GOSPODARKA SUROWCAMI MINERALNYMI

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On some randomness phenomena occuring during the process of the formation of the post mining subsidence

Key words

GPS method, Itô's Equation, Kolmogorov's Equation, measurement error, mining exploitation, randomness, subsidence trough, surveying

Abstract

The dislocation process of the points located on a surface in the area that was under the influence of underground mining was observed in a small time scale. It exhibited highly irregular behaviour (e.g. the observed points during the time of their vertical subsidence oscilated in a random way). In the paper is analysed vertical subsidence of three points. The results of the analysis show in an evident way that the nature of the observed irregularities is inherently random. Thus the observed phenomen proves that the underlying mechanism which rules the formation process of the post mining subsidence is of the purely random nature.

Introduction

In the papers (Bugiel, Piwowarski 2003a, 2003b) was proposed Itô's Equation and, derivable from it, Kolmogorov's Equation to describe mathematically the process of the formation of the post mining subsidence. The authors were motivated mainly by the observed random phenomena which occured in a small time scale (short period of time between two

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consecutive surveys) during the dislocation process of points located on a surface in the area that was under the influence of underground mining. In the above two cited papers the observed random phenomena were sketchy illustrated by vertical subsidence of only one point.

The purpose of this paper is to provide the reader with an extensive analysis of the vertical subsidence of points in a small time scale. Namely, we analyse the results of surveys of the vertical subsidence of three points whose location is shown by Figure 1. For each of these three points were carried out fifty two surveys.

Measurements were carried out with the aid of GPS technology over the period of time from 15.02.1999 to 04.03.1999. Approximately one measurement was performed every eight hours. Such a small time scale makes it possible to observe distinctly the oscillatory nature of the subsidence process in contrast to a long time scale (when the period of time between two consecutive observations of the surface is not too small, e.g. 3 months or more) during which the random phenomena are hardly observed and therefore were not usually analysed up to now.

As was said, the surveys of the displacements of the points located on the surface were carried out with the aid of GPS technology. Thus the error of the method of the measurement interfered with the inherent randomness of the subsidence process in an essential way. To overcome this interference we applied a method of analysis which enabled one to show inherent randomness of the subsidence process.

In conclusion we would like to note that we have included for the reader's convenience one complete set of numerical results i.e., both measured and calculated results. The numerical results which concern Point No 4 are gathered in Table 1. In Sect. 1 we describe the method of the analysis. The results of the analysis of the surveys are presented in Sect. 2.

1. The method of the analysis

We explain below how one can justify that the results of the measurements of the process $t \rightarrow W(t)$ of the vertical subsidence of the Points Nos 4, 5 and 8 are perturbed by a factor of purely random nature. Consequently we show that these results can be regarded as a sample paths of a stochastic process of the form: $t \rightarrow W(t) = W_d(t) + \xi(t)$. Here $W_d(t)$ and $\xi(t)$ stand, respectively, for the deterministic and purely random parts of $W(t)$.

Denote by $W_m(t_i)$ and $W(t_i)$ respectively, the measured and the theoretical values of the vertical subsidence process $t \rightarrow W(t)$ at the moment t_i of measurement. Thus

$$W_m(t_i) = W(t_i) + \varepsilon(t_i) \tag{1}$$

where $\varepsilon(t_i)$ stands for the errors of the measurements at the moments t_i $(i = 1, 2, ..., 52)$.

As was above noted we claim that

$$W(t) = W_d(t) + \xi(t) \tag{2}$$

where $W_d(t)$ and $\xi(t)$ denote the deterministic and the random parts of $W(t)$, respectively. Accordingly, at each moment t_i of the measurement one can write

$$W(t_i) = W_d(t_i) + \xi(t_i) \tag{3}$$

To evaluate the values $W_d(t_i)$ at the moments t_i ($i = 1, 2, ..., 52$) of measurements we proceed as follows. Firstly, $W_m(t_1)$ and $W_m(t_{52})$ are exactly surveyed. Therefore one can assume that they are unbiased by the error of measurement. Secondly, we assume that the vertical deterministic part of the subsidence of the points in question during the whole period of quasi-continuous measurements had a constant rate.

