, we will capture the aggregate jurors' reports by a simple notion of *consensus* whose strength is reflected in the steepness of the final score vector. Formally, the steepness of the final score vector, or the consensus, is measured by Euclidean distance between the average score vector – that is, the total score vector divided by the number of jurors whose vote is taken into account – and the hypothetical flat score vector which represents a tie among all candidates, or, equivalently, no consensus between jurors. For example, if the number of candidates is 4, then the flat score vector assigns 2.5 – that is, $(1+2+3+4)/4$ to each candidate. Below, we present a worked–through example that illustrates the key difference in incentives regarding consensus in the two analyzed games.

Example 1 (Incentives in Standard Borda and JE Borda Games). *Consider the standard Borda game with 5 jurors and 4 candidates. Suppose juror j_5 has true preferences $c_2 \succ c_1 \succ c_3 \succ c_4$, and knows the score vectors submitted by the other jurors, given in Table 1. If j_5 reports truthfully, that is reports the score of vector of 3421, his preferred candidate c_2 will lose. The optimal strategy (left panel of Table 1) is to give c_2 the highest score and assign the lowest scores to candidate c_1 posing the greatest threat to c_2.*

Table 1: Optimal strategy of juror j_5 under standard Borda (left) and JE Borda (right)

Standard Borda Game					JE Borda Game					
	c_1	c_2	c_3	c_4		c_1	c_2	c_3	c_4	Distance
j_1	4	3	2	1	j_1	4	3	2	1	0.4
j_2	4	3	2	1	j_2	4	3	2	1	0.4
j_3	4	3	1	2	j_3	4	3	1	2	1.6
j_4	3	4	2	1	j_4	3	4	2	1	0.8
j_5	1	4	3	2	j_5	3	4	2	1	0.8
Borda Sum	16	17	10	7	JEB Sum	14	14	8	4	

Notes: The excluded juror in JE Borda is highlighted. JEB sum is a sum of candidate scores after excluding the score of an outlier juror.

Note that the strategic voting of juror j_5 flattens the score vector as compared to truthful voting, that is, decreases score differences between candidates.

Under JE Borda (right panel), j_5's optimal strategy is to report truthfully, and closer to the consensus. Lowering the score for c_1 too much would risk removal due to larger deviation from the average score vector. In this case, the final score vector does not flatten.

The above example suggests that JE Borda discourages manipulation in at least some cases by penalizing deviation from other jurors' scores. Moreover, manipulation in JE Borda is inherently more complex than under standard Borda due to the dual effects described above: a juror must evaluate both the direct impact on scores and the indirect effect on average scores and the identity of the excluded juror. These effects cannot be easily anticipated (as we know from computer simulations); multiple deviations may need to be tested, making manipulation computationally demanding. We now turn to additional examples to explore whether this insight can be generalized.

3 Nash equilibrium analysis

In this section, we present a few preference examples that would give us some basic intuition about why juror exclusion might be plausible, and under what conditions. In all the examples, we shall compare the standard Borda count with the JE Borda method, with exactly one juror excluded from the poll. The method excludes the full score of the juror whose report deviates the most from the average report, in terms of the Euclidean distance, i.e. the sum of squared deviations from the mean.
The Nash equilibrium in this context is defined in a standard way:

Definition 2. *A strategy profile $s = (s_j)_{j\in\mathcal{J}}$ forms a Nash equilibrium of the game $\big(J, (\mathfrak{S}_\mathcal{C})^J, (u_j)_{j\in\mathcal{J}}\big)$ if for every player*

$$u_j(s_j, s_{-j}) \geq u_j(s'_j, s_{-j}) \quad \forall s'_j \in \mathfrak{S}_\mathcal{C},$$

where s_{-j} denotes the strategy profile of all players except j.

Given the richness of the strategy space – every juror has $C!$ distinct strategies – the game, either with the Borda count or the JE Borda, has a plethora of equilibria. For example, an equilibrium emerges when all jurors uniformly report a specific score vector, regardless of its specific values provided that the number of jurors (and the ratio of jurors to candidates) is not too small. More specifically, in the game with 5 juror if all jurors had the same preferences 4321, but each reported 1234, the equilibrium would result in the worst outcome for each juror, much worse than under truthful reporting. Even though it is an equilibrium, it is not clear why and how it would emerge in practice.
One way to rule out implausible equilibria, that is those that arise solely because no individual juror can unilaterally affect the outcome, is to allow for deviations by coalitions rather than individuals, leading to the concept of a Strong Nash equilibrium (Aumann, 1959). Considering deviations by juror coalitions rather than single jurors yields a more restrictive (and often more desirable) set of equilibria, given the abundance of standard Nash equilibria. However, coalition deviations require jurors to freely communicate and form binding agreements. The rules of many competitions severely restrict this possibility. Nevertheless, Strong Nash

equilibria provide a valuable theoretical benchmark, which we discuss in Section 4.1.

Equilibrium refinement strategy Since our primary focus is on a noncooperative framework where jurors act independently, we adopt an alternative criterion for ruling out implausible equilibria – specifically, we exclude any equilibrium in which all jurors are strictly worse off than under truthful reporting (see Jackson et al., 1994). We do it as follows. We start with the matrix of truthful reports, a natural and salient benchmark, and proceed by iteratively changing the score matrix via unilateral, profitable deviations of individual jurors. This process continues until no further deviation is possible, yielding a Nash equilibrium. Obviously, there are usually more than one paths that this algorithm can go, leading to many alternative equilibria. However, we never end up in an equilibrium such as the one described above in which all jurors are worse off than under truthful reporting.
The important question is whether this algorithm may end up in a cycle and thus produce no equilibrium. The answer is that it can end up in a cycle. However, for any cycle that we found, there has always been an alternative path leading to a pure-strategy NE. therefore, the cycle path is not unique among the best-response paths. Assuming that a juror randomly chooses among multiple available best-response paths, the players is guaranteed to escape the cycle.
For example, consider a JE Borda game with 5 jurors and 3 candidates with the following true preferences (already matched to scores): j_1, j_2 : 321, j_3 : 312, j_4, j_5 : 231. Under true reporting juror j_3 is excluded and candidate c_1, c_2 have equal chance of winning. It is straightforward to verify that the following sequence of strictly profitable deviations leads to a cycle: $j_1 : 213 \to j_4 : 132 \to j_1 : 321 \to j_4 : 231$. However, an equally beneficial alternative path of $j_1 : 312$ leads to an immediate NE.

Analyzed preference profiles We consider three simple preference profiles involving 5 jurors and 4 candidates. They are given in Table 2. We assume a strong initial consensus among jurors, who differ in at most 1, 2, or 3 pairwise candidate comparisons. In all three examples preferences of jurors j_1–j_4 are fixed and strongly aligned and only the preferences of juror j_5 will differ. Jurors j_1 and j_2, each have preferences 4321 (recall that such score corresponds to the following ranking of candidates: $c_1 \succ c_2 \succ c_3 \succ c_4$), and jurors j_3 and j_4, each have preferences 3421. Thus, these two pairs of jurors disagree only on the relative classification of c_1 and c_2.
Note that in Example 2 juror j_5 agrees with jurors j_1–j_2 on all pairwise comparisons among candidates. In Example 3, he only agrees with them on the first-best candidate, while in Example 4, he agrees on all comparisons except c_1 vs. c_3 and c_2 vs. c_3. Based on these three examples, we illustrate the difference between equilibria in the standard Borda and JE Borda games. In particular, we show that in Example 2 truthful reporting is a NE under JE Borda game but not under standard Borda, in Example 3 it is the opposite (truthful reporting is a NE under standard Borda but

Table 2: True preferences in Examples 2–4

	c_1	c_2	c_3	c_4
j_1	4	3	2	1
j_2	4	3	2	1
j_3	3	4	2	1
j_4	3	4	2	1
A: j_5	4	3	2	1
B: j_5	4	1	2	3
C: j_5	3	2	4	1

not under JE Borda), while in Example 4 truthful reporting is neither a NE under standard Borda nor under JE Borda. We will show reports that are NE in one but not in the other game, and comment on all NE of these games.