Finally, we additionally assume (for the sake of the simplicity) that both the values $\xi(t_1)$ and $\xi(t_{52})$ of the random part $\xi(t)$ are equal to zero.

Results $W_m(t_i)$ of the surveys of the subsidence $W(t)$ and the pieces of the straight lines joining the two points (t_1, $W_m(t_1)$) and (t_{52}, $W_m(t_{52})$) are plotted for all three points in Figures 2, 4 and 6 in the next section.

The measured values $W_m(t_i)$ of the vertical subsidence and the calculated values $W_d(t_i)$ and $W_m(t_i) - W_d(t_i)$ at the moments t_i ($i = 1, 2, ..., 52$) are given for Point No 4 in the second, third and fourth columns of Table 1, respectively.

For all the three points their calculated values $W_m(t_i) - W_d(t_i)$ are plotted in Figures 3, 5 and 7.

Note that by the relations (1) and (3) the calculated values $W_m(t_i) - W_d(t_i)$ satisfy relation:

$$W_m(t_i) - W_d(t_i) = \xi(t_i) - \varepsilon(t_i) \tag{4}$$

This means that the calculated values $W_m(t_i) - W_d(t_i)$ are equal to the values $\xi(t_i)$ of the random part of $W(t)$, given by (2), which however are biased by $\varepsilon(t_i)$ the errors of the measurements. It follows from the precise evaluation of the error of the used instrument that each result may be biased by the measurement error $\delta_{max} \leq 6$ mm.

Since $|\varepsilon(t_i)| \leq \delta_{max}$, those of the measured vertical subsidence $W_m(t_i)$ which satisfy the relation

$$|W_m(t_i) - W_d(t_i)| > \delta_{max} \tag{5}$$

reveal the genuine random nature of the process under discussion.

TABLE 1

The results W_m (t_i) of the surveys and the calculated values of the deterministic $W_d(t_i)$ and random $W_m(t_i)$ – $W_d(t_i)$ components for Point No 4

TABELA 1

Wyniki pomiarów obniżeń W_m (t_i) oraz obliczone wartości części deterministycznej $W_d(t_i)$ i losowej $W_m(t_i)$ – $W_d(t_i)$ dla punktu nr 4

Survey's number	$W_m(t_i)$ Survey [mm]	$W_d(t_i)$ deterministic component [mm]	$W_m(t_i) - W_d(t_i)$ random component [mm]	Survey's number	$W_m(t_i)$ Survey [mm]	$W_d(t_i)$ deterministic component [mm]	$W_m(t_i) - W_d(t_i)$ random component [mm]
1	2	3	4	1	2	3	4
1	–29.60	–29.60	*0.00*	27	–57.60	–58.56	*0.96*
2	–32.40	–30.71	*–1.69*	28	–58.50	–59.67	*1.17*
3	–29.70	–31.83	*2.13*	29	–60.90	–60.78	*–0.12*
4	–41.70	–32.94	*–8.76*	30	–56.40	–61.90	*5.50*
5	–51.90	–34.05	*–17.85*	31	–48.70	–63.01	*14.31*
6	–34.00	–35.17	*1.17*	32	–66.10	–64.13	*–1.97*
7	–29.50	–36.28	*6.78*	33	–51.10	–65.24	*15.14*
8	–33.40	–37.40	*4.00*	34	–64.80	–66.35	*1.55*
9	–37.30	–38.51	*1.21*	35	–70.20	–67.47	*–2.73*
10	–41.60	–39.62	*–1.98*	36	–68.30	–68.58	*0.28*
11	–34.40	–40.74	*6.34*	37	–71.00	–69.69	*–1.31*
12	–35.40	–41.85	*6.45*	38	–74.30	–70.81	*–3.49*
13	–42.50	–42.96	*0.46*	39	–66.50	–71.92	*5.42*
14	–37.70	–44.08	*6.38*	40	–76.10	–73.04	*–3.06*
15	–42.00	–45.19	*3.19*	41	–82.30	–74.15	*–8.15*
16	–37.00	–46.31	*9.31*	42	–78.50	–75.26	*–3.24*
17	–39.30	–47.42	*8.12*	43	–67.00	–76.38	*9.38*
18	–59.20	–48.53	*–10.67*	44	–87.40	–77.49	*–9.91*
19	–50.30	–49.65	*–0.65*	45	–81.50	–78.60	*–2.90*
20	–42.10	–50.76	*8.66*	46	–82.70	–79.72	*–2.98*
21	–68.30	–51.87	*–16.43*	47	–88.80	–80.83	*–7.97*
22	–53.50	–52.99	*–0.51*	48	–76.90	–81.95	*5.05*
23	–54.60	–54.10	*–0.50*	49	–79.70	–83.06	*3.36*
24	–55.10	–55.22	*0.12*	50	–85.90	–84.17	*–1.73*
25	–59.60	–56.33	*–3.27*	51	–78.18	–85.29	*6.39*
26	–59.70	–57.44	*–2.26*	52	–86.40	–86.40	*0.0*