Example 2 (**True reporting is a Nash Equilibrium under the JE Borda but not under the standard Borda game**). *Consider the ranking arising from true preferences, presented in Table 3.*

Table 3: True preferences A

	c_1	c_2	c_3	c_4
j_1	4	3	2	1
j_2	4	3	2	1
j_3	3	4	2	1
j_4	3	4	2	1
j_5	4	3	2	1
Σ	**18**	17	10	5

Truthful reporting (each juror reports according to their preferences), as shown in the left panel of Table 4, is a Nash equilibrium under the JE Borda game. Outliers, i.e., juror score vectors that are furthest from the mean score vector, are highlighted in gray. We assume, however, that only one of these vectors – chosen at random – is excluded. Specifically, jurors j_3 and j_4 are outliers, so one of them will be excluded, resulting in c_1 winning. In this case, the outlying jurors do not have a profitable deviation, since the only way they can move closer to the mean is by reporting the same as jurors j_1–j_2 and j_5, which would only worsen the outcome for their preferred candidate.

By contrast, truthful reporting is not a NE under the standard Borda game, because

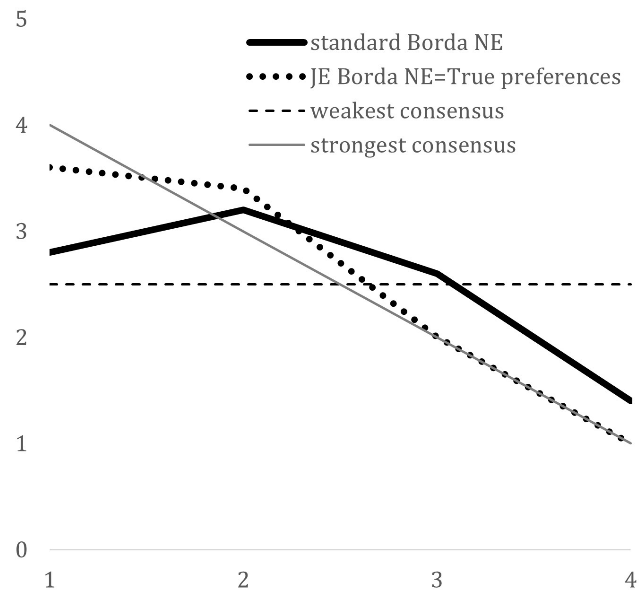
Table 4: Equilibria under standard Borda and JE Borda games for preferences from Table 3

A NE under JE Borda						A NE under standard Borda				
	c_1	c_2	c_3	c_4	distance		c_1	c_2	c_3	c_4
j_1	4	3	2	1	0.32	j_1	4	3	2	1
j_2	4	3	2	1	0.32	j_2	4	3	2	1
j_3	3	4	2	1	0.72	j_3	1	4	3	2
j_4	3	4	2	1	0.72	j_4	1	4	3	2
j_5	4	3	2	1	0.32	j_5	4	3	2	1
JEB sum	**15**	13	8	4		Σ	14	**17**	12	7

jurors j_3 or j_4 can shift the outcome in favor of c_2. For example, conditional on jurors j_1–j_2 and j_4–j_5 reporting truthfully, juror j_3 benefits by strategically reporting 1432, leading to a total score vector of $(16, 17, 11, 6)$ and resulting in candidate c_2 winning. On the other hand, the right panel of Table 4 illustrates one NE under the standard Borda rule. Here, none of the jurors j_1–j_2 or j_5 – who would prefer to change the outcome – can do so individually. By comparison, this profile is not a NE under JE Borda, because jurors j_3–j_4 are again the outliers. After excluding one of their score vectors, the resulting total becomes $(13, 13, 9, 5)$, producing a tie between c_1 and c_2. However, if juror j_5 instead reported 4231, jurors j_3–j_4 would still be outliers, but the score of c_1 would improve relative to c_2. The new JE Borda sum would be $(13, 12, 10, 5)$. Thus, this move by j_5 constitutes a profitable deviation.
This example illustrates a key feature of the JE Borda game compared to the standard Borda game: when there is strong consensus among jurors – that is, the total score vector under truthful reporting is steep – the JE Borda game tends to preserve this consensus. It does so by penalizing any deviation from it. In contrast, the standard Borda game lacks this property and thus tends to erode consensus.
This difference can be visualized by examining the steepness of the average score vector, as shown in Figure 1. Notably, compared to the JE Borda Nash equilibrium, the average score vector under the standard Borda Nash equilibrium is closer to the weakest possible consensus, represented by the completely flat vector $(2.5, 2.5, 2.5, 2.5)$, and farther from the strongest consensus, in which every juror reports identically. Note that the Euclidean distance between the average score vector and the flat vector equals 2.12 under standard Borda NE and 4.52 under true reporting and under JE Borda NE. If we calculate the distance under JE Borda NE after excluding the most deviating juror, the distance increases to 4.63.

In the next example, we consider preferences for which there are incentives to report strategically under the JE Borda game.

Figure 1: The average score vectors for Nash Equilibria from Table 4, Example 1



Notes: The JE Borda game preserves the consensus of truthful preferences, while the standard Borda game weakens it.

Example 3 (True reporting is a Nash Equilibrium under the standard Borda game but not under the JE Borda game). *Consider the ranking arising from true preferences, presented in Table 5. Truthful reporting, presented in the right*

Table 5: True preferences B

	c_1	c_2	c_3	c_4
j_1	4	3	2	1
j_2	4	3	2	1
j_3	3	4	2	1
j_4	3	4	2	1
j_5	4	1	2	3
Σ	**18**	15	10	7

panel of Table 6, is a NE under the standard Borda game. The reason is that jurors j_3 and j_4, who are not satisfied with the outcome under truthful reporting, cannot unilaterally change it. They can at most lower the score of c_1 from 3 to 1, which is not sufficient to increase the winning chance of their preferred candidate c_2.

Table 6: Equilibria under standard and JE Borda games

A NE under JE Borda						A NE under standard Borda				
	c_1	c_2	c_3	c_4	distance		c_1	c_2	c_3	c_4
j_1	4	1	3	2	0.32	j_1	4	3	2	1
j_2	4	3	2	1	0.32	j_2	4	3	2	1
j_3	3	4	2	1	0.72	j_3	3	4	2	1
j_4	3	4	2	1	0.72	j_4	3	4	2	1
j_5	4	3	2	1	0.32	j_5	4	1	2	3
JEB sum	**15**	13	8	4		Σ	**18**	15	10	7

Truthful reporting is not a NE under JE Borda. The reason is that under truthful reporting j_5's score is an outlier score. After excluding it, the total score vector is $(14, 14, 8, 4)$, which means that candidates c_1 and c_2 each have a $1/2$ chance of winning. A profitable deviation is e.g. juror j_5 reporting 4321 instead of her true 4123. If she does so, j_5 will not be an outlier anymore, but j_3–j_4. This would result in a NE under which c_1 is the only winner. This is shown in the left panel of Table 6. We will call such an equilibrium a nearest Nash Equilibrium as it is achievable from the true preferences in only one step (i.e. requires only one profitable deviation). One might wonder whether NNE always exists in pure strategies. As long as the set of any

PSNE reachable from a given truthful profile is non-empty, a NNE is guaranteed to exist – it is simply the subset of minimal elements among all reachable PSNE.

Finally, we analyze the case where there are incentives to vote strategically in both the standard Borda as well as the JE Borda games.

Example 4 (**Truthful reporting is neither a Nash Equilibrium under the standard Borda nor under the JE Borda**). *Consider the ranking arising from true preferences, presented in Table 7. Truthful reporting does not constitute a NE*

Table 7: True preferences C

	c_1	c_2	c_3	c_4
j_1	4	3	2	1
j_2	4	3	2	1
j_3	3	4	2	1
j_4	3	4	2	1
j_5	3	2	4	1
Σ	**17**	16	12	5