Analysis of the case complementary to the relation (5) requires an assumption concerning the probability distribution of the error of the measurement. Since $|\varepsilon(t_i)| \le \delta_{max}$, one has to exclude the normal distribution. However due to the fact that the measurements were carried out with the aid of GPS technology it is reasonable to assume uniform distribution of the errors of measurements. Since the very same instrument was exploited during the whole period of the measurements, therefore at any moment t we have:

$$P(\alpha \le \varepsilon(t) \le \beta) = \frac{1}{b-a} \int_{\beta}^{a} dx$$

where $-a = b = \delta$, and $\alpha < \beta$, $\alpha, \beta \in [a, b]$ are arbitrary.

Hence the measured vertical subsidences $W_m(t_i)$ which satisfy the relation

$$\sigma < |W_m(t_i) - W_d(t_i)| \le \delta_{max} \tag{6}$$

can also be counted among those ones which reveal the random nature of the process. Here $\sigma = \delta_{max} / \sqrt{3} \approx 3.46$ stands for the standard deviation.

As for the part of the measured vertical subsidence $W_m(t_i)$ which satisfy

$$|W_m(t_i) - W_d(t_i)| \le \sigma \tag{7}$$

one cannot give a conclusive opinion.

The results of the above analysis for Points Nos 4, 5 and 8 are gathered by the below Tables 2, 3 and 4, respectively.

TABLE 2

Classification of fifty two surveys $W_m(t_i)$ — point No 4

TABELA 2

Klasyfikacja pięćdziesięciu dwu pomiarów $W_m(t_i)$ — punkt nr 4

Point No	Relation	Number of $W_m(t_i)$'s satisfying relation	Does true randomness occur
4	$\|W_m(t_i) - W_d(t_i)\| > \delta_{max}$	18	decisively yes
	$\sigma < \|W_m(t_i) - W_d(t_i)\| \le \delta_{max}$	5	yes
	$\|W_m(t_i) - W_d(t_i)\| \le \sigma$	29	cannot excluded

TABLE 3

Classification of fifty two surveys $W_m(t_i)$ — point No 5

TABELA 3

Klasyfikacja pięćdziesięciu dwu pomiarów $W_m(t_i)$ — punkt nr 5

Point No	Relation	Number of $W_m(t_i)$'s satisfying relation	Does true randomness occur
5	$\lvert W_m(t_i) - W_d(t_i)\rvert > \delta_{max}$	17	decisively yes
	$\sigma < \lvert W_m(t_i) - W_d(t_i)\rvert \le \delta_{max}$	13	yes
	$\lvert W_m(t_i) - W_d(t_i)\rvert \le \sigma$	22	cannot excluded