under either the standard Borda game or the JE Borda game. For instance, under JE Borda, juror j_5 has a profitable deviation by reporting the ranking 4321 *instead of his true preference* 3241*, as illustrated in the left panel of Table 8. Under truthful reporting, juror j_5's score is excluded, and candidates c_1 and c_2 tie for victory. By reporting* 4321*, matching the ballots of jurors j_1–j_2, juror j_5 moves closer to the consensus, and his score is no longer excluded. Consequently, one of the outlying votes, from either j_3 or j_4, is excluded instead, leading to a clear victory for candidate c_1. Since j_5 prefers c_1 over c_2, this manipulation is strictly beneficial for him.*
In simpler terms, juror j_5 sacrifices his favorite candidate c_3, who cannot win, in favor of his second-best candidate c_1, in order to reduce the chances of an even less preferred candidate c_2. Note that this type of strategic voting does not "flatten" the aggregate (or average) scores – that is, it does not reduce the aggregate score differences between candidates. Rather, it makes the ranking even "steeper", with wider margins between candidates. This property may be referred to as a strengthening of the consensus. Moreover, this profitable deviation results in the nearest NE under JE Borda.
Truthful reporting is also not a NE under the standard Borda. A profitable deviation is for example juror j_3 reporting 1432 *instead of her true preferences* 3421*. This leads to the total score vector of* $(15, 16, 13, 6)$*. Thus, c_2 wins instead of c_1, which is strictly better for j_3. The resulting score matrix is still not a NE. The following is an example sequence of profitable deviations (starting from true preferences) that leads to a NE that is depicted in the right panel of Table 8:*

Table 8: Equilibria under standard and JE Borda games for preferences from Table 3

A NE under JE Borda						A NE under standard Borda				
	c_1	c_2	c_3	c_4	distance		c_1	c_2	c_3	c_4
j_1	4	3	2	1	0.32	j_1	4	2	1	3
j_2	4	3	2	1	0.32	j_2	4	1	3	2
j_3	3	4	2	1	0.72	j_3	1	4	3	2
j_4	3	4	2	1	0.72	j_4	1	4	2	3
j_5	4	3	2	1	0.32	j_5	2	1	4	3
JEB sum	**15**	13	8	4		Σ	12	12	**13**	**13**

$$\begin{array}{llll} j_3 & 3421 & \rightarrow & 1432 \\ j_2 & 4321 & \rightarrow & 4132 \\ j_4 & 3421 & \rightarrow & 1423 \\ j_1 & 4321 & \rightarrow & 4213 \\ j_5 & 3241 & \rightarrow & 2143 \end{array}$$

Note that even though jurors unanimously agree that candidate c_4 is the worst among all candidates, he nevertheless has a 1/2 chance of winning in the equilibrium. Moreover, candidates c_1 and c_2, each of whom is the top choice for two jurors, have zero chance of winning in the equilibrium. One might conclude that this equilibrium is very far from the consensus implied by the true preferences. An objection may arise that the comparison of equilibria under the standard and JE Borda games is not entirely fair. In the JE Borda case, we have selected a nearest NE – requiring only one profitable deviation from the true preferences – while in the standard Borda case, the NE considered is five profitable deviations away from truth. This raises the question of whether there also exist Nash equilibria in the JE Borda game that require many steps of profitable deviations from true preferences.

Consider, for example, the NE of the JE Borda game presented in Table 9. This NE is achieved by the following sequence of profitable deviations starting from true preferences:

$$\begin{array}{llll} j_3 & 3421 & \rightarrow & 2431 \\ j_2 & 4321 & \rightarrow & 4231 \\ j_4 & 3421 & \rightarrow & 2431 \end{array}$$

In this equilibrium, which among all equilibria of this game is furthest from the original consensus (measured, for example, by the Euclidean distance from the average score vector in the reported matrix to the average score vector of the true preference profile), candidates $c_1 - c_3$ have an equal chance of winning. This is clearly closer to the original consensus than the equilibrium under the standard Borda game presented in the right panel of Table 8.

Table 9: The worst-case NE under the JE Borda

	c_1	c_2	c_3	c_4	distance
j_1	4	3	2	1	2
j_2	4	2	3	1	2
j_3	2	4	3	1	2
j_4	2	4	3	1	2
j_5	3	2	4	1	2
Σ	15	15	15	5	

Notes: The worst-case NE under the JE Borda game is closer to the original juror consensus than under the NE in the standard Borda game given in the right panel of Table 8.

4 Discussion and conclusions

In this section, we discuss the difference between excluding full juror scores vs. excluding most deviating individual scores. We analyze the possibility of collusion among the jurors and the resulting notion of strong Nash Equilibrium. We close by offering some policy advice based on the insights gained in this paper.

4.1 Collusion

Another approach for identifying possible outcomes of the Borda game involves examining the dynamics of coalitions among players, that is, Strong Nash equilibria, as defined by Aumann (1959). In this context, for any subset of players $A \subset \mathcal{J}$ denote by $s_A = (s_j)_{j \in A}$ their joint strategy profile, and by $s_{-A} = (s_j)_{j \in \mathcal{J} \setminus A}$ the profile of the remaining players. A Strong Nash Equilibrium is a profile, for which no coalition can find a joint deviation that strictly benefits all its members. Notice there are two distinct features of this refinement: first, the players must form a coalition, and second, the coalition deviation must be *strictly*, rather than weakly, beneficial for its members.

Definition 3. *A strategy profile $s = (s_j)_{j \in \mathcal{J}}$ forms a Strong Nash equilibrium of the game $\left(\mathcal{J}, (\mathfrak{S}_{\mathcal{C}})^J, (u_j)_{j \in \mathcal{J}}\right)$ if there does not exist a coalition $A \subset \mathcal{J}$ and deviation s'_A, such that:*

$$u_j(s'_A, s_{-A}) > u_j(s) \quad \forall_{j \in A}.$$

The notion of SNE is particularly useful in the context of voting games, as players' payoffs are non-rivalrous. Specifically, there is no need to specify how the coalition's gains are distributed among its members.

In our analysis of the game in Table 4, we can identify two natural player coalitions: jurors j_1–j_2 and j_5 (referred to as group A), who favor candidate c_1, and jurors j_3–j_4 (referred to as group B), who prefer candidate c_2.

Which group of jurors has the ability to secure victory for their preferred candidate? It is quite evident that group A possesses this capability, while group B does not. For group A to ensure victory for candidate c_1, it is sufficient for them to collectively allocate a total of 12 points to c_1 and to refrain from awarding more than 5 points to candidate c_2. Group A also needs to ensure that c_3 or c_4 do not receive too many points, which could lead to an unintended victory due to a flattening of the ranking. This coordination is relatively easy if the jurors can communicate, as all members of the group prefer c_1 or c_2 to win over c_3 or c_4. However, such coordination becomes less likely in scenarios where communication or cooperation among jurors (or groups of jurors) is not feasible.

Although individual jurors have no incentive to unilaterally deviate from truthful reporting, the situation changes when coalitions of jurors are considered. The Nash Equilibrium of the game determined by the true preferences in Table 4 is not a Strong Nash Equilibrium. To see how coalitional considerations might influence the outcome, consider the scenario where jurors j_3 and j_4 report their preferences as illustrated in the right panel of Table 10.

Table 10: Collusion in JE Borda

True preferences A					Reports					
	c_1	c_2	c_3	c_4		c_1	c_2	c_3	c_4	distance
j_1	4	3	2	1	j_1	4	3	2	1	2.16
j_2	4	3	2	1	j_2	4	3	2	1	2.16
j_3	3	4	2	1	j_3	1	2	4	3	8.16
j_4	3	4	2	1	j_4	1	4	3	2	4.56
j_5	4	3	2	1	j_5	4	3	2	1	2.16
Borda sum	**18**	17	10	5	JEB sum	**13**	**13**	9	5	

The idea is that since only one juror is excluded, the coalition of jurors j_3 and j_4 may agree to vote radically and sacrifice one of them in order to lower the score of c_1 while keeping the score of candidate c_2 high. In the case depicted in the right panel of Table 10, juror j_3 gets excluded, but j_4 is able to influence the final result sufficiently to raise c_2's chances of winning from 0 under true reporting to 0.5. Note that the threat of j_3, j_4 reporting as indicated will change the strategy of jurors j_1, j_2 and j_5. Collectively, they can prevent j_3 and j_4 from attempting to promote the victory of candidate c_2 by diminishing the score assigned to c_2. However, they need to coordinate their actions to maintain a score close to the mean, thus avoiding exclusion.

Note that the strategy of j_3, j_4 presented in Table 10 relies on "sacrificing" j_3, whose strategy concentrates solely on shifting the mean. This enables j_4 to promote c_2 by lowering the score of c_1. This strategy would be ineffective if two jurors were excluded instead of one, or if the number of excluded jurors were endogenous, e.g., all jurors whose deviation is above a certain threshold. This is a common technique in the literature on outlier detection (Wilcox, 2011). This issue is outside the scope of this paper, but in general, it presents a challenge to determine a threshold value that would not only lead to distortion of the final result, but also allow for detecting and addressing manipulation by jurors (Kontek and Kenner, 2023).