TABLE 4

Classification of fifty two surveys $W_m(t_i)$ — point No 8

TABELA 4

Klasyfikacja pięćdziesięciu dwu pomiarów $W_m(t_i)$ — punkt nr 8

Point No	Relation	Number of $W_m(t_i)$'s satisfying relation	Does true randomness occur
8	$\lvert W_m(t_i) - W_d(t_i)\rvert > \delta_{max}$	15	decisively yes
	$\sigma < \lvert W_m(t_i) - W_d(t_i)\rvert \le \delta_{max}$	11	yes
	$\lvert W_m(t_i) - W_d(t_i)\rvert \le \sigma$	26	cannot excluded

2. Analysis of the surveys

The discussed results of surveys concern vertical subsidences of Points Nos 4, 5 and 8 which were located on the surface of an area that was under the influence of underground mining of the layer No 612 of Coal Mining "Jowisz". Figure 1 shows the location of the points and the face advance of the layer No 612 during the time of the quasi-continuous surveys.

The following parameters characterize the uderground mining exploitation:

— the duration of the exploitation: 01.06.1998—01.07.1999,

— retarded caving: collapse,

— average depth of the exploitation: $H_{av} = 370$ m,

— average thickness of the seam: $g_{av} = 1.4$ m,

— the duration of the observation of the process: 27.01.1999—10.04.1999,

— the duration of the quasi-continuous measurements of the process (by GPS): 15.02.1999—04.03.1999.