4.2 Excluding jurors vs. excluding individual scores

Excluding most deviating jurors incentivizes jurors to align more with the mean and thus decreases incentives to report extreme scores. If jurors' true preferences are sufficiently aligned with each other, which is to be expected in most competitions in which subjective taste does not dominate the objective capabilities, then JE Borda works pretty well in discouraging manipulation. We now ask a question: can we achieve similar effects by excluding individual outlier scores instead of the whole juror's score vector?

Table 11: Excluding jurors vs. excluding individual scores

Excluding jurors						Excluding individual scores				
	c_1	c_2	c_3	c_4	distance		c_1	c_2	c_3	c_4
j_1	4	3	2	1	1.44	j_1	4	3	2	1
j_2	4	3	2	1	1.44	j_2	4	3	2	1
j_3	1	4	3	2	5.44	j_3	1	4	3	2
j_4	3	4	2	1	1.04	j_4	3	4	2	1
j_5	3	2	4	1	3.44	j_5	3	2	4	1
JEB sum	14	12	10	4		I SE sum	14	14	9	6

Notes: Excluding jurors has two effects: it eliminates the outlier score and punishes the juror by neutralizing all his scores. Excluding individual scores only has the first effect. ISE sum denotes a sum of candidate scores after excluding outlier scores (shaded in grey).

To make things interesting, consider preferences presented in Table 7 and juror j_3's decision to misreport his true preferences. Instead of his true 3421 he reports 1432. This deviation is profitable for j_3 under standard Borda, because it forces her best candidate c_2 to win. However, under JE Borda or under Borda with excluding the most deviating individual score, this deviation is not profitable anymore, as it results in the deviating scores being excluded. This is presented in Table 11. The left panel shows the effect of excluding the most deviating score (here the score of j_3) and the right panel shows the effect of excluding the most deviating score separately for each candidate. Note that JE Borda successfully discourages j_3 from executing the deviation, because excluding his whole score neutralizes his move completely. This is different under excluding individual scores. Even though, the decreased score for candidate c_1 given by juror j_3 is excluded, the score for candidate c_2 given by juror j_3 is not. In both cases, the effect of manipulation is indeed reduced, both under juror exclusion and score exclusion. However, while the individual score exclusion method results in a draw, the juror exclusion method leads to c_1 winning the competition. Hence, in this case the latter method restores the true ranking while the former does not. Moreover, the juror exclusion method changes the incentives to manipulate, preventing extreme reports in the first place.

4.3 Closing remarks

The juror exclusion method has been put forward as a potential countermeasure to deter manipulation. We show that when applied to the Borda count, excluding jurors may reduce manipulation. Based on a few key examples, we hypothesize that juror exclusion is particularly effective when jurors' preferences exhibit a high degree of alignment, resulting in only minor discrepancies among their rankings. However, if jurors' preferences are highly heterogeneous, excluding jurors may distort the truth and create incentives to align with the mean.

Generally, one can think of jurors' preferences as consisting of an objective part plus individual taste. The objective evaluation of any candidate is a stochastic realization of some common "true" distribution, imperfectly, but correctly observed by expert jurors. Such a framework is well-known in social sciences under the umbrella term of Thurstonian models (Thurstone, 1994). Suppose that the underlying preferences of jurors are rankings derived from some latent normally distributed variables, all drawn from the same distribution. Then one can expect the jurors' truthful rankings to be highly correlated, with little strong disagreement in evaluating particular candidates. In such a case, juror exclusion Borda may be a useful tool to prevent potential or eliminate existing manipulation, helping to recover the underlying common, i.e. objective evaluation of the candidates.