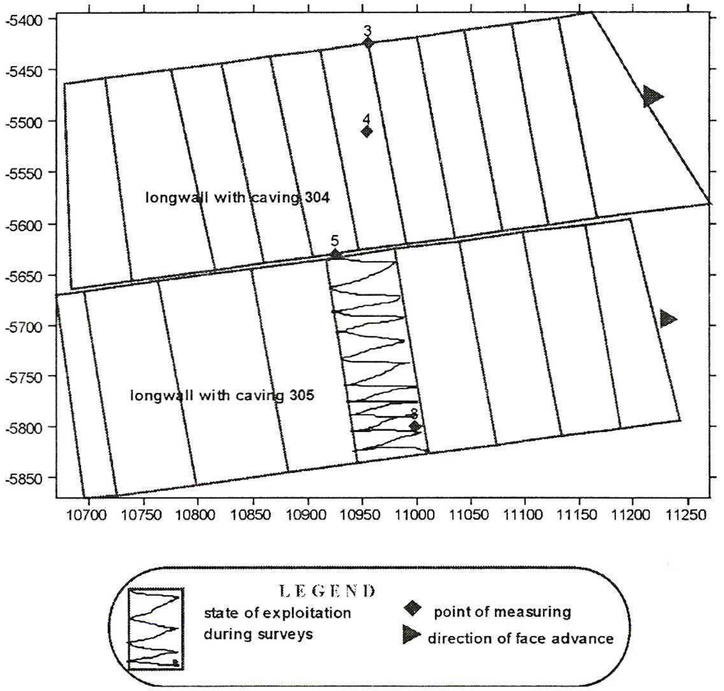

Fig. 1. Scheme of the development of underground mining exploitation

Rys. 1. Schemat rozwoju eksploatacji podziemnej

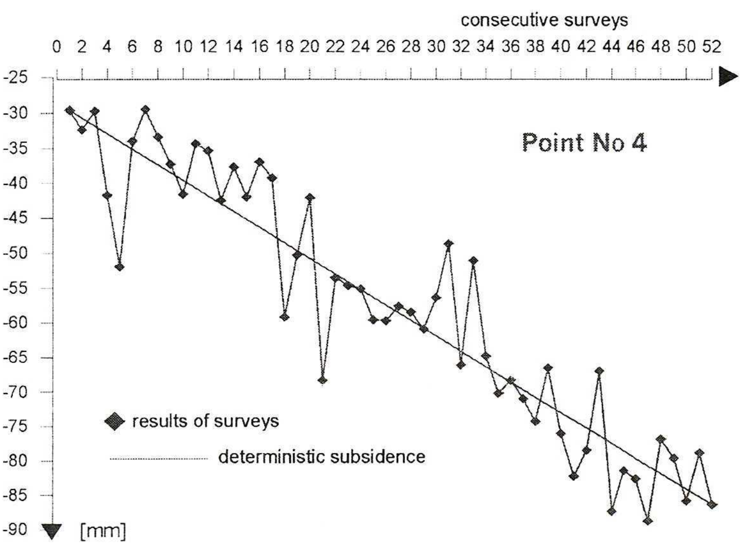

Fig. 2. Results $W_m(t_i)$ of the surveys of the subsidence $W(t)$ of Point No 4 under the influence of underground mining exploitation (obtained by GPS technology)

Rys. 2. Wyniki $W_m(t_i)$ pomiarów osiadania $W(t)$ punktu nr 4 pod wpływem podziemnej eksploatacji górniczej (uzyskane w technologii GPS)

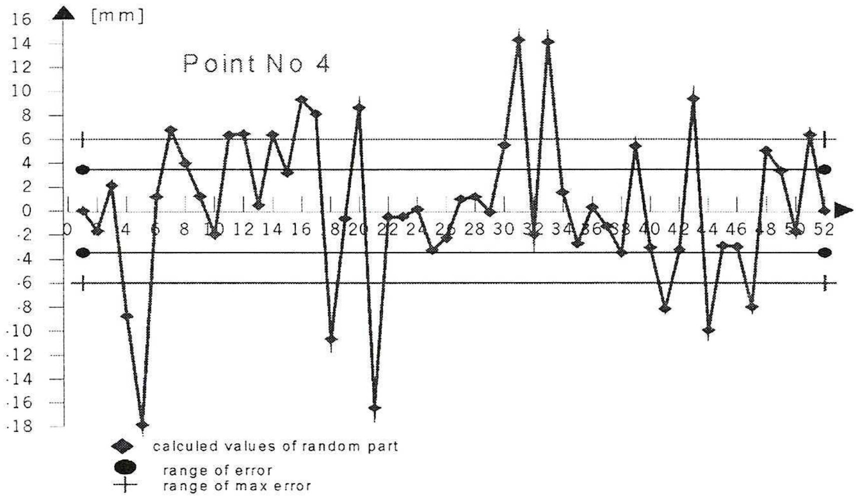

Fig. 3. Calculated values of the random part $W_m(t_i) - W_d(t_i)$ of subsidence, biased by $\varepsilon(t_i)$ the error of measurment

Rys. 3. Obliczone wartości $W_m(t_i) - W_d(t_i)$ losowej części obniżeń obciążone $\varepsilon(t_i)$ błędem pomiaru

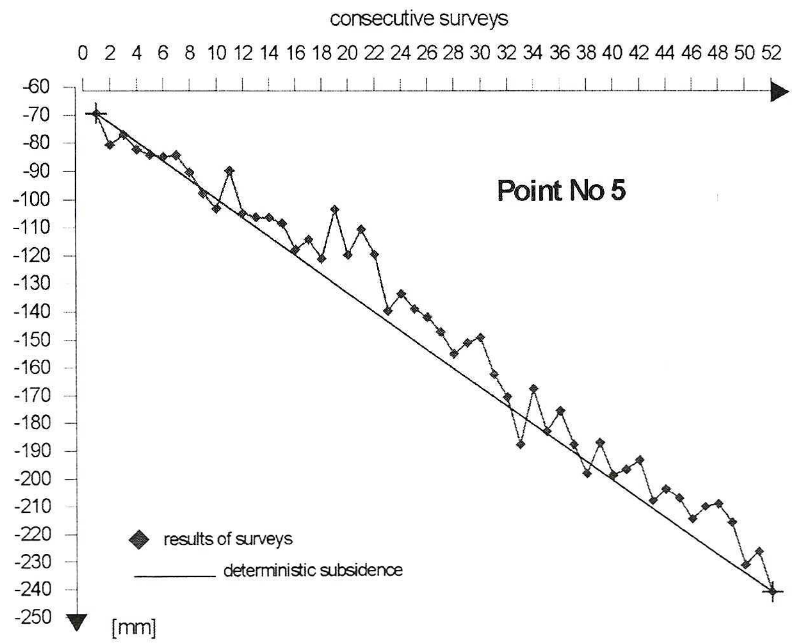

Fig. 4. Results $W_m(t_i)$ of the surveys of the subsidence $W(t)$ of Point No 5 under the influence of underground mining exploitation (obtained by GPS technology)

Rys. 4. Wyniki $W_m(t_i)$ pomiarów osiadania $W(t)$ punktu nr 5 pod wpływem podziemnej eksploatacji górniczej (uzyskane w technologii GPS)

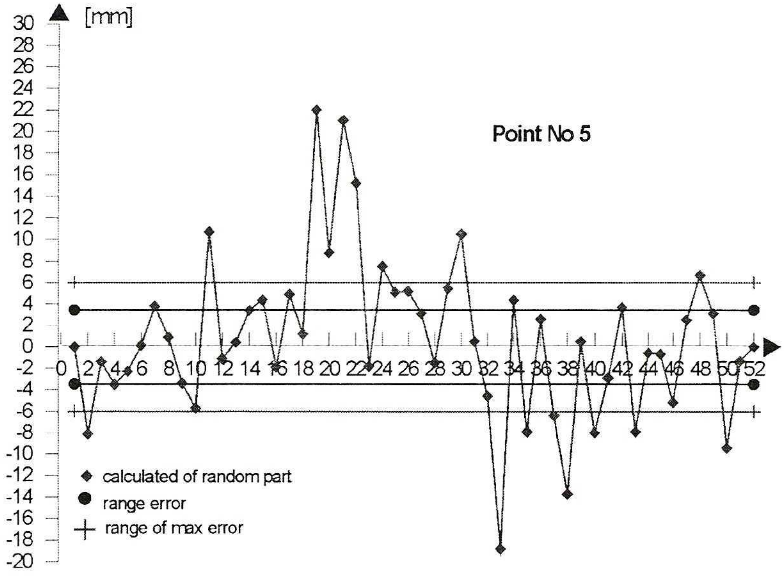

Fig. 5. Calculated values of the random part $W_m(t_i) - W_d(t_i)$ of subsidence, biased by $\varepsilon(t_i)$ the error of measurment

Rys. 5. Obliczone wartości $W_m(t_i) - W_d(t_i)$ losowej części obniżeń obciążone $\varepsilon(t_i)$ błędem pomiaru