It is, however, crucial to observe that the Thurstonian model implicitly assumes the existence of an objective truth. In this scenario, jurors are just experts who use their competence to evaluate the genuine "quality" of a candidate. This stands in stark contrast to models centred solely on preference, where each juror's ranking merely reflects their personal taste. Following the intuition from Kontek and Sosnowska (2020) (who analyse 2016 Henryk Wieniawski International Violin Competition and argue that excluding 20% of jurors is a decent rule of thumb), we argue that juror exclusion Borda may be particularly effective in classical music competitions, where jurors might indeed be perceived as experts, evaluating the musical proficiency of a given candidate. In such contests, the objective part of jurors' preferences carries significant weight, while the taste component is of lesser significance. More generally, juror exclusion might help achieve the "correct" outcome in the setups with a "deserving winner" (Amorós, 2013, 2011; Adachi, 2014).

Conversely, juror exclusion may yield unfavorable outcomes and encourage manipulation in contests where subjective taste plays a more substantial role, such as popular music competitions – as an example, votes in the Eurovision contest were shown to be influenced by cultural and geographical proximity (Yair, 2018). We recommend the use of the juror exclusion Borda in competitions where the objective component in jurors' preferences holds greater significance relative to the subjective component.

To conclude, the paper examines incentives in a novel modification of the Borda count, namely the juror exclusion (JE) Borda, introduced in Kontek and Sosnowska (2020). In the modified scoring system, the juror with the largest deviation from the average juror ranking is removed from the poll. This modification has a strong effect on the voting outcome, both directly by removing potentially misreported scores and indirectly by changing incentives to manipulate.

We show that the threat of being excluded can reduce the juror's incentives to manipulate. Indeed, the "classic" misreporting strategy that works well in the Borda count, may easily lead to being excluded as the most deviating juror. The threat of being excluded may prevent strong manipulation and flattening of the final ranking. While juror exclusion gets rid of both the result of potential manipulation and the source, the ultimate effect of the modification depends on the full profile of preferences. In particular, there are profiles for which the JE Borda does not, in fact, reduce manipulation. We have presented examples in which JE Borda incentivizes the juror with truly deviating preferences to strategically misreport to align with the mean. We briefly compare the exclusion of jurors to the exclusion of individual scores. While the two methods are similar in their attempt to counteract apparent manipulation, they work differently. In particular, the juror exclusion method *prevents* manipulation, as the strategizing juror faces a threat of her full score being excluded. If such exclusion happens, this effectively buries the chance of the juror's preferred outcome. This result does not hold in the score exclusion method.

We have also presented some Nash equilibria and Strong Nash equilibria under the Borda count and JE Borda method. We show that if we fail to counter manipulation in the standard Borda ballot, the final ranking becomes flatter than true preferences. JE Borda may act not only as a remedy but also as a preventive measure. However, for more heterogenous preferences, the JE Borda may create false incentives for jurors to converge toward the mean.

In the discussion section, we conclude that, based on our examples, the JE Borda method can be particularly effective in revealing the unobservable truthful ranking, especially when one can anticipate the jurors' preferences to be highly correlated. Such preferences arise e.g. in the Thurstonian model, where jurors' preferences represent imperfect signals of the underlying "true" distribution of candidates' quality.

This article serves as an initial exploration of the topic. Future work will focus on conducting numerical experiments to identify all Nash equilibria, including those attainable from truthful reporting, as well as the nearest equilibria for every preference

profile in games involving up to 5 jurors and 5 candidates (larger games may be too complex to enumerate all equilibria and the paths leading to them.) These experiments will support the testing of theoretical hypotheses and the development of predictive insights. For example, we aim to examine whether Nash equilibria accessible from truthful preferences in JE Borda games preserve the level of consensus seen under truthful voting, whereas in standard Borda games, such equilibria may not enhance consensus.
We also plan to investigate alternative specifications of jurors' utility functions. Under the current model, certain implausible equilibria arise because individual jurors exert minimal influence over the final candidate ranking. To address this, we propose a modified utility structure in which a juror's move is deemed beneficial if it increases the score gap between their top-ranked candidate and that candidate's closest competitors. In this revised framework, a move can be considered profitable even if it does not change the overall ranking. We also intend to investigate theoretical properties of the nearest Nash equilibrium concept, along with a proposed refinement specific to JE Borda. In this modified version, the sequence of jurors' moves at each iteration is determined by their distance from the overall score vector, giving precedence to those jurors whose current strategies deviate most significantly.

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