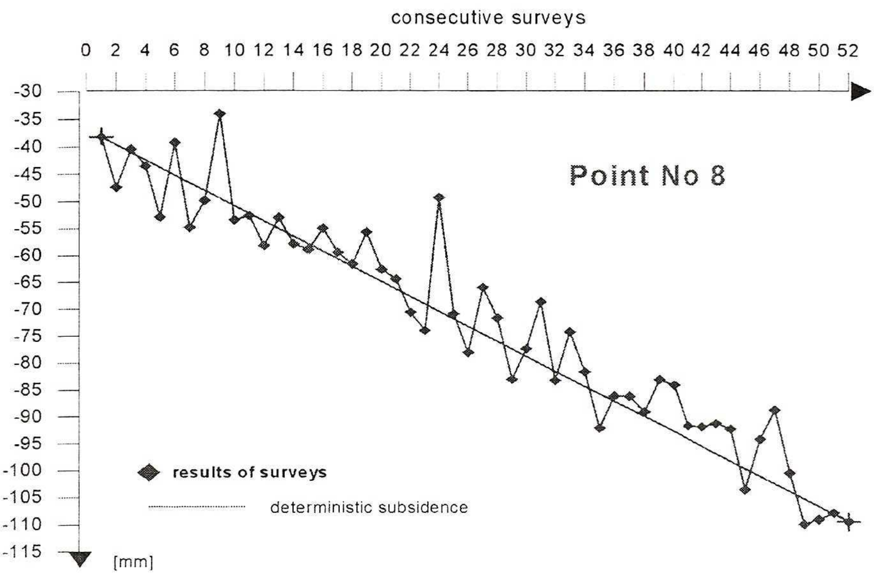

Fig. 6. Results $W_m(t_i)$ of the surveys of the subsidence $W(t)$ of Point No 8 under the influence of underground mining exploitation (obtained by GPS technology)

Rys. 6. Wyniki $W_m(t_i)$ pomiarów $W(t)$ osiadania punktu nr 8 pod wpływem podziemnej eksploatacji górniczej (uzyskane w technologii GPS)

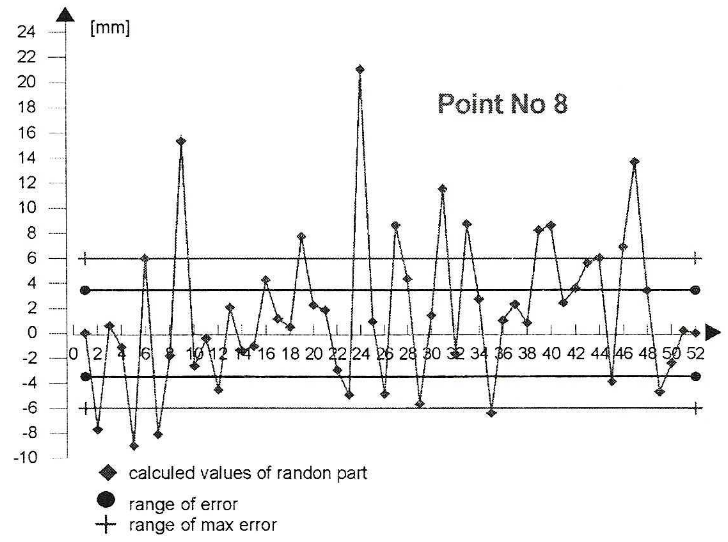

Fig. 7. Calculated values of the random part $W_m(t_i) - W_d(t_i)$ of subsidence, biased by $\varepsilon(t_i)$ the error of measurment

Rys. 7. Obliczone wartości $W_m(t_i) - W_d(t_i)$ losowej części obniżeń obciążone $\varepsilon(t_i)$ błędem pomiaru

Measurements were carried out with the aid of GPS technology over the period of time from 15.02.1999 to 04.03.1999. Approximately one measurement was performed every eight hours.

In conclusion we would like to note the following. In the paper we have presented an original method of the analysis of the surveys. The method enables one to show clearly the inherent randomness of the surveys which are biased in an essential way by the errors of the measurements.

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O PEWNYCH ZJAWISKACH LOSOWYCH WYSTĘPUJĄCYCH W CZASIE PROCESU FORMOWANIA SIĘ GÓRNICZEJ NIECKI EKSPLOATACYJNEJ

Słowa kluczowe

Błąd pomiaru, eksploatacja górnicza, geodezyjny pomiar górniczy, losowość, niecka osiadania, równanie Itô', równanie Kolmogorowa, technologia GPS

Streszczenie

Proces przemieszczeń punktów powierzchni w obszarze oddziaływania podziemnej eksploatacji górniczej był obserwowany w małej skali czasowej (krótki przedział czasu pomiędzy dwoma kolejnymi pomiarami). Proces ten wykazuje nieregularne zachowanie (np. obserwowane punkty oscylowały w sposób losowy w czasie ich obniżania się). W pracy zanalizowano osiadanie trzech punktów.

Rezultaty analizy ukazują w sposób wyraźny, że charakter zaobserwowanych nieregularności jest z natury rzeczy losowy. Zaobserwowane zjawisko odsłania zatem losowy mechanizm rządzący procesem formowania się niecki pogórniczej.